THIRD EDITION

Data Structures and Abstractions with Java™

FRANK M. CARRANO

Reserved Words

Reserved words are also called **keywords**. You may not redefine any of these reserved words. Their meanings are determined by the Java language and cannot be changed. In particular, you cannot use any of these reserved words for variable names, method names, or class names.

```
abstract
assert

boolean
break
byte

case
catch
char
class
const
continue

default
do
double

else
enum
extends

false
final
finally
float
for

goto

if
implements
import
instanceof
int
interface

long

native
new
null

package
private
protected
public

return

short
static
strictfp
super
switch
synchronized

this
throw
throws
transient
true
try

void
volatile

while
```

This page intentionally left blank

Operator Precedence

In the following list, operators on the same line are of equal precedence. As you move down the list, each line is of lower precedence. When the order of operations is not dictated by parentheses, the operator of higher precedence executes before an operator of lower precedence. When operators have equal precedence, binary operators execute in left-to-right order, and unary operators execute in right-to-left order.

Highest Precedence
The unary operators `+`, `-`, `++`, `--`, `!`, `~`
The unary operators `new` and `(`*type*`)`
The binary operators `*`, `/`, `%`
The binary operators `+`, `-`
The binary (shift) operators `<<`, `>>`, `>>>`
The binary operators `<`, `>`, `<=`, `>=`
The binary operators `==`, `!=`
The binary operator `&`
The binary operator `^`
The binary operator `|`
The binary operator `&&`
The binary operator `||`
The ternary (conditional) operator `? :`
Assignment operators `=`, `*=`, `/=`, `%=`, `+=`, `-=`, `<<=`, `>>=`, `>>>=`, `&=`, `^=`, `|=`
Lowest Precedence

Primitive Data Types

Type	**Size**	**Values**
Integer		
`byte`	1 byte	-128 to 127
`short`	2 bytes	-32,768 to 32,767
`int`	4 bytes	-2,147,483,648 to 2,147,483,647
`long`	8 bytes	-9,223,372,036,854,775,808 to 9,223,372,036,854,775,807
Real		
`float`	4 bytes	-3.402824 x 10^{38} to 3.402824 x 10^{38}
`double`	8 bytes	-1.79769313486232 x 10^{308} to 1.79769313486232 x 10^{308}
Character (Unicode)		
`char`	2 bytes	All Unicode values between 0 and 65,535
Boolean		
`boolean`	1 bit	`true`, `false`

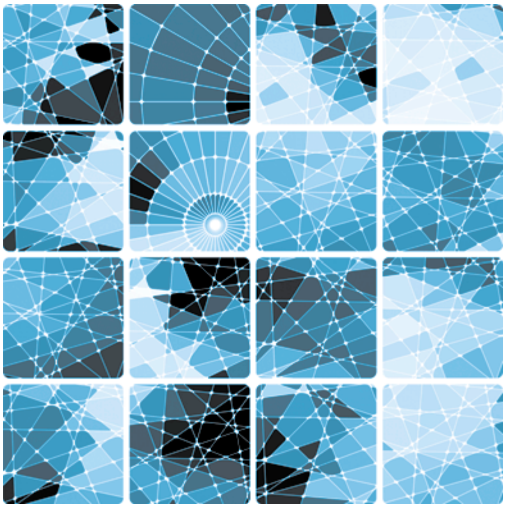
Unicode Character Codes

The printable characters shown are a subset of the Unicode character set known as the ASCII character set. The numbering is the same whether the characters are considered to be members of the Unicode character set or members of the ASCII character set. (Character number 32 is the blank.)

32		56	8	80	P	104	h
33	!	57	9	81	Q	105	i
34	"	58	:	82	R	106	j
35	#	59	;	83	S	107	k
36	$	60	<	84	T	108	l
37	%	61	=	85	U	109	m
38	&	62	>	86	V	110	n
39	'	63	?	87	W	111	o
40	(	64	@	88	X	112	p
41	)	65	A	89	Y	113	q
42	*	66	B	90	Z	114	r
43	+	67	C	91	[	115	s
44	,	68	D	92	\	116	t
45	-	69	E	93	]	117	u
46	.	70	F	94	^	118	v
47	/	71	G	95	_	119	w
48	0	72	H	96	‘	120	x
49	1	73	I	97	a	121	y
50	2	74	J	98	b	122	z
51	3	75	K	99	c	123	{
52	4	76	L	100	d	124	|
53	5	77	M	101	e	125	}
54	6	78	N	102	f	126	~
55	7	79	O	103	g		

Data Structures and Abstractions with Java™

Third Edition



Frank M. Carrano

University of Rhode Island

Prentice Hall

Boston Columbus Indianpolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto
Delhi Mexico City Sao Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Editorial Director: Marcia Horton
Editor in Chief: Michael Hirsch
Acquisitions Editor: Tracy Dunkelberger
Editorial Assistant: Stephanie Sellinger
Director of Marketing: Patrice Jones
Marketing Manager: Yezan Alayan
Marketing Coordinator: Kathryn Ferranti
Vice President, Production: Vince O'Brien
Managing Editor: Jeff Holcomb
Associate Managing Editor: Robert Engelhardt
Manufacturing Manager: Nick Sklitsis
Operations Specialist: Lisa McDowell
Cover Designer: Anthony Gemmellaro
Photo Researcher: AV Manager, Rights and Permissions: Karen Sanatar
Cover Image Credit: © Color Symphony/Shutterstock
Media Editor: Dan Sandin
Full-Service Project Management: GEX Publishing Services
Composition: GEX Publishing Services
Printer/Binder: Edwards Brothers
Cover Printer: Lehigh-Phoenix Color, Hagerstown

Library of Congress Cataloging-in-Publication Data
Carrano, Frank M.
Data structures and abstractions with Java / Frank M. Carrano. -- 3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-0-13-610091-1 (alk. paper)
ISBN-10: 0-13-610091-0 (alk. paper)
1. Data structures (Computer science) 2. Java (Computer program language) I. Title.
QA76.9.D33C37 2012
005.13'3--dc23
2011029581

10 9 8 7 6 5 4 3 2 1

Prentice Hall
is an imprint of

PEARSON

www.pearsonhighered.com

ISBN-10: 0-13-610091-0
ISBN-13: 978-0-13-610091-1

From the Author

Welcome to the third edition of *Data Structures and Abstractions with Java*, a book for an introductory course in data structures, typically known as CS-2. Readers of my book *Imagine! Java* can consider this one as a sequel.

I wrote this book with you in mind—whether you are an instructor or a student—based upon my experiences during more than three decades of teaching undergraduate computer science. I wanted my book to be reader friendly so that students could learn more easily and instructors could teach more effectively. To this end, you will find the material covered in small pieces—I call them "segments"—that are easy to digest and facilitate learning. Numerous examples that mimic real-world situations provide a context for the new material and help to make it easier for students to learn and retain abstract concepts. Many simple figures illustrate and clarify complicated ideas. Included are over 60 video tutorials to supplement the instruction and help students when their instructor is unavailable.

I hope that you enjoy reading this book. Like many others before you, you can learn—or teach—data structures in an effective and sustainable way.

Warm regards,
Frank M. Carrano

P. S. I am always available to connect with instructors and students who use my books. Here are a few ways you can reach me:

Find me on Facebook:	`www.facebook.com/makingitreal`
Follow me on Twitter:	`twitter.com/Frank_M_Carrano`
Send me an e-mail:	`carrano@acm.org`
Post on my blog:	`frank-m-carrano.com/makingitreal`

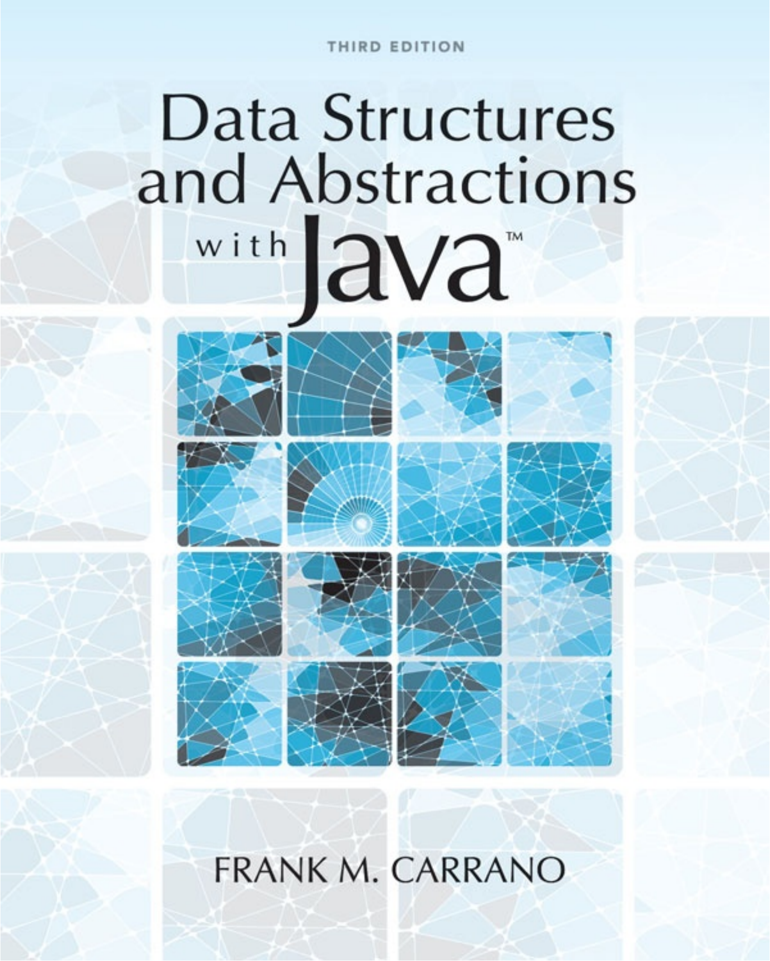
making
it
real
Frank M. Carrano

A Note to Students

The topics that we cover in this book deal with the various ways of organizing data so that a given application can access and manipulate data in an efficient way. These topics are fundamental to your future study of computer science, as they provide you with the foundation of knowledge required to create complex and reliable software. Whether you are interested in designing video games or software for robotic-controlled surgery, the study of data structures is vital to your success. Even if you do not study all of the topics in this book now, you are likely to encounter them later. I hope that you will enjoy reading the book, and that it will serve as a useful reference tool for your future courses.

After looking over this preface, you should read the Introduction. There you will quickly see what this book is about and what you need to know about Java before you begin. Appendices A through G review Java basics, classes, inheritance, exceptions, files, and `javadoc` comments. Note that inside the front and back covers you will find Java's reserved words, its primitive data types, the precedence of its operators, and a list of Unicode characters.

Please be sure to browse the rest of this preface to see the features that will help you in your studies.



Organization and Structure

This book's organization, sequencing, and pace of topic coverage make learning and teaching easier by focusing your attention on one concept at a time, by providing flexibility in the order in which you can cover topics, and by clearly distinguishing between the specification and implementation of abstract data types, or ADTs. To accomplish these goals, I have organized the material into 30 chapters, composed of small, numbered segments that deal with one concept at a time. Each chapter focuses on either the specification and use of an ADT or its various implementations. You can choose to cover the specification of an ADT followed by its implementations, or you can treat the specification and use of several ADTs before you consider any implementation issues. The book's organization makes it easy for you to choose the topic order that you prefer.

Table of Contents at a Glance

The following list of chapter titles shows the overall composition of the book. A further chapter-by-chapter description appears later. Note that highlighted sections are available online.

Appendices

What's New?

Based on comments from readers and reviewers, I have reorganized some of the material. Many students are familiar with stacks and queues, and so the coverage of these data organizations is much earlier in this edition. Moreover, the reorganization makes the difficult topic of linked data more accessible to students. Adding or removing the first node in a chain of linked nodes is the easiest operation. By introducing the bag, the book uses these simple operations on a linked chain in the bag's implementation. That data collection is followed by the stack, a more useful organization that has the same simple chain in one of its definitions. Queue implementations provide the opportunity to discuss adding and removing the last node in a chain. Finally, the treatment of lists looks at the more involved operations of adding and removing a node that lies between existing nodes.

You will notice that algorithm efficiency—including improved motivation—recursion, and sorting also are covered earlier in this edition than in the previous one. To maintain the focus on data structures, I have moved the first three chapters—*Java Classes*, *Creating Classes from Other Classes*, and *Designing Classes*—to the appendices. The presentation now moves from the introduction immediately to the first data collection, the bag. However, readers who need to study Java classes before embarking on the main topic of this book will find the original coverage intact in the appendices.

Finally, I have added some new features. Extensive examples are presented in the form of "A Problem Solved," in which a problem is posed and its solution is discussed and implemented. An occasional "Design Decision" explores various design choices of a solution. These two new elements help students to think about important aspects of program design and to consider concepts in a situational context. Another new feature is the availability online of over 60 VideoNotes that provide additional instruction in a more dynamic form than a static textbook. The Notes, Programming Tips, and Questions—with answers—that were featured in the previous edition have been retained. And you will find an introduction to the interface `Deque` and the class `ArrayDeque`, as well as additional programming projects.

Here is a summary of what is new:

- Earlier introduction of abstract data types, resizable arrays, and linked data.
- More gradual coverage of linked data.
- Earlier coverage of algorithm efficiency, stacks, recursion, sorting, and queues.
- Better motivation of the need for algorithm efficiency.
- Chapters 1 through 3—*Bags*, *Bag Implementations That Use Arrays*, and *A Bag Implementation That Links Data*—introduce and implement the ADT bag.
- New elements, including A Problem Solved and Design Decision.
- Review of Java classes that appeared in the initial chapters of the second edition is now in the appendices.
- VideoNotes—short instructional tutorials—reinforce key concepts presented in the book.
- Coverage of the standard interface `Deque` and the class `ArrayDeque`.
- Additional Programming Projects.
- Answers to Self-Test Questions appear at the end of each chapter instead of an appendix.

Features to Enhance Learning

Each chapter begins with a table of contents, a list of prerequisite chapters or appendices that students should have read, and the learning objectives for the material to be covered. Other pedagogical elements appear throughout the book, as follows:

Notes Important ideas are presented or summarized in highlighted paragraphs and are meant to be read in line with the surrounding text.

Programming Tips Suggestions to improve or facilitate programming are featured as soon as they become relevant.

Examples Numerous examples illuminate new concepts.

A Problem Solved Large examples are presented in the form of "A Problem Solved," in which a problem is posed and its solution is discussed, designed, and implemented.

Design Decisions To give readers insight into the design choices that one could make when formulating a solution, "Design Decision" elements lay out such options, along with the rationale behind the choice made for a particular example. These discussions are often in the context of one of the A Problem Solved examples.

Self-Test Questions Questions are posed throughout each chapter, integrated within the text, that reinforce the concept just presented. These "self-test" questions help readers to understand the material, since answering them requires pause and reflection. Solutions to these questions are provided at the end of each chapter.

VideoNotes Online tutorials are a Pearson feature that provides visual and audio support to the presentation given throughout the book. They offer students another way to recap and reinforce key concepts. VideoNotes allow for self-paced instruction with easy navigation, including the ability to select, play, rewind, fast-forward, and stop within each video. Unique VideoNote icons appear throughout this book whenever a video is available for a particular concept or problem. A detailed list of the VideoNotes for this text and their associated locations in the book can be found on page xxiv. VideoNotes are free with the purchase of a new textbook. To purchase access to VideoNotes, please go to

`pearsonhighered.com/carrano`

Exercises and Programming Projects Further practice is available by solving the exercises and programming projects at the end of each chapter. Unfortunately, we cannot give readers the answers to these exercises and programming projects, even if they are not enrolled in a class. Only instructors who adopt the book can receive selected answers from the publisher. For help with these exercises and projects, you will have to contact your instructor.

Accessing Instructor and Student Resource Materials

The following items are available on the publisher's website at `pearsonhighered.com/carrano`:

- Java code as it appears in the book
- A link to any misprints that have been discovered since the book was published
- Links to additional online content, which is described next

Instructor Resources

The following protected material is available to instructors who adopt this book by logging onto Pearson's Instructor Resource Center, accessible from `pearsonhighered.com/carrano`:

- PowerPoint lecture slides
- Instructor solutions manual
- Figures from the book

Additionally, instructors can access the book's Companion Website for the following online premium content, also accessible from `pearsonhighered.com/carrano`:

- Instructional VideoNotes
- Chapter 30
- A glossary of terms
- Exercises and projects for Appendices B, C, and D

Please contact your Pearson sales representative for an instructor access code. Contact information is available at `pearsonhighered.com/replocator`.

Student Resources

The following material is available to students by logging onto the Companion Website accessible from `pearsonhighered.com/carrano`:

- Instructional VideoNotes
- Chapter 30
- A glossary of terms
- Exercises and projects for Appendices B, C, and D

Students must use the access card located in the front of the book to register for and then enter the Companion Website. Students without an access code can purchase access from the Companion Website by following the instructions listed there.

Note that the Java Class Library is available at `download.oracle.com/javase/7/docs/api/`.

Chapter Overview

Readers of this book should have completed a programming course, preferably in Java. The appendices cover the essentials of Java that we assume readers will know. You can use these appendices as a review or as the basis for making the transition to Java from another programming language. The book itself begins with the Introduction, which sets the stage for the data organizations that we will study.

- **Chapters 1 through 3:** We introduce the bag as an abstract data type (ADT). By dividing the material across several chapters, we clearly separate the specification, use, and implementation of the bag. For example, Chapter 1 specifies the bag and provides several examples of its use. Chapter 2 covers implementations that use arrays and vectors, while Chapter 3 introduces chains of linked nodes and uses one in the definition of a class of bags.

  In a similar fashion, we separate specification from implementation throughout the book when we discuss various other ADTs. You can choose to cover the chapters that specify and use the ADTs and then later cover the chapters that implement them. Or you can cover the chapters as they appear, implementing each ADT right after studying its specification and use. A list of chapter prerequisites appears later in this preface to help you plan your path through the book.

  Chapter 2 does more than simply implement the ADT bag. It shows how to approach the implementation of a class by initially focusing on core methods. When defining a class, it is often useful to implement and test these core methods first and to leave definitions of the other methods for later.
- **Chapter 4:** Here we introduce the complexity of algorithms, a topic that we integrate into future chapters.
- **Chapters 5 and 6:** Chapter 5 discusses stacks, giving examples of their use, and Chapter 6 implements the stack using an array, a vector, and a chain.
- **Chapters 7 through 9:** Next, we present recursion as a problem-solving tool and its relationship to stacks. Recursion, along with algorithm efficiency, is a topic that is revisited throughout the book. For example, Chapters 8 and 9 discuss various sorting techniques and their relative complexities. We consider both iterative and recursive versions of these algorithms.
- **Chapters 10 and 11:** Chapter 10 discusses queues, deques, and priority queues, and Chapter 11 considers their implementations. It is in this latter chapter that we introduce circularly linked and doubly linked chains.
- **Chapters 12, 13, and 14:** The next three chapters introduce the ADT list. We discuss this collection abstractly and then implement it by using an array, a vector, and finally a chain of linked nodes.
- **Chapter 15:** Next, we discuss iterators in the context of a list. This chapter considers and implements Java's iterator interfaces `Iterator` and `ListIterator`. The chapter also introduces the interface `Iterable`.
- **Chapters 16 and 17:** Continuing the discussion of a list, Chapter 16 introduces the sorted list, looking at two possible implementations and their efficiencies. Chapter 17 shows how to use the list as a superclass for the sorted list and discusses the general design of a superclass.
- **Chapter 18:** We then examine some strategies for searching an array or a chain in the context of a list or a sorted list. This discussion is a good basis for the sequence of chapters that follows.
- **Chapters 19 through 22:** Chapter 19 covers the specification and use of the ADT dictionary. Chapter 20 presents implementations of the dictionary that are linked or that use arrays. Chapter 21 introduces hashing, and Chapter 22 uses it as a dictionary implementation.
- **Chapters 23 through 27:** Chapter 23 discusses trees and their possible uses. Included among the several examples of trees is an introduction to the binary search tree and the heap. Chapter 24 considers implementations of the binary tree and the general tree, and Chapter 25 focuses on the implementation of the binary search tree. Chapter 26 shows how to use an array to implement the heap. Chapter 27

introduces balanced search trees. Included in this chapter are the AVL, 2-3, 2-4, and red-black trees, as well as B-trees.

- **Chapters 28 and 29:** Next, we discuss graphs and look at several applications and two implementations.
- **Chapter 30:** The final chapter expands on the notion of mutable objects and immutable objects, and introduces cloning. If a client can maintain a reference to the data within an ADT, it can change that data without using the class's public methods, if the data is mutable. We consider steps that you can take to prevent the client from doing so.
- **Appendices A through G:** The appendices provide supplemental coverage of Java. As we mentioned earlier, Appendix A reviews Java up to but not including classes. However, this appendix also covers the `Scanner` class, enumerations, boxing and unboxing, and the for-each loop. Appendix B discusses Java classes, Appendix C expands this topic by looking at composition and inheritance, and Appendix D focuses on class design. Appendix E covers exception handling, and Appendix F discusses files. Appendix G considers programming style and comments. It introduces `javadoc` comments and defines the tags that we use in this book.

Acknowledgments

My sincere appreciation and thanks go to the following reviewers for carefully reading the previous edition and making candid comments and suggestions that greatly improved the work.

Steven Andrianoff—*St. Bonaventure University*
Brent Baas—*LeTourneau University*
Timothy Henry—*University of Rhode Island*
Ken Martin—*University of North Florida*
Bill Siever—*Northwest Missouri State University*
Lydia Sinapova—*Simpson College*
Lubomir Stanchev—*Indiana University*
Judy Walters—*North Central College*
Xiaohui Yuan—*University of North Texas*

Special thanks go to my support team during the lengthy process of revising this book. My editor, Tracy Dunkelberger, gave me her constant enthusiasm, encouragement, wisdom, and guidance. Melinda Haggerty and Allison Michael coordinated the review process, and Stephanie Sellinger oversaw the development of the book and its supplements. My long-time copy editor, Rebecca Pepper, ensured that my presentation is clear, correct, and grammatical. Jeff Holcomb, Bob Engelhardt, and Louise Capulli directed the production of the book. This team was there for me whenever I needed them at a moment's notice. Thank you so much!

My gratitude for the previously mentioned people does not diminish my appreciation for the help provided by many others. Tim Henry, my colleague at URI, created over 60 VideoNotes that provide extra instruction for every chapter of the book. Steve Armstrong produced the lecture slides for this edition and previous editions of the book. Thank you again to the reviewers of the first two editions of the book:

Reviewers for the second edition:

Harold Anderson—*Marist College*
Razvan Andonie—*Central Washington University*
Tom Blough—*Rensselaer Polytechnic Institute*

Chris Brooks—*University of San Francisco*
Adrienne Decker—*University at Buffalo, SUNY*
Henry Etlinger—*Rochester Institute of Technology*
Derek Harter—*Texas A&M University*
Timothy Henry—*University of Rhode Island*
Robert Holloway—*University of Wisconsin, Madison*
Charles Hoot—*Oklahoma City University*
Teresa Leyk—*Texas A&M University*
Robert McGlinn—*Southern Illinois University, Carbondale*
Edward Medvid—*Marymount University*
Charles Metzler—*City College of San Francisco*
Daniel Zeng—*University of Arizona*

Reviewers for the first edition:

David Boyd—*Valdosta State University*
Dennis Brylow—*Purdue University*
Michael Croswell—*Industry trainer/consultant*
Matthew Dickerson—*Middlebury College*
Robert Holloway—*University of Wisconsin, Madison*
John Motil—*California State University, Northridge*
Bina Ramamurthy—*University at Buffalo, SUNY*
David Surma—*Valparaiso University*

I continue to appreciate the many others who helped during previous editions. They include Alan Apt, James Blanding, Lianne Dunn, Mike Giacobbe, Toni Holm, Charles Hoot, Brian Jepson, Rose Kernan, Christianna Lee, Patrick Lindner, John Lovell, Vince O’Brien, Patty Roy, Walt Savitch, Ben Schomp, Heather Scott, Carole Snyder, Chirag Thakkar, Camille Trentacoste, Nate Walker, and Xiaohong Zhu.

Finally, I thank my family and friends—Doug, Ted, Vandee, Nancy, Sue, Tom, Maybeth, Marge, and Lorraine—for giving me a life away from computers.

Thank you, everyone, for your expertise and good cheer.

Frank M. Carrano

This page intentionally left blank

Contents

Appendices

VideoNotes Directory

VideoNote

This table lists the VideoNotes that are available online. The page numbers indicate where in the book each VideoNote has relevance.

Chapter Prerequisites

Each chapter and appendix assumes that the reader has studied certain previous material. This list indicates those prerequisites. Numbers represent chapter numbers, and letters reference appendices. You can use this information to plan a path through the book.

		Prerequisites
Chapter 1	Bags	C, D
Chapter 2	Bag Implementations That Use Arrays	1, D
Chapter 3	A Bag Implementation That Links Data	1, 2
Chapter 4	The Efficiency of Algorithms	2, 3, B
Chapter 5	Stacks	1, D
Chapter 6	Stack Implementations	2, 3, 4, 5
Chapter 7	Recursion	2, 3, 4, 5, B
Chapter 8	An Introduction to Sorting	3, 4, 7
Chapter 9	Faster Sorting Methods	4, 7, 8
Chapter 10	Queues, Deques, and Priority Queues	5, 8, D
Chapter 11	Queue, Deque, and Priority Queue Implementations	2, 3, 6, 10
Chapter 12	Lists	6, B, D, E
Chapter 13	List Implementations That Use Arrays	2, 4, 12, D
Chapter 14	A List Implementation That Links Data	3, 11, 12, 13
Chapter 15	Iterators	13, 14, E
Chapter 16	Sorted Lists	4, 7, 12, 14
Chapter 17	Inheritance and Lists	12, 13, 14, 16, C
Chapter 18	Searching	4, 7, 12, 13, 14, 16
Chapter 19	Dictionaries	12, 15, 18
Chapter 20	Dictionary Implementations	3, 4, 12, 13, 14, 15, 18, 19
Chapter 21	Introducing Hashing	19, 20
Chapter 22	Hashing as a Dictionary Implementation	4, 13, 14, 15, 19, 20, 21
Chapter 23	Trees	5, 7, 14, 15, 18
Chapter 24	Tree Implementations	5, 10, 14, 23, C, E
Chapter 25	A Binary Search Tree Implementation	7, 19, 23, 24, C
Chapter 26	A Heap Implementation	2, 13, 23
Chapter 27	Balanced Search Trees	23, 24, 25
Chapter 28	Graphs	5, 10, 23
Chapter 29	Graph Implementations	5, 10, 12, 15, 19, 23, 28
Chapter 30	Mutable, Immutable, and Cloneable Objects	13, 14, 15, 16, 17, C, E
Appendix A	Java Essentials	Knowledge of a programming language
Appendix B	Java Classes	A
Appendix C	Creating Classes from Other Classes	B
Appendix D	Designing Classes	A, B, C, G
Appendix E	Handling Exceptions	A, B, C
Appendix F	File Input and Output	A, D, E
Appendix G	Documentation and Programming Style	A

This page intentionally left blank

Introduction

Look around and you will see ways that people organize things. When you stopped at the store this morning, you went to the back of a line to wait for the cashier. The line organized people chronologically. The first person in the line was the first to be served and to leave the line. Eventually, you reached the front of the line and left the store with a bag containing your purchases. The items in the bag were in no particular order, and some of them were the same.

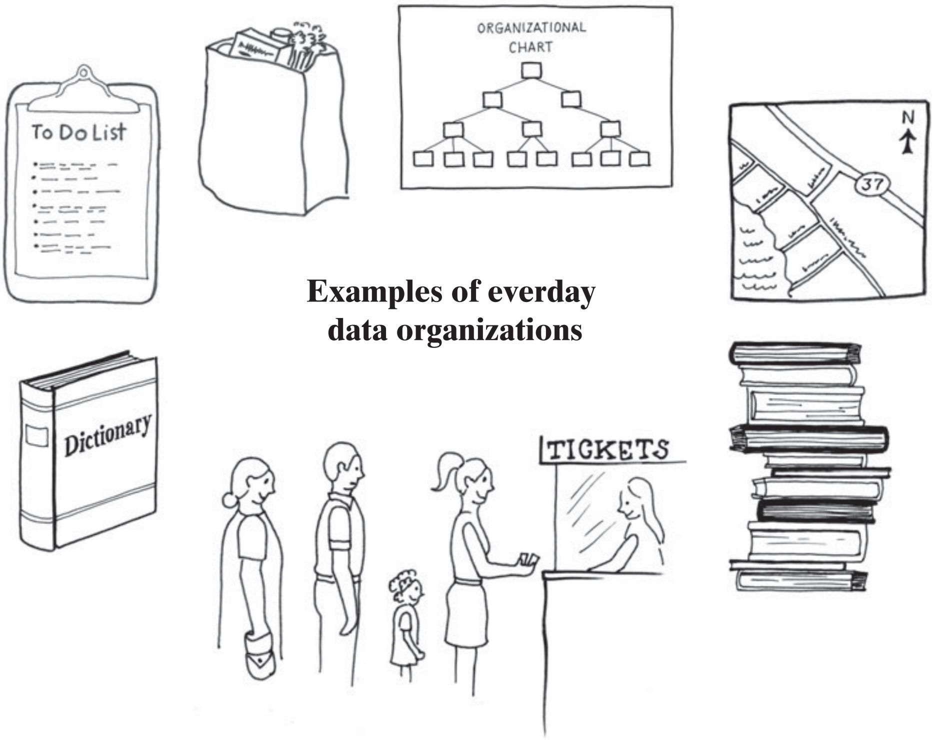
Do you see a stack of books or a pile of papers on your desk? It's easy to look at or remove the top item of the stack or to add a new item to the top of the stack. The items in a stack also are organized chronologically, with the item added most recently on top and the item added first on the bottom.

At your desk, you see your to-do list. Each entry in the list has a position that might or might not be important to you. You may have written them either as you thought of them, in their order of importance, or in alphabetical order. You decide the order; the list simply provides places for your entries.

Your dictionary is an alphabetical list of words and their definitions. You search for a word and get its definition. If your dictionary is printed, the alphabetical organization helps you to locate a word quickly. If your dictionary is computerized, its alphabetical organization is hidden, but it still speeds the search.

Speaking of your computer, you have organized your files into folders, or directories. Each folder contains several other folders or files. This type of organization is hierarchical. If you drew a picture of it, you would get something like a family tree or a chart of a company's internal departments. These data organizations are similar and are called trees.

Finally, notice the road map that you are using to plan your weekend trip. The diagram of roads and towns shows you how to get from one place to another. Often, several ways are possible. One way might be shorter, another faster. The road map has an organization known as a graph.



Examples of everday data organizations

Computer programs also need to organize their data. They do so in ways that parallel the examples we just cited. That is, programs can use a list, a stack, a dictionary, and so on. These ways of organizing data are represented by abstract data types. An **abstract data type**, or **ADT**, is a specification that describes a data set and the operations on that data. Each ADT specifies what data is stored and what the operations on the data do. Since an ADT does not indicate how to store the data or how to implement the operations, we can talk about ADTs independently of any programming language. In contrast, a **data structure** is an implementation of an ADT within a programming language.

A **collection** is a general term for an ADT that contains a group of objects. Some collections allow duplicate items, some do not. Some collections arrange their contents in a certain order, while others do not. A **container** is a class that implements a collection. Some people use the terms “container” and “collection” interchangeably.

We might create an ADT **bag** consisting of an unordered collection that allows duplicates. It is like a grocery bag, a lunch bag, or a bag of potato chips. Suppose you remove one chip from a bag of chips. You don’t know when the chip was placed into the bag. You don’t know whether the bag contains another chip shaped exactly like the one you just removed. But you don’t really care. If you did, you wouldn’t store your chips in a bag!

A bag does not order its contents, but sometimes you do want to order things. ADTs can order their items in a variety of ways. The ADT **list**, for example, simply numbers its items. A list, then, has a first item, a second item, and so on. Although you can add an item to the end of a list, you can also insert an item at the beginning of the list or between existing items. Doing so renumbers the items after the new item. Additionally, you can remove an item at a particular position within a list.

Thus, the position of an item in the list does not necessarily indicate when it was added. Notice that the list does not decide where an item is placed; you make this decision.

In contrast, the ADTs **stack** and **queue** order their items chronologically. When you remove an item from a stack, you remove the one that was added most recently. When you remove an item from a queue, you remove the one that was added the earliest. Thus, a stack is like a pile of books. You can remove the top book or add another book to the top of the pile. A queue is like a line of people. People leave a line from its front and join it at its end.

Some ADTs maintain their entries in sorted order, if the items can be compared. For instance, strings can be organized in alphabetical order. When you add an item to the ADT **sorted list**, for example, the ADT determines where to place the item in the list. You do not indicate a position for the item, as you would with the ADT list.

The ADT **dictionary** contains pairs of items, much as a language dictionary contains a word and its definition. In this example, the word serves as a **key** that is used to locate the entries. Some dictionaries sort their entries and some do not.

The ADT **tree** organizes its entries according to some hierarchy. For example, in a family tree, people are associated with their children and their parents. The ADT **binary search tree** has a combined hierarchical and sorted organization that makes locating a particular entry easier.

The ADT **graph** is a generalization of the ADT tree that focuses on the relationship among its entries instead of any hierarchical organization. For example, a road map is a graph that shows the existing roads and distances between towns.

This book shows you how to use and implement these data organizations. Before we begin, you need to know Java. Appendix A reviews the basic statements in Java. Appendix B discusses the basic construction of classes and methods. You can choose to glance at this material, read it carefully, or come back to it as necessary. Appendices C and D also focus on Java, but some or all of the material might be new to you. Appendix C covers techniques, including composition and inheritance, for creating new classes from existing classes. Appendix D discusses how to design classes, specify methods, and write Java interfaces. Using interfaces and writing comments to specify methods are essential to our presentation of ADTs. Appendix E reviews how to handle exceptions, Appendix F presents reading and writings external files, and Appendix G gives an overview of writing comments suitable for `javadoc`.

This page intentionally left blank

Chapter 1

Bags

Contents

Prerequisites

Appendix C	Creating Classes from Other Classes
Appendix D	Designing Classes

Objectives

After studying this chapter, you should be able to

- Describe the concept of an abstract data type (ADT)
- Describe the ADT bag
- Use the ADT bag in a Java program

This chapter builds on the concepts of encapsulation and data abstraction presented in Appendix D, and it develops the notion of an abstract data type. As you probably know, a **data type** such as `int` or `double` is a group of values and operations on those values that is defined within a specific programming language. In contrast, an **abstract data type**, or **ADT**, is a specification for a group of values and the operations on those values that is defined conceptually and independently of any programming language. A **data structure** is an implementation of an ADT within a programming language.

This chapter also begins to generalize the idea of grouping objects. A **collection** is an object that groups other objects and provides various services to its client. In particular, a typical collection enables a client to add, remove, retrieve, and query the objects it represents. Various collections exist for different purposes. Their behaviors are specified abstractly and can differ in purpose according to the collection. Thus, a collection is an abstraction and is an abstract data type. However, an ADT is not necessarily a collection.

To provide an example of a collection and of an abstract data type, we will specify and use the ADT bag. In doing so we will provide a Java interface for our bag. Knowing just this interface, you will be able to use a bag in a Java program. You do not need to know how the entries in the bag are represented or how the bag operations are implemented. Indeed, your program will not depend on these specifics. As you will see, this important program characteristic is what data abstraction is all about.

The Bag

1.1 Imagine a paper bag, a reusable cloth bag, or even a plastic bag. People use bags when they shop, pack a lunch, or eat potato chips. Bags contain things. In everyday language, a bag is a kind of container. In Java, however, a **container** is an object whose class extends the standard class `Container`. Such containers are used in graphics programs. Rather than being considered a container, a **bag** in Java is a kind of collection.

What distinguishes a bag from other collections? A bag doesn't do much more than contain its items. It doesn't order them in a particular way, nor does it prevent duplicate items. Most of its behaviors could be performed by other kinds of collections. While describing the behaviors for the collection that we'll design in this chapter, let's keep in mind that we are specifying an abstraction inspired by an actual physical bag. For example, a paper bag holds things of various dimensions and shapes in no particular order and without regard for duplicates. Our abstract bag will hold unordered and possibly duplicate objects, but let's insist that these objects have the same or related types.

Note: A bag is a finite collection of objects in no particular order. A bag can contain duplicate items.

A Bag's Behaviors

1.2 Since a bag contains a finite number of objects, reporting how many objects it contains could be one of a bag's behaviors:

Get the number of items currently in the bag

Two related behaviors detect whether a bag is full or empty:

See whether the bag is full
See whether the bag is empty

1.3 We should be able to add and remove objects:

Add a given object to the bag
Remove an unspecified object from the bag
Remove an occurrence of a particular object from the bag, if possible
Remove all objects from the bag

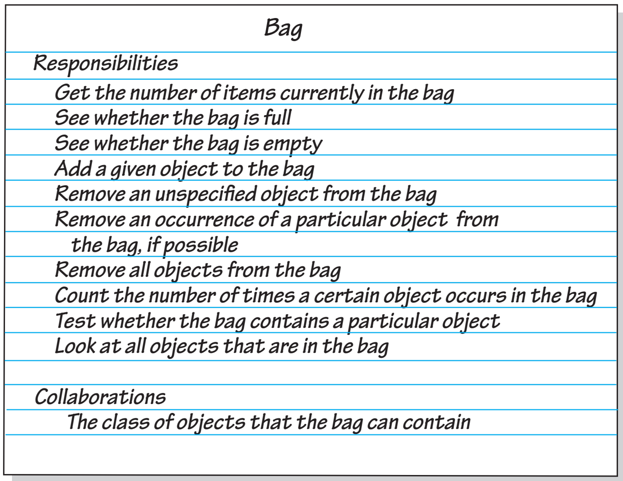
While you hope that the bagger at the grocery store does not toss six cans of soup into a bag on top of your bread and eggs, our add operation does not indicate where in the bag an object should go. Remember that a bag does not order its contents. Likewise, the first remove operation just removes any object it can. This operation is like reaching into a grab bag and pulling something out. On the other hand, the second remove operation looks for a particular item in the bag. If you find it, you take it out. If the bag contains several equal objects that satisfy your search, you remove any one of them. If you can't find the object in the bag, you can't remove it, and you just say so. Finally, the last remove operation simply empties the bag of all objects.

1.4 How many cans of dog food did you buy? Did you remember to get anchovy paste? Just what is in that bag? The answers to these questions can be answered by the following operations:

Count the number of times a certain object occurs in the bag
Test whether the bag contains a particular object
Look at all objects that are in the bag

We have enough behaviors for now. At this point, we would have written all 10 behaviors on a piece of paper or on the class-responsibility-collaboration (CRC) card pictured in Figure 1-1, as suggested in Appendix D.

FIGURE 1-1 A CRC card for a class `Bag`



Bag

Responsibilities
Get the number of items currently in the bag
See whether the bag is full
See whether the bag is empty
Add a given object to the bag
Remove an unspecified object from the bag
Remove an occurrence of a particular object from
the bag, if possible
Remove all objects from the bag
Count the number of times a certain object occurs in the bag
Test whether the bag contains a particular object
Look at all objects that are in the bag

Collaborations
The class of objects that the bag can contain

1.5 Since a bag is an abstract data type, we only describe its data and specify its operations. We do not indicate how to store the data or how to implement its operations. Don't think about arrays, for example. You first need to clearly know what the bag operations do: Focus on *what* the operations do, not on *how* they do them. That is, you need a detailed set of specifications before you can use a bag in a program. In fact, you should specify the bag operations before you even decide on a programming language.

Note: Since an abstract data type, or ADT, describes a data organization independently of a programming language, you have a choice of programming languages for its implementation.

Specifying a Bag

VideoNote
Designing an ADT

Before we can implement a bag in Java, we need to describe its data and specify in detail the methods that correspond to the bag's behaviors. We'll name the methods, choose their parameters, decide their return types, and write comments to fully describe their effect on the bag's data. Our eventual goal, of course, is to write a Java header and comments for each method, but first we will express the methods in pseudocode and then in Unified Modeling Language (UML) notation.

1.6 The first behavior on our CRC card gives rise to a method that returns a count of the current number of entries in the bag. The corresponding method has no parameters and returns an integer. In pseudocode, we have the following specification:

```
// Returns the current number of entries in the bag.
getCurrentSize()
```

We can express this method using UML as

```
+getCurrentSize(): integer
```

and add this line to a class diagram.

We can test whether the bag is full or empty by using two boolean-valued methods, again without parameters. Their specifications in pseudocode and UML are

```
// Returns true if the bag is full.
isFull()

// Returns true if the bag is empty.
isEmpty()
```

and

```
+isFull(): boolean
+isEmpty(): boolean
```

We add these two lines to our class diagram.

1.7 We now want to add a given object to the bag. We can name the method `add` and give it a parameter to represent the new entry. We could write the following pseudocode:

```
// Adds a new entry to the bag.
add(newEntry)
```

We might be tempted to make `add` a void method, but if the bag is full, we cannot add a new entry to it. What should we do in this case?

Design Decision: What should the method `add` do when it cannot add a new entry?

Here are two options that we can take when `add` cannot complete its task:

- Do nothing. We cannot add another item, so we ignore it and leave the bag unchanged.
- Leave the bag unchanged, but signal the client that the addition is impossible.

The first option is easy, but it leaves the client wondering what happened. Of course, we could state as a precondition of `add` that the bag must not already be full. Then the client has the responsibility to avoid adding a new entry to a full bag.

The second option is the better one, and it is not too hard to specify or implement. How can we indicate to the client whether the addition was successful? The standard Java interface `Collection` specifies that an exception should occur if the addition is not successful. We will leave this approach for later and use another way. Displaying an error message is not a good choice, as you should let the client dictate all written output. Since the addition is either successful or not, we can simply have the method `add` return a boolean value.

Thus, we can specify the method `add` in UML as

```
+add(newEntry: T): boolean
```

where `newEntry`'s data type is the generic type[1] `T`.

[1] Appendix C reviews generic types in Java.

Question 1 Suppose aBag represents an empty bag that has a finite capacity. Write some pseudocode statements to add user-supplied strings to the bag until it becomes full.

1.8 Three behaviors involve removing entries from a bag: remove all entries, remove any one entry, and remove a particular entry. Suppose we name the methods and any parameters and specify them in pseudocode as follows:

```
// Removes all entries from the bag.
clear()

// Removes one unspecified entry from the bag.
remove()

// Removes one occurrence of a particular entry from the bag, if possible.
remove(anEntry)
```

What return types are these methods?

1.9 The method `clear` can be a void method: We just want to empty the bag, not retrieve any of its contents. Thus, we write

```
+clear(): void
```

in UML.

If the first `remove` method removes an entry from the bag, the method can easily return the object it has removed. Its return type is then the generic type `T`. In UML, we have

```
+remove(): T
```

Notice that we can respond to an attempt to remove an object from an empty bag by returning `null`.

The second `remove` method won't be able to remove a particular entry from the bag if the bag does not contain that entry. We could have the method return a boolean value, much as `add` does, so it can indicate success or not. Or the method could return either the removed object or `null` if it can't remove the object. Here are the specifications for these two possible versions of the method in UML—we must choose one:

```
+remove(anEntry: T): boolean
```

or

```
+remove(anEntry: T): T
```

If `anEntry` equals an entry in the bag, the first version of this method would remove that entry and return true. Even though the method would not return the removed entry, the client would have the method's argument, `anEntry`, which is equal to the removed entry. We will choose this first version, to be consistent with the interface `Collection`.

Question 2 Is it legal to have both versions of `remove(anEntry)`, which were just described, in one class? Explain.

Question 3 Is it legal to have two versions of `remove`, one that has no parameter and one that has a parameter, in the same class? Explain.

Question 4 Given the full bag `aBag` that you created in Question 1, write some pseudocode statements that remove and display all of the strings in the bag.

1.10 The remaining behaviors do not change the contents of the bag. One of these behaviors counts the number of times a given object occurs within the bag. We specify it first in pseudocode and then in UML, as follows:

```
// Counts the number of times a given entry appears in the bag.
getFrequencyOf(anEntry)
```

```
+getFrequencyOf(anEntry: T): integer
```

Another method tests whether the bag contains a given object. Its specifications in pseudocode and UML are

```
// Tests whether the bag contains a given entry.
contains(anEntry)
```

```
+contains(anEntry: T): boolean
```

Question 5 Given the full bag `aBag` that you created in Question 1, write some pseudocode statements to find the number of times, if any, that the string `"Hello"` occurs in `aBag`.

1.11 Finally, we want to look at the contents of the bag. Rather than providing a method that displays the entries in the bag, we will define one that returns an array of these entries. The client is then free to display any or all of them in any way desired. Here are the specifications for our last method:

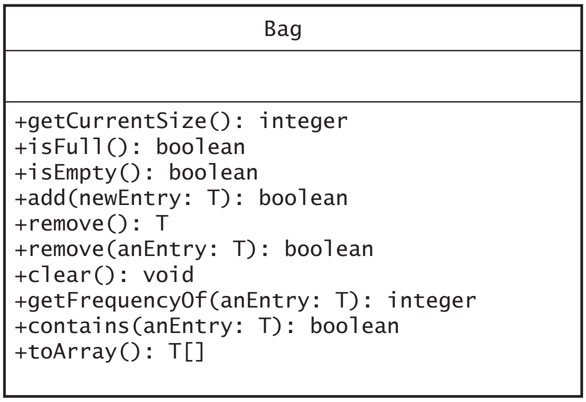
```
// Looks at all entries in the bag.
toArray()
```

```
+toArray(): T[]
```

When a method returns an array, it usually should define a new one to return. We will note that detail for this method.

1.12 As we developed the previous specifications for the bag's methods, we represented them using UML notation. Figure 1-2 shows the result of doing so.

FIGURE 1-2 UML notation for the class `Bag`



```
Bag

+getCurrentSize(): integer
+isFull(): boolean
+isEmpty(): boolean
+add(newEntry: T): boolean
+remove(): T
+remove(anEntry: T): boolean
+clear(): void
+getFrequencyOf(anEntry: T): integer
+contains(anEntry: T): boolean
+toArray(): T[]
```

Notice that the CRC card and the UML do not reflect all of the details, such as assumptions and unusual circumstances, that we mentioned in our previous discussion. However, after you have identified such conditions, you should specify how your methods will behave under each one. You should write down your decisions about how you want your methods to behave, as we have done in the following table. Later, you can incorporate these informal descriptions into the Java comments that document your methods.

Abstract Data Type: Bag

Data

- A finite number of objects, not necessarily distinct, in no particular order, and having the same data type
- The number of objects in this collection

Operations

Pseudocode	UML	Description
`getCurrentSize()`	`+getCurrentSize(): integer`	Task: Reports the current number of objects in the bag. Input: None. Output: The number of objects currently in the bag.
`isFull()`	`+isFull(): boolean`	Task: Sees whether the bag is full. Input: None. Output: True or false according to whether the bag is full.
`isEmpty()`	`+isEmpty(): boolean`	Task: Sees whether the bag is empty. Input: None. Output: True or false according to whether the bag is empty.
`add(newEntry)`	`+add(newEntry: T): boolean`	Task: Adds a given object to the bag. Input: `newEntry` is an object. Output: True or false according to whether the addition succeeds.
`remove()`	`+remove(): T`	Task: Removes an unspecified object from the bag, if possible. Input: None. Output: Either the removed object, if the removal was successful, or `null`.

`remove(anEntry)`	`+remove(anEntry: T): boolean`	Task: Removes an occurrence of a particular object from the bag, if possible. Input: `anEntry` is an object. Output: True or false according to whether the removal succeeds.
`clear()`	`+clear(): void`	Task: Removes all objects from the bag. Input: None. Output: None.
`getFrequencyOf(anEntry)`	`+getFrequencyOf(anEntry: T): integer`	Task: Counts the number of times an object occurs in the bag. Input: `anEntry` is an object. Output: The integer number of times `anEntry` occurs in the bag.
`contains(anEntry)`	`+contains(anEntry: T): boolean`	Task: Tests whether the bag contains a particular object. Input: `anEntry` is an object. Output: True or false according to whether `anEntry` occurs in the bag.
`toArray()`	`+toArray(): T[]`	Task: Looks at all objects in the bag. Input: None. Output: A new array of entries currently in the bag.

Design Decision: **What should happen when an unusual condition occurs?**

You as class designer need to make decisions about how to treat unusual conditions and include these decisions in your specifications. The documentation for the ADT bag should reflect both these decisions and the details in the previous discussion.

In general, you can address unusual situations in several ways. Your method could

- Assume that the invalid situations will not occur. This assumption is not as naive as it might sound. A method could state as an assumption—that is, a precondition—restrictions to which a client must adhere. It is then up to the client to check that the precondition is satisfied before invoking the method. For example, a precondition for the method `remove` might be that the bag is not empty. Notice that the client can use other methods of the ADT bag, such as `isEmpty` and `getCurrentSize`, to help with this task. As long as the client obeys the restriction, the invalid situation will not occur.

- Ignore the invalid situations. A method could simply do nothing when given invalid data. Doing absolutely nothing, however, leaves the client without knowledge of what happened.
- Guess at the client's intention. Like the previous option, this choice can cause problems for the client.
- Return a value that signals a problem. For example, if a client tries to remove an entry from an empty bag, the `remove` method could return `null`. The value returned must be something that cannot be in the bag.
- Return a boolean value that indicates the success or failure of an operation.
- Throw an exception.

Note: Throwing an exception is often a desirable way for a Java method to react to unusual events that occur during its execution. The method can simply report a problem without deciding what to do about it. The exception enables each client to do what is needed in its own particular situation. However, the method invocation in the client must appear within a `try` block. For simplicity right now, we will adopt the philosophy that methods should throw exceptions only in truly exceptional circumstances, when no other reasonable solution exists.

Note: A first draft of an ADT's specifications often overlooks or ignores situations that you really need to consider. You might intentionally make these omissions to simplify this first draft. Once you have written the major portions of the specifications, you can concentrate on the details that make the specifications complete.

An Interface

1.13 As your specifications become more detailed, they increasingly should reflect your choice of programming language. Ultimately, you can write Java headers for the bag's methods and organize them into a Java interface for the class that will implement the ADT. The Java interface in Listing 1-1 contains the methods for an ADT bag and detailed comments that describe their behaviors. Recall that a class interface does not include data fields, constructors, private methods, or protected methods.

For now, the items in the bag will be objects of the same class. For example, we could have a bag of strings. To accommodate entries of any one class type, the bag methods use a generic type `T` for each entry. To give meaning to the identifier `T`, we must write <T> after the name of the interface. Once the actual data type is chosen within a client, the compiler will use that data type wherever `T` appears.

As you examine the interface, notice the decisions that were made to address the unusual situations mentioned in the previous segment. In particular, each of the methods `add`, `remove`, and `contains` returns a value. Since our programming language is Java, notice that one of the `remove` methods returns a reference to an entry, not the entry itself.

Although writing an interface before implementing a class is certainly not required, doing so enables you to document your specifications in a concise way. You then can use the code in the interface as an outline for the actual class. Having an interface also provides a data type for a bag that is independent of a particular class definition. The next two chapters will develop different implementations of a class of bags. Code written with respect to an interface allows us to more easily replace one implementation of a bag with another.

LISTING 1-1 A Java interface for a class of bags

```
/**
   An interface that describes the operations of a bag of objects.
   @author Frank M. Carrano
*/
public interface BagInterface<T>
{
   /** Gets the current number of entries in this bag.
       @return the integer number of entries currently in the bag */
   public int getCurrentSize();

   /** Sees whether this bag is full.
       @return true if the bag is full, or false if not */
   public boolean isFull();

   /** Sees whether this bag is empty.
       @return true if the bag is empty, or false if not */
   public boolean isEmpty();

   /** Adds a new entry to this bag.
       @param newEntry  the object to be added as a new entry
       @return true if the addition is successful, or false if not */
   public boolean add(T newEntry);

   /** Removes one unspecified entry from this bag, if possible.
       @return either the removed entry, if the removal
               was successful, or null */
   public T remove();

   /** Removes one occurrence of a given entry from this bag,
       if possible.
       @param anEntry  the entry to be removed
       @return true if the removal was successful, or false if not */
   public boolean remove(T anEntry);

   /** Removes all entries from this bag. */
   public void clear();

   /** Counts the number of times a given entry appears in this bag.
       @param anEntry  the entry to be counted
       @return the number of times anEntry appears in the bag */
   public int getFrequencyOf(T anEntry);

   /** Tests whether this bag contains a given entry.
       @param anEntry  the entry to locate
       @return true if the bag contains anEntry, or false otherwise */
   public boolean contains(T anEntry);

   /** Creates an array of all entries that are in this bag.
       @return a newly allocated array of all the entries in the bag */
   public T[] toArray();
} // end BagInterface
```

1.14 After specifying an ADT and writing a Java interface for its operations, you should write some Java statements that use the ADT. Although we cannot execute these statements yet—after all, we have not written a class that implements `BagInterface`—we can use them to confirm or revise both our decisions about the design of the methods and the accompanying documentation. In this way, you check both the suitability and your understanding of the specifications. It is better to revise the design or documentation of the ADT now, instead of after you have written its implementation. An added benefit of doing this task carefully is that you can use these same Java statements later to test your implementation.

SELF-TEST

Question 6 Given the full bag `aBag` that you created in Question 1, write some Java statements that display all of the strings in `aBag`. Do not alter the contents of `aBag`.

Programming Tip **Write a test program before you implement a class**

Writing Java statements that test a class's methods will help you to fully understand the specifications for the methods. Obviously, you must understand a method before you can implement it correctly. If you are also the class designer, your use of the class might help you see desirable changes to your design or its documentation. You will save time if you make these revisions before you have implemented the class. Since you must write a program that tests your implementation sometime, why not get additional benefits from the task by writing it now instead of later?

Note: Although we said that the entries in a bag belong to the same class, those entries can also belong to classes related by inheritance. For example, assume `Bag` is a class that implements the interface `BagInterface`. If we create a bag of class `C` objects by writing

```
BagInterface<C> aBag = new Bag<C>();
```

`aBag` can contain objects of class `C`, as well as objects of any subclass of `C`.

The following section looks at two examples that use a bag. Later, these examples can be part of a test of your implementation.

Using the ADT Bag

1.15 Imagine that we hire a programmer to implement the ADT bag in Java, given the interface and specifications that we have developed so far. If we assume that these specifications are clear enough for the programmer to complete the implementation, we can use the ADT's operations in a program without knowing the details of the implementation. That is, we do not need to know *how* the programmer implemented the bag to be able to use it. We only need to know *what* the ADT bag does. This section assumes that we have a Java class, `Bag`, that implements the Java interface `BagInterface` given in Listing 1-1. The simple examples demonstrate how we can use `Bag`.

VideoNote
Designing a test for an ADT

Notice that once we choose the data type of the objects to be in a bag, that data type is enclosed in brackets that follow the interface name and the class name. All entries in the bag then must have either that data type or a subtype of that data type. The compiler will enforce this restriction for us. For primitive types, you can place instances of an appropriate wrapper class into a bag. For example, instead of instances of the primitive type `int`, you could use instances of the wrapper class `Integer`.

1.16 **Example: Shopping online.** When you shop online, your selections are saved in a shopping cart, or bag, until you are ready to check out. The program that implements the shopping website can use the class `Bag` to maintain the shopping cart. After all, the order in which you choose items to purchase is not important. Listing 1-2 shows a simple example of such a program.

LISTING 1-2 A program that maintains a bag for online shopping

```java
/**
    A class that maintains a shopping cart for an online store.
    @author Frank M. Carrano
*/
public class OnlineShopper
{
   public static void main(String[] args)
   {
      Item[] items = {new Item("Bird feeder", 2050),
                      new Item("Squirrel guard", 1547),
                      new Item("Bird bath", 4499),
                      new Item("Sunflower seeds", 1295)};
      BagInterface<Item> shoppingCart = new Bag<Item>();
      int totalCost = 0;

      // statements that add selected items to the shopping cart:
      for (int index = 0; index < items.length; index++)
      {
         Item nextItem = items[index]; // simulate getting item from
                                       // shopper
         shoppingCart.add(nextItem);
         totalCost = totalCost + nextItem.getPrice();
      } // end for

      // simulate checkout
      while (!shoppingCart.isEmpty())
         System.out.println(shoppingCart.remove());

      System.out.println("Total cost: " +
                         "\t$" + totalCost / 100 + "." +
                         totalCost % 100);
   } // end main
} // end OnlineShopper
```

Output

```
    Sunflower seeds $12.95
    Bird bath       $44.99
    Squirrel guard  $15.47
    Bird feeder     $20.50
    Total cost:     $93.91
```

To keep the example simple, we create an array of `Item` objects to represent the choices made by the shopper. The class `Item`, which is available to you in this book's online resources, defines data fields for an item's description and price, accessor methods for these fields, and the method `toString`.

Initially, we create an empty bag for `Item` objects by using `Bag`'s default constructor. Notice that the data type of `shoppingCart` is `BagInterface<Item>`. This declaration obliges `shoppingCart` to receive only calls to methods declared in `BagInterface`. Moreover, we could replace the class `Bag` with another class that also implements `BagInterface` without modifying the subsequent statements in the program.

Notice the loop that adds the chosen items to the bag and the loop that removes them one at a time during checkout.

Question 7 In the previous example, a `while` loop executes during the checkout process until the bag is empty. What `for` statement could replace the `while` statement? Use only the existence of `shoppingCart`, not the array `items`.

1.17 **Example: A piggy bank.** You might have a piggy bank, jar, or some other receptacle to hold your spare coins. The piggy bank holds the coins but gives them no other organization. And certainly the bank can contain duplicate coins. A piggy bank is like a bag, but it is simpler, as it has only three operations: You can add a coin to the bank, remove one (you shake the bank, so you have no control over what coin falls out), or see whether the bank is empty.

Assuming that we have the class `Coin` to represent coins, we can create the class `PiggyBank` given in Listing 1-3. A `PiggyBank` object stores its coins in a bag, that is, in an instance of a class that implements the interface `BagInterface`. The `add`, `remove`, and `isEmpty` methods of `PiggyBank` each call the respective bag method to achieve their results. The class `PiggyBank` is an example of an adapter class. See Appendix C for more on adapter classes.

LISTING 1-3 A class of piggy banks

```
/**
   A class that implements a piggy bank by using a bag.
   @author Frank M. Carrano
*/
public class PiggyBank
{
   private BagInterface<Coin> coins;

   public PiggyBank()
   {
      coins = new Bag<Coin>();
   } // end default constructor

   public boolean add(Coin aCoin)
   {
      return coins.add(aCoin);
   } // end add

   public Coin remove()
   {
      return coins.remove();
   } // end remove

   public boolean isEmpty()
   {
      return coins.isEmpty();
   } // end isEmpty
} // end PiggyBank
```

1.18 Listing 1-4 provides a brief demonstration of the class `PiggyBank`. The program adds some coins to the bank and then removes all of them. Since the program does not keep a record of the coins it adds to the bank, it has no control over which coins are removed. Although the output indicates that the coins leave the bank in the opposite order from how they entered it, that order depends on the bag's implementation. We'll consider these implementations in the next chapters.

Notice that, in addition to the `main` method, the program defines another method, `addCoin`. Since `main` is static and calls `addCoin`, it must be static as well. The method `addCoin` accepts as its arguments a `Coin` object and a `PiggyBank` object. The method then adds the coin to the bank.

LISTING 1-4 A demonstration of the class `PiggyBank`

```
/**
    A class that demonstrates the class PiggyBank.
    @author Frank M. Carrano
*/
public class PiggyBankExample
{
   public static void main(String[] args)
   {
      PiggyBank myBank = new PiggyBank();

      addCoin(new Coin(1, 2010), myBank);
      addCoin(new Coin(5, 2011), myBank);
      addCoin(new Coin(10, 2000), myBank);
      addCoin(new Coin(25, 2012), myBank);
      System.out.println("Removing all the coins:");
      int amountRemoved = 0;

      while (!myBank.isEmpty())
      {
         Coin removedCoin = myBank.remove();
         System.out.println("Removed a " + removedCoin.getCoinName() +
                            ".");
         amountRemoved = amountRemoved + removedCoin.getValue();
      } // end while

      System.out.println("All done. Removed " + amountRemoved +
                         " cents.");
   } // end main

   private static void addCoin(Coin aCoin, PiggyBank aBank)
   {
      if (aBank.add(aCoin))
         System.out.println("Added a " + aCoin.getCoinName() + ".");
      else
         System.out.println("Tried to add a " + aCoin.getCoinName() +
                            ", but couldn't");
   } // end addCoin
} // end PiggyBankExample
```

Output

```
   Added a PENNY.
   Added a NICKEL.
   Added a DIME.
   Added a QUARTER.
   Removing all the coins:
   Removed a QUARTER.
   Removed a DIME.
   Removed a NICKEL.
   Removed a PENNY.
   All done. Removed 41 cents.
```

Note: A method can change the state of an object passed to it as an argument

You pass two arguments to the method addCoin: a coin and a piggy bank. Both of these arguments are references to objects that exist in the main method. The method addCoin stores copies of these references in its parameters, which, as you will recall, behave as local variables. Although addCoin cannot change the references, because they exist in the main method, it can alter the state of the referenced objects. In particular, it changes the piggy bank—that is, the PiggyBank object—by adding coins to it. That bank, remember, is local to main and is outside of addCoin.

Note: As soon as we implement a class of bags in the next chapters, you can actually run the programs shown in the previous listings. You just need to reconcile the class name `Bag` that these examples use with the names of the classes in the next chapters.

SELF-TEST

Question 8 Consider the program in Listing 1-4. After creating the instance `myBank` of the class `PiggyBank`, suppose that we add several unknown coins to `myBank`. Write some code that will remove coins from the bank until either you remove a penny or the bank becomes empty.

Using an ADT Is Like Using a Vending Machine

1.19 Imagine that you are in front of a vending machine, as Figure 1-3 depicts; or better yet, take a break and go buy something from one!

FIGURE 1-3 A vending machine

When you look at the front of a vending machine, you see its interface. By inserting coins and pressing buttons, you are able to make a purchase. Here are some observations that we can make about the vending machine:

- You can perform only the specific tasks that the machine's interface presents to you.
- You must understand these tasks—that is, you must know what to do to buy a soda.
- You cannot access the inside of the machine, because a locked shell encapsulates it.
- You can use the machine even though you do not know what happens inside.
- If someone replaced the machine's inner mechanism with an improved version, leaving the interface unchanged, you could still use the machine in the same way.

You, as the user of a vending machine, are like the client of the ADT bag that you saw earlier in this chapter. The observations that we just made about the user of a vending machine are similar to the following observations about a bag's client:

- The client can perform only the operations specific to the ADT bag. These operations often are declared within a Java interface.
- The client must adhere to the specifications of the operations that the ADT bag provides. That is, the programmer of the client must understand how to use these operations.
- The client cannot access the data within the bag without using an ADT operation. The principle of encapsulation hides the data representation within the ADT.
- The client can use the bag, even though the programmer does not know how the data is stored.
- If someone changed the implementation of the bag's operations, the client could still use the bag in the same way, as long as the interface did not change.

1.20 In the examples of the previous section, each bag is an instance of a class that implements the ADT bag. That is, each bag is an object whose behaviors are the operations of the ADT bag. You can think of each such object as being like the vending machine we just described. Each object encapsulates the bag's data and operations, just as the vending machine encapsulates its product (soda cans) and delivery system.

Some ADT operations have inputs analogous to the coins you insert into a vending machine. Some ADT operations have outputs analogous to the change, soda cans, messages, and warning lights that a vending machine provides.

Now imagine that you are the designer of the front, or interface, of the vending machine. What can the machine do, and what should a person do to use the machine? Will it help you or hinder you to think about how the soda cans will be stored and transported within the machine? We maintain that you should ignore these aspects and focus solely on how someone will use the machine—that is, you focus on designing the interface. Ignoring extraneous details makes your task easier and increases the quality of your design.

Recall that abstraction as a design principle asks you to focus on *what* instead of *how*. When you design an ADT, and ultimately a class, you use data abstraction to focus on what you want to do with or to the data, without worrying about how you will accomplish these tasks. We practiced data abstraction at the beginning of this chapter when we designed the ADT bag. As we chose the methods that a bag would have, we did not consider how we would represent the bag. Instead, we focused on what each method should do.

Ultimately, we wrote a Java interface that specified the methods in detail. We were then able to write a client that used the bag, again without knowledge of its implementation. If someone wrote the implementation for us, our program would presumably run correctly. If someone else gave us a better implementation, we could use it without changing our already-written client. This feature of the client is a major advantage of abstraction.

Java Class Library: The Interface `Set`

As we mentioned at the end of Appendix B, the Java Class Library is a collection of classes and interfaces that Java programmers use as a matter of course. From time to time, we will present members of the Java Class Library that are like or relevant to our current discussion. The **Java Collections Framework** is a subset of this library that provides us with a uniform way of representing and working with collections. Many of the classes and interfaces in the Java Class Library that we will note are a part of this framework, although we usually will not point out this fact.

1.21 The ADT **set** is a bag that does not allow duplicate entries. Although we leave the specification and implementation of the set as exercises in this and subsequent chapters, we do want to present the standard interface `Set`, which belongs to the package `java.util` within the Java Class Library. Sets that adhere to the specifications of this interface do not contain a pair of objects `x` and `y` such that `x.equals(y)` is true.

The following method headers declared in the interface `Set` are similar to the methods within our `BagInterface`. The differences between a method in `Set` and a corresponding method in `BagInterface` are highlighted.

```
public boolean add(T newEntry)
public boolean remove(Object anEntry)
public void clear()
public boolean contains(Object anEntry)
public boolean isEmpty()
public int size()
public Object[] toArray()
```

Each of the interfaces `Set` and `BagInterface` declare methods that are not in the other.

Chapter Summary

- An abstract data type, or ADT, is a specification of a data set and the operations on that data. This specification does not indicate how to store the data or how to implement the operations, and it is independent of any programming language.
- When you use data abstraction to design an ADT, you focus on what you want to do with or to the data without worrying about how you will accomplish these tasks. That is, you ignore the details of how you represent data and how you manipulate it.
- The manifestation of the ADT in a programming language encapsulates the data and operations. As a result, the particular data representations and method implementations are hidden from the client.
- A collection is an object that holds a group of other objects.
- A bag is a finite collection whose entries are in no particular order.
- A client manipulates or accesses a bag's entries by using only the operations defined for the ADT bag.
- When you add an object to a bag, you cannot indicate where in the bag it will be placed.
- You can remove from a bag an object having either a given value or one that is unspecified. You also can remove all objects from a bag.
- A bag can report whether it contains a given object. It can also report the number of times a given object occurs within its contents.
- A bag can tell you the number of objects it currently contains and can provide an array of those objects.
- Carefully specify the methods for a proposed class before you begin to implement them, using tools such as CRC cards and UML notation.
- After designing a draft of an ADT, confirm your understanding of the operations and their design by writing some pseudocode that uses the ADT.
- You should specify the action a method should take if it encounters an unusual situation.
- Writing a Java interface is a way to organize a specification for an ADT.
- Writing a program that tests a class before it is defined is a way to see whether you fully understand and are satisfied with the specification of the class's methods.

Programming Tip

- Writing Java statements that test a class's methods will help you to fully understand the specifications for the methods. Obviously, you must understand a method before you can implement it correctly. If you are also the class designer, your use of the class might help you see desirable changes to your design or its documentation. You will save time if you make these revisions before you have implemented the class. Since you must write a program that tests your implementation sometime, why not get additional benefits from the task by writing it now instead of later?

Exercises

1. Specify each method of the class `PiggyBank`, as given in Listing 1-3, by stating the method's purpose; by describing its parameters; and by writing preconditions, postconditions, and a pseudocode version of its header. Then write a Java interface for these methods that includes `javadoc`-style comments.

2. Suppose that `groceryBag` is a bag filled to its capacity with 10 strings that name various groceries. Write Java statements that remove and count all occurrences of `"soup"` in `groceryBag`. Do not remove any other strings from the bag. Report the number of times that `"soup"` occurred in the bag. Accommodate the possibility that `groceryBag` does not contain any occurrence of `"soup"`.

3. Given `groceryBag`, as described in Exercise 2, what effect does the operation `groceryBag.toArray()` have on `groceryBag`?

4. Given `groceryBag`, as described in Exercise 2, write some Java statements that create an array of the distinct strings that are in this bag. That is, if `"soup"` occurs three times in `groceryBag`, it should only appear once in your array. After you have finished creating this array, the contents of `groceryBag` should be unchanged.

5. The *union* of two collections consists of their contents combined into a new collection. Add a method `union` to the interface `BagInterface` for the ADT bag that returns as a new bag the union of the bag receiving the call to the method and the bag that is the method's one argument. Include sufficient comments to fully specify the method.

   Note that the union of two bags might contain duplicate items. For example, if object *x* occurs five times in one bag and twice in another, the union of these bags contains *x* seven times. Specifically, suppose that `bag1` and `bag2` are `Bag` objects, where `Bag` implements `BagInterface`; `bag1` contains the `String` objects `a`, `b`, and `c`; and `bag2` contains the `String` objects `b`, `b`, `d`, and `e`. After the statement

   ```
   BagInterface<String> everything = bag1.union(bag2);
   ```

   executes, the bag `everything` contains the strings `a`, `b`, `b`, `b`, `c`, `d`, and `e`. Note that `union` does not affect the contents of `bag1` and `bag2`.

6. The *intersection* of two collections is a new collection of the entries that occur in both collections. That is, it contains the overlapping entries. Add a method `intersection` to the interface `BagInterface` for the ADT bag that returns as a new bag the intersection of the bag receiving the call to the method and the bag that is the method's one argument. Include sufficient comments to fully specify the method.

   Note that the intersection of two bags might contain duplicate items. For example, if object *x* occurs five times in one bag and twice in another, the intersection of these bags contains *x* twice. Specifically, suppose that `bag1` and `bag2` are `Bag` objects, where `Bag` implements `BagInterface`; `bag1` contains the `String` objects `a`, `b`, and `c`; and `bag2` contains the `String` objects `b`, `b`, `d`, and `e`. After the statement

   ```
   BagInterface<String> commonItems = bag1.intersection(bag2);
   ```

   executes, the bag `commonItems` contains only the string `b`. If `b` had occurred in `bag1` twice, `commonItems` would have contained two occurrences of `b`, since `bag2` also contains two occurrences of `b`. Note that `intersection` does not affect the contents of `bag1` and `bag2`.

7. The *difference* of two collections is a new collection of the entries that would be left in one collection after removing those that also occur in the second. Add a method `difference` to the interface `BagInterface` for the ADT bag that returns as a new bag the difference of the bag receiving the call to the method and the bag that is the method's one argument. Include sufficient comments to fully specify the method.

   Note that the difference of two bags might contain duplicate items. For example, if object *x* occurs five times in one bag and twice in another, the difference of these bags contains *x* three times. Specifically, suppose that `bag1` and `bag2` are `Bag` objects, where `Bag` implements `BagInterface`; `bag1` contains the `String` objects `a`, `b`, and `c`; and `bag2` contains the `String` objects `b`, `b`, `d`, and `e`. After the statement

   ```
   BagInterface leftOver1 = bag1.difference(bag2);
   ```

   executes, the bag `leftOver1` contains the strings `a` and `c`. After the statement

   ```
   BagInterface leftOver2 = bag2.difference(bag1);
   ```

   executes, the bag `leftOver2` contains the strings `b`, `d`, and `e`. Note that `difference` does not affect the contents of `bag1` and `bag2`.

8. Write code that accomplishes the following tasks: Consider two bags that can hold strings. One bag is named `letters` and contains several one-letter strings. The other bag is empty and is named `vowels`. One at a time, remove a string from `letters`. If the string contains a vowel, place it into the bag `vowels`; otherwise, discard the string. After you have checked all of the strings in `letters`, report the number of vowels in the bag `vowels` and the number of times each vowel appears in the bag.

9. Write code that accomplishes the following tasks: Consider three bags that can hold strings. One bag is named `letters` and contains several one-letter strings. Another bag is named `vowels` and contains five strings, one for each vowel. The third bag is empty and is named `consonants`. One at a time, remove a string from `letters`. Check whether the string is in the bag `vowels`. If it is, discard the string. Otherwise, place it into the bag `consonants`. After you have checked all of the strings in `letters`, report the number of consonants in the bag `consonants` and the number of times each consonant appears in the bag.

PROJECTS

1. As we mentioned in Segment 1.21, a set is a special bag that does not allow duplicates.

   a. Specify each operation for a set of objects by stating its purpose; by describing its parameters; and by writing preconditions, postconditions, and a pseudocode version of its header. Then write a Java interface, `SetInterface<T>`, for the set. Include `javadoc`-style comments in your code.

   b. Suppose the class `Set<T>` implements `SetInterface<T>`. Given an empty set that is an object of `Set<String>` and an object of the class `Bag<String>` that contains several strings, write statements at the client level that create a set from the given bag.

2. Imagine a pile of books on your desk. Each book is so large and heavy that you can remove only the top one from the pile. You cannot remove a book from under another one. Likewise, you can add another book to the pile only by placing it on the top of the pile. You cannot add a book beneath another one.

   If you represent books by their titles alone, design a class that you can use to track the books in the pile on your desk. Specify each operation by stating its purpose, by describing its parameters, and by writing a pseudocode version of its header. Then write a Java interface for the pile's methods. Include `javadoc`-style comments in your code.

3. A *ring* is a collection of items that has a reference to a current item. An operation—let's call it `advance`—moves the reference to the next item in the collection. When the reference reaches the last item, the next `advance` operation will move the reference back to the first item. A ring also has operations to get the current item, add an item, and remove an item. The details of where an item is added and which one is removed are up to you.

   Design an ADT to represent a ring of objects. Specify each operation by stating its purpose, by describing its parameters, and by writing a pseudocode version of its header. Then write a Java interface for a ring's methods. Include `javadoc`-style comments in your code.

4. A *shoe* of playing cards contains some number of standard decks of cards. Cards in the shoe can be shuffled together and dealt one at a time. The number of cards in the shoe can also be calculated.

   After a hand is complete, you should be able to return all cards to the shoe and shuffle them. Some card games require that the discard pile be returned to the shoe when the shoe becomes empty. Then the cards in the shoe can be shuffled. In this case, not all cards are in the shoe; some are held by the players.

   Design an ADT for a shoe, assuming that you have the class `PlayingCard`, which was described in Project 6 of the online projects for Appendix C. You do not need an ADT deck, since a deck is a shoe whose number of decks is 1.

   Specify each ADT operation by stating its purpose, by describing its parameters, and by writing a pseudocode version of its header. Then write a Java interface for a shoe's methods. Include `javadoc`-style comments in your code.

5. A bid for installing an air conditioner consists of the name of the company, a description of the unit, the performance of the unit, the cost of the unit, and the cost of installation.

   Design an ADT that represents any bid. Then design another ADT to represent a collection of bids. The second ADT should include methods to search for bids based on price and performance. Also note that a single company could make multiple bids, each with a different unit.

   Specify each ADT operation by stating its purpose, by describing its parameters, and by writing a pseudocode version of its header. Then write a Java interface for a bid's methods. Include `javadoc`-style comments in your code.

6. A *matrix* is a rectangular array of numerical values. You can add or multiply two matrices to form a third matrix. You can multiply a matrix by a scalar, and you can transpose a matrix. Design an ADT that represents a matrix that has these operations.

   Specify each ADT operation by stating its purpose, by describing its parameters, and by writing a pseudocode version of its header. Then write a Java interface for the methods of a matrix. Include `javadoc`-style comments in your code.

Answers to Self-Test Questions

1.
```
// aBag is empty
do
{
   entry = next string read from user
   aBag.add(entry)
} while (!aBag.isFull())
// aBag is full
```

2. No. The two methods have identical signatures. Recall that a method's return type is not a part of its signature. These methods have the same name and parameter list.

3. Yes. The two methods have different signatures. They are overloaded methods.

4.
```
// aBag is full
while (!aBag.isEmpty())
{
   entry = aBag.remove()
   Display entry
}
// aBag is empty
```

5.
```
Display "The string Hello occurs in aBag " + aBag.getFrequencyOf("Hello") + " times."
```

6.
```
String[] contents = aBag.toArray();
for (int index = 0; index < contents.length; index++)
   System.out.print(contents[index] + " ");
System.out.println();
```

7.
```
int itemCount = shoppingCart.getCurrentSize();
for (int counter = 0; counter < itemCount; counter++)
   System.out.println(shoppingCart.remove());
```

8.
```
boolean lookingForPenny = true;
while (!myBank.isEmpty() && lookingForPenny)
{
   Coin removedCoin = myBank.remove();
   System.out.println("Removed a " + removedCoin.getCoinName() + ".");
   if (removedCoin.getCoinName() == CoinName.PENNY)
// if (removedCoin.getValue() == 1) // ALTERNATE
   {
      System.out.println("Found a penny. All done!");
      lookingForPenny = false; // penny is found
   }
} // end while

if (lookingForPenny)
   System.out.println("No penny was found. Sorry!");
```

Chapter

2

Bag Implementations That Use Arrays

Contents

Prerequisites

Appendix	D	Designing Classes
Chapter	1	Bags

Objectives

After studying this chapter, you should be able to

- Implement the ADT bag by using a fixed-size array or an array that you expand dynamically
- Discuss the advantages and disadvantages of the two implementations presented

You have seen several examples of how to use the ADT bag in a program. This chapter presents two different ways—each involving an array—to implement a bag in Java. When you use an array to organize data, the implementation is said to be **array based**. You will see a completely different approach in the next chapter.

We begin by using an ordinary Java array to represent the entries in a bag. With this implementation, your bag could become full, just as a grocery bag does. We then offer another implementation that does not suffer from this problem. When you use all of the space in an array, Java enables you to move the data to a larger array. The effect is to have an array that apparently expands to meet your needs. Thus, we can have a bag that is never full.

Using a Fixed-Size Array to Implement the ADT Bag

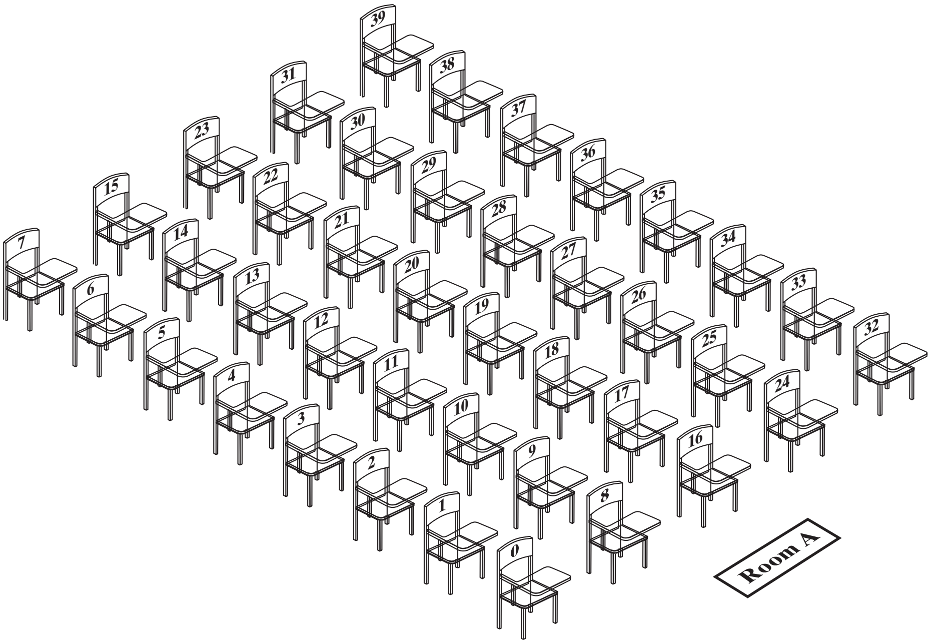
Our task is to define the methods we specified in the previous chapter when we wrote the interface `BagInterface`. We begin by using an analogy to describe how a fixed-size array could contain the entries in a bag. In doing so, we show how the `add` and `remove` methods would work. Subsequently, we present a corresponding Java implementation for the bag.

An Analogy

2.1 Imagine a classroom—call it room A—containing 40 desks in fixed positions. If a course is restricted to 30 students, 10 desks are idle and wasted. If we lift the enrollment restriction, we can accommodate only 10 more students, even if 20 more want to take the course.

An array is like this classroom, and each desk is like one array location. Suppose that we number the 40 desks in the room sequentially, beginning with zero, as Figure 2-1 illustrates. Although desks are arranged in rows in typical classrooms, we will ignore this detail and treat the desks as a one-dimensional array.

FIGURE 2-1 A classroom that contains desks in fixed positions



2.2 **Adding a new student.** Suppose that the instructor asks arriving students to occupy consecutively numbered desks. Thus, the first student who arrives at the classroom sits at desk 0; the second student sits at desk 1, and so on. The instructor's request that consecutively numbered desks be occupied is arbitrary and simply for his or her convenience. As you will see, we will fill an array of bag entries in an analogous way.

Imagine that 30 students in room A occupy the desks numbered sequentially from 0 to 29, and a new student wants to join those students. Since 40 desks are in the room, the desk numbered 30 is available. We can simply assign the new student to desk 30. When all 40 desks are occupied, we can no longer accommodate more students. The room is full.

2.3 **Removing a particular student.** Now imagine that the student in desk 5 of room A drops the course. Desk 5 stays in its fixed location within the room and will be vacant. If we still want students to sit in consecutively numbered desks, however, one student will need to move to desk 5. Since the students are not in any particular order, if the student in the highest-numbered desk moves to desk 5, no one else need move. For example, if 30 students are seated in the room in desks 0 to 29, the student in desk 29 would move to desk 5. Desks 29 and above would be vacant.

? SELF-TEST

Question 1 What is an advantage of moving a student as just described so that the vacated desk does not remain vacant?

Question 2 What is an advantage of leaving the vacated desk vacant?

Question 3 If a student were to drop the course, which one could do so without forcing another to change desks?

A Group of Core Methods

2.4 The Java array-based implementation for the ADT bag incorporates some of the ideas that our classroom example illustrates. The result is the class `ArrayBag`, which implements the interface `BagInterface` that you saw in Listing 1-1 of Chapter 1. Each public method within the interface corresponds to an ADT bag operation. Recall that the interface defines a generic type `T` for the objects in a bag. We use this same generic type in the definition of `ArrayBag`.

VideoNote
An array-based bag

The definition for the class `ArrayBag` could be fairly involved. The class certainly will have quite a few methods. For such classes, you should not define the entire class and then attempt to test it. Instead, you should identify a group of **core methods** to both implement and test before continuing with the rest of the class definition. By leaving the definitions of the other methods for later, you can focus your attention and simplify your task. But what methods should be part of this group? In general, such methods should be central to the purpose of the class and allow reasonable testing. We sometimes will call a group of core methods a **core group**.

When dealing with a collection such as a bag, you cannot test most methods until you have created the collection. Thus, adding objects to the collection is a fundamental operation. If the method `add` does not work correctly, testing other methods such as `remove` would be pointless. Thus, the bag's `add` method is part of the group of core methods that we implement first.

To test whether `add` works correctly, we need a method that allows us to see the bag's contents. The method `toArray` serves this purpose, and so it is a core method. The constructors are also fundamental and are in the core group. Similarly, any methods that a core method might call are part of the core group as well. For example, since we cannot add an entry to a full bag, the method `add` will need to call `isFull`.

2.5 **The core methods.** We have identified the following core methods to be a part of the first draft of the class `ArrayBag`:

- Constructors
- `public boolean add(T newEntry)`
- `public T[] toArray()`
- `public boolean isFull()`

With this core, we will be able to construct a bag, add objects to it, and look at the result. We will not implement the remaining methods until these core methods work correctly.

Note: Methods such as `add` and `remove` that can alter the underlying structure of a collection are likely to have the most involved implementations. In general, you should define such methods before the others in the class. But since we can't test `remove` before `add` is correct, we will delay implementing it until after `add` is completed and thoroughly tested.

Programming Tip: When defining a class, implement and test a group of core methods. Begin with methods that add to a collection of objects and/or have involved implementations.

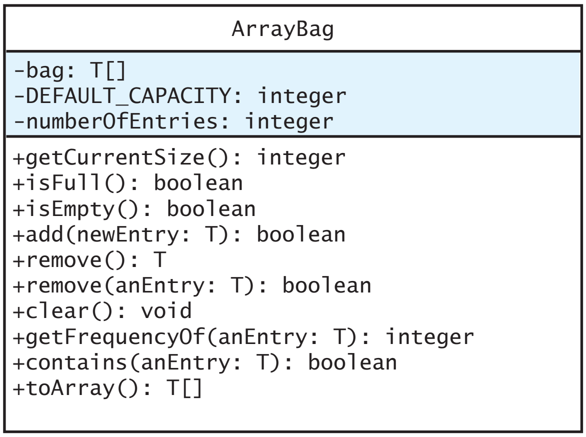
Implementing the Core Methods

2.6 **The data fields.** Before we define any of the core methods, we need to consider the class's data fields. Since the bag will hold a group of objects, one field can be an array of these objects. The length of the array defines the bag's capacity. We can let the client specify this capacity, and we can also provide a default capacity. In addition, we will want to track the current number of entries in the bag. Thus, we can define the following data fields for our class,

```
private final T[] bag;
private static final int DEFAULT_CAPACITY = 25;
private int numberOfEntries;
```

and add them to our earlier UML representation of the class in Figure 1-2 of the previous chapter. The resulting notation appears in Figure 2-2.

FIGURE 2-2 UML notation for the class `ArrayBag`, including the class's data fields



```
ArrayBag
-bag: T[]
-DEFAULT_CAPACITY: integer
-numberOfEntries: integer
+getCurrentSize(): integer
+isFull(): boolean
+isEmpty(): boolean
+add(newEntry: T): boolean
+remove(): T
+remove(anEntry: T): boolean
+clear(): void
+getFrequencyOf(anEntry: T): integer
+contains(anEntry: T): boolean
+toArray(): T[]
```

2.7 **About the constructors.** A constructor for this class must create the array `bag`. Notice that the declaration of the data field `bag` in the previous segment does not create an array. Forgetting to create an array in a constructor is a common mistake. To create the array, the constructor must specify the array's length, which is the bag's capacity. And since we are creating an empty bag, the constructor should also initialize the field `numberOfEntries` to zero.

The decision to use a generic data type in the declaration of the array bag affects how we allocate this array within the constructor. A statement such as

```
bag = new T[capacity]; // SYNTAX ERROR
```

is syntactically incorrect. You cannot use a generic type when allocating an array. Instead, we allocate an array of objects of type Object, as follows:

```
new Object[capacity];
```

However, problems arise when we try to assign this array to the data field bag. The statement

```
bag = new Object[capacity]; // SYNTAX ERROR: incompatible types
```

causes a syntax error because you cannot assign an array of type Object[] to an array of type T[]. That is, the types of the two arrays are not compatible.

A cast is necessary but creates its own problem. The statement

```
bag = (T[])new Object[capacity];
```

produces the compiler warning

```
ArrayBag.java uses unchecked or unsafe operations.
Note: Recompile with -Xlint:unchecked for details.
```

If you compile the class again and use the option -Xlint, the messages will be more detailed, beginning as follows:

```
ArrayBag.java:24: warning: [unchecked] unchecked cast
found   : java.lang.Object[]
required: T[]
      bag = (T[])new Object[capacity];
                 ^
```

The compiler wants you to ensure that casting each entry in the array from type Object to the generic type T is safe. Since the array has just been allocated, it contains null entries. Thus, the cast is safe, and so we instruct the compiler to ignore the warning by writing the annotation

```
@SuppressWarnings("unchecked")
```

before the offending statement. This instruction to the compiler can only precede a method definition or a variable declaration. Since the assignment

```
bag = (T[])new Object[capacity];
```

does not declare bag—bag has already been declared—we revise it as follows:

```
// the cast is safe because the new array contains null entries
@SuppressWarnings("unchecked")
T[] tempBag = (T[])new Object[capacity]; // unchecked cast
bag = tempBag;
```

Note: Suppressing compiler warnings

To suppress an unchecked-cast warning from the compiler, you precede the flagged statements with the instruction

```
@SuppressWarnings("unchecked")
```

Note that this instruction can precede only a method definition or a variable declaration.

2.8 **The constructors.** The following constructor performs the previous steps, using a capacity given as an argument:

```
/** Creates an empty bag having a given capacity.
    @param capacity  the integer capacity desired */
```

```
public ArrayBag(int capacity)
{
   numberOfEntries = 0;
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempBag = (T[])new Object[capacity]; // unchecked cast
   bag = tempBag;
} // end constructor
```

The default constructor can invoke the previous one, passing it the default capacity as an argument, as follows:

```
/** Creates an empty bag whose capacity is 25. */
public ArrayBag()
{
   this(DEFAULT_CAPACITY);
} // end default constructor
```

Recall that a constructor can invoke another constructor in the same class by using the keyword `this` as a method name.

2.9 **An outline of the class.** Let's look at the class as we have defined it so far. After you complete the initial portion of the class—that is, the header, data fields, and constructors—you can add the comments and headers for the public methods simply by copying them from `BagInterface`. You then write empty bodies after each of those headers. Listing 2-1 shows the result of these steps. Our next task is to implement our three core methods.

LISTING 2-1 An outline of the class `ArrayBag`

```
/**
    A class of bags whose entries are stored in a fixed-size array.
    @author Frank M. Carrano
*/
public class ArrayBag<T> implements BagInterface<T>
{
   private final T[] bag;
   private static final int DEFAULT_CAPACITY = 25;
   private int numberOfEntries;

   /** Creates an empty bag whose initial capacity is 25. */
   public ArrayBag()
   {
      this(DEFAULT_CAPACITY);
   } // end default constructor

   /** Creates an empty bag having a given initial capacity.
       @param capacity  the integer capacity desired */
   public ArrayBag(int capacity)
   {
      numberOfEntries = 0;
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempBag = (T[])new Object[capacity]; // unchecked cast
      bag = tempBag;
   } // end constructor
   /** Adds a new entry to this bag.
       @param newEntry  the object to be added as a new entry
       @return true if the addition is successful, or false if not */
   public boolean add(T newEntry)
   {
      < Body to be defined >
   } // end add

   /** Retrieves all entries that are in this bag.
       @return a newly allocated array of all the entries in the bag */
   public T[] toArray()
   {
      < Body to be defined >
   } // end toArray

   /** Sees whether this bag is full.
       @return true if the bag is full, or false if not */
   public boolean isFull()
   {
      < Body to be defined >
   } // end isFull

   < Similar partial definitions are here for the remaining methods
     declared in BagInterface. >
   . . .
} // end ArrayBag
```

Design Decision: When the array `bag` is partially full, which array elements should contain the bag's entries?

When you add a first entry to an array, you typically place it in the array's first element, that is, the element whose index is 0. Doing so, however, is not a requirement, especially for arrays that implement collections. For example, some collection implementations can benefit by ignoring the array element whose index is 0 and using index 1 as the first element in the array. Sometimes you might want to use the elements at the end of the array before the ones at its beginning. For the bag, we have no reason to be atypical, and so the objects in our bag will begin at index 0 of the array.

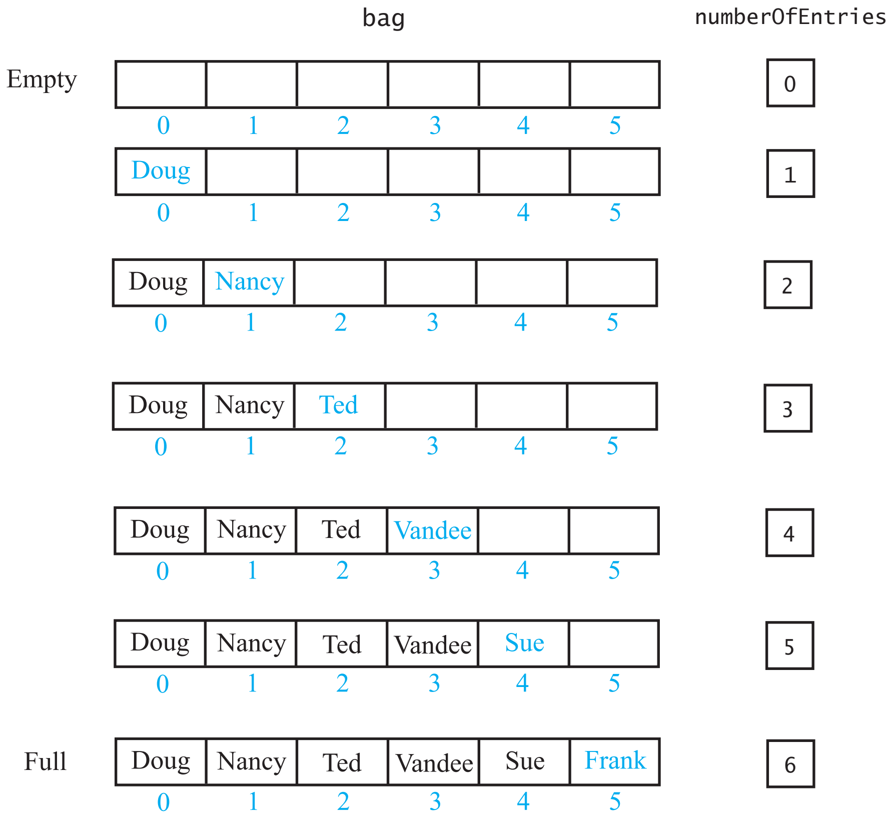
Another consideration is whether the bag's objects should occupy consecutive elements of the array. Requiring the `add` method to place objects into the array `bag` consecutively is certainly reasonable, but why should we care, and is this really a concern? We need to establish certain truths, or assertions, about our planned implementation so that the action of each method is not detrimental to other methods. For example, the method `toArray` must "know" where `add` has placed the bag's entries. Our decision now also will affect what must happen later when we remove an entry from the bag. Will the method `remove` ensure that the array entries remain in consecutive elements? It must, because for now at least, we will insist that bag entries occupy consecutive array elements.

2.10 **The method `add`.** If the bag is full, we cannot add anything to it. In that case, the method `add` should return false. Otherwise, we simply add `newEntry` immediately after the last entry in the array `bag` by writing the following statement:

```
bag[numberOfEntries] = newEntry;
```

If we are adding to an empty bag, `numberOfEntries` would be zero, and the assignment would be to `bag[0]`. If the bag contained one entry, an additional entry would be assigned to `bag[1]`, and so on. After each addition to the bag, we increase the counter `numberOfEntries`. These steps are illustrated in Figure 2-3 and accomplished by the definition of the method `add` that follows the figure.

FIGURE 2-3 Adding entries to an array that represents a bag, whose capacity is six, until it becomes full



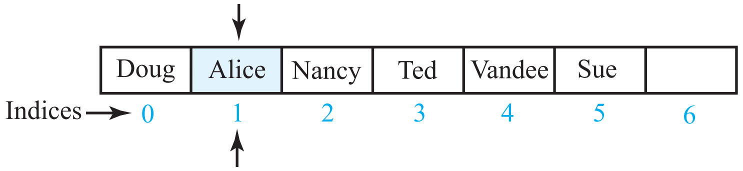
```
/** Adds a new entry to this bag.
    @param newEntry  the object to be added as a new entry
    @return true if the addition is successful, or false if not */
public boolean add(T newEntry)
{
   boolean result = true;
   if (isFull())
   {
      result = false;
   }
   else
   {  // assertion: result is true here
      bag[numberOfEntries] = newEntry;
      numberOfEntries++;
   } // end if

   return result;
} // end add
```

Notice that we call `isFull` as if it has been defined already. Had we not considered `isFull` as a core method earlier, its use now would indicate to us that it should be in the core group.

Note: The entries in a bag have no particular order. Thus, the method `add` can place a new entry into a convenient element of the array `bag`. In the previous definition of `add`, that element is the one immediately after the last element used.

Note: For simplicity, our figures and discussion portray arrays as if they actually contained objects. In reality, Java arrays contain references to objects, as Figure 2-4 illustrates.

FIGURE 2-4 An array of objects contains references to those objects



2.11 **The method `isFull`.** A bag is full when it contains as many objects as the array `bag` can accommodate. That situation occurs when `numberOfEntries` is equal to the capacity of the array. Thus, `isFull` has the following straightforward definition:

```
/** Sees whether this bag is full.
    @return true if the bag is full, or false if not */
public boolean isFull()
{
   return numberOfEntries == bag.length;
} // end isFull
```

2.12 **The method `toArray`.** The last method, `toArray`, in our initial core group retrieves the entries that are in a bag and returns them to the client within a newly allocated array. The length of this new array can equal the number of entries in the bag—that is, `numberOfEntries`—rather than the length of the array `bag`. However, we have the same problems in allocating an array that we had in defining the constructor, so we take the same steps as for the constructor.

After `toArray` creates the new array, a simple loop can copy the references in the array `bag` to this new array before returning it. Thus, the definition of `toArray` can appear as follows:

```
/** Retrieves all entries that are in this bag.
    @return a newly allocated array of all the entries in the bag */
public T[] toArray()
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] result = (T[])new Object[numberOfEntries]; // unchecked cast
   for (int index = 0; index < numberOfEntries; index++)
   {
      result[index] = bag[index];
   } // end for

   return result;
} // end toArray
```

Design Decision: Should the method toArray return the array bag instead of a copy?

Suppose that we define toArray as follows:

```
public String[] toArray()
{
   return bag;
} // end toArray
```

This simple definition would certainly return an array of the bag's contents to a client. For example, the statement

```
String[] bagArray = myBag.toArray();
```

provides a reference to an array of the entries in myBag. A client could use the variable bagArray to display the contents of myBag.

The reference bagArray, however, is to the array bag itself. That is, bagArray is an alias for the private instance variable bag within the object myBag, and therefore it gives the client direct access to this private data. Thus, a client could change the contents of the bag without calling the class's public methods. For instance, if myBag is the full bag pictured in Figure 2-3, the statement

```
bagArray[2] = null;
```

would change the entry Ted to null. Although this approach might sound good to you if the intent is to remove Ted from the bag, doing so would destroy the integrity of the bag. In particular, the entries in the array bag would no longer be consecutive, and the count of the number of entries in the bag would be incorrect.

Programming Tip

A class should not return a reference to an array that is a private data field.

Note: A variable whose declared data type is Object can reference an object of any data type. A collection whose entries are referenced by variables of type Object can contain objects of various unrelated classes. In contrast, a variable having a generic data type can reference only an object of specific data types. A collection whose entries are referenced by variables of a generic type can contain only objects of classes related by inheritance. Generics enable you to restrict the data types of the entries in your collections.

SELF-TEST

Question 4 In the previous method toArray, does the value of numberOfEntries equal bag.length in general?

Question 5 Suppose that the previous method toArray gave the new array result the same length as the array bag. How would a client get the number of entries in the returned array?

Question 6 Suppose that the previous method toArray returned the array bag instead of returning a new array such as result. If myBag is a bag of five entries, what effect would the following statements have on the array bag and the field numberOfEntries?

```
Object[] bagArray = myBag.toArray();
bagArray[0] = null;
```

Question 7 The body of the method toArray could consist of one return statement if you call the method Arrays.copyOf. Make this change to toArray.

Testing the Core Methods

2.13 **Getting ready.** Now that we have defined the three core methods, we can test them. But what about the other methods in BagInterface? Since ArrayBag—as given in Listing 2-1—implements BagInterface, Java's syntax checker will look for a definition of each method declared in this interface. Should we wait until we complete their definitions to begin testing? Absolutely not! Testing methods as you write them makes finding logical errors easier. However, instead of writing a complete implementation of each method in BagInterface, we can provide incomplete definitions of the methods we choose to temporarily ignore.

An incomplete definition of a method is called a **stub**. The stub needs only to keep the syntax checker happy. For example, for each method that returns a value, you can avoid syntax errors by adding a return statement that returns a dummy value. Methods that return a boolean value could return true. Methods that return an object could return null. On the other hand, void methods can simply have an empty body.

For instance, the method remove ultimately will return the removed entry, so its stub must contain a return statement and could appear as follows:

```
public T remove()
{
   return null; // STUB
} // end remove
```

A stub for the void method clear could be

```
public void clear()
{
   // STUB
} // end clear
```

Note that if you plan to call a stub within your test program, the stub should report that it was invoked by displaying a message.

Programming Tip: Do not wait until you complete the implementation of an ADT before testing it. By writing stubs, which are incomplete definitions of required methods, you can begin testing early in the process.

2.14 **A test program.** Listing 2-2 shows a program to test the core methods add, isFull, and toArray of the class ArrayBag[1] at this stage of its development. Initially, the main method creates an empty bag by using the default constructor. Since the capacity of this bag is 25, it should not be full if you add fewer than 25 entries to it. Thus, isFull should return false after these additions. The program's descriptive output, in fact, indicates that the tested methods are correct.

Next in the main method, we consider a full bag by creating a bag whose capacity is seven and then adding seven strings to it. This time, isFull should return true. Again, the program's output shows that our methods are correct.

LISTING 2-2 A program that tests three core methods of the class ArrayBag

```
/**
   A test of the methods add, toArray, and isFull, as defined
   in the first draft of the class ArrayBag.
```

[1] Note that this version of the class ArrayBag is available online at the book's website and is named ArrayBag1.

```
    @author Frank M. Carrano
*/
public class ArrayBagDemo1
{
   public static void main(String[] args)
   {
      // a bag that is not full
      BagInterface<String> aBag = new ArrayBag<String>();

      // tests on an empty bag
      testIsFull(aBag, false);

      // adding strings
      String[] contentsOfBag1 = {"A", "A", "B", "A", "C", "A"};
      testAdd(aBag, contentsOfBag1);
      testIsFull(aBag, false);

      // a bag that will be full
      aBag = new ArrayBag<String>(7);
      System.out.println("\nA new empty bag:");

      // tests on an empty bag
      testIsFull(aBag, false);

      // adding strings
      String[] contentsOfBag2 = {"A", "B", "A", "C", "B", "C", "D"};
      testAdd(aBag, contentsOfBag2);
      testIsFull(aBag, true);
   } // end main

   // Tests the method add.
   private static void testAdd(BagInterface<String> aBag,
                               String[] content)
   {
      System.out.print("Adding to the bag: ");
      for (int index = 0; index < content.length; index++)
      {
         aBag.add(content[index]);
         System.out.print(content[index] + " ");
      } // end for
      System.out.println();

      displayBag(aBag);
   } // end testAdd

   // Tests the method isFull.
   // correctResult indicates what isFull should return.
   private static void testIsFull(BagInterface<String> aBag,
                                  boolean correctResult)
   {
      System.out.print("\nTesting the method isFull with ");
      if (correctResult)
         System.out.println("a full bag:");
      else
         System.out.println("a bag that is not full:");

      System.out.print("isFull finds the bag ");
      if (correctResult && aBag.isFull())
         System.out.println("full: OK.");
      else if (correctResult)
         System.out.println("not full, but it is full: ERROR.");
      else if (!correctResult && aBag.isFull())
         System.out.println("full, but it is not full: ERROR.");
      else
         System.out.println("not full: OK.");
   } // end testIsFull

   // Tests the method toArray while displaying the bag.
   private static void displayBag(BagInterface<String> aBag)
   {
      System.out.println("The bag contains the following string(s):");
      Object[] bagArray = aBag.toArray();
      for (int index = 0; index < bagArray.length; index++)
      {
         System.out.print(bagArray[index] + " ");
      } // end for

      System.out.println();
   } // end displayBag
} // end ArrayBagDemo1
```

Output

```
Testing the method isFull with a bag that is not full:
isFull finds the bag not full: OK.
Adding to the bag: A A B A C A
The bag contains the following string(s):
A A B A C A

Testing the method isFull with a bag that is not full:
isFull finds the bag not full: OK.

A new empty bag:

Testing the method isFull with a bag that is not full:
isFull finds the bag not full: OK.
Adding to the bag: A B A C B C D
```

```
The bag contains the following string(s):
A B A C B C D

Testing the method isFull with a full bag:
isFull finds the bag full: OK.
```

2.15 Notice that, in addition to the `main` method, `ArrayBagDemo1` has three other methods. Since `main` is static and calls these other methods, they must be static as well. The method `testAdd` accepts as its arguments a bag and an array of strings. The method uses a loop to add each string in the array to the bag. The method `testIsFull` takes a bag as its argument and a boolean value that indicates the value we expect `isFull` to return if its logic is correct. Finally, the method `displayBag` takes a bag as its argument and uses the bag's method `toArray` to access its contents. Once we have an array of the bag's entries, a simple loop can display them.

SELF-TEST

Question 8 What is the result of executing the following statements within the `main` method of `BagDemo1`?

```
ArrayBag<String> aBag = new ArrayBag<String>();
displayBag(aBag);
```

Implementing More Methods

Now that we can add objects to a bag, we can implement the remaining methods, beginning with the easiest ones. We will postpone the definitions of `remove` momentarily until we see how to search a bag.

2.16 **The methods `isEmpty` and `getCurrentSize`.** The methods `isEmpty` and `getCurrentSize` have straightforward definitions, as you can see:

```
/** Sees whether this bag is empty.
    @return true if the bag is empty, or false if not */
public boolean isEmpty()
{
   return numberOfEntries == 0;
} // end isEmpty

/** Gets the current number of entries in this bag.
    @return the integer number of entries currently in the bag */
public int getCurrentSize()
{
   return numberOfEntries;
} // end getCurrentSize
```

Note: The definitions of some methods are almost as simple as the stubs you might use to define them in an early version of a class. Such is the case for the bag methods `isEmpty` and `getCurrentSize`. Although these two methods are not in our first group of core methods, they could have been. That is, we could have defined them earlier instead of writing stubs.

2.17 **The method `getFrequencyOf`.** To count the number of times a given object occurs in a bag, we count the number of times the object occurs in the array `bag`. Using a `for` loop to cycle through the array's indices from 0 to `numberOfEntries` – 1, we compare the given object to every object in the array.

Each time we find a match, we increment a counter. When the loop ends, we simply return the value of the counter. Note that we must use the method `equals` to compare objects. That is, we must write

```
anEntry.equals(bag[index])
```

and not

```
anEntry == bag[index] // WRONG!
```

We assume that the class to which the objects belong defines its own version of `equals`.

The method definition follows:

```
/** Counts the number of times a given entry appears in this bag.
    @param anEntry  the entry to be counted
    @return the number of times anEntry appears in the bag */
public int getFrequencyOf(T anEntry)
{
   int counter = 0;

   for (int index = 0; index < numberOfEntries; index++)
   {
      if (anEntry.equals(bag[index]))
      {
         counter++;
      } // end if
   } // end for

   return counter;
} // end getFrequencyOf
```

2.18 **The method `contains`.** To see whether a bag contains a given object, we once again search the array `bag`. The loop we need here is similar to the one in the method `getFrequencyOf`, but it should stop as soon as it finds the first occurrence of the desired entry. The following pseudocode describes this logic:

```
while (anEntry is not found and we have more array elements to check)
{
   if (anEntry equals the next array entry)
      anEntry is found in the array
}
```

This loop terminates under one of two conditions: Either `anEntry` has been found in the array or the entire array has been searched without success.

Here, then, is our definition of the method `contains`:

```
/** Tests whether this bag contains a given entry.
    @param anEntry  the entry to locate
    @return true if the bag contains anEntry, or false otherwise */
public boolean contains(T anEntry)
{
   boolean found = false;

   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
      {
         found = true;
      } // end if
   } // end for

   return found;
} // end contains
```

SELF-TEST

Question 9 The method `contains` could call `getFrequencyOf` instead of executing a loop. That is, you could define the method as follows:

```
public boolean contains(T anEntry)
{
   return getFrequencyOf(anEntry) > 0;
} // end contains
```

What is an advantage and a disadvantage of this definition as compared to the one given in the previous segment?

Note: Two kinds of loops

To count how many times an entry occurs in an array, the method `getFrequencyOf` uses a loop that cycles through all of the array's entries. In fact, the body of the loop executes `numberOfEntries` times. In contrast, to indicate whether a given entry occurs in an array, the loop in the method `contains` ends as soon as the desired entry is discovered. The body of this loop executes between one and `numberOfEntries` times. You should be comfortable writing loops that execute either a definitive or a variable number of times.

2.19 **Testing the additional methods.** As you define additional methods for the class `ArrayBag`, you should test them. The program `ArrayBagDemo2`, which is available online from the book's website, focuses only on these additional methods. However, you should form a test program incrementally so that it tests all the methods you have defined so far. The tests in `ArrayBagDemo2` are performed on a bag that is not full and on a full bag, as we did in `ArrayBagDemo1`. The version of the class `ArrayBag` to date is named `ArrayBag2` within the source code available online.

Methods That Remove Entries

We have postponed the three methods that remove entries from a bag until now because one of them is somewhat difficult and involves a search much like the one we performed in the method `contains`. We begin with the two methods that are easier to define.

2.20 **The method `clear`.** The method `clear` removes all entries from a bag, one at a time. The following definition of `clear` calls the method `remove` until the bag is empty:

```
/** Removes all entries from this bag. */
public void clear()
{
   while (!isEmpty())
      remove();
} // end clear
```

Exactly which entry is removed by each cycle of the loop is unimportant. Thus, we call the `remove` method that removes an unspecified entry. Moreover, we do not save the entry that the method returns.

Note: We can write the definition of the method `clear` in terms of the as yet undefined method `remove`. However, we cannot test `clear` completely until `remove` is defined.

Question 10 Revise the definition of the method `clear` so that it does not call `isEmpty`. *Hint*: The `while` statement should have an empty body.

Question 11 Consider the following definition of `clear`:

```
public void clear()
{
   numberOfEntries = 0;
} // end clear
```

What is a disadvantage of this definition as compared to the one shown in Segment 2.20?

2.21 **Removing an unspecified entry.** The method `remove` that has no parameter removes an unspecified entry from a bag, as long as the bag is not empty. Recall from the method's specification given in the interface in Listing 1-1 of the previous chapter that the method returns the entry it removes:

```
/** Removes one unspecified entry from this bag, if possible.
    @return either the removed entry, if the removal was successful,
            or null otherwise */
public T remove()
```

If the bag is empty before the method executes, `null` is returned.

Removing an entry from a bag involves removing it from an array. Although we can access any entry in the array `bag`, the last one is easy to remove. To do so, we

- Access the entry so it can be returned
- Set the entry's array element to `null`
- Decrement `numberOfEntries`

Decrementing `numberOfEntries` causes the last entry to be ignored, meaning that it is effectively removed, even if we did not set its location in the array to `null`.

A literal translation of the previous steps into Java leads to the following definition of the method:

```
public T remove()
{
   T result = null;
   if (numberOfEntries > 0)
   {
      result = bag[numberOfEntries - 1];
      bag[numberOfEntries - 1] = null;
      numberOfEntries--;
   } // end if

   return result;
} // end remove
```

Note that this method computes `numberOfEntries - 1` three times. The following refinement avoids this repetition:

```
public T remove()
{
   T result = null;
   if (numberOfEntries > 0)
   {
      numberOfEntries--;
      result = bag[numberOfEntries];
      bag[numberOfEntries] = null;
   } // end if
```

```
      return result;
   } // end remove
```

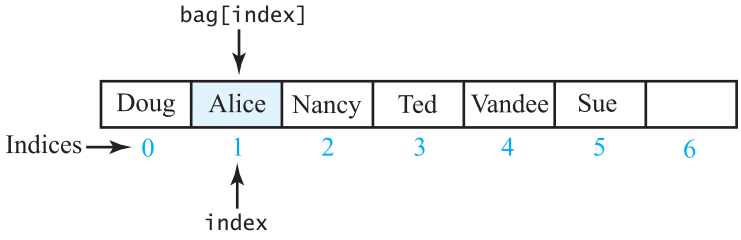
Question 12 Why does the method `remove` set `bag[numberOfEntries]` to `null`?

Question 13 The previous `remove` method removes the last entry in the array `bag`. Why might removing a different entry be more difficult to accomplish?

2.22 **Removing a given entry.** Our third method that removes an entry from the bag involves removing a given entry—call it `anEntry`. If the entry occurs more than once in the bag, we will remove only one occurrence. Exactly which occurrence is removed is unspecified. We will simply remove the first occurrence of `anEntry` that we encounter while searching for it. As we discussed in Segment 1.9 of Chapter 1, we will return either true or false, according to whether we find the entry in the bag.

Assuming that the bag is not empty, we search the array `bag` much as the method `contains` did in Segment 2.18. If `anEntry` equals `bag[index]`, we note the value of `index`. Figure 2-5 illustrates the array after a successful search.

FIGURE 2-5 The array `bag` after a successful search for the string `"Alice"`

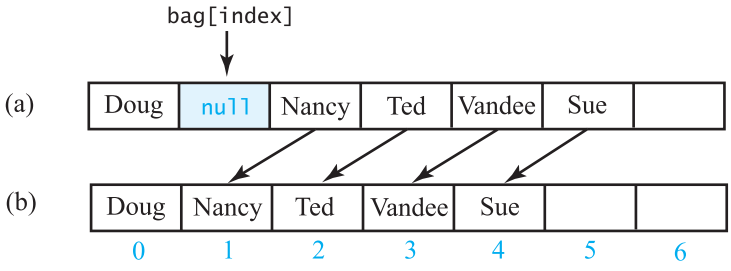


We now need to remove the entry in `bag[index]`. If we simply write

```
bag[index] = null;
```

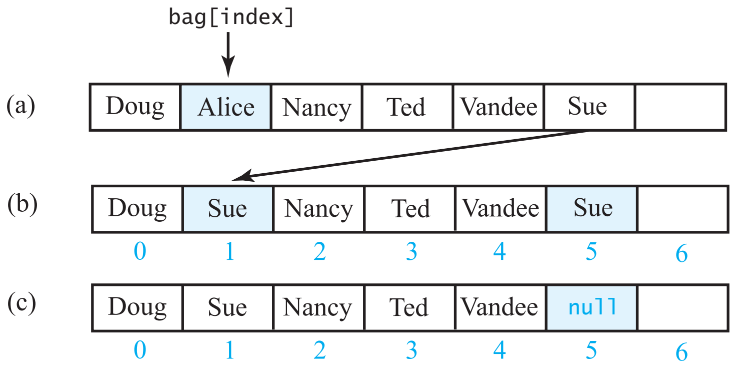
the reference in `bag[index]` to the entry will be removed, but we will have a gap in the array. That is, the contents of the bag will no longer be in consecutive array locations, as Figure 2-6a illustrates. We could get rid of that gap by shifting the subsequent entries, as shown in Figure 2-6b. This time-consuming approach is not necessary, however.

FIGURE 2-6 (a) A gap in the array `bag` after setting the entry in `bag[index]` to `null`; (b) the array after shifting subsequent entries to avoid a gap



Remember that we are not required to maintain any particular order for a bag's entries. So instead of shifting array entries after removing an entry, we can replace the entry being removed with the last entry in the array, as follows. After locating `anEntry` in `bag[index]`, as Figure 2-7a indicates, we copy the entry in `bag[numberOfEntries - 1]` to `bag[index]` (Figure 2-7b). We then replace the entry in `bag[numberOfEntries - 1]` with `null`, as Figure 2-7c illustrates, and finally we decrement `numberOfEntries`.

FIGURE 2-7 Avoiding a gap in the array while removing an entry



2.23 **Pseudocode for removing a given entry.** Let's organize our discussion by writing some pseudocode to remove the given entry, `anEntry`, from a bag that contains it:

```
Locate anEntry in the array bag; assume it occurs at bag[index]
bag[index] = bag[numberOfEntries - 1]
bag[numberOfEntries - 1] = null
Decrement the counter numberOfEntries
return true
```

This pseudocode assumes that the bag contains `anEntry`.

After we add some details to the pseudocode to accommodate the situation in which `anEntry` is not in the bag, and to avoid computing `numberOfEntries - 1` more than once, as we did in Segment 2.21, the pseudocode appears as follows:

```
Search the array bag for anEntry
if (anEntry is in the bag at bag[index])
{
   Decrement the counter numberOfEntries
   bag[index] = bag[numberOfEntries]
   bag[numberOfEntries] = null
   return true
}
else
   return false
```

2.24 **Avoiding duplicate effort.** We can easily translate this pseudocode into the Java method `remove`. However, if we were to do so, we would see much similarity between our new method and the `remove` method we wrote earlier in Segment 2.21. In fact, if `anEntry` occurs in `bag[numberOfEntries - 1]`, both `remove` methods will have exactly the same effect. To avoid this duplicate effort, both `remove` methods can call a private method that performs the removal. We can specify such a method as follows:

```
// Removes and returns the entry at a given array index.
// If no such entry exists, returns null.
private T removeEntry(int givenIndex)
```

Before we implement this private method, let's see if we can use it by revising the `remove` method in Segment 2.21. Since that method removes and returns the last entry in the array `bag`, that is, `bag[numberOfEntries - 1]`, its definition can make the call `removeEntry(numberOfEntries - 1)`. Proceeding as if `removeEntry` were defined and tested, we can define `remove` as follows:

```
/** Removes one unspecified entry from this bag, if possible.
    @return either the removed entry, if the removal was successful,
    or null otherwise */
public T remove()
{
   T result = removeEntry(numberOfEntries - 1);
   return result;
} // end remove
```

This definition looks good; let's implement the second `remove` method.

2.25 **The second `remove` method.** The first `remove` method does not search for the entry to remove, as it removes the last entry in the array. The second `remove` method, however, does need to perform a search. Rather than thinking about the details of locating an entry in an array right now, let's delegate that task to another private method, which we specify as follows:

```
// Locates a given entry within the array bag.
// Returns the index of the entry, if located, or -1 otherwise.
private int getIndexOf(T anEntry)
```

Assuming that this method is defined and tested, we can define our public method as follows:

```
/** Removes one occurrence of a given entry from this bag.
    @param anEntry  the entry to be removed
    @return true if the removal was successful, or false if not */
public boolean remove(T anEntry)
{
   int index = getIndexOf(anEntry);
   T result = removeEntry(index);
   return anEntry.equals(result);
} // end remove
```

Notice that `removeEntry` returns either the entry it removes or `null`. That is exactly what the first `remove` method needs, but the second `remove` method has to return a boolean value. Thus, in the second method we need to compare the entry we want to remove with the one `removeEntry` returns to get the desired boolean value.

? SELF-TEST

Question 14 Can the `return` statement in the previous definition of `remove` be written as follows?

a. `return result.equals(anEntry);`
b. `return result != null;`

Question 15 The array `bag` in `ArrayBag` contains the entries in the bag `aBag`. If `bag` contains the strings `"A", "A", "B", "A", "C"`, why does `aBag.remove("B")` change the contents of `bag` to `"A", "A", "C", "A", null` instead of either `"A", "A", "A", "C", null` or `"A", "A", null, "A", "C"`?

2.26 **The definition of the private method `removeEntry`.** Let's look back at the pseudocode we wrote in Segment 2.23 for removing a particular entry from the bag. The private method `removeEntry` assumes that the search for the entry is done already, so we can ignore the first step of the pseudocode. The rest of the pseudocode, however, gives the basic logic for removing an entry. We can revise the pseudocode as follows:

```
// Removes and returns the entry at a given index within the arraybag.
// If no such entry exists, returns null.
if (the bag is not empty and the given index is not negative)
{
   result = bag[givenIndex]
   Decrement the counter numberOfEntries
   bag[givenIndex] = bag[numberOfEntries]
   bag[numberOfEntries] = null
   return result
}
else
   return null
```

The definition of the method `remove` given in the previous segment passes the integer returned by `getIndexOf` to `removeEntry`. Since `getIndexOf` can return –1, `removeEntry` must watch for such an argument. Thus, if the bag is not empty—that is, if `numberOfEntries` is greater than zero—and `givenIndex` is greater than or equal to zero, `removeEntry` removes the array entry at `givenIndex` by replacing it with the last entry and decrementing `numberOfEntries`. The method then returns the removed entry. If, however, the bag is empty, the method returns `null`.

The code for the method is

```
// Removes and returns the entry at a given index within the arraybag.
// If no such entry exists, returns null.
private T removeEntry(int givenIndex)
{
   T result = null;
   if (!isEmpty() && (givenIndex >= 0))
   {
      result = bag[givenIndex];                 // entry to remove
      numberOfEntries--;
      bag[givenIndex] = bag[numberOfEntries];   // replace entry with last entry
      bag[numberOfEntries] = null;              // remove last entry
   } // end if

   return result;
} // end removeEntry
```

2.27 **Locating the entry to remove.** We now need to think about locating the entry to remove from the bag so we can pass its index to `removeEntry`. The method `contains` performs the same search that we will use to locate `anEntry` within the definition of `remove`. Unfortunately, `contains` returns true or false; it does not return the index of the entry it locates in the array. Thus, we cannot simply call that method within our method definition.

Design Decision: Should the method `contains` return the index of a located entry?

Should we change the definition of `contains` so that it returns an index instead of a boolean value? No. As a public method, `contains` should not provide a client with such implementation details. The client should have no expectation that a bag's entries are in an array, since they are in no particular order. Instead of changing the specifications for `contains`, we will follow our original plan to define a private method to search for an entry and return its index.

The definition of getIndexOf will be like the definition of contains, which we recall here:

```
public boolean contains(T anEntry)
{
   boolean found = false;

   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
      {
         found = true;
      } // end if
   } // end for

   return found;
} // end contains
```

The structure of the loop is suitable for the method getIndexOf, but we must save the value of index when the entry is found. The method will return this index instead of a boolean value.

2.28 **The definition of getIndexOf.** To revise the loop in contains for use in getIndexOf, we define an integer variable where to record the value of index when anEntry equals bag[index]. Thus, the definition of getIndexOf looks like this:

```
// Locates a given entry within the array bag.
// Returns the index of the entry, if located, or -1 otherwise.
private int getIndexOf(T anEntry)
{
   int where = -1;
   boolean found = false;

   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
      {
         found = true;
         where = index;
      } // end if
   } // end for

   // Assertion: If where > -1, anEntry is in the array bag, and it
   // equals bag[where]; otherwise, anEntry is not in the array

   return where;

   } // end getIndexOf
```

The method getIndexOf returns the value of where. Notice that we initialize where to –1, which is the value to return if anEntry is not found.

Question 16 What assert statement can you add to the definition of the method getIndexOf just before the return statement to indicate the possible values that the method can return?

Question 17 Revise the definition of the method getIndexOf so that it does not use a boolean variable.

Aside: Thinking positively

Unlike the method contains, the method getIndexOf uses the boolean variable found only to control the loop and not as a return value. Thus, we can modify the logic somewhat to avoid the use of the not operator !.

Let's use a variable stillLooking instead of found and initialize it to true. Then we can replace the boolean expression !found with stillLooking, as you can see in the following definition of the method getIndexOf:

```
// Locates a given entry within the array bag.
// Returns the index of the entry, if located, or -1 otherwise.
private int getIndexOf(T anEntry)
{
   int where = -1;
   boolean stillLooking = true;

   for (int index = 0; stillLooking && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
      {
         stillLooking = false;
         where = index;
      } // end if
   } // end for

   return where;
} // end getIndexOf
```

If anEntry is found within the array, stillLooking is set to false to end the loop. Some programmers prefer to think positively, as in this revision, while others find !found to be perfectly clear.

2.29 **A revised definition for the method contains.** Having completed the definitions of remove and the private methods they call, we realize that the method contains can call the private method getIndexOf, resulting in a simpler definition than the one given in Segment 2.18. Recall that the expression getIndexOf(anEntry) returns an integer between 0 and numberOfEntries – 1 if anEntry is in the bag, or –1 otherwise. That is, getIndexOf(anEntry) is greater than –1 if anEntry is in the bag. Thus, we can define contains as follows:

```
/** Tests whether this bag contains a given entry.
    @param anEntry  the entry to locate
    @return true if the bag contains anEntry, or false otherwise */
public boolean contains(T anEntry)
{
   return getIndexOf(anEntry) > -1;
} // end contains
```

Since we have changed the definition of contains, we should test it again. By doing so, we are also testing the private method getIndexOf.

Note: Both the method contains and the second remove method must perform similar searches for an entry. By isolating the search in a private method that both contains and remove can call, we make our code easier to debug and to maintain. This strategy is the same one we used when we defined the removal operation in the private method removeEntry that both remove methods call.

Programming Tip

Even though you might have written a correct definition of a method, do not hesitate to revise it if you think of a better implementation.

2.30 **Testing.** Our class `ArrayBag` is essentially complete. We can use the previously tested methods—which we assume are correct—in the tests for `remove` and `clear`. Starting with a bag that is not full, the online program `ArrayBagDemo3` removes the bag's entries until it is empty. It also includes similar tests beginning with a full bag. Finally, we should consolidate our previous tests and run them again. The source code available on the book's website identifies our test program as `ArrayBagDemo` and the complete version of the class as `ArrayBag`.

Using Array Resizing to Implement the ADT Bag

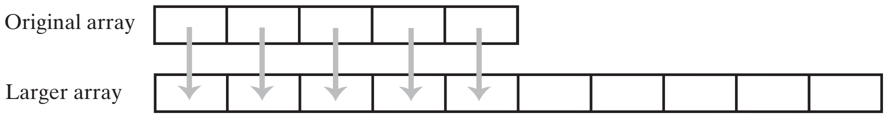
2.31 An array has a fixed size, which is chosen by either the programmer or the user before the array is created. A fixed-size array is like a classroom. If the room contains 40 desks but only 30 students, we waste 10 desks. If 40 students are taking the course, the room is full and cannot accommodate anyone else. Likewise, if we do not use all of the locations in an array, we waste memory. If we need more, we are out of luck.

Using a fixed-size array to implement the ADT bag, therefore, limits the size of the bag. When the array, and hence the bag, becomes full, the method `isFull` returns true and subsequent calls to the `add` method return false. Some applications can use a bag or other collection that has a limited capacity. For other applications, however, we need the size of a collection to grow without bound. We will now show you how a group of items can be as large as you want—within the limits of your computer's memory—but still be in an array.

Resizing an Array

2.32 **The strategy.** When a classroom is full, one way to accommodate additional students is to move to a larger room. In a similar manner, when an array becomes full, you can move its contents to a larger array. This process is called **resizing** an array. Figure 2-8 shows two arrays: an original array of five consecutive memory locations and another array—twice the size of the original array—that is in another part of the computer's memory. If you copy the data from the original smaller array to the beginning of the new larger array, the result will be like expanding the original array. The only glitch in this scheme is the name of the new array: You want it to be the same as the name of the old array. You will see how to accomplish this momentarily.

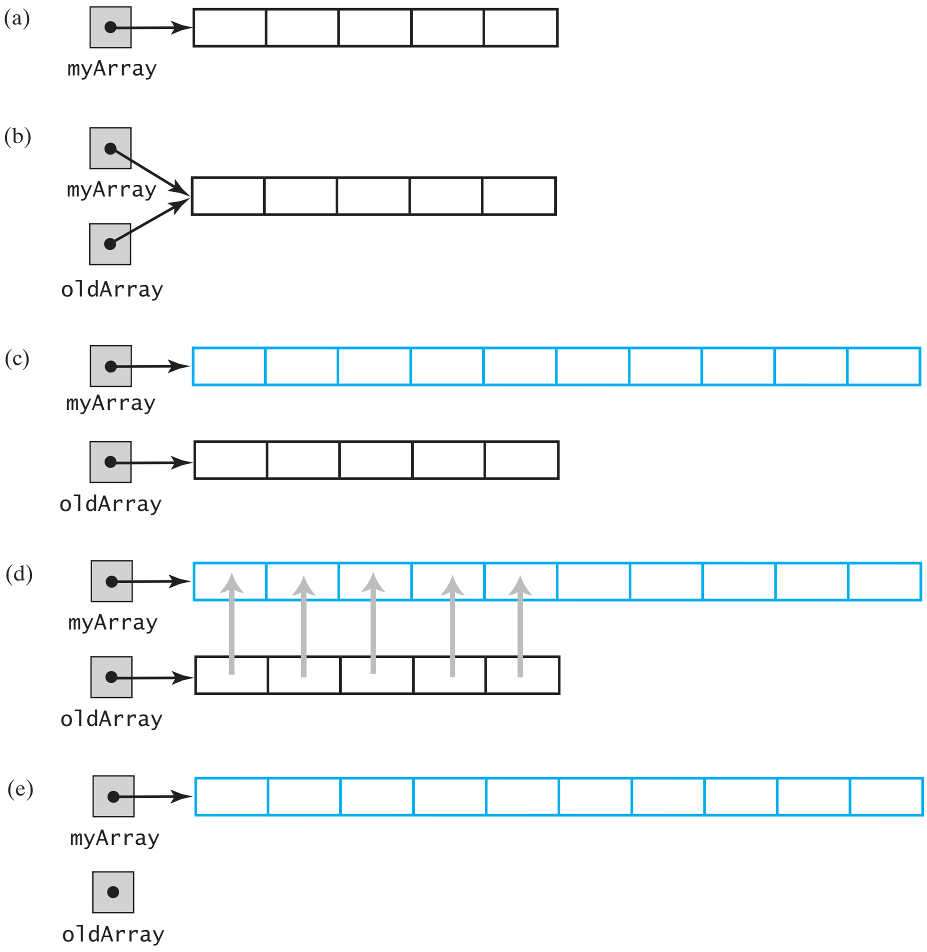
FIGURE 2-8 Resizing an array copies its contents to a larger second array



2.33 **The details.** Suppose we have an array that `myArray` references, as Figure 2-9a illustrates. We first define an alias `oldArray` that also references the array, as Figure 2-9b shows. The next step is to create a new array that is larger than the original array and let `myArray` reference this new array. As pictured in Figure 2-9c, the new array typically doubles the size of the original array. The final step copies the contents of the original array to the new array (Figure 2-9d) and then discards the original array (Figure 2-9e). The following pseudocode summarizes these steps:

```
oldArray = myArray
myArray = a new array whose length is 2 * oldArray.length
Copy entries from the original array—oldArray–to the new array—myArray
oldArray = null // discard old array
```

FIGURE 2-9 (a) An array; (b) two references to the same array; (c) the original array variable now references a new, larger array; (d) the entries in the original array are copied to the new array; (e) the original array is discarded



Note: When an array is no longer referenced, its memory is recycled during garbage collection, just as occurs with any other object.

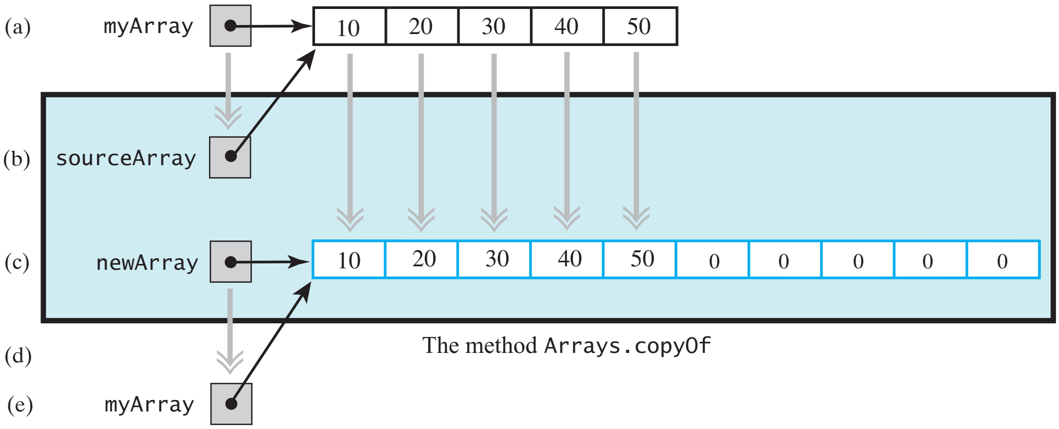
2.34 **The code.** While we could simply translate the previous pseudocode into Java, much of the work can be done by using the method `Arrays.copyOf`, which is in the Java Class Library. For example, let's work with a simple array of integers:

```
int[] myArray = {10, 20, 30, 40, 50};
```

At this point, `myArray` references the array, as Figure 2-10a shows. Next, we'll call `Arrays.copyOf`. The method's first parameter, `sourceArray`, is assigned the reference in the variable `myArray`, as Figure 2-10b implies. Next the method creates a new, larger array and copies the entries in the argument array to it (Figure 2-10c). Finally, the method returns a reference (Figure 2-10d) to the new array, and we assign this reference to `myArray` (Figure 2-10e). The following statement performs these steps:

```
myArray = Arrays.copyOf(myArray, 2 * myArray.length);
```

FIGURE 2-10 The effect of the statement
`myArray = Arrays.copyOf(myArray, 2 * myArray.length);`
(a) The argument array; (b) the parameter that references the argument array; (c) a new, larger array that gets the contents of the argument array; (d) the return value that references the new array; (e) the argument variable is assigned the return value



2.35 Resizing an array is not as attractive as it might first seem. Each time you expand the size of an array, you must copy its contents. If you were to expand an array by one element each time you needed additional space in the array, the process would be expensive in terms of computing time. For example, if a 50-element array is full, accommodating another entry would require you to copy the array to a 51-element array. Adding yet another entry would require that you copy the 51-element array to a 52-element array, and so on. Each addition would cause the array to be copied. If you added 100 entries to the original 50-entry array, you would copy the array 100 times.

However, expanding the array by *m* elements spreads the copying cost over *m* additions instead of just one. Doubling the size of an array each time it becomes full is a typical approach. For example, when you add an entry to a full array of 50 entries, you copy the 50-element array to a 100-element array before completing the addition. The next 49 additions then can be made quickly without copying the array. Thus, you will have copied the array only once.

Programming Tip: When increasing the size of an array, you copy its entries to a larger array. You should expand the array sufficiently to reduce the impact of the cost of copying. A common practice is to double the size of the array.

Note: Importing a class

The definition of a class that uses a class from the Java Class Library must be preceded by a `import` statement. For example, to use the class `Arrays`, you would write the following statement prior to your class definition and its descriptive comments:

```
import java.util.Arrays;
```

Some programmers replace `Arrays` in this statement with an asterisk to make all classes in the package `java.util` available to their program.

Note: To say that we "resize" an array is really a misnomer, since an array's length cannot be changed. The process of resizing an array involves creating a completely new array that contains the entries of the original array. The new array is given the name of the original array—in other words, a reference to the new array is assigned to the variable that had referenced the original array. The original array is then discarded.

SELF-TEST

Question 18 Consider the array of strings that the following statement defines:

```
String[] text = {"cat", "dog", "bird", "snake"};
```

What Java statements will increase the capacity of the array `text` by five elements without altering its current contents?

Question 19 Consider an array `text` of strings. If the number of strings placed into this array is less than its length (capacity), how could you decrease the array's length without altering its current contents? Assume that the number of strings is in the variable `size`.

A New Implementation of a Bag

2.36 **The approach.** We can revise the previous implementation of the ADT bag by resizing the array `bag` so that the bag's capacity is limited only by the amount of memory available on your computer. If we look at the outline of the class `ArrayBag` in Listing 2-1, we can see what we need to revise. Let's itemize these tasks:

VideoNote
A resizable bag

- Change the name of the class to `ResizableArrayBag` so we can distinguish between our two implementations.
- Remove the modifier `final` from the declaration of the array `bag` to enable it to be resized.
- Change the name of the constant `DEFAULT_CAPACITY` to `DEFAULT_INITIAL_CAPACITY`. Although unnecessary, this change clarifies the new purpose of the constant, since the bag's capacity will increase as necessary. Make the same change in the default constructor, which uses the constant.
- Change the names of the constructors to match the new class name.

- Revise the definition of the method add to always accommodate a new entry. The method will never return false.
- Revise the definition of the method isFull to always return false. A bag will never become full.

Revising the method add is the only substantial task in this list. The rest of the class will remain unchanged.

2.37 **The method add.** Here is the original definition of the method add, as it appears in Segment 2.10:

```
/** Adds a new entry to this bag.
    @param newEntry  the object to be added as a new entry
    @return true if the addition is successful, or false if not */
public boolean add(T newEntry)
{
   boolean result = true;
   if (isFull())
   {
      result = false;
   }
   else
   {  // assertion: result is true here
      bag[numberOfEntries] = newEntry;
      numberOfEntries++;
   } // end if

   return result;
} // end add
```

Since the bag will never be full, the method isFull will always return false. Thus, we can no longer call isFull to see whether the array bag is full. Instead, we can define a private method to both make this check and resize the array bag, if necessary. Let's name the method ensureCapacity and specify it as follows:

```
// Doubles the size of the array bag if it is full.
private void ensureCapacity()
```

Assuming that we have defined this private method, we can revise the method add as follows:

```
public boolean add(T newEntry)
{
   ensureCapacity();
   bag[numberOfEntries] = newEntry;
   numberOfEntries++;

   return true;
} // end add
```

2.38 **The private method ensureCapacity.** The array bag is full when numberOfEntries equals the array's length, bag.length. When that is the case, we will resize bag using the technique described earlier in Segment 2.34. Thus, the definition of ensureCapacity is straightforward:

```
// Doubles the size of the array bag if it is full.
private void ensureCapacity()
{
   if (numberOfEntries == bag.length)
      bag = Arrays.copyOf(bag, 2 * bag.length);
} // end ensureCapacity
```

2.39 **The class ResizableArrayBag.** Our new class is available online from the book's website. You should examine its details.

Design Decision: You might wonder about some of the decisions we made while defining the class `ResizableArrayBag`, with questions such as the following:

- Why is the method `add` a boolean method and not a void method? It always returns true!
- Why did we bother to define `isFull`? The bag is never full!
- Why did we define the private method `ensureCapacity`? Only one method, `add`, calls it!

The answers to the first two questions are the same: The class implements the interface `BagInterface`, so we followed its specifications. As a result, we have two different implementations, `ArrayBag` and `ResizableArrayBag`, each of which can be used by the same client. Our answer to the third question reflects our approach to problem solving. To implement `add`, we needed to answer two questions: When is an array full, and how do we expand a full array? Rather than risking the distraction of answering these questions while we were defining the method `add`, we chose to specify a private method to provide those answers. Admittedly, the definition of this private method turned out to be short. We could now integrate the body of the private method into that of `add`, but we have no pressing reason to do so.

Question 20 What is the definition of a constructor that you could add to the class `ResizableArrayBag` to initialize the bag to the contents of a given array?

Question 21 In the definition of the constructor described in the previous question, is it necessary to copy the entries from the argument array to the array `bag`, or would a simple assignment (`bag = contents`) be sufficient?

Question 22 What is an advantage of using an array to organize data? What is a disadvantage?

2.40 **Testing the class.** A program that tests the class `ResizableArrayBag` can create a bag whose initial capacity is small—3, for example. This choice allows us to easily test the bag's ability to increase its capacity. For instance, when the fourth item is added, the bag's capacity is doubled to 6. At the seventh addition, the capacity is doubled again, this time to 12. Such a program, `ResizableArrayBagDemo`, is available online at the book's website.

Programming Tip: A class implementing a single interface that declares the operations of an ADT should define the methods declared in the interface as its only public methods. However, the class can also define private methods and protected methods.

The Pros and Cons of Using an Array to Implement the ADT Bag

2.41 This chapter discussed two implementations of the ADT bag that use an array to store a bag's entries. An array is simple to use and enables you to access any element immediately, if you know its index. Since we know the index of the last entry in the array, removing it is easy and fast. Similarly, adding an entry at the end of the array is equally easy and fast. On the other hand, removing a particular entry, if it occurs between other entries, requires us to avoid a gap within the array. To do so, we replace the removed entry with the last entry in the array. This is an insignificant increase in execution time, as it is overshadowed by the time it takes to locate the desired entry. We will talk more about such a search later in this book.

Using a fixed-size array limits the capacity of a bag, which is usually a disadvantage. Resizing an array dynamically enables you to increase the array's size but requires copying data. You should realize that the array entries that we copy are references, and so do not occupy much space nor take much time to move. Some languages other than Java store the data itself within the array. In such cases, moving large, complex objects can be quite time-consuming.

Note: When you use an array to implement the ADT bag,

- Adding an entry to the bag is fast
- Removing an unspecified entry is fast
- Removing a particular entry requires time to locate the entry
- Increasing the size of the array requires time to copy its entries

Chapter Summary

- You can use a Java array to define a relatively simple implementation of the ADT bag, but other implementations are possible.
- Adding an entry right after the last entry in an array does not disturb the position of existing entries. Likewise, deleting the last entry from an array does not disturb the position of existing entries.
- Because a bag does not maintain its entries in a specific order, deleting an entry does not require you to move all subsequent array entries to the next lower position. Instead, you can replace the entry that you want to delete with the last entry in the array and replace the last entry with `null`.
- Identifying and implementing a class's central, or core, methods before any others is a good strategy to use when you expect the class to be lengthy or complex. Use stubs for the remaining methods.
- Test a class at each stage of its development, particularly after adding a significant method.
- Using a fixed-size array can result in a full bag.
- Resizing an array makes it appear to change size. To do so, you allocate a new array, copy the entries from the original array to the new array, and use the original variable to reference the new array.
- Resizing an array enables you to implement collections whose contents are limited in number only by the size of the computer's memory.

Programming Tips

- When defining a class, implement and test a group of core methods. Begin with methods that add to a collection of objects and/or have involved implementations.
- A class should not return a reference to an array that is a private data field.
- Do not wait until you complete the implementation of an ADT before testing it. By writing stubs, which are incomplete definitions of required methods, you can begin testing early in the process.
- Even though you might have written a correct definition of a method, do not hesitate to revise it if you think of a better implementation.

- When increasing the size of an array, you copy its entries to a larger array. You should expand the array sufficiently to reduce the impact of the cost of copying. A common practice is to double the size of the array.
- A class implementing a single interface that declares the operations of an ADT should define the methods declared in the interface as its only public methods. However, the class can also define private methods and protected methods.

Exercises

1. Why are the methods `getIndexOf` and `removeEntry` in the class `ArrayBag` private instead of public?

2. Implement a method `replace` for the ADT bag that replaces and returns any object currently in a bag with a given object.

3. Revise the definition of the method `remove`, as given in Segment 2.24, so that it removes a random entry from a bag. Would this change affect any other method within the class `ArrayBag`?

4. Define a method `removeEvery` for the class `ArrayBag` that removes all occurrences of a given entry from a bag.

5. An instance of the class `ArrayBag` has a fixed size, whereas an instance of `ResizableArrayBag` does not. Give some examples of situations where a bag would be appropriate if its size is

a. Fixed.
b. Resizable.

6. Suppose that you wanted to define a class `PileOfBooks` that implements the interface described in Project 2 of the previous chapter. Would a bag be a reasonable collection to represent the pile of books? Explain.

7. Consider an instance `myBag` of the class `ResizableArrayBag`, as discussed in Segments 2.36 to 2.40. Suppose that the initial capacity of `myBag` is 10. What is the length of the array `bag` after

a. Adding 145 entries to `myBag`?
b. Adding an additional 20 entries to `myBag`?

8. Define a method at the client level that accepts as its argument an instance of the class `ArrayBag` and returns an instance of the class `ResizableArrayBag` that contains the same entries as the argument bag.

9. Suppose that a bag contains `Comparable` objects. Implement the following methods for the class `ArrayBag`:

- The method `getMin` that returns the smallest object in a bag
- The method `getMax` that returns the largest object in a bag
- The method `removeMin` that removes and returns the smallest object in a bag
- The method `removeMax` that removes and returns the largest object in a bag

10. Suppose that a bag contains `Comparable` objects. Define a method for the class `ArrayBag` that returns a new bag of items that are less than some given item. The header of the method could be as follows:

```
public ArrayBag<T> getAllLessThan(Comparable<T> anObject)
```

Make sure that your method does not affect the state of the original bag.

11. Define an `equals` method for the class `ArrayBag` that returns true when the contents of two bags are the same. Note that two equal bags contain the same number of entries, and each entry occurs in each bag the same number of times.

12. The class `ResizableArrayBag` has an array that can grow in size as objects are added to the bag. Revise the class so that its array also can shrink in size as objects are removed from the bag. Accomplishing this task will require two new private methods, as follows:
    - The first new method checks whether we should reduce the size of the array:

      ```
      private boolean isTooBig()
      ```

      This method returns true if the number of entries in the bag is less than half the size of the array and the size of the array is greater than 20.
    - The second new method creates a new array that is three quarters the size of the current array and then copies the objects in the bag to the new array:

      ```
      private void reduceArray()
      ```

    Implement each of these two methods, and then use them in the definitions of the two `remove` methods.

13. Consider the two private methods described in the previous exercise.

    a. The method `isTooBig` requires the size of the array to be greater than 20. What problem could occur if this requirement is dropped?

    b. The method `reduceArray` is not analogous to the method `ensureCapacity` in that it does not reduce the size of the array by one half. What problem could occur if the size of the array is reduced by one half instead of three quarters?

14. Define the method `union`, as described in Exercise 5 of the previous chapter, for the class `ResizableArrayBag`.

15. Define the method `intersection`, as described in Exercise 6 of the previous chapter, for the class `ResizableArrayBag`.

16. Define the method `difference`, as described in Exercise 7 of the previous chapter, for the class `ResizableArrayBag`.

PROJECTS

1. Define a class `ArraySet` that represents a set and implements the interface described in Project 1a of the previous chapter. Use the class `ResizableArrayBag` in your implementation. Then write a program that adequately demonstrates your implementation.

2. Repeat the previous project, but use a resizable array instead of the class `ResizableArrayBag`.

3. Define a class `PileOfBooks` that implements the interface described in Project 2 of the previous chapter. Use a resizable array in your implementation. Then write a program that adequately demonstrates your implementation.

4. Define a class `Ring` that represents a ring and implements the interface described in Project 3 of the previous chapter. Use a resizable array in your implementation. Then write a program that adequately demonstrates your implementation.

5. You can use either a set or a bag to create a spell checker. The set or bag serves as a dictionary and contains a collection of correctly spelled words. To see whether a word is spelled correctly, you see whether it is contained in the dictionary. Use this scheme to create a spell checker for the words in an external file. To simplify your task, restrict your dictionary to a manageable size.

6. Repeat the previous project to create a spell checker, but instead place the words whose spelling you want to check into a bag. The difference between the dictionary (the set or bag containing the correctly spelled words) and the bag of words to be checked is a bag of incorrectly spelled words.

Answers to Self-Test Questions

1. The students remain in consecutively numbered desks. You do not have to keep track of the locations of the empty desks.

2. Time is saved by not moving a student.

3. The student in the highest-numbered desk.

4. No. The two values are equal only when a bag is full.

5. If the client contained a statement such as

```
Object[] bagContents = myBag.toArray();
```

myBag.getCurrentSize() would be the number of entries in the array bagContents. With the proposed design, bagContents.length could be larger than the number of entries in the bag.

6. The statements set the first element of bag to null. The value of numberOfEntries does not change, so it is 5.

7.

```
public T[] toArray()
{
   return Arrays.copyOf(bag, bag.length);
} // end toArray
```

8. The bag aBag is empty. When displayBag is called, the statement

```
Object[] bagArray = aBag.toArray();
```

executes. When toArray is called, the statement

```
T[] result = (T[])new Object[numberOfEntries];
```

executes. Since aBag is empty, numberOfEntries is zero. Thus, the new array, result, is empty. The loop in toArray is skipped and the empty array is returned and assigned to bagArray. Since bagArray.length is zero, the loop in displayBag is skipped. The result of the call displayBag(aBag) is simply the line

```
The bag contains
```

9. Advantage: This definition is easier to write, so you are less likely to make a mistake.
Disadvantage: This definition takes more time to execute, if the bag contains more than one occurrence of anEntry. Note that the loop in the method getFrequencyOf cycles through all of the entries in the bag, whereas the loop in the method contains, as given in Segment 2.18, ends as soon as the desired entry is found.

10.

```
public void clear()
{
   while (remove() != null)
   {
   } // end while
} // end clear
```

11. Although the bag will appear empty to both the client and the other methods in ArrayBag, the references to the removed objects will remain in the array bag. Thus, the memory associated with these objects will not be deallocated.

12. By setting bag[numberOfEntries] to null, the method causes the memory assigned to the deleted entry to be recycled, unless another reference to that entry exists in the client.

13. An entry in the array bag, other than the last one, would be set to null. The remaining entries would no longer be in consecutive elements of the array. We could either rearrange the entries to get rid of the null entry or modify other methods to skip any null entry.

14. **a.** No. If `result` were `null`—and that is quite possible—a `NullPointerException` would occur.
b. Yes.

15. After locating `"B"` in the bag, the `remove` method replaces it with the last relevant entry in the array `bag`, which is `"C"`. It then replaces that last entry with `null`. Although we could define `remove` to result in either of the two other possibilities given in the question, both choices are inferior. For example, to get `"A", "A", "A", "C", null`, `remove` would shift the array elements, requiring more execution time. Leaving a gap in the array, such as `"A", "A", null, "A", "C"`, is easy for `remove` to do but complicates the logic of the remaining methods.

16.
```
assert ((where >= 0) && (where < numberOfEntries)) || (where == -1);
```

17.
```
private int getIndexOf(T anEntry)
{
   int where = -1;
   for (int index = 0; (where == -1) && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
         where = index;
   } // end for

   return where;
} // end getIndexOf
```

or

```
private int getIndexOf(T anEntry)
{
   int where = numberOfEntries - 1;
   while ((where > -1) && !anEntry.equals(bag[where]))
      where--;

   return where;
} // end getIndexOf
```

18.
```
text = Arrays.copyOf(text, text.length + 5);
```

or

```
String[] origText = text;
text = new String[text.length + 5];
System.arraycopy(origText, 0, text, 0, origText.length);
```

19.
```
text = Arrays.copyOf(text, size);
```

20.
```
/** Creates a bag containing the given array of entries.
    @param contents  an array of objects */
public ResizableArrayBag(T[] contents)
{
   bag = Arrays.copyOf(contents, contents.length);
   numberOfEntries = contents.length;
} // end constructor
```

21. A simple assignment statement would be a poor choice, since then the client could corrupt the bag's data by using the reference to the array that it passes to the constructor as an argument. Copying the argument array to the array `bag` is necessary to protect the integrity of the bag's data.

22. Advantage: You can access any array location directly if you know its index.
Disadvantages: The array has a fixed size, so you will either waste space or run out of room. Resizing the array avoids the latter disadvantage, but requires you to copy the contents of the original array to a larger array.

A Bag Implementation That Links Data

Chapter
3

Contents

Prerequisites

Chapter	1	Bags
Chapter	2	Bag Implementations That Use Arrays

Objectives

After studying this chapter, you should be able to

- Describe a linked organization of data
- Describe how to add a new node to the beginning of a chain of linked nodes
- Describe how to remove the first node in a chain of linked nodes
- Describe how to locate a particular piece of data within a chain of linked nodes
- Implement the ADT bag by using a chain of linked nodes
- Describe the differences between the array-based and linked implementations of the ADT bag

Using an array to implement the ADT bag has both advantages and disadvantages, as you saw in Chapter 2. An array has a fixed size, and so it can either become full or have several unused elements. You can resize an array when it becomes full by moving its entries to a larger array. Although resizing an array can provide as much space as a bag needs, you must move data each time you expand the array.

This chapter introduces a data organization that uses memory only as needed for a new entry and returns the unneeded memory to the system after an entry is removed. By linking data, this new organization avoids moving data when adding or removing bag entries. These features make this way of implementing a bag an important alternative to array-based approaches.

Linked Data

3.1 In Chapter 2, we used the analogy of a classroom to describe how data is stored in an array. Here we use a classroom to show you another way to organize data.

Imagine an empty classroom—room L—that is assigned to a course. All available desks are in the hallway. Any student who registers for the course receives a desk, takes it into the room, and sits at it. Assume that the room can accommodate all of the desks that are in the hall.

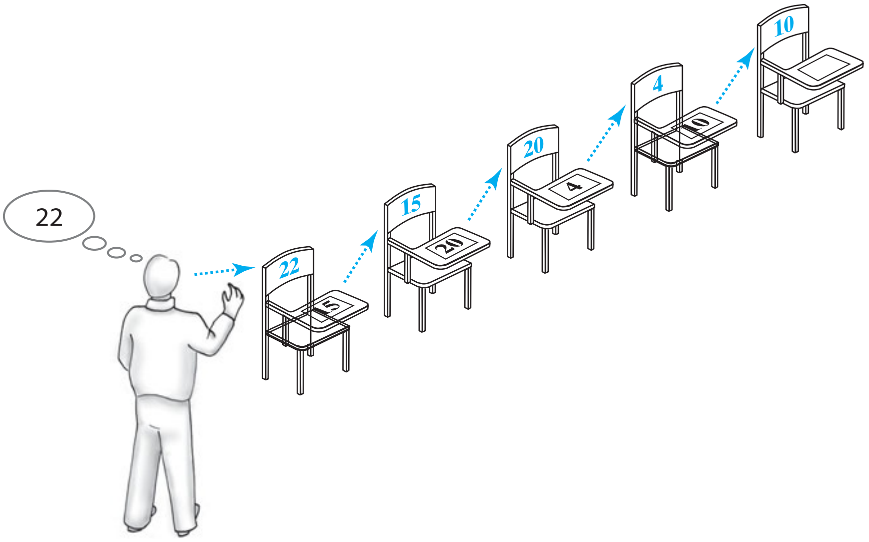
VideoNote
Linked data

Each desk in the hallway has a number stamped on its back. This number—called an **address**—never changes and is not considered when desks are given to students. Thus, the room will eventually contain desks whose addresses are not sequential.

Now imagine that Jill is among 30 students who are seated in room L at exactly 30 desks. Taped to each desktop is a piece of paper. As Jill entered the room, we wrote on her paper the desk number (address) of another desk in the room. For example, the paper on Jill's desk might contain the number 20. If her desk is desk 15, we say that desk 15 **references** desk 20 and that desks 15 and 20 are **linked**. Since all of the desks are linked to one another in this way, we say that they form a **chain** of desks.

Figure 3-1 shows a chain of five desks. No desk references the first desk in the chain, but the instructor knows its desk number, 22. Notice that the last desk in the chain does not reference another desk; the piece of paper on this desk is blank.

FIGURE 3-1 A chain of five desks



3.2 The chain of desks provides an order for the desks. Suppose that first in the chain is the student who arrived most recently. Written on this student's desk is the desk number of the student who arrived just before. With one exception, everyone's desk references the desk of the student who arrived just before. The exception is the person who arrived first. That person sits at the last desk, which does not reference another desk.

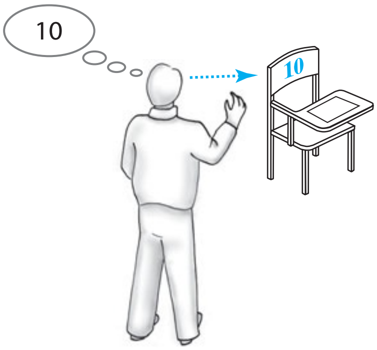
The instructor knows the address of the first desk in the chain and so can ask questions of the student at that first desk. Then, by looking at the address, or desk number, that is written on the paper on the first desk, the instructor can locate the second desk in the chain and can question its occupant. Continuing in this way, the instructor can visit every desk in the order in which they appear in the chain. Ultimately, the instructor reaches the last desk in the chain, which references no other desk. Note that the only way the instructor can locate the student in this last desk is to begin at the first desk. Also note that the instructor can traverse this chain in only one order. In our similar example in Chapter 2, the instructor in room A was able to ask questions of any student in any order.

Forming a Chain by Adding to Its Beginning

3.3 How did we form the chain of desks in the first place? Let's return to the time when room L was empty and all available desks were in the hallway.

Suppose that Matt arrives first. He gets a desk from the hallway and enters the room. The instructor notes Matt's desk number (address), and we leave the paper on his desk blank to indicate that no other student has arrived. The room appears as in Figure 3-2.

FIGURE 3-2 One desk in the room



3.4 When the second student arrives, we write Matt's desk number on the new desk's paper and give the instructor the number of the new desk to remember. Let's assume that the instructor can remember only one desk number at a time. The room now appears as in Figure 3-3. The new desk is at the beginning of the chain.

When the third student arrives, we write the instructor's memorized desk number, which is that of the desk at the beginning of the chain, on the new desk's paper. We then tell the instructor to remember the number of the new desk, which is now at the beginning of the chain. The room now appears as in Figure 3-4.

After all the students have arrived, the instructor knows only the desk number of the student who arrived most recently. On that student's desk is the desk number of the student who arrived just previously. In general, written on each student's desk is the number of the desk that belongs to the previous student who arrived. Since Matt was the first student to arrive, the paper on his desk is still blank. In Figures 3-1 through 3-4, desk 10 belongs to Matt.

FIGURE 3-3 Two linked desks, with the newest desk first

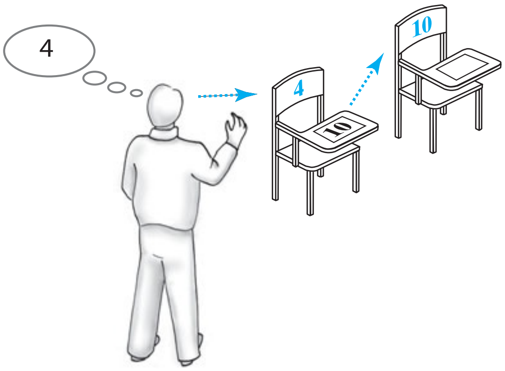
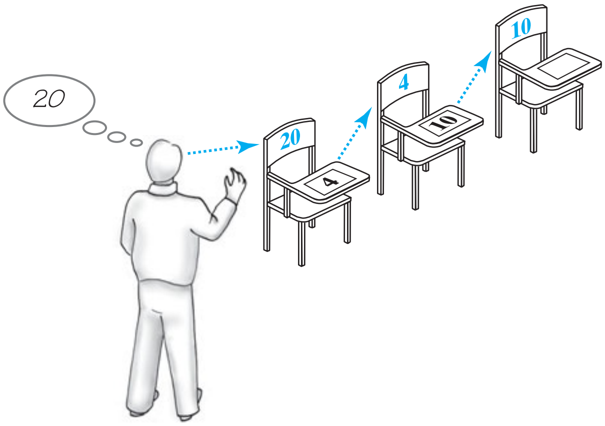


FIGURE 3-4 Three linked desks, with the newest desk first



Question 1 The instructor knows the address of only one desk.

a. Where in the chain is that desk: first, last, or somewhere else?
b. Who is sitting at that desk: the student who arrived first, the student who arrived last, or someone else?

Question 2 Where in the chain of desks is a new desk added: at the beginning, at the end, or somewhere else?

3.5 The following pseudocode details the steps taken to form a chain of desks by adding new desks to the beginning of the chain:

```
// Process the first student
newDesk represents the new student's desk
New student sits at newDesk
Instructor memorizes the address of newDesk
```

```
// Process the remaining students
while (students arrive)
{
    newDesk represents the new student's desk
    New student sits at newDesk
    Write the instructor's memorized address on newDesk
    Instructor memorizes the address of newDesk
}
```

A Linked Implementation of the ADT Bag

The previous section described how you can organize data by linking it together. This section expresses these ideas in Java by beginning the implementation of the ADT bag.

The Private Class Node

3.6 We begin by defining the Java equivalent of a desk, called a **node**. Nodes are objects that you typically link together to form a data structure. Our particular nodes have two data fields each: one to reference a piece of data—presently, an entry in a bag—and one to reference another node. An entry in a bag is analogous to a person who sits at a desk. The reference to another node is analogous to the address written on the paper that is on each desk.

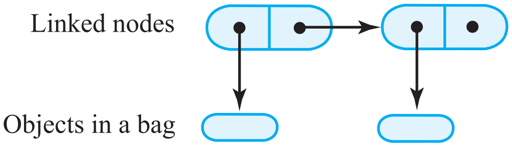
The class that represents these nodes can have the following form:

```
class Node
{
    private T    data; // entry in bag
    private Node next; // link to next node

    < Constructors >
    . . .
    < Accessor and mutator methods: getData, setData, getNextNode, setNextNode >
    . . .
} // end Node
```

3.7 Let's focus on the data fields. The field `data` contains a reference to one of the objects in the bag. Sometimes we will call this field the **data portion** of the node. The data type of `data` is represented here by the generic type `T`. Soon, you will see that `T` is the same generic type that the class of bags will declare.

The field `next` contains a reference to another node. Notice that its data type is `Node`, which is the class that we are currently defining! Such a circular definition might surprise you, but it is perfectly legal in Java. It also enables one node to reference another node, just as one desk references another desk in our example. Notice that a desk does not reference a student in another desk. Likewise, a node does not reference the data in another node, but rather references the entire other node. Sometimes we will call the field `next` the **link portion** of the node. Figure 3-5 illustrates two nodes that are linked and contain references to objects in the bag.

FIGURE 3-5 Two linked nodes that each reference object data



3.8 The rest of the definition of the class `Node` is uneventful. Constructors to initialize the node are useful, and since the data fields are private, methods to access and alter their contents are provided. But are they really necessary? If we intend `Node` to be for public use, like our other classes, such methods are necessary; however, `Node` is a detail of this implementation of the ADT bag that should be hidden from the bag's client. One way to hide `Node` from the world is to define it within a package that also contains the class that implements the bag. Another way—the way we will use here—is to define `Node` within an **outer class**, the one that implements the bag. Because of its placement within another class, `Node` is an example of an **inner class**. We declare it to be private. An outer class can access the data fields of an inner class directly by name without the need for accessor and mutator methods. Thus, we write the simpler definition of `Node` shown in Listing 3-1.

LISTING 3-1 The private inner class `Node`

```
private class Node
{
   private T    data; // entry in bag
   private Node next; // link to next node

   private Node(T dataPortion)
   {
      this(dataPortion, null);
   } // end constructor

   private Node(T dataPortion, Node nextNode)
   {
      data = dataPortion;
      next = nextNode;
   } // end constructor
} // end Node
```

We did not include a default constructor because we will not need one.

Because `Node` will be an inner class, the generic type `T` will be the same as the generic type declared by the outer class that contains `Node`. Thus, we do not write <T> after `Node`. If, however, `Node` was not an inner class but instead had package access or public access, you would write `Node`<T>. In that case, `Node` would also require set and get methods for its data fields.

Note: Terminology
A **nested class** is defined entirely within another class definition. Nested classes can be static, although we will not encounter any in this book. An inner class is a nested class that is not static. An outer class, or **enclosing class**, contains a nested class. A **top-level class** is one that is not nested.

An Outline of the Class `LinkedBag`

3.9 For this implementation of the ADT bag, we will use a chain of linked nodes to contain the bag's entries. In our earlier classroom example, the instructor remembered the address of the first desk in a chain of desks. Similarly, our implementation must "remember" the address of the first node in the chain of nodes. We use a data field called the **head reference** to record a reference to this first node. A second data field can track the number of entries in the bag, that is, the number of nodes in the chain.

Listing 3-2 contains an outline of the class `LinkedBag` that implements the ADT bag and contains the class `Node` as an inner class. Recall that Chapter 1 introduced the interface `BagInterface` in Listing 1-1. It and the classes that implement it define a generic type for the objects in a bag. The identifier `T` that we use for this generic type must match the one that we use within the class `Node`.

VideoNote
Beginning the class `LinkedBag`

LISTING 3-2 An outline of the class `LinkedBag`

```
/**
    A class of bags whose entries are stored in a chain of linked nodes.
    The bag is never full.
    @author Frank M. Carrano
*/
public class LinkedBag<T> implements BagInterface<T>
{
   private Node firstNode;       // reference to first node
   private int  numberOfEntries;

   public LinkedBag()
   {
      firstNode = null;
      numberOfEntries = 0;
   } // end default constructor

   < Implementations of the public methods declared in BagInterface go here. >

   . . .

   private class Node // private inner class
   {
      < See Listing 3-1. >
   } // end Node
} // end LinkedBag
```

The data field `firstNode` is the head reference of the chain of nodes. Just like the instructor who knew the address of the first desk in the chain of desks, `firstNode` references the first node in the chain of nodes. Another data field, `numberOfEntries`, records the number of entries in the current bag. This number is also the number of nodes in the chain. Initially, a bag is empty, so the default constructor simply initializes the data fields `firstNode` to `null` and `numberOfEntries` to zero.

Defining Some Core Methods

As we stated in the previous chapter, implementing and testing a core group of methods often is advantageous when you write a class. Any method that adds an entry to a collection typically is a core method for a class that implements a collection, such as a bag. Moreover, to verify that additions to a collection are made correctly, we need a way to look at the collection's entries. The method `toArray` can serve this purpose, and so it also is a core method. Such was the case for the class ArrayBag in the previous chapter, and it is true of our present class LinkedBag. Before we do anything else, let's define the bag's methods add and toArray.

3.10 **The method add: Beginning a chain of nodes.** In Segment 3.3, the room was empty when the first student arrived. As we noted in Segment 3.5, we took the following steps to begin a chain of desks:

```
newDesk represents the new student's desk
New student sits at newDesk
Instructor memorizes the address of newDesk
```

Here are the analogous steps that the method add must take to add the first entry to an initially empty bag. Note that the desk in the previous pseudocode is analogous to a node defined within LinkedBag, the student is analogous to a bag entry—that is, the data within the node—and the instructor is analogous to firstNode.

```
newNode references a new instance of Node
Place data in newNode
firstNode = address of newNode
```

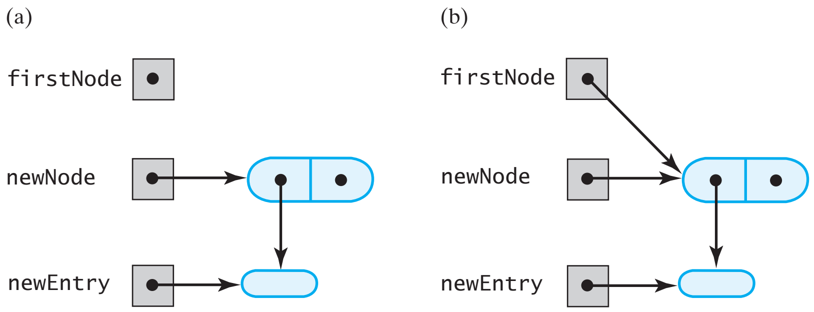
Thus, when the method add adds the first entry to an initially empty bag, it creates a new node and makes it a one-node chain.

In Java, these steps appear as follows, where newEntry references the entry to be added to the bag:

```
Node newNode = new Node(newEntry);
firstNode = newNode;
```

Figure 3-6 illustrates these two steps. Part *a* of this figure shows the empty chain and the node created by the first statement. Part *b* shows the result of the second statement. Notice that in Part *b*, both firstNode and newNode reference the same node. After the insertion of the new node is complete, only firstNode should reference it. We could set newNode to null, but as you will see shortly, newNode is a local variable of the method add. As such, newNode will not exist after add ends its execution. The same is true of the parameter newEntry, which behaves like a local variable.

FIGURE 3-6 (a) An empty chain and a new node; (b) after adding a new node to a chain that was empty



3.11 **The method add: Adding to the chain of nodes.** Just as we added new desks to the beginning of an existing chain in Segment 3.5, the method add will add new nodes to the beginning of its chain. In the context of desks in a room, the necessary steps are

```
newDesk represents the new student's desk
New student sits at newDesk
Write the instructor's memorized address on newDesk
Instructor memorizes the address of newDesk
```

As a result of these steps, the new desk references the current first desk in the chain and becomes the new first desk.

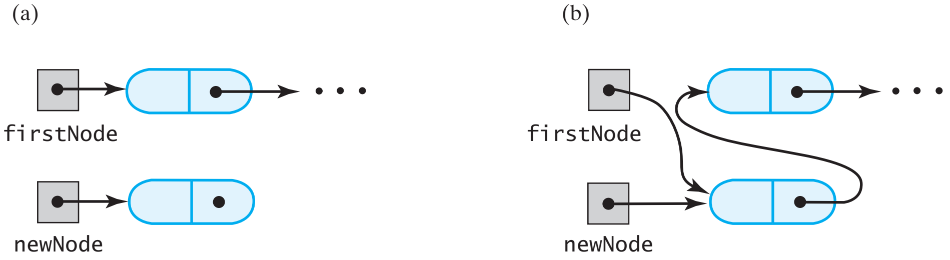
Here are the analogous steps that add takes:

```
newNode references a new instance of Node
Place data in newNode
Set newNode's link to firstNode
Set firstNode to newNode
```

That is, we make the new node reference the first node in the chain, making it the new first node. Figure 3-7 illustrates these steps, and the following Java statements implement them:

```
Node newNode = new Node(newEntry);
newNode.next = firstNode;
firstNode = newNode;
```

FIGURE 3-7 A chain of nodes (a) just prior to adding a node at the beginning; (b) just after adding a node at the beginning



Adding a node to an empty chain, as Figure 3-6 depicts, is actually the same as adding a node to the beginning of a chain. Question 3 asks you to think about this fact.

SELF-TEST

Question 3 The code that we developed in Segment 3.10 to add a node to an empty chain is

```
Node newNode = new Node(newEntry);
firstNode = newNode;
```

The code that we just developed to add to the beginning of a chain is

```
Node newNode = new Node(newEntry);
newNode.next = firstNode;
firstNode = newNode;
```

Why do these three statements also work correctly when the chain is empty?

3.12 **The method add.** As you have seen, although it might appear that an empty bag is a special case when adding a new entry to a bag, it really is not. The following definition of the method add uses this conclusion:

```
/** Adds a new entry to this bag.
    @param newEntry  the object to be added as a new entry
    @return true */
public boolean add(T newEntry) // OutOfMemoryError possible
```

```
{
   // add to beginning of chain:
   Node newNode = new Node(newEntry);
   newNode.next = firstNode;   // make new node reference rest of chain
                               // (firstNode is null if chain is empty)
   firstNode = newNode;        // new node is at beginning of chain
   numberOfEntries++;

   return true;
} // end add
```

3.13 **An out-of-memory error.** With a linked implementation, the bag cannot become full. Anytime you add a new entry, you create a new node for that entry. Thus, the method add always returns true. It is possible, however, for your program to use all of your computer's memory. If this occurs, your request for a new node will cause the error OutOfMemoryError. You might interpret this condition as a full bag, but an OutOfMemoryError is fatal, and the client will not have the opportunity to react to it.

Note: Allocating memory
When you use the new operator, you create, or instantiate, an object. At that time, the Java run-time environment **allocates**, or assigns, memory to the object. When you create a node for a linked chain, we sometimes say that you have allocated the node.

3.14 **The method toArray.** The method toArray returns an array of the entries currently in a bag. By implementing this method, we will be able to test whether the add method works before we complete the rest of the class LinkedBag. To access the bag's entries, we need to access each node in a chain, beginning with the first one. This action is called a **traversal**, and it is analogous to visiting each desk in a chain of desks, as we described in Segment 3.2.

The data field firstNode contains a reference to the first node in the chain. That node contains a reference to the second node in the chain, the second node contains a reference to the third node, and so on. To traverse the chain, the method toArray needs a temporary, local variable currentNode to reference each node in turn. When currentNode references the node whose data we want to access, that data is at currentNode.data.

Initially, we want currentNode to reference the first node in the chain, so we set it to firstNode. After accessing the data at currentNode.data, we move to the next node by executing

```
currentNode = currentNode.next;
```

We again access the data at currentNode.data and then move to the next node by executing

```
currentNode = currentNode.next;
```

once again. We continue in this manner until currentNode becomes null.

The following method toArray uses these ideas:

```
/** Retrieves all entries that are in this bag.
    @return a newly allocated array of all the entries in the bag */
public T[] toArray()
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] result = (T[])new Object[numberOfEntries]; // unchecked cast

   int index = 0;
```

```
      Node currentNode = firstNode;
      while ((index < numberOfEntries) && (currentNode != null))
      {
         result[index] = currentNode.data;
         index++;
         currentNode = currentNode.next;
      } // end while

      return result;
   } // end toArray
```

Programming Tip

If `ref` is a reference to a node in a chain, be sure that `ref` is not `null` before you use it to access `ref.data` or `ref.next`. Otherwise, if `ref` is `null`, a `NullPointerException` will occur.

Question 4 In the previous definition of `toArray`, the `while` statement uses the boolean expression `(index < numberOfEntries) && (currentNode != null)` to control the loop. Is it necessary to test the values of both `index` and `currentNode`? Explain your answer.

Testing the Core Methods

3.15 Earlier, we realized that the `add` method is fundamental to our class, so it is one of the core methods that we implement and test first. The method `toArray` lets us see whether `add` works correctly, so it too is in our core group. But what about the methods that are not in our core group? Because `LinkedBag` implements the interface `BagInterface`, it must define every method in the interface. As the previous chapter described, we write stubs for methods that are declared in the interface but are not a part of our core group. Since the methods `getCurrentSize`, `isFull`, and `isEmpty` have simple definitions, we will write them instead of stubs in this first draft of the class `LinkedBag`.

Note: The method `isFull` should always return false. As noted in Segment 3.13, the only time a bag whose implementation is linked could appear full is when the system cannot provide memory to the `add` method for a new node. In that case, an `OutOfMemoryError` occurs, which is fatal. A client would not have the opportunity to call `isFull`.

A test program for `LinkedBag`[1] could be just like the one for `ArrayBag`, as given in Listing 2-2 of the previous chapter, except for its name, the class used to create a bag, and one other significant distinction: Although an instance of `ArrayBag` can become full, an instance of `LinkedBag` will not. Listing 3-3 outlines such a test program. Notice that the private static methods here are exactly the same as those given in Listing 2-2 of the previous chapter. This is possible because the methods use `BagInterface` as the data type of a bag.

[1] Note that this version of the class `LinkedBag` is available online at the book's website and is named `LinkedBag1`.

LISTING 3-3 A sample program that tests some methods in the class `LinkedBag`

```
/** A test of the methods add, toArray, isFull, isEmpty, and
    getCurrentSize, as defined in the first draft of the class LinkedBag.
    @author Frank M. Carrano
*/
public class LinkedBagDemo1
{
   public static void main(String[] args)
   {
      System.out.println("Creating an empty bag.");
      BagInterface<String> aBag = new LinkedBag<String>();
      testIsEmpty(aBag, true);
      displayBag(aBag);

      String[] contentsOfBag = {"A", "D", "B", "A", "C", "A", "D"};
      testAdd(aBag, contentsOfBag);
      testIsEmpty(aBag, false);
      testIsFull(aBag, false);
   } // end main

   < The static methods testAdd, testIsFull, and displayBag from Listing 2-2 are here. >
   < The static method testIsEmpty is analogous to testIsFull and is here. >
} // end LinkedBagDemo1
```

The Method `getFrequencyOf`

3.16 To count the number of times a given entry appears in a bag, we must traverse the chain of nodes and look at the entry in each one. The traversal is much like the one we used in the method `toArray`. Thus, if `currentNode` will reference the node that we want to examine, we set it initially to `firstNode`—the first node in the chain—and then use the statement

```
currentNode = currentNode.next;
```

VideoNote
Completing the class LinkedBag

to advance it to the next node. Using this technique, we can write a loop like the following one:

```
int counter = 0;
Node currentNode = firstNode;
while ((counter < numberOfEntries) && (currentNode != null))
{
   . . .
   counter++;
   currentNode = currentNode.next;
} // end while
```

Although the method `toArray` uses the variable `index`, since it deals with an array, we use the variable `counter` here, as we do not have an array. You should note that `counter` is counting nodes for loop control; it is not counting how many times a given entry occurs in a bag. Moreover, we could omit `counter` entirely, but we retain it as a check on our logic.

Within the body of the loop, we access the data in the current node and compare it with the entry passed to the method as its argument. Each time we find a match, we increment a frequency count. Thus, we have the following definition for the method `getFrequencyOf`:

```
/** Counts the number of times a given entry appears in this bag.
    @param anEntry  the entry to be counted
    @return the number of times anEntry appears in the bag */
public int getFrequencyOf(T anEntry)
{
   int frequency = 0;

   int counter = 0;
   Node currentNode = firstNode;
   while ((counter < numberOfEntries) && (currentNode != null))
   {
      if (anEntry.equals(currentNode.data))
         frequency++;

      counter++;
      currentNode = currentNode.next;
   } // end while

   return frequency;
} // end getFrequencyOf
```

The Method `contains`

3.17 In the previous chapter—where we used an array to represent the bag's entries—we determined whether a bag contained a given entry by examining each array element—starting at index zero—until we either found the desired entry or discovered that it was not in the array. We use an analogous approach here to search a chain for a particular piece of data by looking at the chain's nodes, one at a time. We begin at the first node, and if that does not contain the entry we are seeking, we look at the second node, and so on.

When searching an array, we use an index. To search a chain, we use a reference to a node. So, just as in the method `getFrequencyOf`, we use a local variable `currentNode` to reference the node that we want to examine. Initially, we set `currentNode` to `firstNode` and then to `currentNode.next` as we traverse the chain. However, instead of traversing the entire chain, our loop iterates until either we find the desired entry or `currentNode` becomes `null`—in which case the entry is not in the bag.

Thus, the method `contains` has the following implementation:

```
public boolean contains(T anEntry)
{
   boolean found = false;
   Node currentNode = firstNode;

   while (!found && (currentNode != null))
   {
      if (anEntry.equals(currentNode.data))
         found = true;
      else
         currentNode = currentNode.next;
   } // end while

   return found;
} // end contains
```

SELF-TEST

Question 5 If `currentNode` in the previous method `contains` becomes `null`, what value does the method return when the bag is not empty?

Question 6 Trace the execution of the method `contains` when the bag is empty. What is the result?

Removing an Item from a Linked Chain

3.18 Earlier in this chapter, we used the analogy of a classroom to describe how to form a linked chain of data. Available desks are in the hallway outside of the classroom. Each desk has a number (address) stamped on its back and a blank piece of paper taped to its desktop. As students enter the room, they take a desk from the hall. The number of another desk already in the room is written on the new desk's piece of paper, and the instructor is given the new desk's number. In this way, the desks are linked to one another, forming a chain of desks. As you saw in Figure 3-1, no desk references the first desk in the chain, but the instructor knows its address. The last desk does not reference another desk; its paper is blank.

Students who leave our classroom—room L—return their desks to the hall. Such desks can be reassigned to other students who enter either room L or other rooms that share this hallway. Suppose that you are a student in room L but you want to drop the course. If you simply move your desk to the hallway, you will not actually remove yourself from the chain of desks in the room: Either another desk or the instructor will still reference your desk. We need to disconnect your desk from the chain. The details of how we do this depend on where your desk is in the chain. Here are the possible cases:

- Case 1: Your desk is first in the chain of desks.
- Case 2: Your desk is not first in the chain of desks.

3.19 **Case 1.** Figure 3-8 illustrates Case 1 before we remove the first desk from the chain. The following steps are necessary to remove the first desk:

1. Locate the first desk by asking the instructor for its address.
2. Give the address that is written on the first desk to the instructor. This is the address of the second desk in the chain.
3. Return the first desk to the hallway.

FIGURE 3-8 A chain of desks just prior to removing its first desk

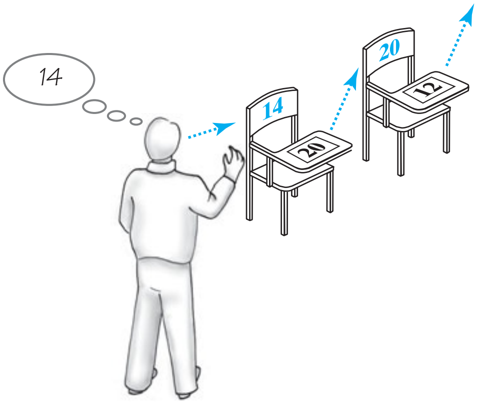
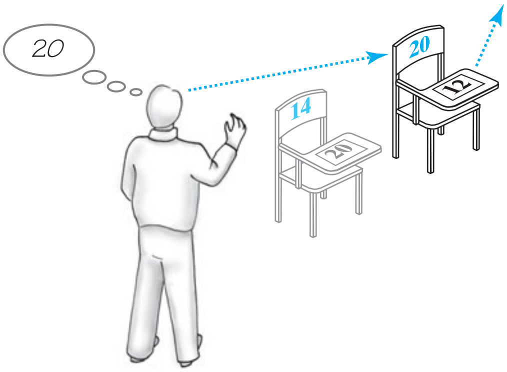


Figure 3-9 shows the chain after the first two steps take place. Notice that the first desk is no longer a part of the chain. Technically, it still references the second desk. But if this desk is ever used again, a new address will be written on its paper.

FIGURE 3-9 A chain of desks just after removing its first desk



3.20 **Case 2.** Remember that a bag does not order its entries in any particular way. Thus, in our analogous classroom, we assume that the students are seated in no particular order. If you want to drop the course and are not seated at the first desk in the chain, we do not have to remove your desk. Instead, we take the following steps:

1. Move the student in the first desk to your former desk.
2. Remove the first desk using the steps described for Case 1.

In effect, we have changed Case 2 into Case 1, which we know how to handle.

SELF-TEST

Question 7 What steps are necessary to remove the first desk in a chain of five desks?

Question 8 What steps are necessary to remove the third desk in a chain of five desks?

The Methods `remove` and `clear`

3.21 **Removing an unspecified entry.** The method `remove` without a parameter removes an unspecified entry from a bag that is not empty. According to the method's specification, as given in the interface in Listing 1-1 of Chapter 1, the method returns the entry it removes:

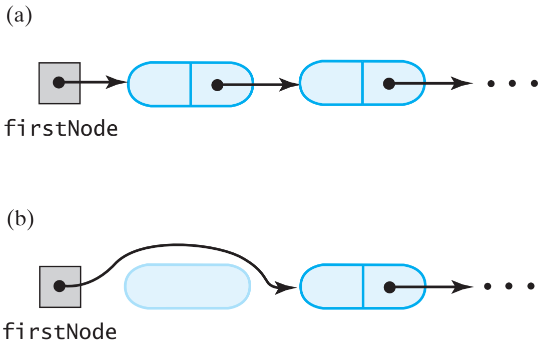
```
/** Removes one unspecified entry from this bag, if possible.
    @return either the removed object, if the removal was successful,
            or null */
public T remove()
```

If the bag is empty before the method executes, `null` is returned.

Removing an entry from a bag involves removing it from a chain of linked nodes. Since the first node is easy to remove from the chain, we can define `remove` so that it removes the entry in this first node. To do so, we take the following steps:

- Access the entry in the first node so it can be returned.
- Set `firstNode` to reference the second node, as Figure 3-10 indicates. If a second node does not exist, set `firstNode` to `null`.
- Decrement `numberOfEntries`.

FIGURE 3-10 A chain of nodes (a) just prior to removing the first node; (b) just after removing the first node



Notice how we implement these steps in the following Java definition of `remove`:

```
public T remove()
{
   T result = null;
   if (firstNode != null)
   {
      result = firstNode.data;
      firstNode = firstNode.next; // remove first node from chain
      numberOfEntries--;
   } // end if

   return result;
} // end remove
```

We first check whether the chain is empty by comparing `firstNode` with `null`. Note that we could have called `isEmpty` instead. While accessing the data in the first node and decrementing the number of entries have straightforward expressions in Java, the entire effect of the statement

```
firstNode = firstNode.next;
```

might not be obvious. It should be clear by now that this statement makes `firstNode` reference the second node in the chain, if such a node exists. But what if it doesn't? That is, what happens when the chain contains only one node? In that case, `firstNode.next` is `null`, so the statement sets `firstNode` to `null`, as required.

3.22 **Removing a given entry.** As the interface in Listing 1-1 of Chapter 1 specifies, a second method `remove` removes a given entry and returns true or false according to the success of the operation:

```
/** Removes one occurrence of a given entry from this bag, if possible.
    @param anEntry  the entry to be removed
    @return true if the removal was successful, or false otherwise */
public boolean remove(T anEntry)
```

If the bag is empty before the method executes, or if `anEntry` is not in the bag, the method returns false.

To remove a specific entry that is in a chain of linked nodes, we first must locate the entry. That is, we must traverse the chain and examine the entries in the nodes. Suppose that we find the desired entry in node *N*. From our previous discussion in Segment 3.20 about a classroom, we can see that if node *N* is not first in the chain, we can remove its entry by taking the following steps:

1. Replace the entry in node *N* with the entry in the first node.
2. Remove the first node from the chain.

Notice that the second step can be accomplished by calling the method `remove`, as given in the previous segment.

What if node *N* is first in the chain? If we do not treat this situation separately, the previous steps will replace the entry in the first node with itself. It will be easier to let this happen than to add logic that asks whether node *N* is the first one.

Thus, we have the following pseudocode for the method `remove`:

```
Locate a node N that contains anEntry
if (node N exists)
{
   Replace the entry in node N with the entry in the first node
   remove()
}
return true or false according to whether the operation succeeds
```

3.23 **Removing a given entry, continued.** The search to locate a node that contains a given entry is the same one done by the method `contains` in Segment 3.17. Rather than repeating this code in the method `remove`, we can place it into a new private method that both `remove` and `contains` can call. The definition of this private method follows:

```
// Locates a given entry within this bag.
// Returns a reference to the node containing the entry, if located,
// or null otherwise.
private Node getReferenceTo(T anEntry)
{
   boolean found = false;
   Node currentNode = firstNode;

   while (!found && (currentNode != null))
   {
      if (anEntry.equals(currentNode.data))
         found = true;
      else
         currentNode = currentNode.next;
   } // end while

   return currentNode;
} // end getReferenceTo
```

The pseudocode given in the previous segment for the method `remove` now translates into Java as follows:

```
public boolean remove(T anEntry)
{
   boolean result = false;
   Node nodeN = getReferenceTo(anEntry);

   if (nodeN != null)
   {
      nodeN.data = firstNode.data; // replace located entry with entry
                                   // in first node
      remove();                    // remove first node
      result = true;
   } // end if

   return result;
} // end remove
```

Question 9 Instead of calling the method `getReferenceTo`, could the method `remove` have called the original definition of `contains`, as given in Segment 3.17? Explain.

Question 10 Revise the definition of the method `contains` so that it calls the private method `getReferenceTo`.

Question 11 Revise the definition of the method `getReferenceTo` so that it controls its loop by using a counter and `numberOfEntries` instead of `currentNode`.

Question 12 What is an advantage of the definition of `getReferenceTo`, as given in the previous segment, over the one that the previous question describes?

3.24 **The method `clear`.** In the class `ArrayBag`, as given in the previous chapter, the method `clear` called the methods `remove` and `isEmpty` to remove all entries from the bag. Since this definition does not depend on how we represent the bag, we can use the same definition here in `LinkedBag`. Thus, `clear`'s definition is

```
public void clear()
{
   while (!isEmpty())
      remove();
} // end clear
```

Note: Deallocating memory
After the method `remove` removes a node from a chain, you have no way to reference the removed node, so you cannot use it. As Segment B.20 in Appendix B noted, the Java run-time environment automatically deallocates and recycles the memory associated with such nodes. No explicit instruction from the programmer is necessary or, in fact, possible to cause deallocation to occur.

A Class `Node` That Has Set and Get Methods

Because `Node` is an inner class of the class `LinkedBag`, `LinkedBag` can access `Node`'s private data fields directly by name. Doing so makes the implementation somewhat easier to write, read, and understand, particularly for novice Java programmers. However, some computer scientists feel that you should access a class's data fields only by calling accessor and mutator (set and get) methods. This section adds these methods to `Node` and explores three ways define this class.

3.25 **As an inner class.** Suppose that we add the methods `getData`, `setData`, `getNextNode`, and `setNextNode` to the inner class `Node`, as it appears in Listing 3-1. The class would then appear as given in Listing 3-4.

LISTING 3-4 The inner class `Node` with set and get methods

```
private class Node
{
   private T    data; // entry in bag
   private Node next; // link to next node

   private Node(T dataPortion)
   {
      this(dataPortion, null);
   } // end constructor

   private Node(T dataPortion, Node nextNode)
   {
      data = dataPortion;
      next = nextNode;
   } // end constructor

   private T getData()
   {
      return data;
   } // end getData

   private void setData(T newData)
   {
      data = newData;
   } // end setData

   private Node getNextNode()
   {
      return next;
   } // end getNextNode

   private void setNextNode(Node nextNode)
   {
      next = nextNode;
   } // end setNextNode
} // end Node
```

3.26 With these additions to Node, we could revise the implementation of LinkedBag by making changes such as the following:

- Change

  ```
  newNode.next = firstNode;
  ```

  to

  ```
  newNode.setNextNode(firstNode);
  ```

- Change

  ```
  currentNode = currentNode.next;
  ```

  to

  ```
  currentNode = currentNode.getNextNode();
  ```

- Change

  ```
  result = firstNode.data;
  ```

  to

  ```
  result = firstNode.getData();
  ```

- Change

  ```
  entryNode.data = firstNode.data;
  ```

to

```
entryNode.setData(firstNode.getData());
```

Project 2 at the end of this chapter asks you to complete these revisions to `LinkedBag`.

3.27 **As a class within a package.** After we modify `Node` and `LinkedBag` as just described, `Node` could remain as a private inner class. Since `Node` is an implementation detail that we want to hide, making it an inner class is appropriate. But if we ever changed our minds and wanted to define `Node` outside of `LinkedBag`, we could do so while retaining the modifications to `LinkedBag` made in the previous segment. We could—with a few changes—make `Node` accessible only within a package, or we could even make it a public class.

To transform `Node`, as given in Listing 3-4, into a class accessible only by other classes in its package, you first omit all the access modifiers except the ones for the data fields. You then add `<T>` after each occurrence of `Node` within the class definition, except when it is used as a constructor name. The revised class appears in Listing 3-5.

LISTING 3-5 The class `Node` with package access

```
package BagPackage;
class Node<T>
{
   private T       data;
   private Node<T> next;

   Node(T dataPortion) // the constructor's name is Node, not Node<T>
   {
      this(dataPortion, null);
   } // end constructor

   Node(T dataPortion, Node<T> nextNode)
   {
      data = dataPortion;
      next = nextNode;
   } // end constructor

   T getData()
   {
      return data;
   } // end getData

   void setData(T newData)
   {
      data = newData;
   } // end setData

   Node<T> getNextNode()
   {
      return next;
   } // end getNextNode

   void setNextNode(Node<T> nextNode)
```

```
      {
         next = nextNode;
      } // end setNextNode
   } // end Node
```

3.28 The class `LinkedBag` can access `Node`, as just given in Listing 3-5, if both classes are in the same package and we modify `LinkedBag` slightly. Each occurrence of `Node` within `LinkedBag` must now appear as `Node<T>`. We begin to make these changes to `LinkedBag` and highlight them in Listing 3-6.

LISTING 3-6 The class `LinkedBag` when `Node` is in the same package

```
package BagPackage;
public class LinkedBag<T> implements BagInterface<T>
{
   private Node<T> firstNode;
   . . .
   public boolean add(T newEntry)
   {
      Node<T> newNode = new Node<T>(newEntry);
      newNode.setNextNode(firstNode);
      firstNode = newNode;
      numberOfEntries++;

      return true;
   } // end add
   . . .
} // end LinkedBag
```

Project 3 at the end of this chapter asks you to complete this revision of `LinkedBag`.

3.29 **As an inner class with a declared generic type.** The version of `LinkedBag` just described in Listing 3-6 could define `Node` as an inner class. `Node` would be similar to the class given in Listing 3-5, but would require the following changes:

- Omit the `package` statement.
- Make the class, constructors, and methods private.
- Replace the generic type `T` with another identifier, such as `S`.

Since both `LinkedBag` and `Node` declare generic types, they must use different identifiers to represent them.

Project 4 at the end of the chapter asks you to revise `Node` and `LinkedBag` as described here.

The Pros and Cons of Using a Chain to Implement the ADT Bag

3.30 You have seen how to use a chain in the implementation of the ADT bag. One of the greatest advantages of this approach is that the chain, and therefore the bag, can grow and shrink in size as necessary. As long as memory is available, you can add as many nodes to a chain as you wish.

Moreover, you can remove and recycle nodes that are no longer needed. Although you can resize an array to allow a bag to grow in size—as the previous chapter describes—each time a larger array is necessary, you must copy the entries from the full array to the new array. No such copying is required when you use a chain.

Adding a new entry to the end of an array or to the beginning of a chain are both relatively simple tasks. Both operations are fast, unless the array needs to be resized. Likewise, removing the entry at the end of an array or the beginning of a chain takes about the same effort. However, removing a specific entry requires a search of the array or chain.

Lastly, a chain requires more memory than an array of the same length. Although both data structures contain references to data objects, each node in a chain also contains a reference to another node. However, an array is often larger than necessary, so memory is wasted. A chain uses memory only as needed.

? SELF-TEST

Question 13 Compare the efforts made by the `contains` methods in the classes `LinkedBag` in this chapter and `ResizableArrayBag` in Chapter 2. Does one take more time to perform its task? Explain.

Chapter Summary

- You can form a chain of linked data by using objects called nodes. Each node has two parts. One part contains a reference to a data object, and the second part references the next node in the chain. The last node, however, references no other node and contains `null`. A head reference external to the chain references the first node.

- You can add a node to the beginning of a chain of linked nodes by changing two references: the one within the node to be added and the chain's head reference.

- You can remove the first node in a chain of linked nodes by setting the chain's head reference to the reference within the first node.

- Locating a particular node in a chain of linked nodes requires a traversal of the chain. Beginning at the first node, you move from node to node sequentially until you reach the desired node.

- The class `Node` can be an inner class of `LinkedBag` or a class within a package that contains `LinkedBag`. In the latter case, `Node` must define set and get methods to provide access to its data fields.

Programming Tip

- If `ref` is a reference to a node in a chain, be sure that `ref` is not `null` before you use it to access `ref.data` or `ref.next`.

Exercises

1. Add a constructor to the class `LinkedBag` that creates a bag from a given array of objects.

2. Consider the definition of `LinkedBag`'s `add` method that appears in Segment 3.12. Interchange the second and third statements in the method's body, as follows:

```
firstNode = newNode;
newNode.next = firstNode;
```

a. What is displayed by the following statements in a client of the modified `LinkedBag`?

```
BagInterface<String> myBag = new LinkedBag<String>();
myBag.add("30");
```

```
      myBag.add("40");
      myBag.add("50");
      myBag.add("10");
      myBag.add("60");
      myBag.add("20");
      int numberOfEntries = myBag.getCurrentSize();
      String[] entries = myBag.toArray();
      for (int index = 0; index < numberOfEntries; index++)
         System.out.print(entries[index] + " ");
```

b. What methods, if any, in `LinkedBag` could be affected by the change to the method `add` when they execute? Why?

3. Repeat Exercise 2 in the previous chapter for the class `LinkedBag`.

4. Revise the definition of the method `remove`, as given in Segment 3.21, so that it removes a random entry from a bag. Would this change affect any other method within the class `LinkedBag`?

5. Define a method `removeEvery` for the class `LinkedBag` that removes all occurrences of a given entry from a bag.

6. Suppose that a bag contains `Comparable` objects. Define the following methods for the class `LinkedBag`:

- The method `getMin` that returns the smallest object in a bag
- The method `getMax` that returns the largest object in a bag

7. Repeat Exercise 10 in the previous chapter for the class `LinkedBag`.

8. Define an `equals` method for the class `LinkedBag`. Consult Exercise 11 in the previous chapter for details about this method.

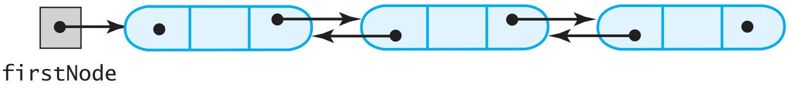
9. Define the method `union`, as described in Exercise 5 of Chapter 1, for the class `LinkedBag`.

10. Define the method `intersection`, as described in Exercise 6 of Chapter 1, for the class `LinkedBag`.

11. Define the method `difference`, as described in Exercise 7 of Chapter 1, for the class `LinkedBag`.

12. In a **doubly linked chain**, each node can reference the previous node as well as the next node. Figure 3-11 shows a doubly linked chain and its head reference. Define a class to represent a node in a doubly linked chain. Write the class as an inner class of a class that implements the ADT bag. You can omit set and get methods.

FIGURE 3-11 A doubly linked chain for Exercises 12, 13, 14, and 15, and Project 7



13. Repeat Exercise 12, but instead write the class within a package that contains an implementation of the ADT bag. Set and get methods will be necessary.

14. List the steps necessary to add a node to the beginning of the doubly linked chain shown in Figure 3-11.

15. List the steps necessary to remove the first node from the beginning of the doubly linked chain shown in Figure 3-11.

Projects

1. Write a program that thoroughly tests the class `LinkedBag`.

2. Listing 3-4 shows the inner class `Node` with set and get methods. Revise the class `LinkedBag` so that it invokes these set and get methods instead of accessing the private data fields `data` and `next` directly by name.

3. Listing 3-5 shows Node as a class within a package containing LinkedBag. Revise LinkedBag to use this version of Node.

4. Revise Node and LinkedBag as described in Segment 3.29.

5. Define a class LinkedSet that represents a set and implements the interface described in Project 1a of Chapter 1. Use the class LinkedBag in your implementation. Then write a program that adequately demonstrates your implementation.

6. Repeat the previous project, but use a chain of linked nodes instead of the class LinkedBag.

7. Define a class DoublyLinkedBag that implements the ADT bag by using a doubly linked chain, as shown in Figure 3-11. Use the inner class of nodes that Exercise 12 defines.

8. Repeat the previous project, but define set and get methods in the inner class of nodes.

9. Use the classes for a set or a bag, as defined in this chapter or described in the previous projects, to create a spell checker. Consult the details given in Projects 5 and 6 of the previous chapter.

Answers to Self-Test Questions

1. **a.** First.
   b. The student who arrived last (most recently).

2. At the beginning.

3. When the chain is empty, firstNode is null. Setting newNode.next to firstNode sets it to null. Since newNode.next already is null, no harm is done by the additional assignment.

4. Testing the values of both index and currentNode is not necessary. Although testing either one of these values is sufficient, testing both values provides a check against mistakes in your code.

5. The method returns false. If currentNode becomes null, the entire chain has been searched without success.

6. Since the bag is empty, firstNode—and hence currentNode—is null. The while loop ends immediately and the method returns false.

7.
   - Locate the first desk by asking the instructor for its address.
   - Give the address that is written on the first desk's paper to the instructor. This is the address of the second desk in the chain.
   - Return the first desk to the hallway.

8.
   - The student in the first desk moves to the third desk.
   - Remove the first desk using the three steps given as the answer to the previous question.

9. No. The method contains returns either true or false. Although remove would be able to tell whether anEntry is in the bag, it would not have a reference to anEntry. Thus, it would not be able to remove anEntry without doing its own search.

10.
```
public boolean contains(T anEntry)
{
   return getReferenceTo(anEntry) != null;
} // end contains
```

11.
```
private Node getReferenceTo(T anEntry)
{
   boolean found = false;
   Node currentNode = firstNode;
   int counter = 0;

   while (!found && (counter < numberOfEntries))
   {
      if (anEntry.equals(currentNode.data))
         found = true;
      else
      {
         currentNode = currentNode.next;
         counter++;
      } // end if
   } // end while

   return currentNode;
} // end getReferenceTo
```

12. The original definition of getReferenceTo ensures that currentNode is not null, and thus avoids a NullPointerException.

13. The effort expended by each of these two methods is about the same. Each method calls a private method that searches for the desired entry. In LinkedBag, contains calls getReferenceTo, which searches at most numberOfEntries nodes for the desired entry. In ResizableArrayBag, contains calls getIndexOf, which searches at most numberOfEntries array elements for the desired entry. The next chapter will discuss these methods and analyze their time requirements in more detail.

This page intentionally left blank

Chapter 4

The Efficiency of Algorithms

Contents

Prerequisites

Appendix B		Java Classes
Chapter	2	Bag Implementations That Use Arrays
Chapter	3	A Bag Implementation That Links Data

Objectives

After studying this chapter, you should be able to

- Assess the efficiency of a given algorithm
- Compare the expected execution times of two methods, given the efficiencies of their algorithms

With amazing frequency, manufacturers introduce new computers that are faster and have larger memories than their recent predecessors. Yet we—and likely your computer science professors—ask you to write code that is efficient in its use of time and space (memory). Admittedly, such efficiency is not as pressing an issue as it was fifty years ago, when computers were much slower and their memory size was much smaller than they are now. (Computers had small memories, but they were physically huge, occupying entire rooms.) Even so, efficiency remains an issue—in some circumstances, a critical issue.

This chapter will introduce you to the terminology and ways that computer scientists use to measure the efficiency of an algorithm. With this background, not only will you have an intuitive feel for efficiency, but you also will be able to talk about efficiency in a quantitative way.

Motivation

4.1 **Example.** Perhaps you think that you are not likely to write a program in the near future whose execution time is noticeably long. You might be right, but we are about to show you some simple Java code that does take a long time to perform its computations.

Consider the problem of computing the sum 1 + 2 + . . . + *n* for any positive integer *n*. Figure 4-1 contains pseudocode showing three ways to solve this problem. Algorithm A computes the sum 0 + 1 + 2 + . . . + *n* from left to right. Algorithm B computes 0 + (1) + (1 + 1) + (1 + 1 + 1) + . . . + (1 + 1 + . . . + 1). Finally, Algorithm C uses an algebraic identity to compute the sum.

FIGURE 4-1 Three algorithms for computing the sum 1 + 2 + . . . + *n* for an integer *n* > 0

Algorithm A	Algorithm B	Algorithm C

sum = 0
for i = 1 *to* n
    sum = sum + i
	sum = 0
	for i = 1 *to* n
	{
	    for j = 1 *to* i
	        sum = sum + 1
	}
		sum = n * (n + 1) / 2

4.2 Let's translate these algorithms into Java code. If we use `long` integers, we could write the following statements:

```java
// Computing the sum of the consecutive integers from 1 to n:
long n = 10000; // ten thousand

// Algorithm A
long sum = 0;
for (long i = 1; i <= n; i++)
   sum = sum + i;
System.out.println(sum);

// Algorithm B
sum = 0;
for (long i = 1; i <= n; i++)
{
   for (long j = 1; j <= i; j++)
      sum = sum + 1;
} // end for
System.out.println(sum);

// Algorithm C
sum = n * (n + 1) / 2;
System.out.println(sum);
```

If you execute this code with n equal to ten thousand (10000), you will get the right answer of 50005000 for each of the algorithms. Now change the value of n to one hundred thousand (100000), and execute the code again. Once more, you will get the correct answer, which this time is 5000050000. However, you should notice a delay in seeing the result for Algorithm B. Now try one million (1000000) for the value of n. Again you will get the correct answer—500000500000—but you will have to wait even longer for the result from Algorithm B. The wait might be long enough for you to suspect that something is broken. If not, try a larger value of n.

The previous simple code for Algorithm B takes a noticeably long time to execute, much longer than either of the other two algorithms. If it were the only algorithm you tried, what should you do? Use a faster computer? While that might be a solution, it's clear that we should use a different algorithm.

Note: As the previous example shows, even a simple program can be noticeably inefficient.

Note: If an algorithm takes longer to execute than is practical, try to reformulate it to make it more efficient of time.

Measuring an Algorithm's Efficiency

4.3 The previous section should have convinced you that a program's efficiency matters. How can we measure efficiency so that we can compare various approaches to solving a problem? In the previous section, we computed the sum of the first n consecutive integers in three different ways. We then observed that one was noticeably slower than the others as the value of n increased. In general, however, implementing several ideas before you choose one requires too much work to be practical. Besides, a program's execution time depends in part on the particular computer and the programming language used. It would be much better to measure an *algorithm's* efficiency before you implement it.

For example, suppose that you want to go to a store downtown. Your options are to walk, drive your car, ask a friend to take you, or take a bus. What is the best way? First, what is your concept of best? Is it the way that saves money, your time, your friend's time, or the environment? Let's say that the best option for you is the fastest one. After defining your criterion, how do you evaluate your options? You certainly do not want to try all four options so you can discover which is fastest. That would be like writing four different programs that perform the same task so you can measure which one is fastest. Instead you would investigate the "cost" of each option, considering the distance, the speed at which you can travel, the amount of other traffic, the number of stops at traffic lights, the weather, and so on. That is, you would consider the factors that have the most impact on the cost.

VideoNote
Measuring efficiency

4.4 The same considerations apply when deciding what algorithm is best. Again, we need to define what we mean by best. An algorithm has both time and space requirements, called its **complexity**, that we can measure. When we assess an algorithm's complexity, we are not measuring how involved or difficult it is. Instead, we measure an algorithm's **time complexity**—the time it takes to execute—or its **space complexity**—the memory it needs to execute. Typically we analyze these requirements separately. So a "best" algorithm might be the fastest one or the one that uses the least memory.

Note: What's best?
Usually the "best" solution to a problem balances various criteria such as time, space, generality, programming effort, and so on.

The process of measuring the complexity of algorithms is called the **analysis of algorithms**. We will concentrate on the time complexity of algorithms, because it is usually more important than the space complexity. You should realize that an inverse relationship often exists between an algorithm's time complexity and its space complexity. If you revise an algorithm to save execution time, you usually will need more space. If you reduce an algorithm's space requirement, it likely will require more time to execute. Sometimes, however, you will be able to save both time and space.

Your measure of the complexity of an algorithm should be easy to compute, certainly easier than implementing the algorithm. You should express this measure in terms of the size of the problem. This **problem size** is the number of items that an algorithm processes. For example, if you are searching a collection of data, the problem size is the number of items in the collection. Such a measure enables you to compare the relative cost of algorithms as a function of the size of the problem. Typically, we are interested in large problems; a small problem is likely to take little time, even if the algorithm is inefficient.

4.5 Realize that you cannot compute the actual time requirement of an algorithm. After all, you have not implemented the algorithm in Java and you have not chosen the computer. Instead, you find a function of the problem size that behaves like the algorithm's actual time requirement. Therefore, as the time requirement increases by some factor, the value of the function increases by the same factor, and vice versa. The value of the function is said to be **directly proportional** to the time requirement. Such a function is called a **growth-rate function** because it measures how an algorithm's time requirement grows as the problem size grows. Because they measure time requirements, growth-rate functions have positive values. By comparing the growth-rate functions of two algorithms, you can see whether one algorithm is faster than the other for large-size problems.

4.6 **Example.** Consider again the problem of computing the sum $1 + 2 + . . . + n$ for any positive integer n. Figure 4-1 gives three algorithms—A, B, and C— to perform this computation. Algorithm A computes the sum $0 + 1 + 2 + . . . + n$ from left to right. Algorithm B computes $0 + (1) + (1 + 1) + (1 + 1 + 1) + . . . + (1 + 1 + . . . + 1)$, and Algorithm C uses an algebraic identity to compute the sum. By executing the Java code in Segment 4.2, we found that Algorithm B is the slowest. We now want to predict this behavior without actually running the code.

So how can we tell which algorithm is slowest and which is fastest? We can begin to answer these questions by considering both the size of the problem and the effort involved. The integer n is a measure of the problem size: As n increases, the sum involves more terms. To measure the effort, or time requirement, of an algorithm, we must find an appropriate growth-rate function. To do so, we might begin by counting the number of operations required by the algorithm.

For example, Algorithm A in Figure 4-1 contains the pseudocode statement

```
for i = 1 to n
```

This statement represents the following loop-control logic:

```
i = 1
while (i <= n)
{
   . . .
   i = i + 1
}
```

This logic requires an assignment to i, $n + 1$ comparisons between i and n, n additions to i, and n more assignments to i. In total, the loop-control logic requires $n + 1$ assignments, $n + 1$ comparisons, and n additions. Furthermore, Algorithm A requires for its initialization and loop body another $n + 1$ assignments and n additions. All together, Algorithm A requires $2n + 2$ assignments, $2n$ additions, and $n + 1$ comparisons.

These various operations probably take different amounts of time to execute. For example, if each assignment takes no more than $t_=$ time units, each addition takes no more than t_+ time units, and each comparison takes no more than t_c time units, Algorithm A would require no more than

$$(2n + 2)\ t_= + (2n)\ t_+ + (n + 1)\ t_c \text{ time units}$$

If we replace $t_=$, t_+, and t_c with the largest of the three values and call it t, Algorithm A requires no more than $(5n + 3)\ t$ time units. We conclude that Algorithm A requires time directly proportional to $5n + 3$.

What is important, however, is not the exact count of operations, but the general behavior of the algorithm. The function $5n + 3$ is directly proportional to n. As you are about to see, we do not have to count every operation to see that Algorithm A requires time that increases linearly with n.

Counting Basic Operations

4.7 An algorithm's **basic operation** is the most significant contributor to its total time requirement. For example, Algorithms A and B in Figure 4-1 have addition as their basic operation. An algorithm that sees whether an array contains a particular object has comparison as its basic operation. Realize that the most frequent operation is not necessarily the basic operation. For example, assignments are often the most frequent operation in an algorithm, but they rarely are basic.

Ignoring operations that are not basic, such as initializations of variables, the operations that control loops, and so on, will not affect our final conclusion about algorithm speed. For example, Algorithm A requires n additions of i to `sum` in the body of the loop. We can conclude that Algorithm A requires time that increases linearly with n, even though we ignored operations that are not basic to the algorithm.

Whether we look at the number, n, of basic operations or the total number of operations, $5n + 3$, we can draw the same conclusion: Algorithm A requires time directly proportional to n. Thus, Algorithm A's growth-rate function is n.

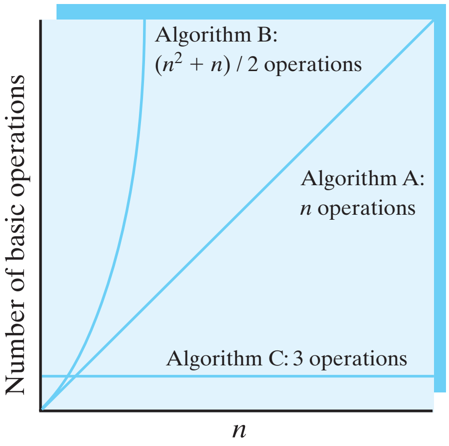
4.8 **Example continued.** Now let's count the number of basic operations required by Algorithms B and C. The basic operation for Algorithm B is addition; for Algorithm C, the basic operations are addition, multiplication, and division. Figure 4-2 tabulates the number of basic operations that Algorithms A, B, and C require. Remember, these counts do not include assignments and the operations that control the loops. Our discussion in the previous segment should have convinced you that we can ignore these operations.

FIGURE 4-2 The number of basic operations required by the algorithms in Figure 4-1

	Algorithm A	**Algorithm B**	**Algorithm C**
Additions	n	$n\ (n + 1) / 2$	1
Multiplications			1
Divisions			1
Total basic operations	$\boldsymbol{n}$	$\boldsymbol{(n^2 + n) / 2}$	**3**

Algorithm B requires time directly proportional to $(n^2 + n) / 2$, and Algorithm C requires time that is constant and independent of the value of n. Figure 4-3 plots these time requirements as a function of n. You can see from this figure that as n grows, Algorithm B requires the most time.

FIGURE 4-3 The number of basic operations required by the algorithms in Figure 4-1 as a function of n



Question 1 For any positive integer n, the identity

$$1 + 2 + . . . + n = n\,(n + 1) / 2$$

is one that you will encounter while analyzing algorithms. Can you derive it? If you can, you will not need to memorize it. *Hint*: Write $1 + 2 + . . . + n$. Under it write $n + (n - 1) + . . . + 1$. Then add the terms from left to right.

Question 2 Can you derive the values in Figure 4-2? *Hint*: For Algorithm B, use the identity given in Question 1.

Note: Useful identities

$1 + 2 + . . . + n = n\,(n + 1) / 2$

$1 + 2 + . . . + (n - 1) = n\,(n - 1) / 2$

4.9 Typical growth-rate functions are algebraically simple. Why? Recall that since you are not likely to notice the effect of an inefficient algorithm when the problem is small, you should focus on large problems. Thus, if we care only about large values of n when comparing the algorithms, we can consider only the dominant term in each growth-rate function.

For example, $(n^2 + n) / 2$ behaves like n^2 when n is large. First, n^2 is much larger than n for large values of n, so $(n^2 + n) / 2$ behaves like $n^2 / 2$. Moreover, $n^2 / 2$ behaves like n^2 when n is large. In other words, for large n, the difference between the value of $(n^2 + n) / 2$ and that of n^2 is relatively small and can be ignored. So instead of using $(n^2 + n) / 2$ as Algorithm B's growth-rate function, we can use n^2—the term with the largest exponent—and say that Algorithm B requires time proportional to n^2. On the other hand, Algorithm C requires time that is independent of n, and we saw earlier that Algorithm A requires time proportional to n. We conclude that Algorithm C is the fastest and Algorithm B is the slowest.

Note: The relative magnitudes of common growth-rate functions
The growth-rate functions that you are likely to encounter grow in magnitude as follows when $n > 10$:

$$1 < \log(\log n) < \log n < \log^2 n < n < n \log n < n^2 < n^3 < 2^n < n!$$

The logarithms given here are base 2. As you will see later in Segment 4.16, the choice of base does not matter.

Figure 4-4 tabulates the magnitudes of these functions for increasing values of the problem size n. From this data you can see that algorithms whose growth-rate functions are log(log n), log n, or $\log^2 n$ take much less time than algorithms whose growth-rate function is n. Although the value of $n \log n$ is significantly larger than n, either of those functions describes a growth rate that is markedly faster than n^2.

FIGURE 4-4 Typical growth-rate functions evaluated at increasing values of n

n	log(log n)	log n	$\log^2 n$	n	$n \log n$	n^2	n^3	2^n	$n!$
10	2	3	11	10	33	10^2	10^3	10^3	10^5
10^2	3	7	44	100	664	10^4	10^6	10^{30}	10^{94}
10^3	3	10	99	1000	9966	10^6	10^9	10^{301}	10^{1435}
10^4	4	13	177	10,000	132,877	10^8	10^{12}	10^{3010}	$10^{19,335}$
10^5	4	17	276	100,000	1,660,964	10^{10}	10^{15}	$10^{30,103}$	$10^{243,338}$
10^6	4	20	397	1,000,000	19,931,569	10^{12}	10^{18}	$10^{301,030}$	$10^{2,933,369}$

Note: When analyzing the time efficiency of an algorithm, consider large problems. For small problems, the difference between the execution times of two solutions to the same problem is usually insignificant.

Best, Worst, and Average Cases

4.10 For some algorithms that operate on a data set, the execution time depends only on the size of the data set. For example, the time needed to find the smallest integer in an array of integers depends only on the number of integers, not on the integers themselves. Finding the smallest of 100 integers takes the same amount of time regardless of the values of the integers.

Other algorithms, however, have time requirements that depend not only on the size of the data set, but also on the data itself. For example, imagine that an array contains a certain value, and we want to know where in the array it occurs. Suppose our search algorithm examines each value in the array until it finds the desired one. If the algorithm finds this desired value in the first array element it examines, it makes only one comparison. In this **best case**, the algorithm takes the least time. The algorithm can do no better than its best-case time. If the best-case time is still too slow, you need another algorithm.

Now suppose that the algorithm locates the desired value after comparing it to every value in the array. This would be the algorithm's **worst case**, since it requires the most time. If you can tolerate this worst-case time, your algorithm is acceptable. For many algorithms, the worst and best cases rarely occur. Thus, we consider an algorithm's **average case**, when it processes a typical data set. The average-case time requirement of an algorithm is more useful, but harder to estimate. Note that the average-case time is not the average of the best-case and worst-case times.

Note: The time requirements of some algorithms depend on the data values given to them. Those times range from a minimum, or best-case, time to a maximum, or worst-case, time. Typically, the best and worst cases do not occur. A more useful measure of such an algorithm's time requirement is its average-case time.

Some algorithms, however, do not have a best, worst, and average case. Their time requirements depend only on the number of data items given them, not on the values of that data.

Big Oh Notation

4.11 Computer scientists use a notation to represent an algorithm's complexity. For example, consider the algorithms A, B, and C given in Figure 4-1 and the number of basic operations that each requires, as shown in Figure 4-2. Instead of saying that Algorithm A has a time requirement proportional to n, we say that A is **O(*n*)**. We call this notation **Big Oh** since it uses the capital letter O. We read O(n) as either "Big Oh of n" or "order of at most n." Similarly, since Algorithm B has a time requirement proportional to n^2, we say that B is O(n^2). Algorithm C always requires three basic operations. Regardless of the problem size n, this algorithm requires the same time. We say that Algorithm C is O(1).

4.12 **Example.** Imagine that you are at a wedding reception, seated at a table of n people. In preparation for the toast, the waiter pours champagne into each of n glasses. That task is O(n). Someone makes a toast. It is O(1), even if the toast seems to last forever, because it is independent of the number of guests. If you clink your glass with everyone at your table, you perform an O(n) operation. If everyone at your table does likewise, a total of O(n^2) clinks are performed.

4.13 Big Oh notation has a formal mathematical meaning that can justify our discussion in the previous sections. You saw that an algorithm's actual time requirement is directly proportional to a function f of the problem size n. For example, $f(n)$ might be $n^2 + n + 1$. In this case, we would conclude that the algorithm is of order at most n^2—that is, O(n^2). We essentially have replaced $f(n)$ with a simpler function—let's call it $g(n)$. In this example, $g(n)$ is n^2.

What does it really mean to say that a function $f(n)$ is of order at most $g(n)$—that is, $f(n)$ is O($g(n)$), or $f(n)$ = O($g(n)$)? Formally, its meaning is described by the following mathematical definition:

Note: **Formal definition of Big Oh**

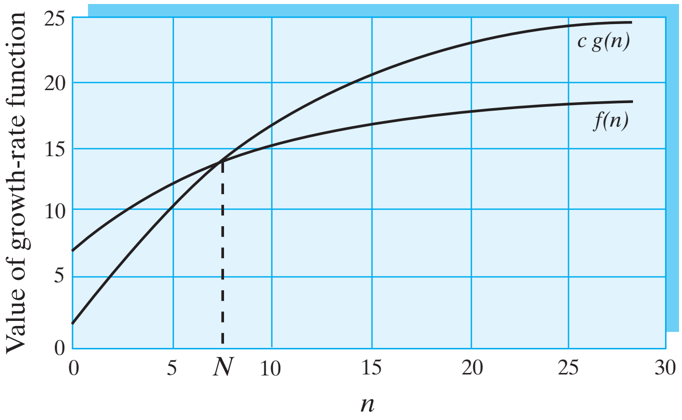
A function $f(n)$ is of order at most $g(n)$—that is, $f(n)$ is O($g(n)$)—if

- A positive real number c and positive integer N exist such that $f(n) \leq c \times g(n)$ for all $n \geq N$. That is, $c \times g(n)$ is an upper bound on $f(n)$ when n is sufficiently large.

In simple terms, $f(n)$ is $O(g(n))$ means that $c \times g(n)$ provides an **upper bound** on $f(n)$'s growth rate when n is large enough. For all data sets of a sufficient size, the algorithm will always require fewer than $c \times g(n)$ basic operations.

Figure 4-5 illustrates the formal definition of Big Oh. You can see that when n is large enough—that is, when $n \geq N$—$f(n)$ does not exceed $c \times g(n)$. The opposite is true for smaller values of n. That is unimportant, since we can ignore these values of n.

FIGURE 4-5 An illustration of the definition of Big Oh



4.14 **Example.** In Segment 4.6, we said that if an algorithm uses $5n + 3$ operations, it requires time proportional to n. We now can show that $5n + 3$ is $O(n)$ by using the formal definition of Big Oh.

When $n \geq 3$, $5n + 3 \leq 5n + n = 6n$. Thus, if we let $f(n) = 5n + 3$, $g(n) = n$, $c = 6$, and $N = 3$, we have shown that $f(n) \leq 6\ g(n)$ for $n \geq 3$, or $5n + 3 = O(n)$. That is, if an algorithm requires time directly proportional to $5n + 3$, it is $O(n)$.

Other values for the constants c and N will also work. For example, $5n + 3 \leq 5n + 3n = 8n$ when $n \geq 1$. Thus, by choosing $c = 8$ and $N = 1$, we have shown that $5n + 3$ is $O(n)$.

You need to be careful when choosing $g(n)$. For example, we just found that $5n + 3 \leq 8n$ when $n \geq 1$. But $8n < n^2$ when $n \geq 9$. So why wouldn't we let $g(n) = n^2$ and conclude that our algorithm is $O(n^2)$? Although this conclusion is correct, it is not as good—or **tight**—as it could be. You want the upper bound on $f(n)$ to be as small as possible.

Note: The upper bound on an algorithm's time requirement should be as small as possible and should involve simple functions like the ones given in Figure 4-4.

4.15 **Example.** Let's show that $4n^2 + 50n - 10$ is $O(n^2)$. It is easy to see that

$$4n^2 + 50n - 10 \leq 4n^2 + 50n \text{ for any } n$$

Since $50n \leq 50n^2$ for $n \geq 50$,

$$4n^2 + 50n - 10 \leq 4n^2 + 50n^2 = 54n^2 \text{ for } n \geq 50$$

Thus, with $c = 54$ and $N = 50$, we have shown that $4n^2 + 50n - 10$ is $O(n^2)$.

Note: To show that $f(n)$ is $O(g(n))$, replace the smaller terms in $f(n)$ with larger terms until only one term is left.

Question 3 Show that $3n^2 + 2^n$ is $O(2^n)$. What values of c and N did you use?

SELF-TEST

4.16 **Example: Show that $\log_b n$ is $O(\log_2 n)$.** Let $L = \log_b n$ and $B = \log_2 b$. From the meaning of a logarithm, we can conclude that $n = b^L$ and $b = 2^B$. Combining these two conclusions, we have

$$n = b^L = (2^B)^L = 2^{BL}$$

Thus, $\log_2 n = BL = B \log_b n$ or, equivalently, $\log_b n = (1 / B) \log_2 n$ for any $n \geq 1$. Taking $c = 1 / B$ and $N = 1$ in the definition of Big Oh, we reach the desired conclusion.

It follows from this example that the general behavior of a logarithmic function is the same regardless of its base. Often the logarithms used in growth-rate functions are base 2. But since the base really does not matter, we typically omit it.

Note: The base of a logarithm in a growth-rate function is usually omitted, since $O(\log_a n)$ is $O(\log_b n)$.

Note: Identities

The following identities hold for Big Oh notation:

$$O(k\, g(n)) = O(g(n)) \text{ for a constant } k$$
$$O(g_1(n)) + O(g_2(n)) = O(g_1(n) + g_2(n))$$
$$O(g_1(n)) \text{ x } O(g_2(n)) = O(g_1(n) \text{ x } g_2(n))$$
$$O(g_1(n) + g_2(n) + \ldots + g_m(n)) = O(\max(g_1(n), g_2(n), \ldots, g_m(n))$$
$$O(\max(g_1(n), g_2(n), \ldots, g_m(n)) = \max(O(g_1(n)), O(g_2(n)), \ldots, O(g_m(n)))$$

By using these identities and ignoring smaller terms in a growth-rate function, you can usually find the order of an algorithm's time requirement with little effort. For example, if the growth-rate function is $4n^2 + 50n - 10$,

$$\begin{aligned} O(4n^2 + 50n - 10) &= O(4n^2) && \text{by ignoring the smaller terms} \\ &= O(n^2) && \text{by ignoring the constant multiplier} \end{aligned}$$

Question 4 If $P_k(n) = a_0\, n^k + a_1 n^{k-1} + \ldots + a_k$ for $k > 0$ and $n > 0$, what is $O(P_k(n))$?

SELF-TEST

The Complexities of Program Constructs

4.17 The time complexity of a sequence of statements in an algorithm or program is the sum of the statements' individual complexities. However, it is sufficient to take instead the largest of these complexities. In general, if $S_1, S_2, \ldots, S_k$ is a sequence of program segments, and if g_i is the growth-rate function for segment S_i, the time complexity of the sequence would be O(max($g_1, g_2, \ldots, g_k$)), which is equivalent to max(O(g_1), O(g_2), . . . , O(g_k)).

The time complexity of the `if` statement

```
if (condition)
   $S_1$
else
   $S_2$
```

is the sum of the complexity of the condition and the complexity of S_1 or S_2, whichever is largest.

The time complexity of a loop is the complexity of its body times the number of times the body executes. Thus, the complexity of a loop such as

```
for i = 1 to m
   $S$
```

is O(m x $g(n)$), or m x O($g(n)$), where $g(n)$ is the growth-rate function for S. Note that the loop variable `i` in this example increments by 1. In the following loop, `i` is doubled at each iteration:

```
for i = 1 to m, i = 2 * i
   $S$
```

The complexity of this loop is O(log(m) x $g(n)$), or O(log(m)) x O($g(n)$).

Note: The complexities of program constructs

Construct	Time Complexity
Consecutive program segments S_1, S_2, . . . , S_k whose growth-rate functions are $g_1, \ldots, g_k$, respectively	max(O(g_1), O(g_2), . . . , O(g_k))
An `if` statement that chooses between program segments S_1 and S_2 whose growth-rate functions are $g_1$and g_2, respectively	O(*condition*) + max(O(g_1), O(g_2))
A loop that iterates m times and has a body whose growth-rate function is g	m x O($g(n)$)

Note: Other notations

Although we will use Big Oh notation most often in this book, other notations are sometimes useful when describing an algorithm's time requirement $f(n)$. We mention them here primarily to expose you to them. Beginning with the definition of Big Oh that you saw earlier, we define **Big Omega** and **Big Theta**.

- **Big Oh.** $f(n)$ is of order at most $g(n)$—that is, $f(n)$ is $O(g(n))$—if positive constants c and N exist such that $f(n) \leq c$ x $g(n)$ for all $n \geq N$. That is, c x $g(n)$ is an upper bound on the time requirement $f(n)$. In other words, $f(n)$ is no larger than c x $g(n)$. Thus, an analysis that uses Big Oh produces a maximum time requirement for an algorithm.
- **Big Omega.** $f(n)$ is of order at least $g(n)$—that is, $f(n)$ is $\Omega(g(n))$—if $g(n)$ is $O(f(n))$. In other words, $f(n)$ is $\Omega(g(n))$ if positive constants c and N exist such that $f(n) \geq c$ x $g(n)$ for all $n \geq N$. The time requirement $f(n)$ is not smaller than c x $g(n)$, its **lower bound**. Thus, a Big Omega analysis produces a minimum time requirement for an algorithm.
- **Big Theta.** $f(n)$ is of order $g(n)$—that is, $f(n)$ is $\Theta(g(n))$—if $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$. Alternatively, we could say that $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$. The time requirement $f(n)$ is the same as $g(n)$. That is, c x $g(n)$ is both a lower bound and an upper bound on $f(n)$. A Big Theta analysis assures us that the time estimate is as good as possible. Even so, Big Oh is the more common notation.

Picturing Efficiency

4.18 Much of an algorithm's work occurs during its repetitive phases, that is, during the execution of loops or—as you will see in Chapter 7—as a result of recursive calls. In this section, we will illustrate the time efficiency of several examples.

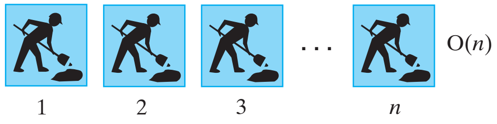
We begin with the loop in Algorithm A of Figure 4-1, which appears in pseudocode as follows:

```
for i = 1 to n
   sum = sum + i
```

The body of this loop requires a constant amount of execution time, and so it is $O(1)$. Figure 4-6 represents that time with one icon, and so a row of n icons represents the loop's total execution time. This algorithm is $O(n)$: Its time requirement grows as n grows.

FIGURE 4-6 An $O(n)$ algorithm

```
for i = 1 to n
   sum = sum + i
```



4.19 Algorithm B in Figure 4-1 contains nested loops, as follows:

```
for i = 1 to n
{
   for j = 1 to i
      sum = sum + 1
}
```

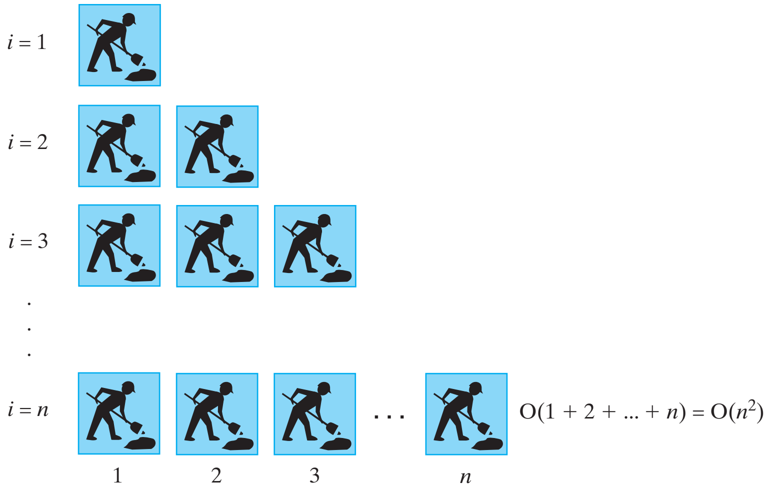
When loops are nested, you examine the innermost loop first. Here, the body of the inner loop requires a constant amount of execution time, and so it is O(1). If we again represent that time with an icon, a row of *i* icons represents the time requirement for the inner loop. Since the inner loop is the body of the outer loop, it executes *n* times. Figure 4-7 illustrates the time requirement for these nested loops, which is proportional to $1 + 2 + . . . + n$. Question 1 asked you to show that

$$1 + 2 + . . . + n = n\,(n + 1) / 2$$

which is $n^2 / 2 + n / 2$. Thus, the computation is $O(n^2)$.

FIGURE 4-7 An $O(n^2)$ algorithm

```
for i = 1 to n
{  for j = 1 to i
      sum = sum + 1
}
```



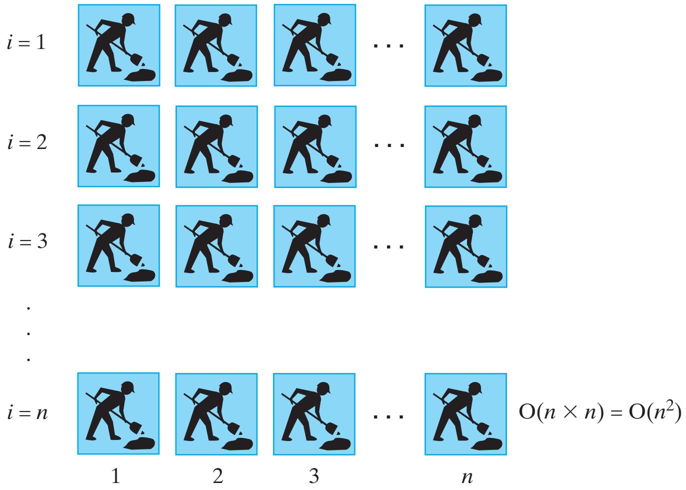
4.20 The body of the inner loop in the previous segment executes a variable number of times that depends on the outer loop. Suppose we change the inner loop so that it executes the same number of times for each repetition of the outer loop, as follows:

```
for i = 1 to n
{
   for j = 1 to n
      sum = sum + 1
}
```

Figure 4-8 illustrates these nested loops and shows that the computation is $O(n^2)$.

FIGURE 4-8 Another O(n^2) algorithm

```
for i = 1 to n
{  for j = 1 to n
        sum = sum + 1
}
```



Question 5 Using Big Oh notation, what is the order of the following computation's time requirement?

```
for i = 1 to n
{
   for j = 1 to 5
      sum = sum + 1
}
```

4.21 Let's get a feel for the growth-rate functions in Figure 4-4. As we mentioned, the time requirement for an O(1) algorithm is independent of the problem size *n*. We can apply such an algorithm to larger and larger problems without affecting the execution time. This situation is ideal, but not typical.

For other orders, what happens if we double the problem size? The time requirement for an O(log *n*) algorithm will change, but not by much. An O(*n*) algorithm will need twice the time, an O(n^2) algorithm will need four times the time, and an O(n^3) algorithm will need eight times the time. Doubling the problem size for an O(2^n) algorithm squares the time requirement. Figure 4-9 tabulates these observations.

Question 6 Suppose that you can solve a problem of a certain size on a given computer in time *t* by using an O(*n*) algorithm. If you double the size of the problem, how fast must your computer be to solve the problem in the same time?

Question 7 Repeat the previous question, but instead use an O(n^2) algorithm.

FIGURE 4-9 The effect of doubling the problem size on an algorithm's time requirement

Growth-Rate Function for Size n Problems	Growth-Rate Function for Size $2n$ Problems	Effect on Time Requirement
1	1	None
$\log n$	$1 + \log n$	Negligible
n	$2n$	Doubles
$n \log n$	$2n \log n + 2n$	Doubles and then adds $2n$
n^2	$(2n)^2$	Quadruples
n^3	$(2n)^3$	Multiplies by 8
2^n	2^{2n}	Squares

4.22 Now suppose that your computer can perform one million operations per second. How long will it take an algorithm to solve a problem whose size is one million? We cannot answer this question exactly without knowing the algorithm, but the computations in Figure 4-10 will give you a sense of how the algorithm's Big Oh would affect our answer. An O($\log n$) algorithm would take a fraction of a second, whereas an O(2^n) algorithm would take many trillions of years! Note that these computations estimate the time requirement of an O($g(n)$) algorithm as $g(n)$. Although this approximation is not universally valid, for many algorithms it is reasonable.

FIGURE 4-10 The time required to process one million items by algorithms of various orders at the rate of one million operations per second

Growth-Rate Function g	$g(10^6) / 10^6$
$\log n$	0.0000199 seconds
n	1 second
$n \log n$	19.9 seconds
n^2	11.6 days
n^3	31,709.8 years
2^n	$10^{301,016}$ years

Note: You can use O(n^2), O(n^3), or even O(2^n) algorithms as long as your problem size is small. For example, at the rate of one million operations per second, an O(n^2) algorithm would take one second to solve a problem whose size is 1000. An O(n^3) algorithm would take one second to solve a problem whose size is 100. And an O(2^n) algorithm would take about one second to solve a problem whose size is 20.

SELF-TEST

Question 8 The following algorithm discovers whether an array contains duplicate entries within its first *n* elements. What is the Big Oh of this algorithm in the worst case?

```
Algorithm hasDuplicates(array, n)
for index = 0 to n - 2
   for rest = index + 1 to n - 1
      if (array[index] equals array[rest])
         return true
return false
```

The Efficiency of Implementations of the ADT Bag

VideoNote
Comparing ADT bag implementations

We now consider the time efficiency of two of the implementations of the ADT bag that we discussed in previous chapters.

An Array-Based Implementation

One of the implementations of the ADT bag given in Chapter 2 used a fixed-size array to represent the bag's entries. We can now assess the efficiency of the bag operations when implemented in this way.

4.23 **Adding an entry to a bag.** Let's begin with the operation that adds a new entry to a bag. Segment 2.10 in Chapter 2 provided the following implementation for this operation:

```
public boolean add(T newEntry)
{
   boolean result = true;
   if (isFull())
   {
      result = false;
   }
   else
   {  // assertion: result is true here
      bag[numberOfEntries] = newEntry;
      numberOfEntries++;
   } // end if

   return result;
} // end add
```

Each step in this method—detecting whether the bag is full, assigning a new entry to an array element, and incrementing the length—is an O(1) operation. It follows then that this method is O(1). Intuitively, since the method adds the new entry right after the last entry in the array, we know the index of the array element that will contain the new entry. Thus, we can make this assignment independently of any other entries in the bag.

4.24 **Searching a bag for a given entry.** The ADT bag has a method `contains` that detects whether a bag contains a given entry. The array-based implementation of the method, as given in Segment 2.29 of Chapter 2, is:

```
public boolean contains(T anEntry)
{
   return getIndexOf(anEntry) > -1;
} // end contains
```

By calling the private method `getIndexOf`, the method locates the first array element, if any, that contains the entry we seek. Let's examine `getIndexOf`, as described in Segment 2.28 of Chapter 2:

```
private int getIndexOf(T anEntry)
{
   int where = -1;
   boolean found = false;

   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(bag[index]))
      {
         found = true;
         where = index;
      } // end if
   } // end for
   return where;
} // end getIndexOf
```

This method searches the array `bag` for the given entry `anEntry`. The basic operation for the method is comparison. As we described earlier in Segment 4.10, the method would make one comparison in the best case and *n* comparisons in the worst case, assuming that the bag contains *n* entries. Typically, the method would make about *n* / 2 comparisons. We can conclude that the method `contains` is O(1) in the best case and O(*n*) in both the worst and average cases.

Note: To simplify our example, we have considered a fixed-size array. Typically, an array-based bag resizes the array as needed. Doubling the size of an array is an O(*n*) operation. As Segment 2.35 of Chapter 2 noted, the next *n* additions would share the cost of this doubling.

Question 9 What is the Big Oh of the bag's `remove` methods? Assume that a fixed-size array represents the bag, and use an argument similar to the one we just made for `contains`.

Question 10 Repeat Question 9, but instead analyze the method `getFrequencyOf`.

Question 11 Repeat Question 9, but instead analyze the method `toArray`.

A Linked Implementation

4.25 **Adding an entry to a bag.** Now consider a linked implementation of the ADT bag as given in Chapter 3. Let's begin with Segment 3.12 and the method `add` that adds an entry to a bag:

```
public boolean add(T newEntry) // OutOfMemoryError possible
{
   // add to beginning of chain:
   Node newNode = new Node(newEntry);
   newNode.next = firstNode;    // make new node reference rest of chain
                                // (firstNode is null if chain is empty)
   firstNode = newNode;         // new node is at beginning of chain
   numberOfEntries++;

   return true;
} // end add
```

All of the statements in this method represent O(1) operations, and so the method is O(1).

4.26 **Searching a bag for a given entry.** The method contains, given in Segment 3.17 of Chapter 3, searches a chain of nodes for a given entry:

```
public boolean contains(T anEntry)
{
   boolean found = false;
   Node currentNode = firstNode;

   while (!found && (currentNode != null))
   {
      if (anEntry.equals(currentNode.data))
         found = true;
      else
         currentNode = currentNode.next;
   } // end while

   return found;
} // end contains
```

The best case occurs when the desired entry is in the first node of the chain of nodes. Since the method has a reference to the chain's first node, no traversal is needed. Thus, the method is O(1) in this case.

In the worst case, the traversal of the chain continues to the last node. The operation is O(*n*) in this case. Finally, in the typical, or average, case, the traversal would examine *n* / 2 nodes, making it an O(*n*) operation.

Note: Searching a bag that has a linked implementation
Searching for an item that is at the beginning of a chain of nodes is an O(1) operation. It takes the least time of any search of the chain, making this case the best case. If the item is in the last node of the chain, searching for it is O(*n*). This search takes the most time among the searches for an item that is in one of the nodes, and so this is the worst case. The actual time required to find an entry in a chain of nodes depends on which node contains it.

SELF-TEST

Question 12 What is the Big Oh of the method contains when it searches for an entry that is not in the bag? Assume that a chain of linked nodes represents the bag.

Question 13 What is the Big Oh of the bag's remove methods? Assume that a chain of linked nodes represents the bag, and use an argument similar to the one you just made for contains.

Question 14 Repeat Question 13, but instead analyze the method getFrequencyOf.

Question 15 Repeat Question 13, but instead analyze the method toArray.

Comparing the Implementations

4.27 Using Big Oh notation, Figure 4-11 summarizes the time complexities of the operations of the ADT bag for the implementations that use a fixed-size array and a chain of linked nodes. For some operations, multiple time requirements indicate the best, worst, and average cases.

FIGURE 4-11 The time efficiencies of the ADT bag operations for two implementations, expressed in Big Oh notation

Operation	Fixed-Size Array	Linked
`add(newEntry)`	O(1)	O(1)
`remove()`	O(1)	O(1)
`remove(anEntry)`	O(1), O(n), O(n)	O(1), O(n), O(n)
`clear()`	O(n)	O(n)
`getFrequencyOf(anEntry)`	O(n)	O(n)
`contains(anEntry)`	O(1), O(n), O(n)	O(1), O(n), O(n)
`toArray()`	O(n)	O(n)
`getCurrentSize(), isEmpty(), isFull()`	O(1)	O(1)

As you can see, all of the operations have the same Big Oh for both implementations. This phenomenon is unusual, but it reflects the simplicity of the ADT bag. Subsequent ADTs will have at least some operations whose time efficiencies differ according to their implementations.

Chapter Summary

- An algorithm's complexity is described in terms of the time and space required to execute it.
- An algorithm's time requirement $f(n)$ is of order at most $g(n)$—that is, $f(n)$ is $O(g(n))$——if positive constants c and N exist such that $f(n) \leq c$ x $g(n)$ for all $n \geq N$.
- The relationships among typical growth-rate functions are as follows:
  $1 < \log(\log n) < \log n < \log^2 n < n < n \log n < n^2 < n^3 < 2^n < n!$
- The time complexity of an ADT bag operation is the same for the fixed-size array implementation and the linked implementation. This situation is atypical of ADTs but reflects the details of the implementations that are possible due to the nature of a bag.

Exercises

1. Using Big Oh notation, indicate the time requirement of each of the following tasks in the worst case. Describe any assumptions that you make.

   a. After arriving at a party, you shake hands with each person there.
   b. Each person in a room shakes hands with everyone else in the room.
   c. You climb a flight of stairs.
   d. You slide down the banister.
   e. After entering an elevator, you press a button to choose a floor.
   f. You ride the elevator from the ground floor up to the n^{th} floor.
   g. You read a book twice.

2. Describe a way to climb from the bottom of a flight of stairs to the top in time that is no better than O(n^2).

3. Using Big Oh notation, indicate the time requirement of each of the following tasks in the worst case.

a. Display all the integers in an array of integers.
b. Display all the integers in a chain of linked nodes.
c. Display the n^{th} integer in an array of integers.
d. Compute the sum of the first n even integers in an array of integers.

4. By using the definition of Big Oh, show that

a. $6n^2 + 3$ is $O(n^2)$
b. $n^2 + 17n + 1$ is $O(n^2)$
c. $5n^3 + 100\ n^2 - n - 10$ is $O(n^3)$
d. $3n^2 + 2^n$ is $O(2^n)$

5. Algorithm X requires $n^2 + 9n + 5$ operations, and Algorithm Y requires $5n^2$ operations. What can you conclude about the time requirements for these algorithms when n is small and when n is large? Which is the faster algorithm in these two cases?

6. Show that $O(\log_a n) = O(\log_b n)$ for $a, b > 1$. *Hint*: $\log_a n = \log_b n / \log_b a$.

7. If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$, use the definition of Big Oh to show that $f(n)$ is $O(h(n))$.

8. Segment 4.9 and the chapter summary showed the relationships among typical growth-rate functions. Indicate where the following growth-rate functions belong in this ordering:

a. $n^2 \log n$
b. $\sqrt{\mathrm{n}}$
c. $n^2 / \log n$
d. 3^n

9. Show that $7n^2 + 5n$ is not $O(n)$.

10. What is the Big Oh of the following computation?

```
int sum = 0;
for (int counter = n; counter > 0; counter = counter - 2)
   sum = sum + counter;
```

11. What is the Big Oh of the following computation?

```
int sum = 0;
for (int counter = 0; counter < n; counter = 2 * counter)
   sum = sum + counter;
```

12. Suppose that your implementation of a particular algorithm appears in Java as follows:

```
for (int pass = 1; pass <= n; pass++)
{
   for (int index = 0; index < n; index++)
   {
      for (int count = 1; count < 10; count++)
      {
         . . .
      } // end for
   } // end for
} // end for
```

The algorithm involves an array of n items. The previous code shows the only repetition in the algorithm, but it does not show the computations that occur within the loops. These computations, however, are independent of n. What is the order of the algorithm?

13. Repeat the previous exercise, but replace 10 with *n* in the inner loop.

14. What is the Big Oh of `method1`? Is there a best case and a worst case?

```
public static void method1(int[] array, int n)
{
   for (int index = 0; index < n - 1; index++)
   {
      int mark = privateMethod1(array, index, n - 1);
      int temp = array[index];
      array[index] = array[mark];
      array[mark] = temp;
   } // end for
} // end method1

public static int privateMethod1(int[] array, int first, int last)
{
   int min = array[first];
   int indexOfMin = first;
   for (int index = first + 1; index <= last; index++)
   {
      if (array[index] < min)
      {
         min = array[index];
         indexOfMin = index;
      } // end if
   } // end for

   return indexOfMin;
} // end privateMethod1
```

15. What is the Big Oh of `method2`? Is there a best case and a worst case?

```
public static void method2(int[] array, int n)
{
   for (int index = 1; index <= n - 1; index++)
      privateMethod2(array[index], array, 0, index - 1);
} // end method2

public static void privateMethod2(int entry, int[] array, int begin, int end)
{
   int index;
   for (index = end; (index >= begin) && (entry < array[index]); index--)
      array[index + 1] = array[index];

   array[index + 1] = entry;
} // end privateMethod2
```

16. Consider two programs, A and B. Program A requires 1000 x n^2 operations and Program B requires $2n$ operations. For which values of *n* will Program A execute faster than Program B?

17. Consider four programs—A, B, C, and D—that have the following performances:

A: O(log *n*)

B: O(*n*)

C: O(n^2)

D: O(2^n)

If each program requires 10 seconds to solve a problem of size 1000, estimate the time required by each program for a problem of size 2000.

18. Suppose that you have a dictionary whose words are not sorted in alphabetical order. As a function of the number, *n*, of words, what is the time complexity of searching for a particular word in this dictionary?

19. Repeat the previous exercise for a dictionary whose words are sorted alphabetically. Compare your results with those for the previous exercise.

20. Consider a football player who runs wind sprints on a football field. He begins at the 0-yard line and runs to the 1-yard line and then back to the 0-yard line. Then he runs to the 2-yard line and back to the 0-yard line, runs to the 3-yard line and back to the 0-yard line, and so on until he has reached the 10-yard line and returned to the 0-yard line.

a. How many total yards does he run?
b. How many total yards does he run if he reaches the *n*-yard line instead of the 10-yard line?
c. How does his total distance run compare to that of a sprinter who simply starts at the 0-yard line and races to the *n*-yard line?

21. Consider the following definition of a sequence *A* of positive integers:

$$A_{i+1} = \begin{cases} A_i / 2 \text{ if } A_i \text{ is even} \\ 3A_i - 1 \text{ if } A_i \text{ is odd} \end{cases}$$

If A_0 has some value *v*, give a Big Oh expression for the

a. Minimum value
b. Maximum value

that A_k can have in terms of *k* and *v*.

22. Chapter 2 describes an implementation of the ADT bag that uses a fixed-size array. Of the following operations, which ones have a constant growth-rate function: `add`, `remove`, `contains`?

23. Chapter 2 describes an implementation of the ADT bag that uses an array that can be resized. Using Big Oh notation, derive the time complexity of the method `ensureCapacity`, as given in Segment 2.38.

24. Consider an array of length *n* containing unique integers in random order and in the range 1 to *n* + 1. For example, an array of length 5 would contain 5 unique integers selected randomly from the integers 1 through 6. Thus, the array might contain 3 6 5 1 4. Of the integers 1 through 6, notice that the 2 was not chosen and is not in the array.

Write Java code that finds the integer that does not appear in such an array. Your solution should use

a. O(n^2) operations
b. O(n) operations

25. Consider an array of length *n* containing positive and negative integers in random order. Write Java code that rearranges the integers so that the negative integers appear before the positive integers. Your solution should use

a. O(n^2) operations
b. O(n) operations

PROJECTS

For the following projects, you should know how to time a section of code in Java. One approach is to use the class `java.util.Date`. A `Date` object contains the time at which it was constructed. This time is stored as a `long` integer equal to the number of milliseconds that have passed since 00:00:00.000 GMT on January 1, 1970. By subtracting the starting time in milliseconds from the ending time in milliseconds, you get the run time—in milliseconds—of a section of code.

For example, suppose that `thisMethod` is the name of a method you wish to time. The following statements will compute the number of milliseconds that `thisMethod` requires to execute:

```
Date current = new Date();                // get current time
long startTime = current.getTime();
thisMethod();                             // code to be timed
current = new Date();                     // get current time
long stopTime = current.getTime();
long elapsedTime = stopTime - startTime; // milliseconds
```

1. Write a Java program that implements the three algorithms in Figure 4-1 and times them for various values of *n*. The program should display a table of the run times of each algorithm for various values of *n*.

2. Consider the following two loops:

```
// Loop A
for (i = 1; i <= n; i++)
   for (j = 1; j <= 10000; j++)
      sum = sum + j;

// Loop B
for (i = 1; i <= n; i++)
   for (j = 1; j <= n; j++)
      sum = sum + j;
```

Although Loop A is O(n) and Loop B is O(n^2), Loop B can be faster than Loop A for small values of *n*. Design and implement an experiment to find a value of *n* for which Loop B is faster.

3. Repeat the previous project, but use the following for Loop B:

```
// Loop B
for (i = 1; i <= n; i++)
   for (j = 1; j <= n; j++)
      for (k = 1; k <= n; k++)
         sum = sum + k;
```

4. Segment 2.12 of Chapter 2 gave the definition of the method `toArray` for the ADT bag, as follows:

```
public T[] toArray()
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] result = (T[])new Object[numberOfEntries]; // unchecked cast
   for (int index = 0; index < numberOfEntries; index++)
   {
      result[index] = bag[index];
   } // end for

   return result;
} // end toArray
```

An alternate definition calls the method `Arrays.copyOf` and appears as follows:

```
public T[] toArray()
{
   return Arrays.copyOf(bag, bag.length);
} // end toArray
```

Compare the execution times of these two methods for bags of various sizes.

5. Suppose that you have several numbered billiard balls on a pool table. At each step you remove a billiard ball from the table. If the ball removed is numbered n, you replace it with n balls whose number is $n / 2$, where the division is truncated to an integer. For example, if you remove the 5 ball, you replace it with five 2 balls.

   Write a program that simulates this process. Use a bag of positive integers to represent the balls on the pool table.

   Using Big Oh notation, predict the time requirement for this algorithm when the initial bag contains only the value n. Then time the actual execution of the program for various values of n and plot its performance as a function of n.

6. Repeat the previous project, but instead replace the n ball with n balls randomly numbered less than n.

7. In mythology, the Hydra was a monster with many heads. Every time the hero chopped off a head, two smaller heads would grow in its place. Fortunately for the hero, if the head was small enough, he could chop it off without two more growing in its place. To kill the Hydra, all our hero needed to do was to chop off all the heads.

   Write a program that simulates the Hydra. Instead of heads, we will use strings. A bag of strings, then, represents the Hydra. Every time you remove a string from the bag, delete the first letter of the string and put two copies of the remaining string back into the bag. For example, if you remove *HYDRA*, you add two copies of *YDRA* to the bag. If you remove a one-letter word, you add nothing to the bag. To begin, read one word from the keyboard and place it into an empty bag. The Hydra dies when the bag becomes empty.

   Using Big Oh notation, predict the time requirement for this algorithm in terms of the number n of characters in the initial string. Then time the actual execution of the program for various values of n and plot its performance as a function of n.

Answers to Self-Test Questions

1. If you follow the hint given in the question, you will get the sum of n occurrences of $n + 1$, which is $(n + 1) + (n + 1) + \ldots + (n + 1)$. This sum is simply the product $n (n + 1)$. To get this sum, we added $1 + 2 + \ldots + n$ to itself. Thus, $n(n + 1)$ is $2 (1 + 2 + \ldots + n)$. The desired conclusion follows immediately from this fact.

2. Algorithm A: The loop iterates n times, so there are n additions and a total of $n + 1$ assignments. We ignore the assignments.
   Algorithm B: For each value of i, the inner loop iterates i times, and so performs i additions and i assignments. The outer loop iterates n times. Together, the loops perform $1 + 2 + \ldots + n$ additions and the same number of assignments. Using the identity given in Question 1, the number of additions is $n (n + 1) / 2$. The additional assignment to set `sum` to zero makes the total number of assignments equal to $1 + n (n + 1) / 2$, which we ignore.

3. $3n^2 + 2^n < 2^n + 2^n = 2 \times 2^n$ when $n \geq 8$. So $3n^2 + 2^n = O(2^n)$, using $c = 2$ and $N = 8$.

4. n^k.

5. The inner loop requires a constant amount of time, and so it is O(1). The outer loop is O(n), and so the entire computation is O(n).

6. Twice as fast.

7. Four times as fast.

8. Let's tabulate the maximum number of times the inner loop executes for various values of `index`:

`index`	**Inner Loop Iterations**
0	$n - 1$
1	$n - 2$
2	$n - 3$
. . .	. . .
$n - 2$	1

As you can see, the maximum number of times the inner loop executes is $1 + 2 + \ldots + n - 1$, which is $n (n - 1) / 2$. Thus, the algorithm is O(n^2) in the worst case.

9. Removing an unspecified entry is O(1). Removing a particular entry is O(1) in the best case and O(*n*) in the worst and average cases.
10. O(*n*).
11. O(*n*).
12. O(*n*).
13. Removing an unspecified entry is O(1). Removing a particular entry is O(1) in the best case and O(*n*) in the worst and average cases.
14. O(*n*).
15. O(*n*).

This page intentionally left blank

Chapter 5

Stacks

Contents

Prerequisites

Appendix	D	Designing Classes
Chapter	1	Bags

Objectives

After studying this chapter, you should be able to

- Describe the operations of the ADT stack
- Use a stack to decide whether the delimiters in an algebraic expression are paired correctly
- Use a stack to convert an infix expression to a postfix expression
- Use a stack to evaluate a postfix expression
- Use a stack to evaluate an infix expression
- Use a stack in a program
- Describe how the Java run-time environment uses a stack to track the execution of methods

In everyday life, a stack is a familiar thing. You might see a stack of books on your desk, a stack of dishes in the cafeteria, a stack of towels in the linen closet, or a stack of boxes in the attic. When you add an item to a stack, you place it on top of the stack. When you remove an item, you take the topmost one. This topmost item is the last one that was added to the stack. So when you remove an item, you remove the item added most recently. That is, the last item added to the stack is the first one removed.

VideoNote
The ADT stack

In spite of our examples of a stack, everyday life usually does not follow this **last-in**, **first-out**, or **LIFO**, behavior. Although the employee hired most recently is often the first one fired during a layoff, we live in a first-come, first-served society. In the computer science world, however, last-in, first-out is exactly the behavior required by many important algorithms. These algorithms often use the abstract data type stack, which is an ADT that exhibits a last-in, first-out behavior. For example, a compiler uses a stack to interpret the meaning of an algebraic expression, and a run-time environment uses a stack when executing a recursive method.

This chapter describes the ADT stack and provides several examples of its use.

Specifications of the ADT Stack

5.1 The ADT **stack** organizes its entries according to the order in which they were added. All additions are to one end of the stack called the **top**. The **top entry**—that is, the entry at the top—is thus the newest item among the items currently in a stack. Figure 5-1 shows some stacks that should be familiar to you.

FIGURE 5-1 Some familiar stacks

Note: Among the items currently in a stack, the one added most recently is at the top of the stack. (Other items might have been added to the stack more recently and then removed.)

The stack restricts access to its entries. A client can look at or remove only the top entry. The only way to look at an entry that is not at the top of the stack is to repeatedly remove items from the stack until the desired item reaches the top. If you were to remove all of a stack's entries, one by one, you would get them in reverse chronological order, beginning with the most recent and ending with the first item added to the stack.

5.2 The operation that adds an entry to a stack is traditionally called `push`. The remove operation is `pop`. The operation that retrieves the top entry without removing it is named `peek`. Typically, you cannot search a stack[1] for a particular entry. The following specifications define a set of operations for the ADT stack.

[1] However, the Java Class Library has a class of stacks that does define a search method, as you will see later in this chapter.

Abstract Data Type: Stack

Date

- A collection of objects in reverse chronological order and having the same data type

Operations

Pseudocode	UML	Description
`push(newEntry)`	`+push(newEntry: T): void`	Task: Adds a new entry to the top of the stack. Input: `newEntry` is the new entry. Output: None.
`pop()`	`+pop(): T`	Task: Removes and returns the stack's top entry. Input: None. Output: Returns either the stack's top entry or, if the stack is empty before the operation, `null`.
`peek()`	`+peek(): T`	Task: Retrieves the stack's top entry without changing the stack in any way. Input: None. Output: Returns either the stack's top entry or, if the stack is empty, `null`.
`isEmpty()`	`+isEmpty(): boolean`	Task: Detects whether the stack is empty. Input: None. Output: Returns true if the stack is empty.
`clear()`	`+clear(): void`	Task: Removes all entries from the stack. Input: None. Output: None.

Note: Alternate names for methods
It is not unusual for a class designer to include aliases for certain methods. For example, you could include the additional methods `add` and `remove` (or `insert` and `delete`) in the ADT stack to mean `push` and `pop`. Moreover, `pull` is sometimes used to mean `pop`, and `getTop` can mean `peek`, so including them as aliases is reasonable.

5.3 The Java interface in Listing 5-1 specifies a stack of objects. The generic type T—which represents any class type—is the data type of the items in the stack.

LISTING 5-1 An interface for the ADT stack

```
public interface StackInterface<T>
{
   /** Adds a new entry to the top of this stack.
       @param newEntry  an object to be added to the stack */
   public void push(T newEntry);

   /** Removes and returns this stack's top entry.
       @return either the object at the top of the stack or, if the
               stack is empty before the operation, null */
   public T pop();

   /** Retrieves this stack's top entry.
       @return either the object at the top of the stack or null if
               the stack is empty */
   public T peek();

   /** Detects whether this stack is empty.
       @return true if the stack is empty */
   public boolean isEmpty();

   /** Removes all entries from this stack */
   public void clear();
} // end StackInterface
```

5.4 **Example: Demonstrating the stack methods.** The following statements add, retrieve, and remove strings from a stack. We assume that the class OurStack implements StackInterface and is available for our use.

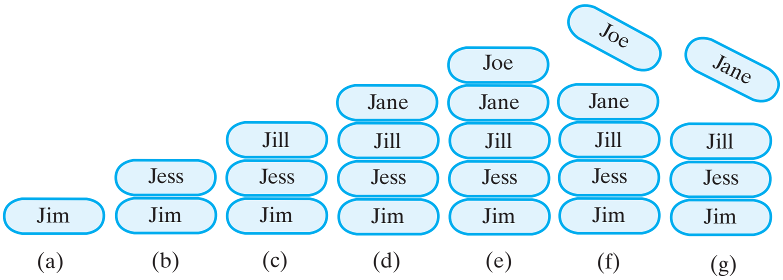
```
StackInterface<String> stringStack = new OurStack<String>();
stringStack.push("Jim");
stringStack.push("Jess");
stringStack.push("Jill");
stringStack.push("Jane");
stringStack.push("Joe");

String top = stringStack.peek(); // returns "Joe"
System.out.println(top + " is at the top of the stack.");

top = stringStack.pop();         // removes and returns "Joe"
System.out.println(top + " is removed from the stack.");

top = stringStack.peek();        // returns "Jane"
System.out.println(top + " is at the top of the stack.");
```

```
top = stringStack.pop();          // removes and returns "Jane"
System.out.println(top + " is removed from the stack.");
```

Parts *a* through *e* of Figure 5-2 show five additions to the stack. At this point, the stack contains—from top to bottom—the strings *Joe*, *Jane*, *Jill*, *Jess*, and *Jim*. The string at the top of the stack is *Joe*; `peek` retrieves it. The method `pop` retrieves *Joe* again and then removes it (Figure 5-2f). A subsequent call to `peek` retrieves *Jane*. Then `pop` retrieves *Jane* and removes it (Figure 5-2g).

Three more calls to `pop` would remove *Jill*, *Jess*, and *Jim*, leaving the stack empty. A subsequent call to either `pop` or `peek` would return `null`.

FIGURE 5-2 A stack of strings after (a) `push` adds *Jim*; (b) `push` adds *Jess*; (c) `push` adds *Jill*; (d) `push` adds *Jane*; (e) `push` adds *Joe*; (f) `pop` retrieves and removes *Joe*; (g) `pop` retrieves and removes *Jane*



Programming Tip: Methods such as `peek` and `pop` must behave reasonably when the stack is empty. Here, we specify that they return `null`. Another possibility is to have them throw an exception.

SELF-TEST

Question 1 After the following statements execute, what string is at the top of the stack and what string is at the bottom?

```
StackInterface<String> stringStack = new OurStack<String>();
stringStack.push("Jim");
stringStack.push("Jess");
stringStack.pop();
stringStack.push("Jill");
stringStack.push("Jane");
stringStack.pop();
```

Question 2 Consider the stack that was created in Question 1, and define a new empty stack `nameStack`.

a. Write a loop that pops the strings from `stringStack` and pushes them onto `nameStack`.

b. Describe the contents of the stacks `stringStack` and `nameStack` when the loop that you just wrote completes its execution.

Using a Stack to Process Algebraic Expressions

5.5 In mathematics, an algebraic expression is composed of operands that are variables or constants and operators, such as + and *. We will use the Java notation +, -, *, and / to indicate addition, subtraction, multiplication, and division. We will use ^ to indicate exponentiation, with the warning that Java has no operator for exponentiation; in Java ^ is the exclusive-or operator.

Operators generally have two operands, and so are called **binary operators**. For example, the + in $a + b$ is a binary operator. The operators + and - can also be **unary operators** when they have one operand. For example, the minus sign in -5 is a unary operator.

When an algebraic expression has no parentheses, operations occur in a certain order. Exponentiations occur first; they take **precedence** over the other operations. Next, multiplications and divisions occur, and then additions and subtractions. For example, the expression

20 - 2 * 2 ^ 3

evaluates as 20 - 2 * 8, then as 20 - 16, and finally as 4.

But what happens when two or more adjacent operators have the same precedence? Exponentiations, such as those in a ^ b ^ c, occur right to left. Thus, 2 ^ 2 ^ 3 means 2 ^ (2 ^ 3), or 2^8, instead of (2 ^ 2) ^ 3, which is 4^3. Other operations occur left to right, such as the multiplication and division in $a * b / c$ or the addition and subtraction in $a - b + c$. Therefore, 8 - 4 + 2 means (8 - 4) + 2, or 6, instead of 8 - (4 + 2), which is 2. Parentheses in an expression override the normal operator precedence.

Ordinarily, we place a binary operator between its operands, as in $a + b$. An expression in this familiar notation is called an **infix expression**. Other notations are possible. For example, you could write a binary operator before its two operands. Thus, $a + b$ becomes + a b. This expression is called a **prefix expression**. Or you could write a binary operator after its two operands, so $a + b$ becomes a b +. This expression is a **postfix expression**. Although infix expressions are more familiar to us, both prefix and postfix expressions are simpler to process because they do not use precedence rules or parentheses. The precedence of an operator in either a prefix expression or a postfix expression is implied by the order in which the operators and operands occur in the expression. We will learn more about these types of expressions later in this chapter.

Our first example looks at ordinary infix expressions.

Note: Algebraic expressions
In an infix expression, each binary operator appears between its operands, as in $a + b$.
In a prefix expression, each binary operator appears before its operands, as in + a b.
In a postfix expression, each binary operator appears after its operands, as in a b +.

Note: The notation in a prefix expression is sometimes called **Polish notation**, because it was invented by the Polish mathematician Jan Lukasiewicz in the 1920s. The notation in a postfix expression is sometimes called **reverse Polish notation**.

A Problem Solved: Checking for Balanced Delimiters in an Infix Algebraic Expression

Although programmers use parentheses when writing arithmetic expressions in Java, mathematicians use parentheses, square brackets, and braces for the same purpose. These delimiters must be paired correctly. For example, an open parenthesis must correspond to a close parenthesis. In addition, pairs of delimiters must not intersect. Thus, an expression can contain a sequence of delimiters such as

{ [( ) ( )] ( ) }

but not

[( ])

For convenience, we will say that a **balanced expression** contains delimiters that are paired correctly, or are **balanced**.

We want an algorithm that detects whether an infix expression is balanced.

5.6 **Example: A balanced expression.** Let's see whether the expression

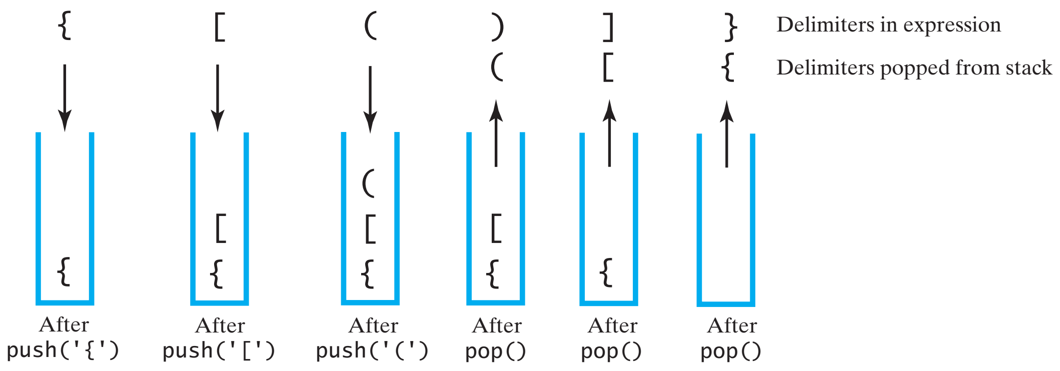
$$a\ \{b\ [c\ (d\ + e)/2 - f\,] + 1\}$$

is balanced. We scan the expression from left to right, looking for delimiters and ignoring any characters that are not delimiters. When we encounter an open delimiter, we must save it. When we find a close delimiter, we must see whether it corresponds to the most recently encountered open delimiter. If it does, we discard the open delimiter and continue scanning the expression. If we are able to scan the entire expression without a mismatch, the delimiters in the expression are balanced.

The ADT that enables us to store objects and then retrieve or remove the most recent one is a stack. Figure 5-3 shows the contents of a stack as we scan the previous expression. Since we ignore all characters that are not delimiters, it is sufficient for us to represent the expression here as

{ [( )] }

FIGURE 5-3 The contents of a stack during the scan of an expression that contains the balanced delimiters { [( )] }



After pushing the first three open delimiters onto the stack, the open parenthesis is at the top of the stack. The next delimiter, the close parenthesis, pairs with the open parenthesis at the top of the stack. We pop the stack and continue by comparing the close bracket with the delimiter now at the top of the stack. They correspond, so we pop the stack again and continue by comparing the close brace with the top entry of the stack. These delimiters correspond, so we pop the stack. We have reached the end of the expression, and the stack is empty. Each open delimiter correctly corresponds to a close delimiter, so the delimiters are balanced.

5.7 **Examples: Unbalanced expressions.** Let's examine some expressions that contain unbalanced delimiters. Figure 5-4 shows a stack during the scan of an expression that contains the delimiters { [( ]) }. This is an example of intersecting pairs of delimiters. After we push the first three open delimiters onto the stack, the open parenthesis at the top of the stack does not correspond to the close bracket that comes next in the expression.

FIGURE 5-4 The contents of a stack during the scan of an expression that contains the unbalanced delimiters { [( ]) }

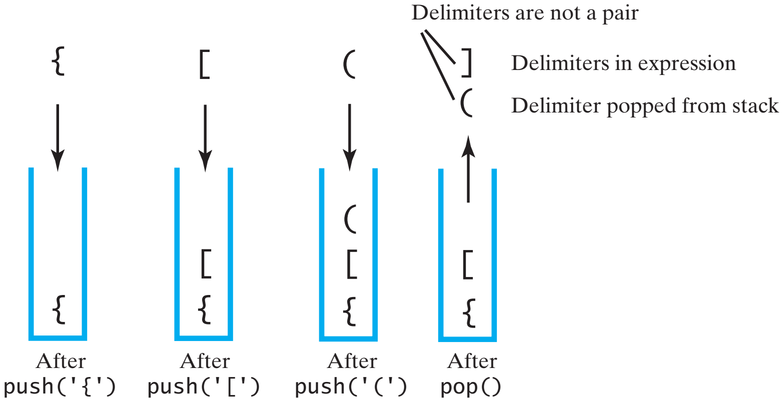


Figure 5-5 shows a stack during the scan of an expression that contains the unbalanced delimiters [( )] }. The close brace does not have a corresponding open brace. When we finally reach the close brace, the stack is empty. Since the stack does not contain an open brace, the delimiters are unbalanced.

FIGURE 5-5 The contents of a stack during the scan of an expression that contains the unbalanced delimiters [( )] }

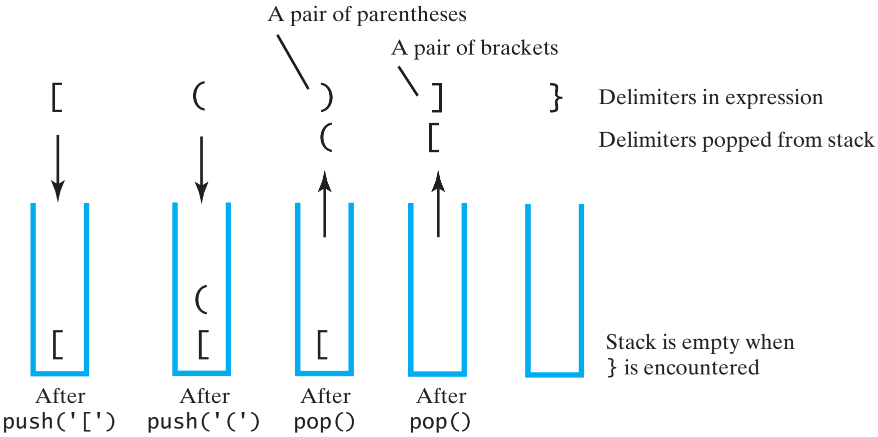
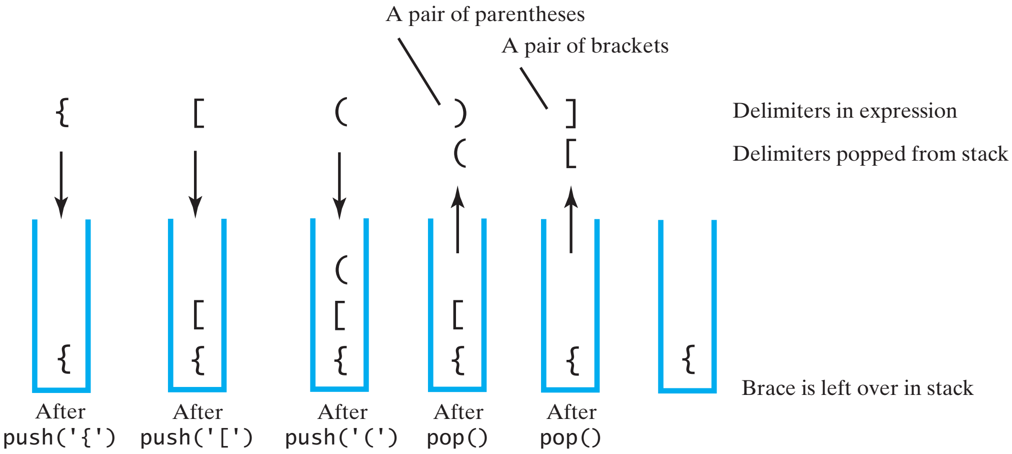


Figure 5-6 shows a stack during the scan of an expression that contains the unbalanced delimiters { [( )]. The open brace does not have a corresponding close brace. When you reach the end of the expression, having processed the brackets and parentheses, the stack still contains the open brace. Since this delimiter is left over, the expression contains unbalanced delimiters.

FIGURE 5-6 The contents of a stack during the scan of an expression that contains the unbalanced delimiters { [( )]



5.8 **The algorithm.** The previous discussion and figures reveal the paths that our algorithm must take. We formalize these observations in the following pseudocode:

```
Algorithm checkBalance(expression)
// Returns true if the parentheses, brackets, and braces in an expression are paired correctly.

isBalanced = true
while ((isBalanced == true) and not at end of expression)
{
   nextCharacter = next character in expression
   switch (nextCharacter)
   {
      case '(': case '[': case '{':
         Push nextCharacter onto stack
         break

      case ')': case ']': case '}':
         if (stack is empty)
            isBalanced = false
         else
         {
            openDelimiter = top entry of stack
            Pop stack
            isBalanced = true or false according to whether openDelimiter and
                         nextCharacter are a pair of delimiters
         }
         break
   }
}

if (stack is not empty)
   isBalanced = false
return isBalanced
```

5.9 Let's examine this algorithm for each of the examples given in the previous figures. For the balanced expression in Figure 5-3, the `while` loop ends with an empty stack and `isBalanced` set to `true`. For the expression in Figure 5-4, the loop ends when it finds that the close bracket does not correspond to the open parenthesis. The flag `isBalanced` is `false`; the fact that the stack is not empty does not affect the outcome of the algorithm.

With the expression in Figure 5-5, the loop ends at the close brace because the stack is empty at that point. Retrieving or popping the top entry of the empty stack results in `null`, so the flag `isBalanced` is set to false. Finally, with the expression in Figure 5-6, the loop ends at the end of the expression with `isBalanced` set to true. But the stack is not empty—it contains an open brace—so after the loop, `isBalanced` becomes false.

SELF-TEST

Question 3 Show the contents of the stack as you trace the algorithm `checkBalance`, as given in Segment 5.8, for each of the following expressions. What does `checkBalance` return in each case?

a. $[a\ \{b\ /\ (c - d) + e/(f + g)\} - h]$
b. $\{a\ [b + (c + 2)/d\,] + e) + f\}$
c. $[a\ \{b + [c\ (d + e) - f\ ] + g\}$

5.10 **Java implementation.** The class `BalanceChecker`, shown in Listing 5-2, implements our algorithm as the static method `checkBalance`. The method has one parameter, the expression as a string. We assume that the class `OurStack` implements `StackInterface` and is available. Since `StackInterface` specifies a stack of objects, but the previous algorithm uses a stack of characters, `checkBalance` uses the wrapper class `Character` to create objects suitable for the stack.

LISTING 5-2 The class `BalanceChecker`

```
public class BalanceChecker
{
   /** Decides whether the parentheses, brackets, and braces
       in a string occur in left/right pairs.
       @param expression  a string to be checked
       @return true if the delimiters are paired correctly */
   public static boolean checkBalance(String expression)
   {
      StackInterface<Character> openDelimiterStack =
                                   new OurStack<Character>();
      int characterCount = expression.length();
      boolean isBalanced = true;
      int index = 0;
      char nextCharacter = ' ';

      for (; isBalanced && (index < characterCount); index++)
      {
         nextCharacter = expression.charAt(index);
         switch (nextCharacter)
         {
            case '(': case '[': case '{':
               openDelimiterStack.push(nextCharacter);
               break;
            case ')': case ']': case '}':
               if (openDelimiterStack.isEmpty())
                  isBalanced = false;
               else
               {
                  char openDelimiter = openDelimiterStack.pop();
                  isBalanced = isPaired(openDelimiter, nextCharacter);
               } // end if
               break;
            default: break;
         } // end switch
      } // end for

      if (!openDelimiterStack.isEmpty())
         isBalanced = false;
      return isBalanced;
   } // end checkBalance

   // Returns true if the given characters, open and close, form a pair
   // of parentheses, brackets, or braces.
   private static boolean isPaired(char open, char close)
   {
      return (open == '(' && close == ')') ||
             (open == '[' && close == ']') ||
             (open == '{' && close == '}');
   } // end isPaired
} // end BalanceChecker
```

The following statements provide an example of how you might use this class:

```
String expression = "a {b [c (d + e)/2 - f] + 1}";
boolean isBalanced = BalanceChecker.checkBalance(expression);
if (isBalanced)
   System.out.println(expression + " is balanced");
else
   System.out.println(expression + " is not balanced");
```

A Problem Solved: Transforming an Infix Expression to a Postfix Expression

Our ultimate goal is to show you how to evaluate infix algebraic expressions, but postfix expressions are easier to evaluate. So we first look at how to represent an infix expression by using postfix notation.

5.11 Recall that in a postfix expression, a binary operator follows its two operands. Here are a few examples of infix expressions and their corresponding postfix forms:

Infix	Postfix
$a + b$	$a\ b\ +$
$(a + b) * c$	$a\ b + c\ *$
$a + b * c$	$a\ b\ c * +$

Notice that the order of the operands a, b, and c in an infix expression is the same in the corresponding postfix expression. However, the order of the operators might change. This order depends on the precedence of the operators and the existence of parentheses. As we mentioned, parentheses do not appear in a postfix expression.

5.12 **A pencil and paper scheme.** One way to determine where the operators should appear in a postfix expression begins with a fully parenthesized infix expression. For example, we write the infix expression $(a + b) * c$ as $((a + b) * c)$. By adding parentheses, we remove the expression's dependence on the rules of operator precedence. Each operator is now associated with a pair of parentheses. We now move each operator to the right so that it appears immediately before its associated close parenthesis to get $((a\ b\ +)\ c\ *)$. Finally, we remove the parentheses to obtain the postfix expression $a\ b + c\ *$.

This scheme should give you some understanding of the order of the operators in a postfix expression. It also can be useful when checking the results of a conversion algorithm. However, the algorithm that we will develop next is not based on this approach.

? SELF-TEST

Question 4 Using the previous scheme, convert each of the following infix expressions to postfix expressions:

a. $a + b * c$
b. $a * b\ /\ (c - d)$
c. $a\ /\ b + (c - d)$
d. $a\ /\ b + c - d$

5.13 **The basics of a conversion algorithm.** To convert an infix expression to postfix form, we scan the infix expression from left to right. When we encounter an operand, we place it at the end of the new expression that we are creating. Recall that operands in an infix expression remain in the same order in the corresponding postfix expression. When we encounter an operator, we must save it until we determine where in the output expression it belongs. For example, to convert the infix expression $a + b$, we append a to the initially empty output expression, save +, and append b to the output expression. We now need to retrieve the + and put it at the end of the output expression to get the postfix expression $a\ b\ +$. Retrieving the operator saved most recently is easy if we have saved it in a stack.

In this example, we saved the operator until we processed its second operand. In general, we hold the operator in a stack at least until we compare its precedence with that of the next operator. For example, to convert the expression $a + b * c$, we append a to the output expression, push + onto a stack, and then append b to the output. What we do now depends on the relative precedences of the next operator, *, and the + at the top of the stack. Since * has a greater precedence than +, b is not the addition's second operand. Instead, the addition waits for the result of the multiplication. Thus, we push * onto the stack and append c to the output expression. Having reached the end of the input expression, we now pop each operator from the stack and append it to the end of the output expression, getting the postfix expression $a\ b\ c * +$. Figure 5-7 illustrates these steps. The stack is shown horizontally; the leftmost element is at the bottom of the stack.

FIGURE 5-7 Converting the infix expression *a* + *b* * *c* to postfix form

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
+	*a*	+
b	*a b*	+
*	*a b*	+ *
c	*a b c*	+ *
	a b c *	+
	a b c * +	

5.14 **Successive operators with the same precedence.** What if two successive operators have the same precedence? We need to distinguish between operators that have a left-to-right association—namely +, -, *, and /—and exponentiation, which has a right-to-left association. For example, consider the expression *a* - *b* + *c*. When we encounter the +, the stack will contain the operator - and the incomplete postfix expression will be *ab*. The subtraction operator belongs to the operands *a* and *b*, so we pop the stack and append - to the end of the expression *ab*. Since the stack is empty, we push the + onto the stack. We then append *c* to the result, and finally we pop the stack and append the +. The result is *a b* - *c* +. Figure 5-8a illustrates these steps.

Now consider the expression *a* ^ *b* ^ *c*. By the time we encounter the second exponentiation operator, the stack contains ^, and the result so far is *ab*. As before, the current operator has the same precedence as the top entry of the stack. But since *a* ^ *b* ^ *c* means *a* ^ (*b* ^ *c*), we must push the second ^ onto the stack, as Figure 5-8b shows.

FIGURE 5-8 Converting an infix expression to postfix form:
(a) *a* - *b* + *c*; (b) *a* ^ *b* ^ *c*

(a)

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
–	*a*	–
b	*a b*	–
+	*a b* –	
	a b –	+
c	*a b* – *c*	+
	a b – *c* +	

(b)

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
^	*a*	^
b	*a b*	^
^	*a b*	^ ^
c	*a b c*	^ ^
	a b c ^	^
	a b c ^ ^	

Question 5 In general, when should you push an exponentiation operator ^ onto the stack?

5.15 **Parentheses.** Parentheses override the rules of operator precedence. We always push an open parenthesis onto the stack. Once it is in the stack, we treat an open parenthesis as an operator with the lowest precedence. That is, any subsequent operator will get pushed onto the stack. When we encounter a close parenthesis, we pop operators from the stack and append them to the forming postfix expression until we pop an open parenthesis. The algorithm continues with no parentheses added to the postfix expression.

Note: Infix-to-postfix conversion

To convert an infix expression to postfix form, you take the following actions, according to the symbols you encounter, as you process the infix expression from left to right:

- Operand

  Append each operand to the end of the output expression.

- Operator ^

  Push ^ onto the stack.

- Operator +, -, *, or /

  Pop operators from the stack, appending them to the output expression, until the stack is empty or its top entry has a lower precedence than the new operator. Then push the new operator onto the stack.

- Open parenthesis

  Push (onto the stack.

- Close parenthesis

  Pop operators from the stack and append them to the output expression until an open parenthesis is popped. Discard both parentheses.

5.16 **The infix-to-postfix algorithm.** The following algorithm encompasses the previous observations about the conversion process. For simplicity, all operands in our expression are single-letter variables.

```
Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression.

operatorStack = a new empty stack
postfix = a new empty string
while (infix has characters left to parse)
{
   nextCharacter = next nonblank character of infix
   switch (nextCharacter)
   {
      case variable:
         Append nextCharacter to postfix
         break

      case '^' :
         operatorStack.push(nextCharacter)
         break

      case '+' : case '-' : case '*' : case '/' :
         while (!operatorStack.isEmpty() and
               precedence of nextCharacter <= precedence of operatorStack.peek())
         {
            Append operatorStack.peek() to postfix
            operatorStack.pop()
         }
         operatorStack.push(nextCharacter)
         break

      case '( ' :
         operatorStack.push(nextCharacter)
         break

      case ')' : // stack is not empty if infix expression is valid
         topOperator = operatorStack.pop()
         while (topOperator != '(')
         {
            Append topOperator to postfix
            topOperator = operatorStack.pop()
         }
         break

      default: break
   }
}

while (!operatorStack.isEmpty())
{
   topOperator = operatorStack.pop()
   Append topOperator to postfix
}
return postfix
```

Figure 5-9 traces this algorithm for the infix expression *a* / *b* * (*c* + (*d* - *e*)). The resulting postfix expression is *a b* / *c d e* - + *.

FIGURE 5-9 The steps in converting the infix expression *a* / *b* * (*c* + (*d* - *e*)) to postfix form

Next Character from Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
/	*a*	/
b	*a b*	/
*	*a b* /	
	a b /	*
(	*a b* /	* (
c	*a b* / *c*	* (
+	*a b* / *c*	* (+
(	*a b* / *c*	* (+ (
d	*a b* / *c d*	* (+ (
–	*a b* / *c d*	* (+ (–
e	*a b* / *c d e*	* (+ (–
)	*a b* / *c d e* –	* (+ (
	a b / *c d e* –	* (+
)	*a b* / *c d e* – +	* (
	a b / *c d e* – +	*
	a b / *c d e* – + *	

SELF-TEST

Question 6 Using the previous algorithm, represent each of the following infix expressions as a postfix expression:

a. $(a + b) / (c - d)$
b. $a / (b - c) * d$
c. $a - (b / (c - d) * e + f) \wedge g$
d. $(a - b * c) / (d * e \wedge f * g + h)$

A Problem Solved: Evaluating Postfix Expressions

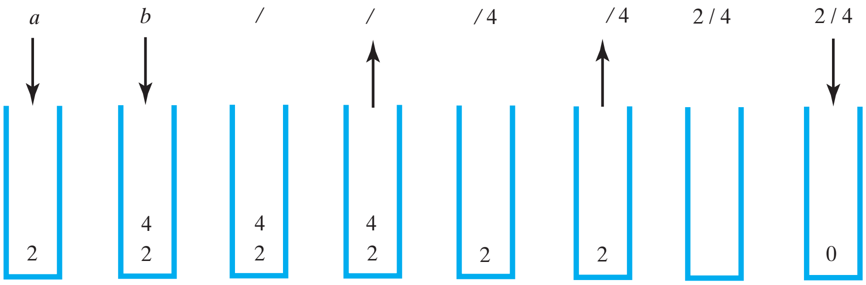
Evaluate a postfix expression that uses the operators +, -, *, /, and ^ to indicate addition, subtraction, multiplication, division, and exponentiation.

5.17 Evaluating a postfix expression requires no rules of operator precedence, since the order of its operators and operands dictates the order of the operations. Additionally, a postfix expression contains no parentheses to complicate the evaluation.

VideoNote
Using the ADT stack

As we scan the postfix expression, we must save operands until we find the operators that apply to them. For example, to evaluate the postfix expression *a b* /, we locate the variables *a* and *b* and save their values.[2] When we identify the operator /, its second operand is the most recently saved value—that is, *b*'s value. The value saved before that—*a*'s value—is the operator's first operand. Storing values in a stack enables us to access the necessary operands for an operator. Figure 5-10 traces the evaluation of *a b* / when *a* is 2 and *b* is 4. The result of 0 assumes integer division.

FIGURE 5-10 The stack during the evaluation of the postfix expression *a b* / when *a* is 2 and *b* is 4

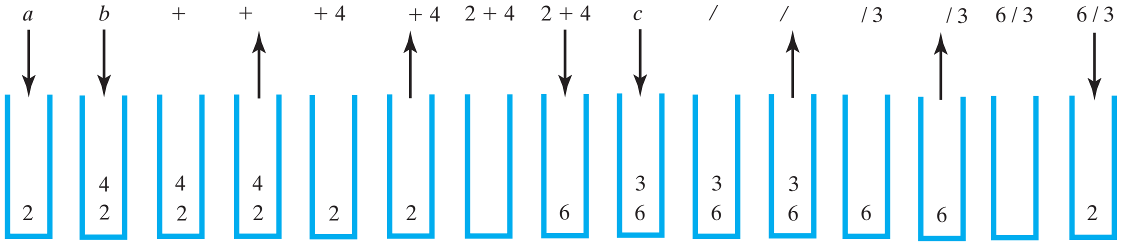


Now consider the postfix expression *a b* + *c* /, where *a* is 2, *b* is 4, and *c* is 3. The expression corresponds to the infix expression (*a* + *b*) / *c*, so its value should be 2. After finding the variable *a*, we push its value 2 onto a stack. Likewise, we push *b*'s value 4 onto the stack. The + operator is next, so we pop two values from the stack, add them, and push their sum 6 onto the stack. Notice that this sum will be the first operand of the / operator. The variable *c* is next in the postfix expression, so we push its value 3 onto the stack. Finally, we encounter the operator /, so we pop two values from the stack and form their quotient, 6 / 3. We push this result onto the stack. We are

[2] Finding the value of a variable is not an easy task, but we will not explore this detail in this book.

at the end of the expression, and one value, 2, is in the stack. This value is the value of the expression. Figure 5-11 traces the evaluation of this postfix expression.

FIGURE 5-11 The stack during the evaluation of the postfix expression *a b* + *c* / when *a* is 2, *b* is 4, and *c* is 3



5.18 The evaluation algorithm follows directly from these examples:

```
Algorithm evaluatePostfix(postfix)
// Evaluates a postfix expression.

valueStack = a new empty stack
while (postfix has characters left to parse)
{
   nextCharacter = next nonblank character of postfix
   switch (nextCharacter)
   {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break

      case '+' : case '-' : case '*' : case '/' : case '^' :
         operandTwo = valueStack.pop()
         operandOne = valueStack.pop()
         result = the result of the operation in nextCharacter and its operands
                  operandOne and operandTwo
         valueStack.push(result)
         break

      default: break
   }
}

return valueStack.peek()
```

We can implement this algorithm and the algorithm `convertToPostfix` given in Segment 5.16 as static methods of a class `Postfix`.The implementations are left as an exercise.

? SELF-TEST

Question 7 Using the previous algorithm, evaluate each of the following postfix expressions. Assume that a = 2, b = 3, c = 4, d = 5, and e = 6.

a. *a e* + *b d* - /
b. *a b c* * *d* * -
c. *a b c* - / *d* *
d. *e b c a* ^ * + *d* -

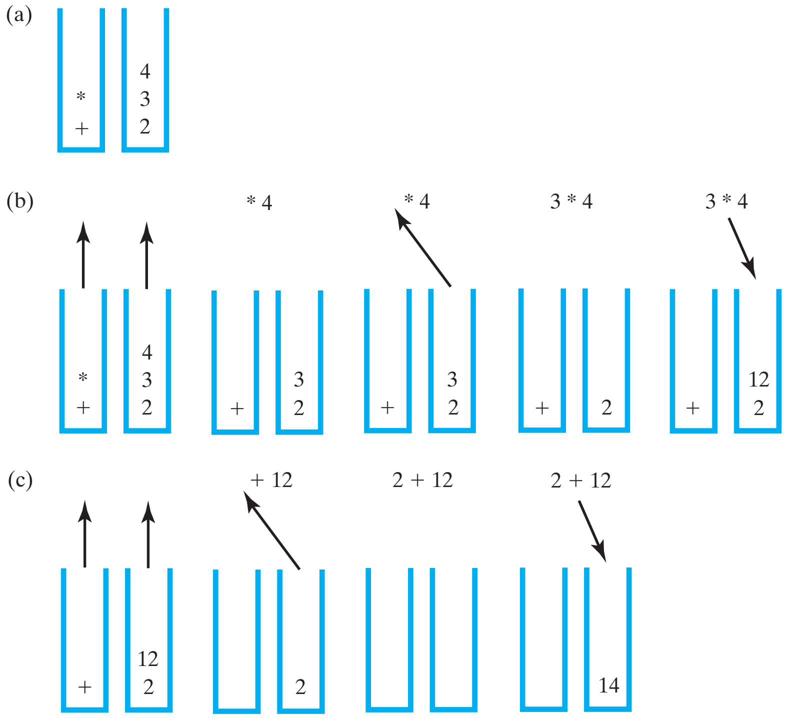
A Problem Solved: Evaluating Infix Expressions

Evaluate an infix expression that uses the operators +, -, *, /, and ^ to indicate addition, subtraction, multiplication, division, and exponentiation.

5.19 Using the two algorithms in Segments 5.16 and 5.18, we could evaluate an infix expression by converting it to an equivalent postfix expression and then evaluating it. We can save some intermediate work, however, by combining the two algorithms into one that evaluates an infix expression directly by using two stacks. This combined algorithm maintains a stack of operators according to the algorithm that converts an infix expression to postfix form. But instead of appending operands to the end of an expression, the new algorithm pushes the value of an operand onto a second stack according to the algorithm that evaluates a postfix expression.

5.20 **Example.** Consider the infix expression *a* + *b* * *c*. When *a* is 2, *b* is 3, and *c* is 4, the expression's value is 14. To compute this result, we push the value of the variable *a* onto a stack of values, push the + onto a stack of operators, and push the value of *b* onto the stack of values. Since * has a higher precedence than the + at the top of the operator stack, we push it onto the stack. Finally, we push the value of *c* onto the stack of values. Figure 5-12a shows the state of the two stacks at this point.

FIGURE 5-12 Two stacks during the evaluation of *a* + *b* * *c* when *a* is 2, *b* is 3, and *c* is 4:
(a) after reaching the end of the expression;
(b) while performing the multiplication;
(c) while performing the addition



We now pop the operator stack and get the *. We get this operator's second and first operands, respectively, by popping the stack of values twice. After computing the product 3 * 4, we push the result 12 onto the stack of values, as Figure 5-12b shows. In a similar fashion, we pop the operator stack once and the value stack twice, compute 2 + 12, and push the result, 14, onto the stack of values. Since the operator stack is now empty, the value of the expression—14—is at the top of the stack of values. Figure 5-12c shows these final steps.

5.21 **The algorithm.** The algorithm to evaluate an infix expression follows. You should recognize aspects of its logic from the previous algorithms.

```
Algorithm evaluateInfix(infix)
// Evaluates an infix expression.

operatorStack = a new empty stack
valueStack = a new empty stack
while (infix has characters left to process)
{
   nextCharacter = next nonblank character of infix
   switch (nextCharacter)
   {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break

      case '^' :
         operatorStack.push(nextCharacter)
         break

      case '+' : case '-' : case '*' : case '/' :
         while (!operatorStack.isEmpty() and
               precedence of nextCharacter <= precedence of operatorStack.peek())
         {
            // Execute operator at top of operatorStack
            topOperator = operatorStack.pop()
            operandTwo = valueStack.pop()
            operandOne = valueStack.pop()
            result = the result of the operation in topOperator and its operands
                     operandOne and operandTwo
            valueStack.push(result)
         }
         operatorStack.push(nextCharacter)
         break

      case '(' :
         operatorStack.push(nextCharacter)
         break

      case ')' : // stack is not empty if infix expression is valid
         topOperator = operatorStack.pop()
         while (topOperator != '(')
         {
            operandTwo = valueStack.pop()
            operandOne = valueStack.pop()
            result = the result of the operation in topOperator and its operands
                     operandOne and operandTwo
            valueStack.push(result)
            topOperator = operatorStack.pop()
         }
         break
```

```
            default: break
        }
    }

    while (!operatorStack.isEmpty())
    {
        topOperator = operatorStack.pop()
        operandTwo = valueStack.pop()
        operandOne = valueStack.pop()
        result = the result of the operation in topOperator and its operands
                  operandOne and operandTwo
        valueStack.push(result)
    }
    return valueStack.peek()
```

Question 8 Using the previous algorithm, evaluate each of the following infix expressions. Assume that $a = 2$, $b = 3$, $c = 4$, $d = 5$, and $e = 6$.

a. $a + b * c - 9$

b. $(a + e) / (b - d)$

c. $a + (b + c * d) - e / 2$

d. $e - b * c \wedge a + d$

The Program Stack

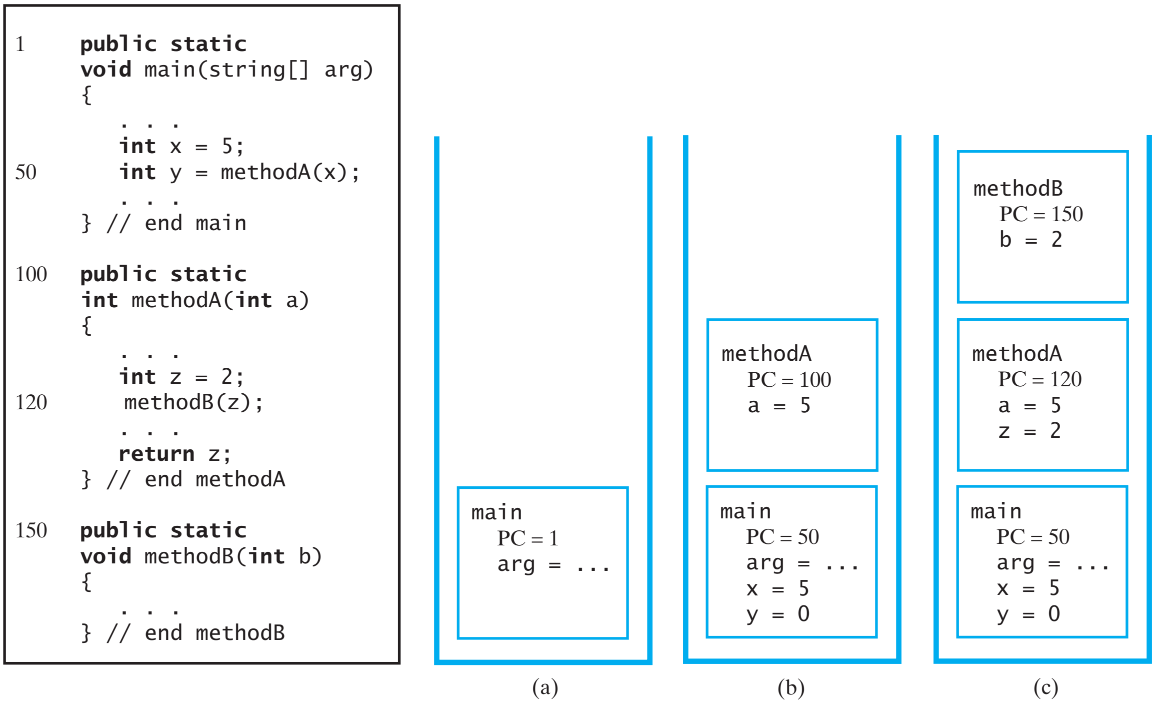
5.22 When a program executes, a special location called the **program counter** references the current instruction. The program counter might be part of an actual computer or, in the case of Java, part of a virtual computer.[3]

When a method is called, the program's run-time environment creates an object called an **activation record**, or **frame**, for the method. The activation record shows the method's state during its execution. In particular, the activation record contains the method's arguments, local variables, and a reference to the current instruction—that is, a copy of the program counter. At the time the method is called, the activation record is pushed onto a stack called the **program stack** or, in Java, the **Java stack**. Since one method can call another, the program stack often contains more than one activation record. The record at the top of the stack belongs to the method that is currently executing. The record just beneath the top record belongs to the method that called the current method, and so on.

Figure 5-13 illustrates a program stack for a `main` method that calls `methodA`, which then calls `methodB`. When `main` begins execution, its activation record is at the top of the program stack (Figure 5-13a). When `main` calls `methodA`, a new record is pushed onto the stack. The program counter is 50 at that time. Figure 5-13b shows the updated record for `main` and the new record for `methodA` just as the method begins execution. When `methodA` calls `methodB`, the program counter is 120. A new activation record is pushed onto the stack. Figure 5-13c shows the unchanged record for `main`, the updated record for `methodA`, and the new record for `methodB` just as it begins execution.

[3] To maintain computer independence, Java runs on a virtual computer called the **Java Virtual Machine** (**JVM**).

FIGURE 5-13 The program stack at three points in time: (a) when `main` begins execution; (b) when `methodA` begins execution; (c) when `methodB` begins execution



```
1     public static
      void main(string[] arg)
      {
         . . .
         int x = 5;
50       int y = methodA(x);
         . . .
      } // end main

100   public static
      int methodA(int a)
      {
         . . .
         int z = 2;
120       methodB(z);
         . . .
         return z;
      } // end methodA

150   public static
      void methodB(int b)
      {
         . . .
      } // end methodB
```

Program

Program stack at three points in time (PC is the program counter)

As `methodB` executes, its activation record is updated, but the records for `main` and `methodA` remain unchanged. The record for `methodA`, for example, represents the method's state at the time it called `methodB`. When `methodB` completes its execution, its record is popped from the stack. The program counter is reset to 120 and then advanced to the next instruction. Thus, `methodA` resumes execution with the values of its argument and local variable as given in its activation record. Ultimately, `methodA` completes its execution, its activation record is popped from the program stack, and `main` continues its execution to completion.

Java Class Library: The Class Stack

5.23 The Java Class Library contains the class `Stack`, which is an implementation of the ADT stack, within the package `java.util`. This class has only one constructor—a default constructor that creates an empty stack. In addition, the following four methods in this class are similar to methods in our `StackInterface`. We have highlighted where they differ from our methods.

```
public T push(T item);
public T pop();
public T peek();
public boolean empty();
```

Stack also defines methods that enable you to search or traverse the entries in the stack, as well as other methods not supported by a traditional stack ADT.

Note: The standard class java.util.Stack is now considered a legacy class. As you will see in Chapter 10, you instead should use the class java.util.ArrayDeque when you do not want to define your own class of stacks. However, you can use Stack for now, and we will define our own stack classes in the next chapter.

Chapter Summary

- The ADT stack organizes its entries on a last-in, first-out basis. The entry at the top of the stack is the one added most recently.
- A stack's major operations—push, pop, and peek—deal only with the top of the stack. The method push adds an entry to the top of the stack; pop removes and returns the top entry, and peek just returns it.
- Arithmetic operators that have two operands are binary operators. When an operator such as + or - has one operand, it is a unary operator.
- An algebraic expression often contains parentheses, square brackets, and braces. You can use a stack to discover whether these delimiters are paired correctly.
- Ordinary algebraic expressions are called infix expressions, because each binary operator appears between its two operands. An infix expression requires rules of operator precedence and can use parentheses to override these rules.
- In a postfix expression, each binary operator appears after its two operands. In a prefix expression, each binary operator appears before its two operands. Postfix and prefix expressions use no parentheses and have no rules of operator precedence.
- You can use a stack of operators when forming a postfix expression that is equivalent to a given infix expression.
- You can use a stack of values to evaluate a postfix expression.
- You can use two stacks—one for operators and one for values—to evaluate an infix expression.
- When a method is called, the Java run-time environment creates an activation record, or frame, to record the status of the method. The record contains the method's arguments and local variables, along with the address of the current instruction. The record is placed in a stack called the program stack.

Programming Tip

- Methods such as peek and pop must behave reasonably when the stack is empty. For example, they could return null or throw an exception.

Exercises

1. If you push the objects x, y, and z onto an initially empty stack, in what order will three pop operations remove them from the stack?

2. What pseudocode statements create a stack of the three strings "Jill", "Jane", and "Joe", in that order with "Jill" at the top?

3. Suppose that s and t are empty stacks and a, b, c, and d are objects. What do the stacks contain after the following sequence of operations executes?

```
s.push(a);
s.push(b);
s.push(c);
t.push(d);
t.push(s.pop());
t.push(s.peek());
s.push(t.pop());
t.pop();
```

4. What are the contents of the stack pile after the following statements execute? Assume that MyStack is a class that implements the interface StackInterface.

```
StackInterface<String> pile = new MyStack<String>();
pile.push("Jane");
pile.push("Jess");
pile.push("Jill");
pile.push(pile.pop());
pile.push(pile.peek());
pile.push("Jim");
String name = pile.pop();
pile.push(pile.peek());
```

5. Consider the following Java statements, assuming that MyStack is a class that implements the interface StackInterface:

```
int n = 4;
StackInterface<Integer> stack = new MyStack<Integer>();
while (n > 0)
{
   stack.push(n);
   n--;
} // end while

int result = 1;
while (!stack.isEmpty())
{
   int integer = stack.pop();
   result = result * integer;
} // end while
System.out.println("result = " + result);
```

   a. What value is displayed when this code executes?
   b. What mathematical function does the code evaluate?

6. Show the contents of the stack as you trace the algorithm `checkBalance`, given in Segment 5.8, for each of the following expressions:

    a. $a \{b [c * (d + e)] - f \}$
    b. $\{a (b * c) / [d + e] / f)- g\}$
    c. $a \{b [c - d] e]) f$

7. Using the algorithm `convertToPostfix`, given in Segment 5.16, convert each of the following infix expressions to postfix expressions:

    a. $a * b / (c - d)$
    b. $(a - b * c) / (d * e * f + g)$
    c. $a / b * (c + (d - e))$
    d. $(a \wedge b * c - d) \wedge e + f \wedge g \wedge h$

8. Using the algorithm `evaluatePostfix`, given in Segment 5.18, evaluate each of the following postfix expressions. Assume that $a = 2$, $b = 3$, $c = 4$, $d = 5$, and $e = 6$.

    a. $a\ b + c * d -$
    b. $a\ b * c\ a - / d\ e * +$
    c. $a\ c - b \wedge d +$

9. What infix expressions are represented by the postfix expressions given in the previous exercise?

10. Show the contents of the two stacks as you trace the algorithm `evaluateInfix`, given in Segment 5.21, to evaluate each of the following infix expressions. Assume that $a = 2$, $b = 3$, $c = 4$, $d = 5$, $e = 6$, and $f = 7$.

    a. $(a + b) / (c - d) - 5$
    b. $(d * f + 1) * e / (a \wedge b - b * c + 1) - 72$
    c. $(a \wedge c - f) \wedge a - a \wedge b \wedge a$

11. A *palindrome* is a string of characters (a word, phrase, or sentence) that is the same regardless of whether you read it forward or backward—assuming that you ignore spaces, punctuation, and case. For example, *Race car* is a palindrome. So is *A man, a plan, a canal: Panama*. Describe how you could use a stack to test whether a string is a palindrome.

12. Suppose that you read a binary string—that is, a string of 0s and 1s—one character at a time. Describe how you could use a stack but no arithmetic to see whether the number of 0s is equal to the number of 1s. When these counts are not equal, state how you could tell which character—0 or 1—occurs most frequently and by how much its count exceeds the other's.

13. Write Java code that displays all the objects in a stack in the order in which they were pushed onto it. After all the objects are displayed, the stack should have the same contents as when you started.

14. Using the class `java.util.Stack`, define a class `OurStack` that implements the interface `StackInterface`, as given in Listing 5-1.

15. Use the class `OurStack` from the previous exercise in a program that demonstrates the class `BalanceChecker`, as given in Listing 5-2.

Projects

Whenever you need a stack for any of the following projects, use the class `OurStack` that Exercise 14 asks you to define.

1. Write a Java program that uses a stack to test whether an input string is a palindrome. Exercise 11 defines "palindrome" and asks you to describe a solution to this problem.

2. Define a class `Postfix` that includes the static methods `convertToPostfix` and `evaluatePostfix`. These methods should implement the algorithms given in Segments 5.16 and 5.18, respectively. Assume that the given algebraic expressions are syntactically correct. The standard class `StringBuilder`, which is in the Java Class Library and is described in Segment A.79 of Appendix A, will be helpful.

3. Define and demonstrate a method that evaluates infix expressions using the algorithm given in Segment 5.21. Assume that expressions are syntactically correct and use single-letter operands.

4. Repeat the previous project, but remove the assumption that the expressions are syntactically correct.

5. Consider the following algorithm to sort the entries in a stack S_1. First create two empty stacks, S_2 and S_3. At any given time, stack S_2 will hold the entries in sorted order, with the smallest at the top of the stack. Move the top entry of S_1 to S_2. Pop and consider the top entry *t* of S_1. Pop entries of stack S_2 and push them onto stack S_3 until you reach the correct place to put *t*. Then push *t* onto S_2. Next move all the entries from S_3 to S_2.

a. Write an iterative implementation of this algorithm.

b. Consider the following revision of this algorithm. After moving the top entry of S_1 to S_2, compare the new top entry *t* of S_1 with the top entry of S_2 and the top entry of S_3. Then either move entries from S_2 to S_3 or from S_3 to S_2 until you locate the correct position for *t*. Push *t* onto S_2. Continue until S_1 is empty. Finally, move any entries remaining in S_3 to S_2. Implement this revised algorithm.

6. In the language Lisp, each of the four basic arithmetic operators appears before an arbitrary number of operands, which are separated by spaces. The resulting expression is enclosed in parentheses. The operators behave as follows:

- `(+ a b c ...)` returns the sum of all the operands, and `(+)` returns 0.
- `(- a b c ...)` returns `a - b - c - ...`, and `(- a)` returns `-a`. The minus operator must have at least one operand.
- `(* a b c ...)` returns the product of all the operands, and `(*)` returns 1.
- `(/ a b c ...)` returns `a / b / c / ...`, and `(/ a)` returns `1 / a`. The divide operator must have at least one operand.

You can form larger arithmetic expressions by combining these basic expressions using a fully parenthesized prefix notation. For example, the following is a valid Lisp expression:

```
(+ (- 6) (* 2 3 4) (/ (+ 3) (*) (- 2 3 1)))
```

This expression is evaluated successively as follows:

```
(+ (- 6) (* 2 3 4) (/ 3 1 -2))
(+ -6 24 -1.5)
16.5
```

Design and implement an algorithm that uses a stack to evaluate a legal Lisp expression composed of the four basic operators and integer values. Write a program that reads such expressions and demonstrates your algorithm.

7. Consider arithmetic expressions like the ones described in the previous project. Allow operands to be either integer values or variable names that are strings of letters. Design and implement an iterative algorithm that uses a stack to test whether an expression is legal in Lisp. Write a program that reads potential expressions and demonstrates your algorithm.

Each expression that your program reads can be split across several lines, which is the style used by typical Lisp programmers. For example, the following expression is legal in Lisp:

```
(+ (- height)
   (* 3 3 4)
   (/ 3 width length)
   (* radius radius)
)
```

In contrast, the following expressions are illegal in Lisp:

```
(+ (-)
   (* 3 3 4)
   (/ 3 width length)
   (* radius radius)
)
```

```
(+ (- height)
   (* 3 3 4) )s
(* (/ 3 width length)
   (* radius radius)
)
```

```
(+ (- height)
   (* 3 3 4)
   (/ 3 width length))
   (* radius radius)
)
```

```
(+ (- height)
   (* 3 3 4)
   ((/ 3 width length))
   (* radius radius)
)
```

8. Write a program that graphically displays a working calculator for simple infix expressions that consist of single-digit operands; the operators +, -, *, and /; and parentheses. Make the following assumptions:

- Unary operators (as in -2) are illegal.
- All operations, including division, are integer operations.
- The input expression contains no embedded spaces and no illegal characters, since it is entered by using a keypad.
- The input expression is a syntactically correct infix expression.
- Division by zero will not occur. (Consider how you can remove this restriction.)

The calculator has a display and a keypad of 20 keys, which are arranged as follows:

```
C < Q /
7 8 9 *
4 5 6 -
1 2 3 +
0 ( ) =
```

As the user presses keys to enter an infix expression, the corresponding characters appear in the display. The C (Clear) key erases all input entered so far; the < (Backspace) key erases the last character entered. When the user presses the = key, the expression is evaluated and the result replaces the expression in the display window. The user can then press C and enter another expression. If the user presses the Q (Quit) key, the calculator ceases operation and is erased from the screen.

Answers to Self-Test Questions

1. *Jill* is at the top, and *Jim* is at the bottom.

2. **a.**

```
StackInterface<String> nameStack = new LinkedStack<String>();
while (!stringStack.isEmpty())
   nameStack.push(stringStack.pop());
```

b. `stringStack` is empty, and `nameStack` contains the strings that were in `stringStack` but in reverse order (*Jim* is at the top, and *Jill* is at the bottom).

3. The following stacks are shown bottom to top when read from left to right:

a.	**b.**	**c.**
[	{	[
[{	{ [	[{
[{ (	{ [(	[{ [
[{	{ [	[{ [(
{ { (	{	[{ [
[{		[{
[		[
empty		

The algorithm `checkBalance` returns true for the expression in Part *a* and false for the other two.

4. **a.** a b c * +
b. a b * c d - /
c. a b / c d - +
d. a b / c + d -

5. Always. Segment 5.14 showed that you push ^ onto the stack if another ^ is already at the top of the stack. But if a different operator is at the top, ^ has a higher precedence, so you push it onto the stack in that situation as well.

6. **a.** a b + c d - /
b. a b c - / d *
c. a b c d - / e * f + g ^ -
d. a b c * - d e f ^ * g * h + /

7. **a.** -4.
b. -58.
c. -10.
d. 49.

8. **a.** 5.
b. -4.
c. 22.
d. -37.

This page intentionally left blank

Chapter 6

Stack Implementations

Contents

Prerequisites

Chapter	2	Bag Implementations That Use Arrays
Chapter	3	A Bag Implementation That Links Data
Chapter	4	The Efficiency of Algorithms
Chapter	5	Stacks

Objectives

After studying this chapter, you should be able to

- Implement the ADT stack by using either a linked chain, an array, or a vector
- Compare and contrast the various implementations and their performance

Two of the implementations of the ADT stack described in this chapter use techniques like the ones we used to implement the ADT bag. We will use, in turn, a chain of linked nodes and an array to store the stack's entries. We also will introduce the standard class `Vector` as a part of the Java Class Library and use an instance of `Vector` to represent a stack. You should be pleasantly surprised by the simplicity and efficiency of these implementations.

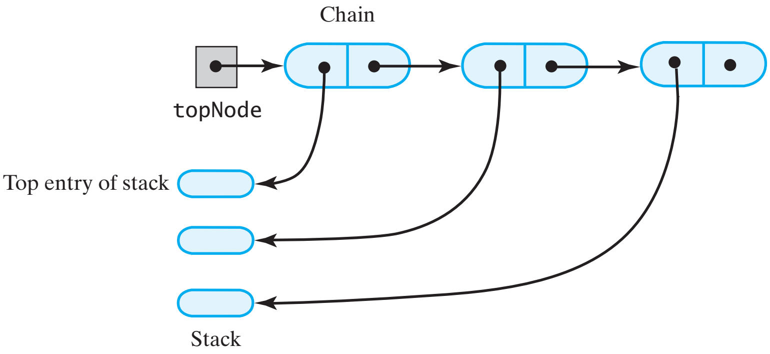
A Linked Implementation

6.1 Each of the operations `push`, `pop`, and `peek` of the ADT stack involve the top of the stack. If we use a chain of linked nodes to implement a stack, where in the chain should we place the stack's top entry? If we have only the chain's head reference, we can add, remove, or access its first node faster than any other node. Thus, the stack operations will execute fastest if the first node in the chain references the top entry in the stack, as Figure 6-1 illustrates.

VideoNote
The Class LinkedStack

Also note in the figure that each node in the chain references one entry in the stack. Nodes are allocated—that is, created—only when needed for a new entry. They are deallocated when an entry is removed. Recall from the note in Segment 3.24 of Chapter 3 that the Java run-time environment automatically reclaims, or deallocates, memory that a program no longer references, without explicit instruction from the programmer.

FIGURE 6-1 A chain of linked nodes that implements a stack



Note: If you use a chain of linked nodes to implement a stack, the first node should reference the stack's top entry.

6.2 **An outline of the class.** The linked implementation of the stack has a data field `topNode`, which is the head reference of the chain of nodes. The default constructor sets this field to `null`. An outline of our class appears in Listing 6-1.

Each node in the chain is an instance of the private class `Node` that is defined within the class `LinkedStack`. This class has set and get methods and is like the one you saw in Listing 3-4 of Chapter 3.

LISTING 6-1 An outline of a linked implementation of the ADT stack

```
/**
    A class of stacks whose entries are stored in a chain of nodes.
    @author Frank M. Carrano
*/
public class LinkedStack<T> implements StackInterface<T>
{
   private Node topNode; // references the first node in the chain

   public LinkedStack()
   {
      topNode = null;
   } // end default constructor
```

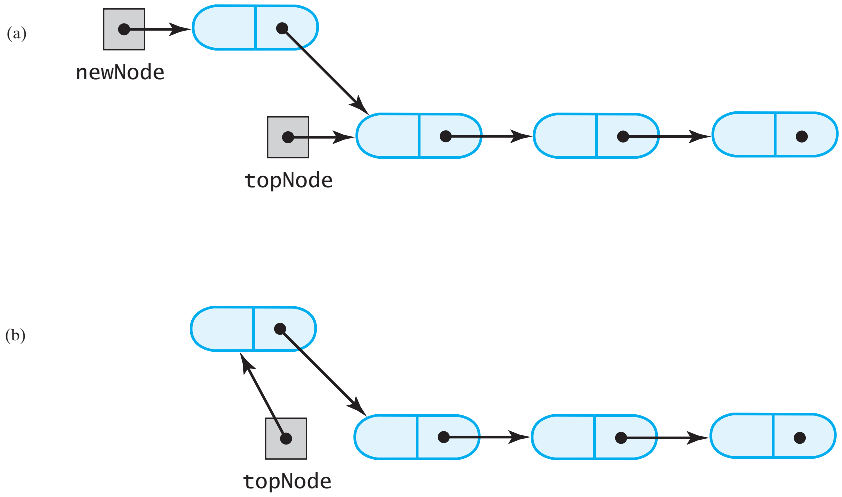
```
    < Implementations of the stack operations go here. >
    . . .

    private class Node
    {
       private T    data; // entry in stack
       private Node next; // link to next node

       < Constructors and the methods getData, setData, getNextNode, and setNextNode
          are here. >
    } // end Node
} // end LinkedStack
```

6.3 **Adding to the top.** We push an entry onto the stack by first allocating a new node that references the stack's existing chain, as Figure 6-2a illustrates. This reference is in `topNode`, the head reference to the chain. We then set `topNode` to reference the new node, as in Figure 6-2b. Thus, the method `push` has the following definition:

```
public void push(T newEntry)
{
   Node newNode = new Node(newEntry, topNode);
   topNode = newNode;
} // end push
```

This operation is independent of the other entries in the stack. Its performance is thus O(1).

FIGURE 6-2 (a) A new node that references the node at the top of the stack; (b) the new node is now at the top of the stack



6.4 **Retrieving the top.** We get the top entry in the stack by accessing the data portion of the first node in the chain. Thus, `peek`, like `push`, is an O(1) operation. Note that if the stack is empty, `peek` returns `null`.

```
public T peek()
{
   T top = null;
```

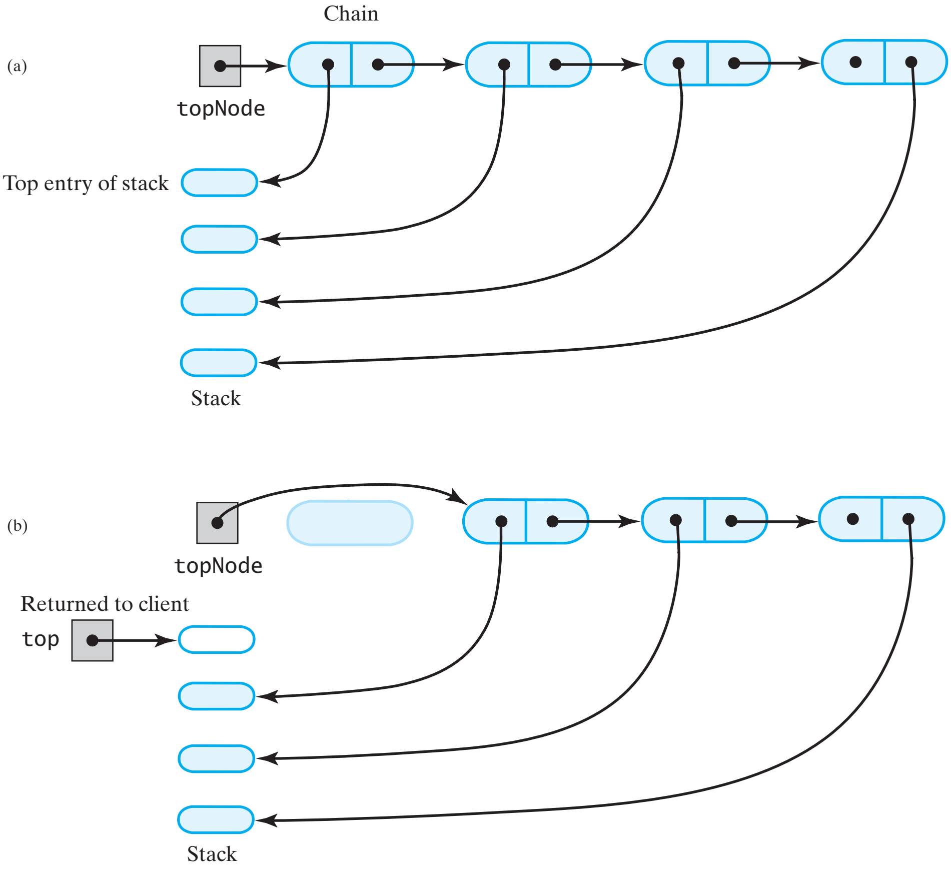
```
      if (topNode != null)
         top = topNode.getData();

      return top;
   } // end peek
```

6.5 **Removing the top.** We pop, or remove, the top entry in the stack by setting `topNode` to the reference in the first node. Thus, `topNode` will reference what was the second node in the chain, as Figure 6-3 shows. Moreover, the original first node will no longer be referenced, so it will be deallocated. Since we also want the operation to return the stack's top entry before it is removed, the method `pop` has the following implementation:

```
public T pop()
{
   T top = peek();

   if (topNode != null)
      topNode = topNode.getNextNode();

   return top;
} // end pop
```

This operation also is O(1).

FIGURE 6-3 The stack (a) before and (b) after the first node in the chain is deleted



SELF-TEST

Question 1 Revise the previous implementation of pop so that it does not call peek.

6.6 **The rest of the class.** The remaining public methods isEmpty and clear involve only topNode:

```
public boolean isEmpty()
{
   return topNode == null;
} // end isEmpty

public void clear()
{
   topNode = null;
} // end clear
```

SELF-TEST

Question 2 Is an implementation of the ADT stack reasonable if the top of the stack is at the end of a chain of linked nodes instead of its beginning? Explain.

An Array-Based Implementation

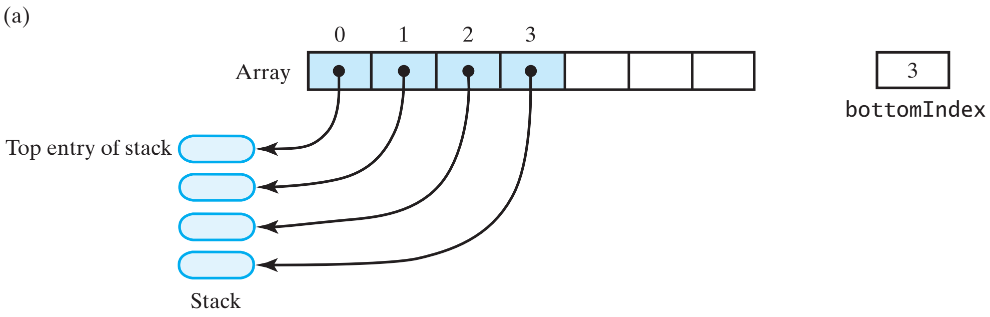
6.7 If we use an array to implement the stack, where should we place the stack's top entry? If the first location of the array references the top entry, as shown in Figure 6-4a, we must move all the entries in the array any time we add or remove a stack entry. We can have more efficient stack operations if the first array location references the bottom entry of the stack. The top entry of the stack is then referenced by the last occupied location in the array, as Figure 6-4b shows. This configuration allows us to add or remove stack entries without moving other array elements. Thus, one disadvantage of a typical array-based implementation does not apply here. The exercises at the end of this chapter consider other ways to place a stack's entries in an array.

VideoNote
The Class ArrayStack

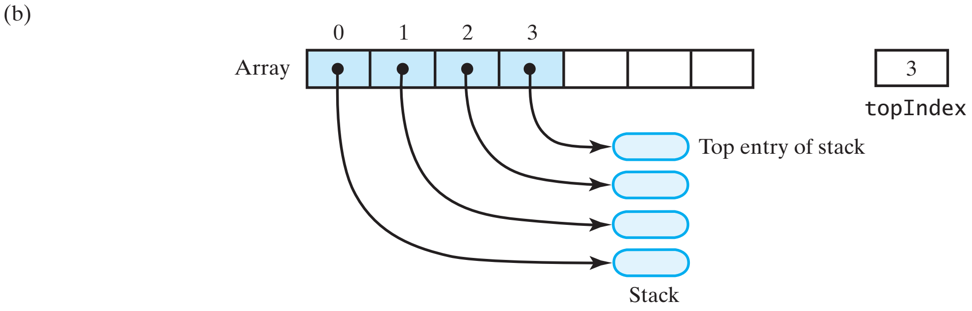
Resizing the array avoids a stack that is too full to accept another entry. However, unlike the linked chain in the previous section, the array in Figure 6-4 contains locations that are unused. If we eventually fill the array with additional stack entries, we can expand the size of the array—but then we will have more unused locations. The chain has its downside as well, in that it uses additional memory for the link portions of its nodes.

Note: If you use an array to implement a stack, the array's first location is the bottom of the stack. The last occupied location in the array, then, references the stack's top entry.

FIGURE 6-4 An array that implements a stack; its first location references (a) the top entry in the stack; (b) the bottom entry in the stack



(Figure 6-4 continued)



6.8 **An outline of the class.** The array-based implementation of the stack has as data fields an array of stack entries and an index to the top entry. The default constructor creates a stack with a default capacity; another constructor lets the client choose the stack's capacity. Listing 6-2 outlines our class.

LISTING 6-2 An outline of an array-based implementation of the ADT stack

```
/**
    A class of stacks whose entries are stored in an array.
    @author Frank M. Carrano
*/
public class ArrayStack<T> implements StackInterface<T>
{
   private T[] stack;    // array of stack entries
   private int topIndex; // index of top entry
   private static final int DEFAULT_INITIAL_CAPACITY = 50;

   public ArrayStack()
   {
      this(DEFAULT_INITIAL_CAPACITY);
   } // end default constructor

   public ArrayStack(int initialCapacity)
   {
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempStack = (T[])new Object[initialCapacity];
      stack = tempStack;
      topIndex = -1;
   } // end constructor

   < Implementations of the stack operations go here. >
   . . .
} // end ArrayStack
```

To indicate an empty stack, we have assigned `-1` to `topIndex` as its initial value. This choice allows `push` to simply increment `topIndex` before using it when placing a new entry in the array.

6.9 **Adding to the top.** The `push` method checks whether the array has room for a new entry by calling the private method `ensureCapacity`. It then places the new entry immediately after the last occupied location in the array:

```
public void push(T newEntry)
{
   ensureCapacity();
   topIndex++;
   stack[topIndex] = newEntry;
} // end push

private void ensureCapacity()
{
   if (topIndex == stack.length - 1) // if array is full,
      // double size of array
      stack = Arrays.copyOf(stack, 2 * stack.length);
} // end ensureCapacity
```

Note that `ensureCapacity` is similar to the `ensureCapacity` method in the class `ResizableArrayBag`, which we encountered in Chapter 2. Both private methods double the size of an array after it becomes full.

When `ensureCapacity` does not resize the array `stack`, `push` is an O(1) operation, since its performance is independent of the size of the stack. However, resizing the array is an O(n) operation, so when the array is full, the performance of `push` degrades to O(n). If this happens, however, the very next `push` is O(1) again. To be fair, all `push` operations should share the cost of the occasional resize of the array. That is, we **amortize** the cost of doubling the array size over all additions to the stack. Unless we must resize the array many times, each `push` is almost O(1).

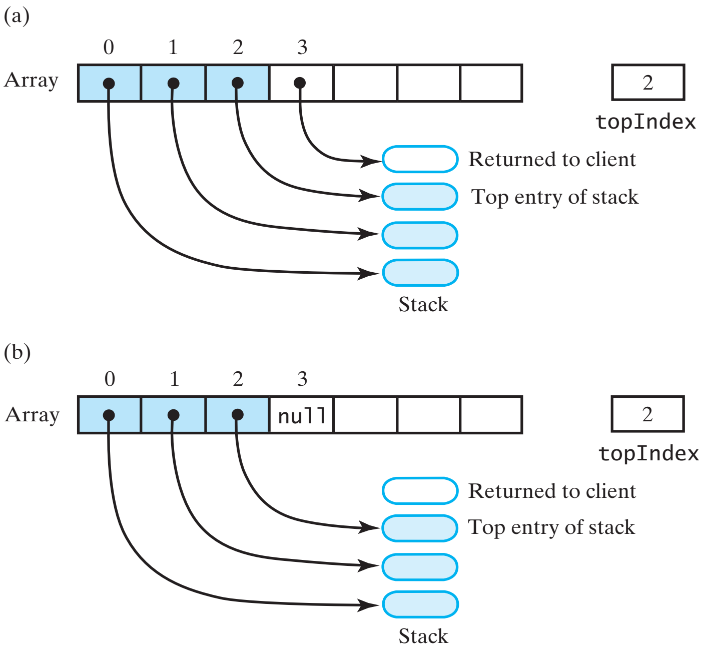
6.10 **Retrieving the top.** The operation `peek` returns either the array entry at `topIndex` or `null` if the stack is empty:

```
public T peek()
{
   T top = null;

   if (!isEmpty())
      top = stack[topIndex];

   return top;
} // end peek
```

This operation is O(1).

6.11 **Removing the top.** The `pop` operation, like `peek`, retrieves the top entry in the stack, but then removes it. To remove the stack's top entry in Figure 6-4b, we could simply decrement `topIndex`, as Figure 6-5a illustrates. This simple step would be sufficient, since the other methods would behave correctly. For example, given the stack pictured in Figure 6-5a, `peek` would return the item that `stack[2]` references. However, the object that previously was the top entry and has now been returned to the client would still be referenced by the array. No harm will come from this situation if our implementation is correct. To be safe, `pop` can set `stack[topIndex]` to `null` before decrementing `topIndex`. Figure 6-5b illustrates the stack in this case.

FIGURE 6-5 An array-based stack after its top entry is removed by (a) decrementing `topIndex`; (b) setting `stack[topIndex]` to `null` and then decrementing `topIndex`



The following implementation of `pop` reflects these comments:

```
public T pop()
{
   T top = null;

   if (!isEmpty())
   {
      top = stack[topIndex];
      stack[topIndex] = null;
      topIndex--;
   } // end if

   return top;
} // end pop
```

Like `peek`, `pop` is an O(1) operation.

Question 3 Revise the previous implementation of `pop` so that it calls `peek`.

Question 4 If we were to implement a stack of primitives instead of a stack of objects, what changes should we make to the method `pop`?

6.12 The methods `isEmpty` and `clear`. The method `isEmpty` involves only `topIndex`:

```
public boolean isEmpty()
{
   return topIndex < 0;
} // end isEmpty
```

The method `clear` could simply set `topIndex` to -1, because the stack methods would behave correctly as though the stack were empty. However, the objects that were in the stack would remain allocated. Just as `pop` sets `stack[topIndex]` to `null`, `clear` should set to `null` each array location that was used for the stack. Alternatively, `clear` could call `pop` repeatedly until the stack is empty. We leave the implementation of `clear` as an exercise.

? SELF-TEST

Question 5 If `stack` is an array that contains the entries in a stack, what is a disadvantage of maintaining the top entry of the stack in `stack[0]`?

Question 6 If you use the locations at the end of an array `stack` for a stack's entries before you use the array's first locations, should the stack's top entry or its bottom entry be in `stack[stack.length - 1]`? Why?

Question 7 Write an implementation of `clear` that sets to `null` each array location that was used for the stack.

Question 8 Write an implementation of `clear` that repeatedly calls `pop` until the stack is empty.

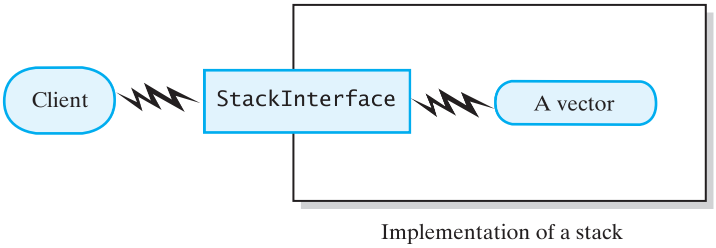
A Vector-Based Implementation

6.13 One way to let a stack grow as needed is to store its entries in an array that you resize, as we did in the implementation of `ArrayStack`. Another way is to use a **vector** instead of an array. A vector is an object that behaves like a high-level array. A vector's entries are indexed beginning with 0, just like an array's entries. But unlike an array, a vector has methods to set or access its entries. You can create a vector of a given size, and it will grow in size as needed. The details of this process are hidden from the client.

If we store a stack's entries in a vector, we can use the vector's methods to manipulate the stack's entries. Figure 6-6 shows a client interacting with a stack by using the methods in `StackInterface`. The implementations of these methods in turn interact with the vector's methods to produce the desired effects on the stack.

A vector is an instance of the standard class `Vector`, which we describe next.

FIGURE 6-6 A client using the methods given in `StackInterface`; these methods interact with a vector's methods to perform stack operations



Java Class Library: The Class Vector

6.14 The Java Class Library contains a class Vector, whose instances—called vectors—behave like a resizable array.

Here are some constructors and methods that we will use to implement the ADT stack:

```
public Vector()
```

Creates an empty vector, or arraylike container, with an initial capacity of 10. When the vector needs to increase its capacity, the capacity doubles.

```
public Vector(int initialCapacity)
```

Creates an empty vector with the specified initial capacity. When the vector needs to increase its capacity, the capacity doubles.

```
public boolean add(T newEntry)
```

Adds a new entry to the end of this vector.

```
public T remove(int index)
```

Removes and returns the entry at the given index in this vector.

```
public void clear()
```

Removes all entries from this vector.

```
public T lastElement()
```

Returns the entry at the end of this vector.

```
public boolean isEmpty()
```

Returns true if this vector is empty.

```
public int size()
```

Returns the number of entries currently in this vector.

You can learn more about Vector at download.oracle.com/javase/7/docs/api/.

Note: Java Class Library: The class Vector
The Java Class Library has the class Vector in the package java.util. A vector is analogous to a resizable array in that its elements are indexed beginning with 0. You work with a vector by using its methods.

Using a Vector to Implement the ADT Stack

6.15 Using a vector to contain a stack's entries is like using an array, but easier. We let the first element of the vector reference the bottom entry of the stack. Thus, the vector looks like the array in Figure 6-4b. We do not need to maintain an index to the top entry of the stack, however, as we can infer this index from the vector's size, which is readily available. Also, the vector expands as necessary, so we do not have to worry about this detail.

Since the implementation of Vector is based on an array that can be resized dynamically, the performance of this implementation of the stack is like that of the array-based implementation given in the previous section.

Note: If you use a vector to implement a stack, the vector's first element should reference the stack's bottom entry. Then the last occupied location in the vector references the stack's top entry.

6.16 **An outline of the class.** The class that implements the stack begins by declaring a vector as a data field and allocating the vector in its constructors. Thus, we must provide an `import` statement prior to the class definition. Listing 6-3 outlines our class.

LISTING 6-3 An outline of a vector-based implementation of the ADT stack

```
import java.util.Vector;
/**
    A class of stacks whose entries are stored in a vector.
    @author Frank M. Carrano
*/
public class VectorStack<T> implements StackInterface<T>
{
   private Vector<T> stack; // last element is the top entry in stack
   private static final int DEFAULT_INITIAL_CAPACITY = 50;

   public VectorStack()
   {
      this(DEFAULT_INITIAL_CAPACITY);
   } // end default constructor

   public VectorStack(int initialCapacity)
   {
      stack = new Vector<T>(initialCapacity);// size doubles as needed
   } // end constructor

   < Implementations of the stack operations go here. >
   . . .
} // end VectorStack
```

6.17 **Adding to the top.** We use `Vector`'s method `add` to add an entry to the end of the vector, that is, to the top of the stack.

```
public void push(T newEntry)
{
   stack.add(newEntry);
} // end push
```

6.18 **Retrieving the top.** We retrieve the stack's top entry by using `Vector`'s method `lastElement`.

```
public T peek()
{
   T top = null;

   if (!isEmpty())
      top = stack.lastElement();

   return top;
} // end peek
```

6.19 **Removing the top.** We can remove the stack's top entry by using `Vector`'s method `remove`. The argument to this method is the index of the last entry in the vector, since that entry is at the top of the stack. This index is 1 less than the vector's current size `stack.size()`.

```
public T pop()
{
   T top = null;

   if (!isEmpty())
      top = stack.remove(stack.size() - 1);

   return top;
} // end pop
```

6.20 **The rest of the class.** The remaining public methods `isEmpty` and `clear` invoke analogous `Vector` methods:

```
public boolean isEmpty()
{
   return stack.isEmpty();
} // end isEmpty

public void clear()
{
   stack.clear();
} // end clear
```

SELF-TEST

Question 9 If a vector contains the entries in a stack, is it reasonable to maintain the stack's top entry in the vector's first element?

Design Decision: Should `VectorStack` extend `Vector`?

The previous implementation of `VectorStack` contains an instance of `Vector`. Suppose that we instead used inheritance to derive `VectorStack` from `Vector`. The resulting class would have all the methods of `Vector` in addition to those in `StackInterface`. However, these `Vector` methods would allow a client to add or remove entries anywhere within the stack, thus violating the premise of the ADT stack. Instead of a stack, we would have an enhanced vector. But a stack is not a vector. Since we do not have an *is-a* relationship between stacks and vectors, we should not use inheritance to define `VectorStack`.

The class `java.util.Stack` in the Java Class Library that we introduced in Segment 5.23 of the previous chapter does extend `Vector`. Thus, an instance of `Stack` is not really a stack.

Chapter Summary

- You can implement a stack by using a chain of linked nodes that has only a head reference. The stack operations execute fastest if the first node in the chain references the stack's top entry. This is true because you can add, remove, or access a chain's first node faster than any other node.
- The stack operations are O(1) for a linked implementation.
- You can implement a stack by using an array. If the first location in the array contains the stack's bottom entry, no array elements will be moved when you add or remove stack entries.
- Resizing an array avoids a stack that is too full to accept another entry. However, the array generally contains locations that are unused.

- The stack operations are O(1) for an array-based implementation. However, when the array is full, `push` doubles the size of the array. In that case, `push` is O(n). If you spread this extra cost over all other pushes, and if doubling the array is not frequent, `push` is almost O(1).
- You can implement a stack by using a vector. You maintain the stack's bottom entry at the beginning of the vector.
- Since the implementation of `Vector` is based on an array that can be resized dynamically, the performance of a vector-based implementation is like that of the array-based implementation.

Exercises

1. Discuss the advantages and disadvantages of an array-based implementation of the ADT stack as compared to a linked implementation.

2. Consider the ADT bag, as described in Chapters 1 through 3.

   a. Would you be able to implement the ADT stack by using a bag to contain its entries? Justify your answer.
   b. Would you be able to implement the ADT bag by using a stack to contain its entries? Justify your answer.

3. Suppose that the ADT stack included the void method `display`, which displays the entries in a stack. Implement this method for each of the following classes:

   a. `LinkedStack`, as outlined in Listing 6-1.
   b. `ArrayStack`, as outlined in Listing 6-2.
   c. `VectorStack`, as outlined in Listing 6-3.
   d. Any client of `LinkedStack`, `ArrayStack`, or `VectorStack`.

4. Repeat the previous exercise, but define the method `toArray` instead of the method `display`.

5. Suppose that the ADT stack included a void method `remove(n)` that removes the topmost `n` entries from a stack. Specify this method by writing comments and a header. Consider the possible outcomes for stacks that do not contain `n` entries.

6. Repeat Exercise 3, but define the method `remove(n)`, as described in the previous exercise, instead of the method `display`.

7. Imagine a linked implementation of the ADT stack that places the top entry of the stack at the end of a chain of linked nodes. Describe how you can define the stack operations `push`, `pop`, and `peek` so that they do not traverse the chain.

8. Segment 6.9 noted that an array-based `push` method is normally O(1), but when a stack needs to be doubled in size, `push` is O(n). This observation is not as bad as it seems, however. Suppose that you double the size of a stack from n elements to $2n$ elements.

   a. How many calls to `push` can you make before the stack must double in size again?
   b. Remembering that each of these calls to `push` is O(1), what is the average cost of all the `push` operations? (The average cost is the total cost of all calls to `push` divided by the number of calls to `push`.)

9. Suppose that instead of doubling the size of an array-based stack when it becomes full, you just increase the size of the array by some positive constant k.

   a. If you have an empty stack that uses an array whose initial size is k, and you perform n pushes, how many resize operations will be performed? Assume that $n > k$.
   b. What is the average cost of the n push operations?

10. Suppose that instead of doubling the size of an array-based stack when it becomes full, you increase the size of the array by the following sequence 3*k*, 5*k*, 7*k*, 9*k*, ... for some positive constant *k*.

    a. If you have an empty stack that uses an array whose initial size is *k*, and you perform *n* pushes, how many resize operations will be performed? Assume that $n > k$.

    b. What is the average cost of the *n* push operations?

11. When an array becomes full, you can double its size or use one of the schemes described in Exercises 9 and 10. What are the advantages and disadvantages of each of these three schemes?

12. Imagine several stack operations on an array-based stack. Suppose that the array doubles in size, but later fewer than half of the array's locations are actually used by the stack. Describe an implementation that halves the size of the array in this case. What are the advantages and disadvantages of such an implementation?

Projects

1. Implement the ADT stack by using an array `stack` to contain its entries. Expand the array dynamically, as necessary. Maintain the stack's bottom entry in `stack[stack.length - 1]`.

2. Repeat Project 1, but maintain the stack's top entry in `stack[stack.length - 1]`.

3. Repeat Project 1, but maintain the stack's top entry in `stack[0]`.

4. Write the implementation of the ADT stack that Exercise 7 describes.

5. The ADT stack lets you peek at its top entry without removing it. For some applications of stacks, you also need to peek at the entry beneath the top entry without removing it. We will call such an operation `peek2`. If the stack has more than one entry, `peek2` returns the second entry from the top without altering the stack. If the stack has fewer than two entries, `peek2` returns `null`. Write a linked implementation of a stack that includes a method `peek2`.

6. When the client attempts to either retrieve or remove an item from an empty stack, our stacks return `null`. An alternative action is to throw an exception.

    a. Modify the interface `StackInterface` so that an `EmptyStackException` is thrown in these cases. (This exception is defined in the package `java.util`.)

    b. Modify the array-based implementation of the stack to conform to your changes to `StackInterface`. Write a program that demonstrates the modifications.

    c. Repeat Part *b* for the linked implementation of the stack.

7. Suppose that we wish to implement the resizing schemes described in Exercises 9 and 10 in addition to the doubling scheme.

    a. Write a new version of the array-based stack that lets the client specify the resizing scheme and the associated constant when a stack is created.

    b. Write a program that demonstrates the modifications.

    c. Discuss the advantages and disadvantages of adding methods that allow the client to change the resize scheme and constant after the stack has been created.

8. Implement the ADT bag by using a vector to contain its entries.

Answers to Self-Test Questions

1.
```
public T pop()
{
   T top = null;

   if (topNode != null)
   {
      top = topNode.getData();
      topNode = topNode.getNextNode();
   } // end if

   return top;
} // end pop
```

2. No. Although maintaining an external reference to the chain's last node, in addition to the chain's head reference, would enable you to either access the stack's top entry or push a new entry onto the stack efficiently, it is not enough to pop the stack. You need a reference to the next-to-last node to remove the chain's last node. To get that reference, you could either traverse the chain or maintain a reference to the next-to-last node in addition to references to the first and last nodes. Thus, placing the stack's top entry at the end of the chain is not as efficient or easy to implement as placing it at the beginning.

3.
```
public T pop()
{
   T top = peek();

   if (!isEmpty()) // or (top != null)
   {
      stack[topIndex] = null;
      topIndex--;
   } // end if

   return top;
} // end pop
```

4. Change `T` to the primitive type and do not assign `null` to `stack[topIndex]`.

5. Each `push` or `pop` would need to move all of the entries currently in the stack.

6. The bottom entry. You can then push entries onto the stack without moving the other entries already in the array.

7.
```
public void clear()
{
   for (; topIndex > -1; topIndex--)
      stack[topIndex] = null;
// Assertion: topIndex is -1
} // end clear
```

8.
```
public void clear()
{
   while (!isEmpty())
      pop();
// Assertion: topIndex is -1
} // end clear
```

9. No. Since `Vector` uses an array to store a vector's entries, each `push` would need to move all of the entries in the vector to vacate its first location, thus making room for the addition to the stack.

This page intentionally left blank

Chapter 7

Recursion

Contents

Prerequisites

Appendix	B	Java Classes
Chapter	2	Bag Implementations That Use Arrays
Chapter	3	A Bag Implementation That Links Data
Chapter	4	The Efficiency of Algorithms
Chapter	5	Stacks

Objectives

After studying this chapter, you should be able to

- Decide whether a given recursive method will end successfully in a finite amount of time
- Write a recursive method
- Estimate the time efficiency of a recursive method
- Identify tail recursion and replace it with iteration

Repetition is a major feature of many algorithms. In fact, repeating actions rapidly is a key ability of computers. Two problem-solving processes involve repetition; they are called iteration and recursion. In fact, most programming languages provide two kinds of repetitive constructs, iterative and recursive.

You know about iteration because you know how to write a loop. Regardless of the loop construct you use—`for`, `while`, or `do`—your loop contains the statements that you want to repeat and a mechanism for controlling the number of repetitions. You might have a counted loop that counts repetitions as 1, 2, 3, 4, 5, or 5, 4, 3, 2, 1. Or the loop might execute repeatedly while a boolean variable or expression is true. Iteration often provides a straightforward and efficient way to implement a repetitive process.

At times, iterative solutions are elusive or hopelessly complex. For some problems, discovering or verifying such solutions is not a simple task. In these cases, recursion can provide an elegant alternative. Some recursive solutions can be the best choice, some provide insight for finding a better iterative solution, and some should not be used at all because they are grossly inefficient. Recursion, however, remains an important problem-solving strategy.

This chapter will show you how to think recursively.

What Is Recursion?

7.1 You can build a house by hiring a contractor. The contractor in turn hires several subcontractors to complete portions of the house. Each subcontractor might hire other subcontractors to help. You use the same approach when you solve a problem by breaking it into smaller problems. In one special variation of this problem-solving process, the smaller problems are identical except for their size. This special process is called **recursion**.

VideoNote
Introducing recursion

Suppose that you can solve a problem by solving an identical but smaller problem. How will you solve the smaller problem? If you use recursion again, you will need to solve an even smaller problem that is just like the original problem in every other respect. How will replacing a problem with another one ever lead to a solution? One key to the success of recursion is that eventually you will reach a smaller problem whose solution you know because either it is obvious or it is given. The solution to this smallest problem is probably not the solution to your original problem, but it can help you reach it. Either just before or just after you solve a smaller problem, you usually contribute a portion of the solution. This portion, together with the solutions to the other, smaller problems, provides the solution to the larger problem.

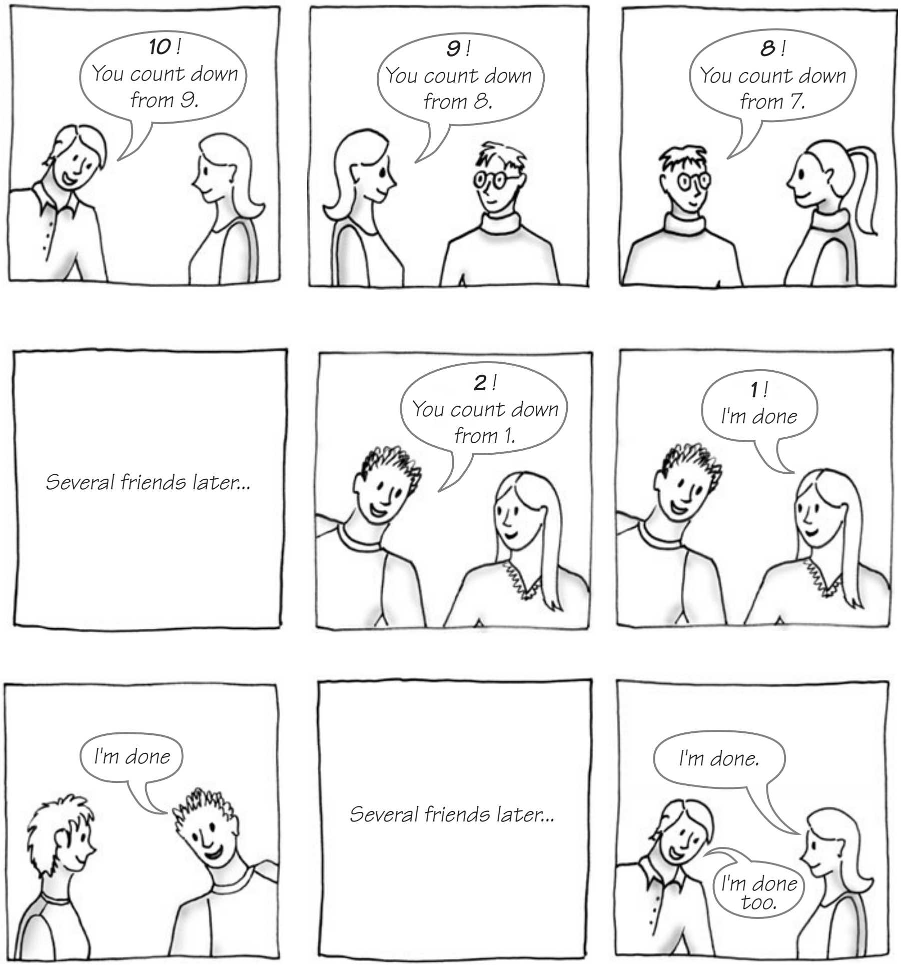
Let's look at an example.

7.2 **Example: The countdown.** It's New Year's Eve and the giant ball is falling in Times Square. The crowd counts down the last 10 seconds: "10, 9, 8, . . ." Suppose that I ask you to count down to 1 beginning at some positive integer like 10. You could shout "10" and then ask a friend to count down from 9. Counting down from 9 is a problem that is exactly like counting down from 10, except that there is less to do. It is a smaller problem.

To count down from 9, your friend shouts "9" and asks a friend to count down from 8. This sequence of events continues until eventually someone's friend is asked to count down from 1. That friend simply shouts "1." No other friend is needed. You can see these events in Figure 7-1.

In this example, I've asked you to complete a task. You saw that you could contribute a part of the task and then ask a friend to do the rest. You know that your friend's task is just like the original task, but it is smaller. You also know that when your friend completes this smaller task, your job will be done. What is missing from the process just described is the signal that each friend gives to the previous person at the completion of a task.

FIGURE 7-1 Counting down from 10


10 ! You count down from 9.
9 ! You count down from 8.
8 ! You count down from 7.
Several friends later...
2 ! You count down from 1.
1 ! I'm done
I'm done
Several friends later...
I'm done.
I'm done too.

To provide this signal, when you count down from 10, I need you to tell me when you are done. I don't care how—or who—does the job, as long as you tell me when it is done. I can take a nap until I hear from you. Likewise, when you ask a friend to count down from 9, you do not care how your friend finishes the job. You just want to know when it is done so you can tell me that you are done. You can take a nap while you are waiting.

Note: Recursion is a problem-solving process that breaks a problem into identical but smaller problems.

Ultimately, we have a group of napping people waiting for someone to say "I'm done." The first person to make that claim is the person who shouts "1," as Figure 7-1 illustrates, since that person needs no help in counting down from 1. At this time in this particular example, the problem is solved, but I don't know that because I'm still asleep. The person who shouted "1" says "I'm done" to the person who shouted "2." The person who shouted "2" says "I'm done" to the person who shouted "3," and so on, until you say "I'm done" to me. The job is done; thanks for your help; I have no idea how you did it, and I don't need to know!

7.3 What does any of this have to do with Java? In the previous example, you play the role of a Java method. I, the client, have asked you, the recursive method, to count down from 10. When you ask a friend for help, you are invoking a method to count down from 9. But you do not invoke another method; you invoke yourself!

Note: A method that calls itself is a **recursive method**. The invocation is a **recursive call** or **recursive invocation**.

The following Java method counts down from a given positive integer, displaying one integer per line.

```
/** Counts down from a given positive integer.
    @param integer  an integer > 0 */
public static void countDown(int integer)
{
   System.out.println(integer);
   if (integer > 1)
      countDown(integer - 1);
} // end countDown
```

Since the given integer is positive, the method can display it immediately. This step is analogous to you shouting "10" in the previous example. Next the method asks whether it is finished. If the given integer is 1, there is nothing left to do. But if the given integer is larger than 1, we need to count down from `integer` - 1. We've already noted that this task is smaller but otherwise identical to the original problem. How do we solve this new problem? We invoke a method, but `countDown` is such a method. It does not matter that we have not finished writing it at this point!

7.4 Will the method `countDown` actually work? Shortly we will trace the execution of `countDown` both to convince you that it works and to show you how it works. But traces of recursive methods are messy, and you usually do not have to trace them. If you follow certain guidelines when writing a recursive method, you can be assured that it will work.

In designing a recursive solution, you need to answer certain questions:

Note: **Questions to answer when designing a recursive solution**

- What part of the solution can you contribute directly?
- What smaller but identical problem has a solution that, when taken with your contribution, provides the solution to the original problem?
- When does the process end? That is, what smaller but identical problem has a known solution, and have you reached this problem, or **base case**?

For the method countDown, we have the following answers to these questions:

- The method countDown displays the given integer as the part of the solution that it contributes directly. This happens to occur first here, but it need not always occur first.
- The smaller problem is counting down from integer - 1. The method solves the smaller problem when it calls itself recursively.
- The if statement asks if the process has reached the base case. Here the base case occurs when integer is 1. Because the method displays integer before checking it, nothing is left to do once the base case is identified.

Note: Design guidelines for successful recursion

To write a recursive method that behaves correctly, you generally should adhere to the following design guidelines:

- The method must be given an input value, usually as an argument, but sometimes as a value read.
- The method definition must contain logic that involves this input value and leads to different cases. Typically, such logic includes an if statement or a switch statement.
- One or more of these cases should provide a solution that does not require recursion. These are the base cases, or **stopping cases**.
- One or more cases must include a recursive invocation of the method. These recursive invocations should in some sense take a step toward a base case by using "smaller" arguments or solving "smaller" versions of the task performed by the method.

Programming Tip: Infinite recursion

A recursive method that does not check for a base case, or that misses the base case, will execute "forever." This situation is known as infinite recursion.

7.5 Before we trace the method countDown, we should note that we could have written it in other ways. For example, a first draft of this method might have looked like this:

```
public static void countDown(int integer)
{
   if (integer == 1)
      System.out.println(integer);
   else
   {
      System.out.println(integer);
      countDown(integer - 1);
   } // end if
} // end countDown
```

Here, the programmer considered the base case first. The solution is clear and perfectly acceptable, but you might want to avoid the redundant println statement that occurs in both cases.

7.6 Removing the redundancy just mentioned could result in either the version given earlier in Segment 7.3 or the following one:

```
public static void countDown(int integer)
{
   if (integer >= 1)
   {
      System.out.println(integer);
      countDown(integer - 1);
   } // end if
} // end countDown
```

When `integer` is 1, this method will produce the recursive call `countDown(0)`. This turns out to be the base case for this method, and nothing is displayed.

All three versions of `countDown` produce correct results; there are probably others as well. Choose the one that is clearest to you.

7.7 The version of `countDown` just given in Segment 7.6 provides us an opportunity to compare it with the following iterative version:

```
// Iterative version.
public static void countDown(int integer)
{
   while (integer >= 1)
   {
      System.out.println(integer);
      integer--;
   } // end while
} // end countDown
```

The two methods have a similar appearance. Both compare `integer` with 1, but the recursive version uses an `if`, and the iterative version uses a `while`. Both methods display `integer`. Both compute `integer - 1`.

Programming Tip: An iterative method contains a loop. A recursive method calls itself. Although some recursive methods contain a loop *and* call themselves, if you have written a `while` statement within a recursive method, be sure that you did not mean to write an `if` statement.

Question 1 Write a recursive void method that skips *n* lines of output, where *n* is a positive integer. Use `System.out.println()` to skip one line.

Question 2 Describe a recursive algorithm that draws a given number of concentric circles. The innermost circle should have a given diameter. The diameter of each of the other circles should be four-thirds the diameter of the circle just inside it.

Tracing a Recursive Method

7.8 Now let's trace the method `countDown` given in Segment 7.3:

```
public static void countDown(int integer)
{
   System.out.println(integer);
   if (integer > 1)
      countDown(integer - 1);
} // end countDown
```

For simplicity, suppose that we invoke this method with the statement

```
countDown(3);
```

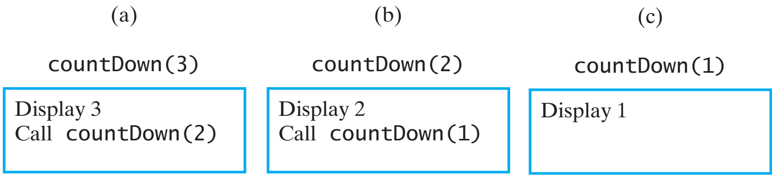
from within a `main` method of the class that defines `countDown`. This call behaves like any other call to a nonrecursive method. The argument 3 is copied into the parameter `integer` and the following statements are executed:

```
System.out.println(3);
if (3 > 1)
   countDown(3 - 1); // first recursive call
```

A line containing 3 is displayed, and the recursive call `countDown(2)` occurs, as Figure 7-2a shows.

Execution of the method is suspended until the results of `countDown(2)` are known. In this particular method definition, no statements appear after the recursive call. So although it appears that nothing will happen when execution resumes, it is here that the method returns to the client.

FIGURE 7-2 The effect of the method call `countDown(3)`



7.9 Continuing our trace, `countDown(2)` causes the following statements to execute:

```
System.out.println(2);
if (2 > 1)
   countDown(2 - 1); // second recursive call
```

A line containing 2 is displayed, and the recursive call `countDown(1)` occurs, as shown in Figure 7-2b. Execution of the method is suspended until the results of `countDown(1)` are known.

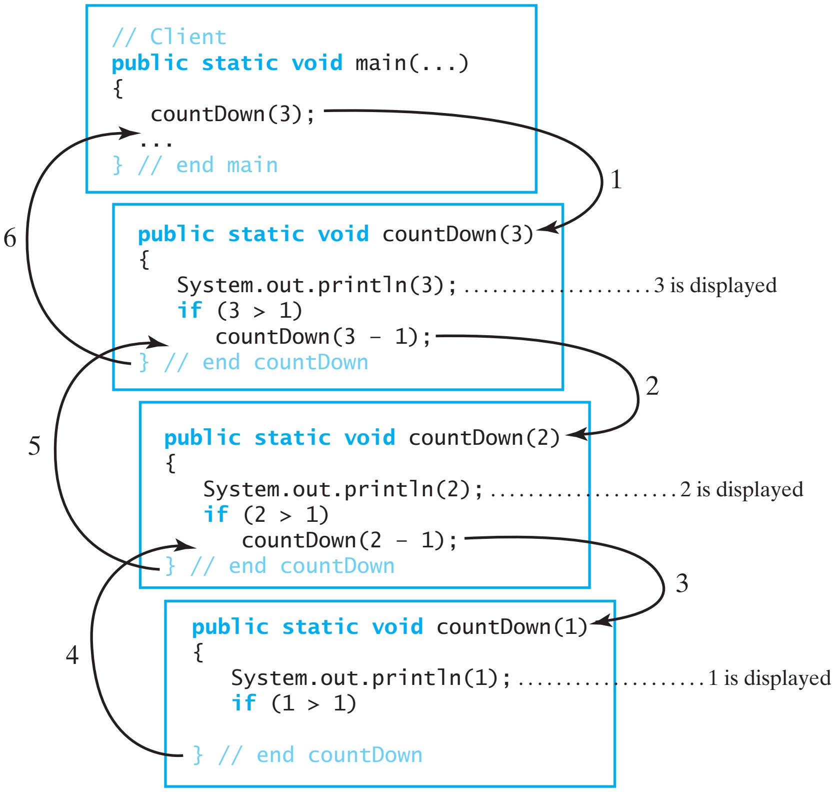
The call `countDown(1)` causes the following statements to execute:

```
System.out.println(1);
if (1 > 1)
```

A line containing 1 is displayed, as Figure 7-2c shows, and no other recursive call occurs.

Figure 7-3 illustrates the sequence of events from the time that `countDown` is first called. The numbered arrows indicate the order of the recursive calls and the returns from the method. After 1 is displayed, the method completes execution and returns to the point (arrow 4) after the call `countDown(2 - 1)`. Execution continues from there and the method returns to the point (arrow 5) after the call `countDown(3 - 1)`. Ultimately, a return to the point (arrow 6) after the initial recursive call in `main` occurs.

FIGURE 7-3 Tracing the recursive call `countDown(3)`



```
// Client
public static void main(...)
{
   countDown(3);
   ...
} // end main
```

```
public static void countDown(3)
{
   System.out.println(3);
   if (3 > 1)
      countDown(3 - 1);
} // end countDown
```

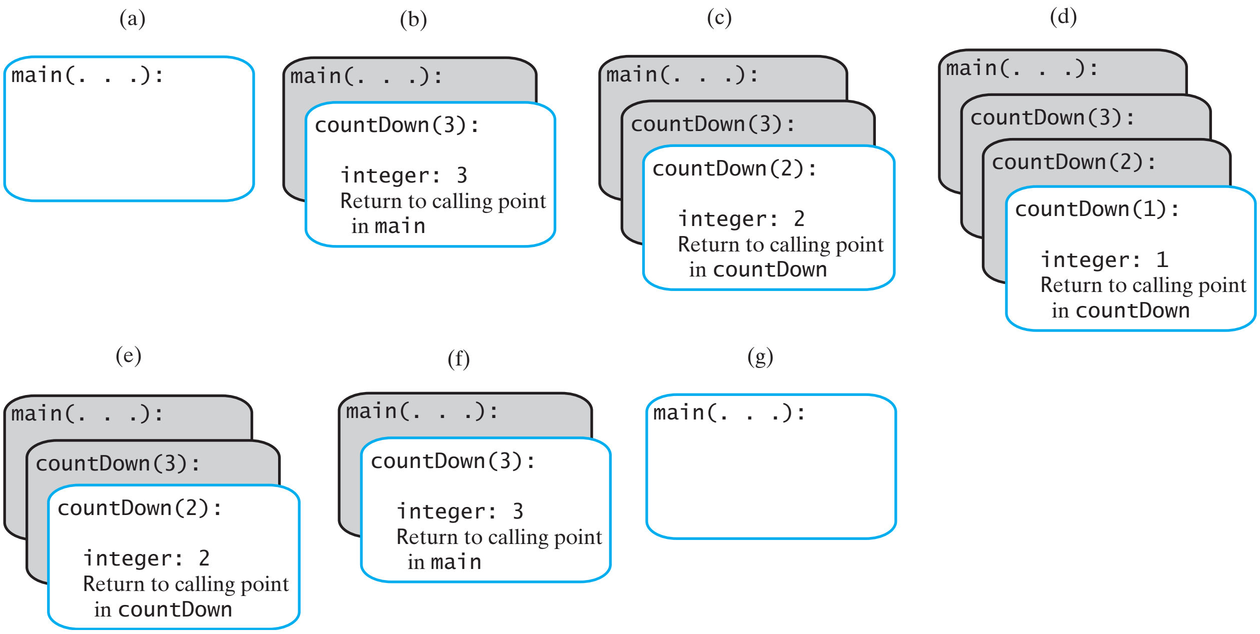
```
public static void countDown(2)
{
   System.out.println(2);
   if (2 > 1)
      countDown(2 - 1);
} // end countDown
```

```
public static void countDown(1)
{
   System.out.println(1);
   if (1 > 1)

} // end countDown
```

Although tracking these method returns seems like a formality that has gained us nothing, it is an important part of any trace because some recursive methods will do more than simply return to their calling method. You will see an example of such a method shortly.

7.10 Figure 7-3 appears to show multiple copies of the method `countDown`. In reality, however, multiple copies do not exist. Instead, for each call to a method—be it recursive or not—Java records the current state of the method's execution, including the values of its parameters and local variables as well as the location of the current instruction. As Segment 5.22 of Chapter 5 described, each record is called an activation record and provides a snapshot of a method's state during its execution. The records are placed into the program stack. The stack organizes the records chronologically, so that the record of the currently executing method is on top. In this way, Java can suspend the execution of a recursive method and invoke it again with new argument values. The boxes in Figure 7-3 correspond roughly to activation records, although the figure does not show them in the order in which they would appear in a stack. Figure 7-4 illustrates the stack of activation records as a result of the call `countDown(3)` in a main method.

FIGURE 7-4 The stack of activation records during the execution of the call `countDown(3)`



Note: The stack of activation records

Each call to a method generates an activation record that captures the state of the method's execution and that is placed into the program stack. Figure 5-13 in Chapter 5 illustrated the program stack when `methodA` calls the distinct method `methodB`. However, these methods need not be distinct. That is, the program stack enables a run-time environment to execute recursive methods. Each invocation of any method produces an activation record that is pushed onto the program stack. The activation record of a recursive method is not special in any way.

Note: A recursive method uses more memory than an iterative method, in general, because each recursive call generates an activation record.

Programming Tip: Stack overflow

A recursive method that makes many recursive calls will place many activation records in the program stack. Too many recursive calls can use all the memory available for the program stack, making it full. As a result, the error message "stack overflow" occurs. Infinite recursion or large-size problems are the likely causes of this error.

Question 3 Write a recursive void method `countUp(n)` that counts up from 1 to *n*, where *n* is a positive integer. *Hint*: A recursive call will occur before you display anything.

Recursive Methods That Return a Value

7.11 The recursive method `countDown` in the previous sections is a void method. Valued methods can also be recursive. The guidelines for successful recursion given in Segment 7.4 apply to valued methods as well, with an additional note. Recall that a recursive method must contain a statement such as an `if` that chooses among several cases. Some of these cases lead to a recursive call, but at least one case has no recursive call. For a valued method, each of these cases must provide a value for the method to return.

7.12 **Example: Compute the sum 1 + 2 + . . . + *n* for any integer *n* > 0.** The given input value for this problem is the integer *n*. Beginning with this fact will help us to find the smaller problem because its input will also be a single integer. The sum always starts at 1, so that can be assumed.

So suppose that I have given you a positive integer *n* and asked you to compute the sum of the first *n* integers. You need to ask a friend to compute the sum of the first *m* integers for some positive integer *m*. What should *m* be? Well, if your friend computes 1 + . . . + (*n* - 1), you can simply add *n* to that sum to get your sum. Thus, if `sumOf(n)` is the method call that returns the sum of the first *n* integers, adding *n* to your friend's sum occurs in the expression `sumOf(n-1) + n`.

What small problem can be the base case? That is, what value of *n* results in a sum that you know immediately? One possible answer is 1. If *n* is 1, the desired sum is 1.

With these thoughts in mind, we can write the following method:

```
/** @param n  an integer > 0
    @return the sum 1 + 2 + ... + n */
public static int sumOf(int n)
{
   int sum;
   if (n == 1)
      sum = 1;                // base case
   else
      sum = sumOf(n - 1) + n; // recursive call

   return sum;
} // end sumOf
```

7.13 The definition of the method `sumOf` satisfies the design guidelines for successful recursion. Therefore, you should be confident that the method will work correctly without tracing its execution. However, a trace will be instructive here because it will not only show you how a valued recursive method works, but also demonstrate actions that occur after a recursive call is complete.

Suppose that we invoke this method with the statement

```
System.out.println(sumOf(3));
```

The computation occurs as follows:

1. `sumOf(3)` is `sumOf(2)` + 3; `sumOf(3)` suspends execution, and `sumOf(2)` begins.
2. `sumOf(2)` is `sumOf(1)` + 2; `sumOf(2)` suspends execution, and `sumOf(1)` begins.
3. `sumOf(1)` returns 1.

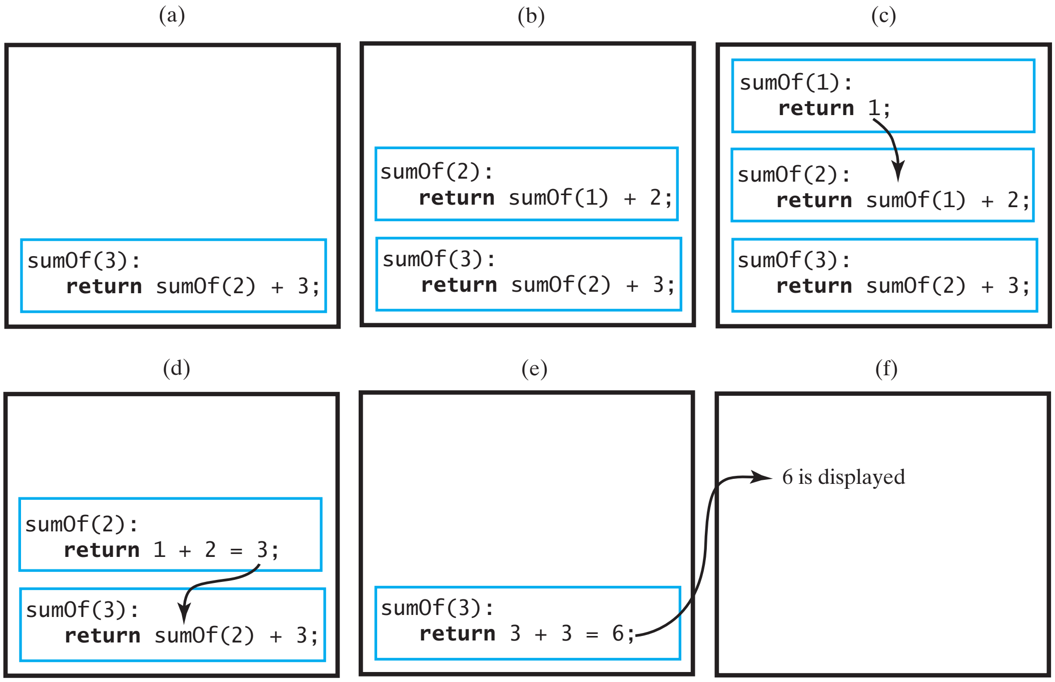
Once the base case is reached, the suspended executions resume, beginning with the most recent. Thus, `sumOf(2)` returns 1 + 2, or 3; then `sumOf(3)` returns 3 + 3, or 6. Figure 7-5 illustrates this computation.

Question 4 Write a recursive valued method that computes the product of the integers from 1 to *n*, where *n* > 0.

Note: Should you trace a recursive method?

We have shown you how to trace the execution of a recursive method primarily to show you how recursion works and to give you some insight into how a typical compiler implements recursion. Should you ever trace a recursive method? Usually no. You certainly should not trace a recursive method while you are writing it. If the method is incomplete, your trace will be, too, and you are likely to become confused. If a recursive method does not work, follow the suggestions given in the next programming tip. You should trace a recursive method only as a last resort.

FIGURE 7-5 Tracing the execution of `sumOf(3)`



Programming Tip: Debugging a recursive method

If a recursive method does not work, answer the following questions. Any “no” answers should guide you to the error.

- Does the method have at least one input value?
- Does the method contain a statement that tests an input value and leads to different cases?
- Did you consider all possible cases?
- Does at least one of these cases cause at least one recursive call?
- Do these recursive calls involve smaller arguments, smaller tasks, or tasks that get closer to the solution?
- If these recursive calls produce or return correct results, will the method produce or return a correct result?
- Is at least one of the cases a base case that has no recursive call?
- Are there enough base cases?
- Does each base case produce a result that is correct for that case?
- If the method returns a value, does each of the cases return a value?

7.14 Our previous examples were simple so that you could study the construction of recursive methods. Since you could have solved these problems iteratively with ease, should you actually use their recursive solutions? Nothing is inherently wrong with these recursive methods. However, given the way that typical present-day systems execute recursive methods, a stack overflow is likely for large values of *n*. Iterative solutions to these simple examples would not have this difficulty and are easy to write. Realize, however, that future computing systems might be able to execute these recursive methods without difficulty.

Recursively Processing an Array

VideoNote
Using recursion to solve problems

Later in this book we will talk about searching an array for a particular item. We will also look at algorithms that **sort**, or arrange, the items in an array into either ascending or descending order. Some of the more powerful searching and sorting algorithms often are stated recursively. In this section, we will process arrays recursively in ways that will be useful to us later. We have chosen a simple task—displaying the integers in an array—for our examples so that you can focus on the recursion without the distraction of the task. We will consider more-complex tasks later in this book and in the exercises at the end of this chapter.

7.15 Suppose that we have an array of integers and we want a method that displays it. So that we can display all or part of the array, the method will display the integers in the array elements whose indices range from `first` through `last`. Thus, we can declare the method as follows:

```
/** Displays the integers in an array.
    @param array  an array of integers
    @param first  the index of the first element displayed
    @param last   the index of the last element displayed,
                  0 <= first <= last < array.length */
public static void displayArray(int[] array, int first, int last)
```

This task is simple and could readily be implemented using iteration. You might not imagine, however, that we could also implement it recursively in a variety of ways. But we can and will.

7.16 **Starting with `array[first]`.** An iterative solution would certainly start at the first element, `array[first]`, so it is natural to have our first recursive method begin there also. If I ask you to display the array, you could display `array[first]` and then ask a friend to display the rest of the array. Displaying the rest of the array is a smaller problem than displaying the entire array. You wouldn't have to ask a friend for help if you had to display only one element—that is, if `first` and `last` were equal. This is the base case. Thus, we could write the method `displayArray` as follows:

```
public static void displayArray(int array[], int first, int last)
{
   System.out.print(array[first] + " ");
   if (first < last)
      displayArray(array, first + 1, last);
} // end displayArray
```

For simplicity, we assume that the integers will fit on one line. Notice that the client would follow a call to `displayArray` with `System.out.println()` to get to the next line.

7.17 **Starting with `array[last]`.** Strange as it might seem, we can begin with the last element in the array and still display the array from its beginning. Rather than displaying the last element right away, you would ask a friend to display the rest of the array. After the elements `array[first]` through `array[last - 1]` had been displayed, you would display `array[last]`. The resulting output would be the same as in the previous segment.

The method that implements this plan follows:

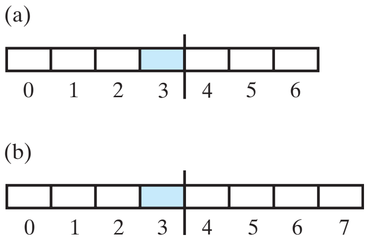
```
public static void displayArray(int array[], int first, int last)
{
   if (first <= last)
   {
      displayArray(array, first, last - 1);
      System.out.print (array[last] + " ");
   } // end if
} // end displayArray
```

7.18 **Dividing the array in half.** A common way to process an array recursively divides the array into two pieces. You then process each of the pieces separately. Since each of these pieces is an array that is smaller than the original array, each defines the smaller problem necessary for recursion. Our first two examples also divided the array into two pieces, but one of the pieces contained only one element. Here we divide the array into two approximately equal pieces. To divide the array, we find the element at or near the middle of the array. The index of this element is

```
int mid = (first + last) / 2;
```

Figure 7-6 shows two arrays and their middle elements. Suppose that we include `array[mid]` in the left "half "of the array, as the figure shows. In Part *b*, the two pieces of the array are equal in length; in Part *a* they are not. This slight difference in length doesn't matter.

FIGURE 7-6 Two arrays with their middle elements within their left halves



Once again, the base case is an array of one element. You can display it without help. But if the array contains more than one element, you divide it into halves. You then ask a friend to display one half and another friend to display the other half. These two friends, of course, represent two recursive calls in the following method:

```
public static void displayArray(int array[], int first, int last)
{
   if (first == last)
      System.out.print(array[first] + " ");
   else
   {
      int mid = (first + last) / 2;
      displayArray(array, first, mid);
      displayArray(array, mid + 1, last);
   } // end if
} // end displayArray
```

Question 5 In Segment 7.18, suppose that the array's middle element is not in either half of the array. Instead you can recursively display the left half, display the middle element, and then recursively display the right half. What is the implementation of `displayArray` if you make these changes?

Note: When you process an array recursively, you can divide it into two pieces. For example, the first or last element could be one piece, and the rest of the array could be the other piece. Or you could divide the array into halves or in some other way.

Note: **Finding an array's midpoint**

To compute the index of an array's middle element, we should use the statement

```
int mid = first + (last - first) / 2;
```

instead of

```
int mid = (first + last) / 2;
```

If we were to search an array of at least 2^{30}, or about one billion, elements, the sum of `first` and `last` could exceed the largest possible `int` value of $2^{31} – 1$. Thus, the computation `first + last` would overflow to a negative integer and result in a negative value for `mid`. If this negative value of `mid` was used as an array index, an `ArrayIndexOutOfBoundsException` would occur. The computation `first + (last - first)/2`, which is algebraically equivalent to `(first + last)/2`, avoids this error.

7.19 **Displaying a bag.** In Chapter 2, we used an array to implement the ADT bag. Suppose that a bag had a method `display` to display its contents. Although we could define this method iteratively, we'll use recursion instead. Since `display` has no parameters, it must call another method—`displayArray`—that has parameters and displays the array of bag entries. The arguments in the call to `displayArray` would be zero for the first index and `numberOfEntries - 1` for the last index, where `numberOfEntries` is a data field of the bag's class. Since the array, `bag`, of bag entries is a data field of the class that implements the bag, it need not be a parameter of `displayArray`. Finally, since `display` is not a static method, `displayArray` is not static.

We can use the approach of any version of `displayArray` given previously. However, we will display objects, one per line, instead of integers on one line. Using the technique shown in Segment 7.16, we revised the methods as follows:

```
public void display()
{
   displayArray(0, numberOfEntries - 1);
} // end display

private void displayArray(int first, int last)
{
   System.out.println(bag[first]);
   if (first < last)
      displayArray(first + 1, last);
} // end displayArray
```

Note: A recursive method that is part of an implementation of an ADT often is private, because its use requires knowledge of the underlying data structure. Such a method is unsuitable as an ADT operation.

Recursively Processing a Linked Chain

7.20 We can illustrate the recursive processing of a chain of linked nodes by performing a simple task such as displaying the data in the chain. Once again, we'll implement the method `display` for the ADT bag, but this time let's use the linked implementation introduced in Chapter 3. That implementation defines the field `firstNode` as a reference to the first node in the chain.

Dividing a linked chain into pieces is not as easy as dividing an array, since we cannot access any particular node without traversing the chain from its beginning. Hence, our first approach displays the data in the first node and then recursively displays the data in the rest of the chain. Thus, as it did in Segment 7.19, `display` will call a private recursive method. We will name that method `displayChain`. As a recursive method, `displayChain` needs an input value. That input should represent the chain, so we give `displayChain` a parameter that references the first node in the chain.

Suppose that we name `displayChain`'s parameter `nodeOne`. Then `nodeOne.getData()` is the data in the first node, and `nodeOne.getNextNode()` is a reference to the rest of the chain. What about the base case? Although a one-element array was a fine base case for `displayArray`, using an empty chain as the base case is easier here because we can simply compare `nodeOne` to `null`. Thus, we have the following implementations for the methods `display` and `displayChain`:

```
public void display()
{
   displayChain(firstNode);
} // end display

private void displayChain(Node nodeOne)
{
   if (nodeOne != null)
   {
      System.out.println(nodeOne.getData()); // display first node
      displayChain(nodeOne.getNextNode());   // display rest of chain
   } // end if
} // end displayChain
```

Note: When you write a method that processes a chain of linked nodes recursively, you use a reference to the chain's first node as the method's parameter. You then can process the first node followed by the rest of the chain.

7.21 **Displaying a chain backwards.** Suppose that you want to traverse a chain of linked nodes in reverse order. In particular, suppose that you want to display the object in the last node, then the one in the next-to-last node, and so on working your way toward the beginning of the chain. Since each node references the next node but not the previous one, using iteration for this task would be difficult. You could traverse to the last node, display its contents, go back to the beginning and traverse to the next-to-last node, and so on. Clearly, however, this is a tedious and time-consuming approach. Alternatively, you could traverse the chain once and save a reference to each node. You could then use these references to display the objects in the chain's nodes in reverse order. A recursive solution would do this for you.

If a friend could display the nodes in reverse order, beginning with the second node, you could display the first node and complete the task. The following recursive solution implements this idea:

```
public void displayBackward()
{
   displayChainBackward(firstNode);
} // end displayBackward

private void displayChainBackward(Node nodeOne)
{
   if (nodeOne != null)
   {
      displayChainBackward(nodeOne.getNextNode());
      System.out.println(nodeOne.getData());
   } // end if
} // end displayChainBackward
```

Note: Traversing a chain of linked nodes in reverse order is easier when done recursively than iteratively.

SELF-TEST

Question 6 Trace the previous method `displayBackward` for a chain of three nodes.

The Time Efficiency of Recursive Methods

Chapter 4 showed you how to measure an algorithm's time requirement by using Big Oh notation. We used a count of the algorithm's major operations as a first step in selecting an appropriate growth-rate function. For the iterative examples we examined, that process was straightforward. We will use a more formal technique here to measure the time requirement of a recursive algorithm and thereby choose the right growth-rate function.

The Time Efficiency of `countDown`

7.22 As a first example, consider the `countDown` method given in Segment 7.3. The size of the problem of counting down to 1 from a given integer is directly related to the size of that integer. Since Chapter 4 used *n* to represent the size of the problem, we will rename the parameter `integer` in `countDown` to `n` to simplify our discussion. Here is the revised method:

```
public static void countDown(int n)
{
   System.out.println(n);
   if (n > 1)
      countDown(n - 1);
} // end countDown
```

When n is 1, `countDown` displays 1. This is the base case and requires a constant amount of time. When $n > 1$, the method requires a constant amount of time for both the `println` statement and the comparison. In addition, it needs time to solve the smaller problem represented by the recursive call. If we let $t(n)$ represent the time requirement of `countDown(n)`, we can express these observations by writing

$t(1) = 1$
$t(n) = 1 + t(n - 1)$ for $n > 1$

The equation for $t(n)$ is called a **recurrence relation**, since the definition of the function t contains an occurrence of itself—that is, a recurrence. What we need is an expression for $t(n)$ that is not given in terms of itself. One way to find such an expression is to pick a value for n and to write out the equations for $t(n)$, $t(n - 1)$, and so on, until we reach $t(1)$. From these equations, we should be able to guess at an appropriate expression to represent $t(n)$. We then need only to prove that we are right. This is actually easier than it sounds.

7.23 **Solving a recurrence relation.** To solve the previous recurrence relation for $t(n)$, let's begin with $n = 4$. We get the following sequence of equations:

$t(4) = 1 + t(3)$
$t(3) = 1 + t(2)$
$t(2) = 1 + t(1) = 1 + 1 = 2$

Substituting 2 for $t(2)$ in the equation for $t(3)$ results in

$t(3) = 1 + 2 = 3$

Substituting 3 for $t(3)$ in the equation for $t(4)$ results in

$t(4) = 1 + 3 = 4$

It appears that

$t(n) = n$ for $n \geq 1$

We can start with a larger value of n, get the same result, and convince ourselves that it is true. But we need to *prove* that this result is true for every $n \geq 1$. This is not hard to do.

7.24 **Proving that $t(n) = n$.** To prove that $t(n) = n$ for $n \geq 1$, we begin with the recurrence relation for $t(n)$, since we know it is true:

$t(n) = 1 + t(n - 1)$ for $n > 1$

We need to replace $t(n - 1)$ on the right side of the equation. Now if $t(n - 1) = n - 1$ when $n > 1$, the following would be true for $n > 1$:

$t(n) = 1 + n - 1 = n$

Thus, if we can find an integer k that satisfies the equation $t(k) = k$, the next larger integer will also satisfy it. By a similar chain of reasoning, the equation is true for all integers larger than k. Since we are given that $t(1) = 1$, all integers larger than 1 will satisfy the equation. This proof is an example of a **proof by induction**.

To conclude, we now know that `countDown`'s time requirement is given by the function $t(n) = n$. Thus, the method is $O(n)$.

Question 7 What is the Big Oh of the method `sumOf` given in Segment 7.12?

Question 8 Computing x^n for some real number x and an integral power $n \geq 0$ has a simple recursive solution:

$$x^n = x\, x^{n - 1}$$
$$x^0 = 1$$

a. What recurrence relation describes this algorithm's time requirement?

b. By solving this recurrence relation, find the Big Oh of this algorithm.

The Time Efficiency of Computing x^n

7.25 We can compute x^n for some real number x and an integral power $n \geq 0$ more efficiently than the approach that Question 8 suggests. To reduce the number of recursive calls and therefore the number of multiplications, we can express x^n as follows:

$$x^n = (x^{n/2})^2 \text{ when } n \text{ is even and positive}$$
$$x^n = x\,(x^{(n - 1)/2})^2 \text{ when } n \text{ is odd and positive}$$
$$x^0 = 1$$

This computation could be implemented by a method `power(x, n)` that contains the recursive call `power(x, n/2)`. Since integer division in Java truncates its result, this call is appropriate regardless of whether n is even or odd. Thus, `power(x, n)` would invoke `power(x, n/2)` once, square the result, and, if n is odd, multiply the square by x. These multiplications are O(1) operations. Thus, the execution time for `power(x, n)` is proportional to the number of recursive calls.

The recurrence relation that represents the number of recursive calls and, therefore, the method's time requirement to compute x^n is then

$$t(n) = 1 + t(n/2) \text{ when } n \geq 2$$
$$t(1) = 1$$
$$t(0) = 1$$

Again, $n/2$ truncates to an integer.

7.26 Since the recurrence relation involves $n/2$, let's choose a power of 2—such as 16—as n's initial value. We then write the following sequence of equations:

$$t(16) = 1 + t(8)$$
$$t(8) = 1 + t(4)$$
$$t(4) = 1 + t(2)$$
$$t(2) = 1 + t(1)$$

By substituting repeatedly, we get the following:

$$t(16) = 1 + t(8) = 1 + (1 + t(4)) = 2 + (1 + t(2)) = 3 + (1 + t(1)) = 4 + t(1)$$

Since $16 = 2^4$, $4 = \log_2 16$. This fact, together with the base case $t(1) = 1$, leads us to guess that

$$t(n) = 1 + \log_2 n$$

7.27 Now we need to prove that this guess is, in fact, true for $n \geq 1$. It is true for $n = 1$, because

$$t(1) = 1 + \log_2 1 = 1$$

For $n > 1$, we know that the recurrence relation for $t(n)$

$$t(n) = 1 + t(n/2)$$

is true. Remember that $n/2$ truncates to an integer.

We need to replace $t(n/2)$. If our guess $t(n) = 1 + \log_2 n$ were true for all values of $n < k$, we would have $t(k/2) = 1 + \log_2 (k/2)$, since $k/2 < k$. Thus,

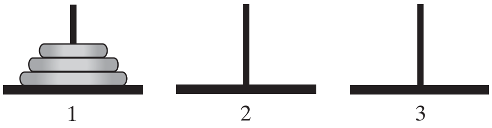
$$\begin{aligned} t(k) &= 1 + t(k/2) \\ &= 1 + (1 + \log_2 (k/2)) \\ &= 2 + \log_2 (k/2) \\ &= \log_2 4 + \log_2 (k/2) \\ &= \log_2 (4k/2) \\ &= \log_2 (2k) \\ &= \log_2 2 + \log_2 k \\ &= 1 + \log_2 k \end{aligned}$$

To summarize, we assumed that $t(n) = 1 + \log_2 n$ for all values of $n < k$ and showed that $t(k) = 1 + \log_2 k$. Thus, $t(n) = 1 + \log_2 n$ for all $n \geq 1$. Since `power`'s time requirement is given by $t(n)$, the method is O(log n).

A Simple Solution to a Difficult Problem

7.28 The Towers of Hanoi is a classic problem in computer science whose solution is not obvious. Imagine three poles and a number of disks of varying diameters. Each disk has a hole in its center so that it can fit over each of the poles. Suppose that the disks have been placed on the first pole in order from largest to smallest, with the smallest disk on top. Figure 7-7 illustrates this initial configuration for three disks.

FIGURE 7-7 The initial configuration of the Towers of Hanoi for three disks.



The problem is to move the disks from the first pole to the third pole so that they remain piled in their original order. But you must adhere to the following rules:

1. Move one disk at a time. Each disk you move must be a topmost disk.
2. No disk may rest on top of a disk smaller than itself.
3. You can store disks on the second pole temporarily, as long as you observe the previous two rules.

7.29 The solution is a sequence of moves. For example, if three disks are on pole 1, the following sequence of seven moves will move the disks to pole 3, using pole 2 temporarily:

Move a disk from pole 1 to pole 3
Move a disk from pole 1 to pole 2
Move a disk from pole 3 to pole 2
Move a disk from pole 1 to pole 3
Move a disk from pole 2 to pole 1
Move a disk from pole 2 to pole 3
Move a disk from pole 1 to pole 3

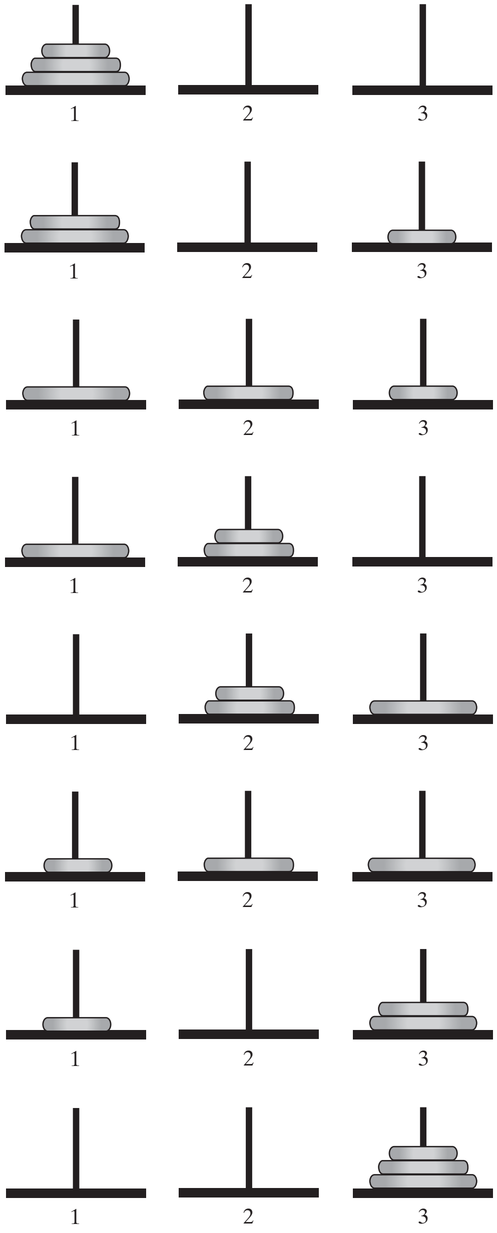
Figure 7-8 illustrates these moves.

? SELF-TEST

Question 9 We discovered the previous solution for three disks by trial and error. Using the same approach, find a sequence of moves that solves the problem for four disks.

With four disks, the problem's solution requires 15 moves, so it is somewhat difficult to find by trial and error. With more than four disks, the solution is much more difficult to discover. What we need is an algorithm that produces a solution for any number of disks. Even though discovering a solution by trial and error is hard, finding a recursive algorithm to produce the solution is fairly easy.

FIGURE 7-8 The sequence of moves for solving the Towers of Hanoi problem with three disks



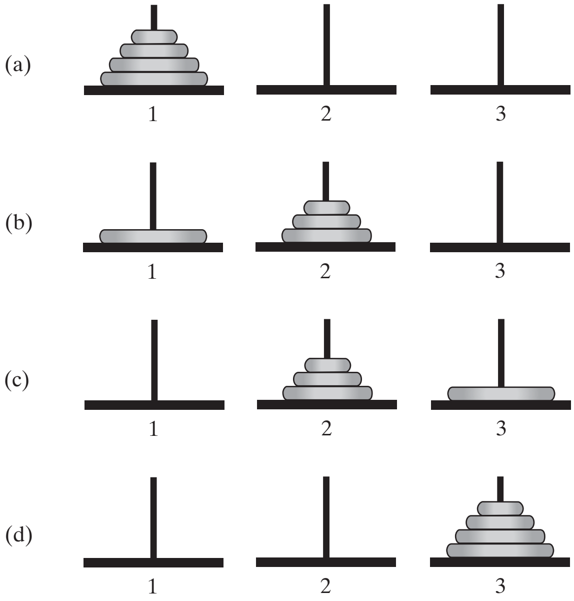
Note: Invented in the late 1800s, the Towers of Hanoi problem was accompanied by this legend. A group of monks was said to have begun moving 64 disks from one tower to another. When they finish, the world will end. When you finish reading this section, you will realize that the monks—or their successors—could not have finished yet. By the time they do, it is quite plausible that the disks, if not the world, will have worn out!

7.30 A recursive algorithm solves a problem by solving one or more smaller problems of the same type. The problem size here is simply the number of disks. So imagine that the first pole has four disks, as in Figure 7-9a, and that I ask you to solve the problem. Eventually, you will need to move the bottom disk, but first you need to move the three disks on top of it. Ask a friend to move these three disks—a smaller problem—according to our rules, but make pole 2 the destination. Allow your friend to use pole 3 as a spare. Figure 7-9b shows the final result of your friend's work.

When your friend tells you that the task is complete, you move the one disk left on pole 1 to pole 3. Moving one disk is a simple task. You don't need help—or recursion—to do it. This disk is the largest one, so it cannot rest on top of any other disk. Thus, pole 3 must be empty before this move. After the move, the largest disk will be first on pole 3. Figure 7-9c shows the result of your work.

Now ask a friend to move the three disks on pole 2 to pole 3, adhering to the rules. Allow your friend to use pole 1 as a spare. When your friend tells you that the task is complete, you can tell me that your task is complete as well. Figure 7-9d shows the final results.

FIGURE 7-9 The smaller problems in a recursive solution for four disks



7.31 Before we write some pseudocode to describe the algorithm, we need to identify a base case. If only one disk is on pole 1, we can move it directly to pole 3 without using recursion. With this as the base case, the algorithm is as follows:

```
Algorithm to move numberOfDisks disks from startPole to endPole using tempPole
    as a spare according to the rules of the Towers of Hanoi problem
if (numberOfDisks == 1)
    Move disk from startPole to endPole
else
{
    Move all but the bottom disk from startPole to tempPole
    Move disk from startPole to endPole
    Move all disks from tempPole to endPole
}
```

At this point, we can develop the algorithm further by writing

```
Algorithm solveTowers(numberOfDisks, startPole, tempPole, endPole)
if (numberOfDisks == 1)
    Move disk from startPole to endPole
else
{
    solveTowers(numberOfDisks - 1, startPole, endPole, tempPole)
    Move disk from startPole to endPole
    solveTowers(numberOfDisks - 1, tempPole, startPole, endPole)
}
```

If we choose zero disks as the base case instead of one disk, we can simplify the algorithm a bit, as follows:

```
Algorithm solveTowers(numberOfDisks, startPole, tempPole, endPole)
// Version 2
if (numberOfDisks > 0)
{
    solveTowers(numberOfDisks - 1, startPole, endPole, tempPole)
    Move disk from startPole to endPole
    solveTowers(numberOfDisks - 1, tempPole, startPole, endPole)
}
```

Although somewhat easier to write, the second version of the algorithm executes many more recursive calls. Both versions, however, make the same moves.

? SELF-TEST

Question 10 For two disks, how many recursive calls are made by each version of the algorithm just given?

Your knowledge of recursion should convince you that both forms of the algorithm are correct. Recursion has enabled us to solve a problem that appeared to be difficult. But is this algorithm efficient? Could we do better if we used iteration?

7.32 **Efficiency.** Let's look at the efficiency of our algorithm. How many moves occur when we begin with n disks? Let $m(n)$ denote the number of moves that `solveTowers` needs to solve the problem for n disks. Clearly,

$$m(1) = 1$$

For $n > 1$, the algorithm uses two recursive calls to solve problems that have $n - 1$ disks each. The required number of moves in each case is $m(n - 1)$. Thus, you can see from the algorithm that

$$m(n) = m(n - 1) + 1 + m(n - 1) = 2\ m(n - 1) + 1$$

From this equation, you can see that $m(n) > 2\ m(n - 1)$. That is, solving the problem with n disks requires more than twice as many moves as solving the problem with $n - 1$ disks.

It appears that $m(n)$ is related to a power of 2. Let's evaluate the recurrence for $m(n)$ for a few values of n:

$$m(1) = 1,\ m(2) = 3,\ m(3) = 7,\ m(4) = 15,\ m(5) = 31,\ m(6) = 63$$

It seems that

$$m(n) = 2^n - 1$$

We can prove this conjecture by using mathematical induction, as follows.

7.33 **Proof by induction that $m(n) = 2^n - 1$.** We know that $m(1) = 1$ and $2^1 - 1 = 1$, so the conjecture is true for $n = 1$. Now assume that it is true for $n = 1, 2, \ldots, k$, and consider $m(k + 1)$.

$$\begin{aligned} m(k + 1) &= 2\ m(k) + 1 && \text{(use the recurrence relation)} \\ &= 2\ (2^k - 1) + 1 && \text{(we assumed that } m(k) = 2^k - 1\text{)} \\ &= 2^{k+1} - 1 \end{aligned}$$

Since the conjecture is true for $n = k + 1$, it is true for all $n \geq 1$.

7.34 **Exponential growth.** The number of moves required to solve the Towers of Hanoi problem grows exponentially with the number of disks n. That is, $m(n) = O(2^n)$. This rate of growth is alarming, as you can see from the following values of 2^n:

$$\begin{aligned} 2^5 &= 32 \\ 2^{10} &= 1024 \\ 2^{20} &= 1{,}048{,}576 \\ 2^{30} &= 1{,}073{,}741{,}824 \\ 2^{40} &= 1{,}099{,}511{,}627{,}776 \\ 2^{50} &= 1{,}125{,}899{,}906{,}842{,}624 \\ 2^{60} &= 1{,}152{,}921{,}504{,}606{,}846{,}976 \end{aligned}$$

Remember the monks mentioned at the end of Segment 7.29? They are making $2^{64} - 1$ moves. It should be clear that you can use this exponential algorithm only for small values of n, if you want to live to see the results.

Before you condemn recursion and discard our algorithm, you need to know that you cannot do any better. Not you, not the monks, not anyone. We demonstrate this observation next by using mathematical induction.

7.35 **Proof that Towers of Hanoi cannot be solved in fewer than $2^n - 1$ moves.** We have shown that our algorithm for the Towers of Hanoi problem requires $m(n) = 2^n - 1$ moves. Since we know that at least one algorithm exists—we found one—there must be a fastest one. Let $M(n)$ represent the number of moves that this optimal algorithm requires for n disks. We need to show that $M(n) = m(n)$ for $n \geq 1$.

When the problem has one disk, our algorithm solves it in one move. We cannot do better, so we have that $M(1) = m(1) = 1$. If we assume that $M(n - 1) = m(n - 1)$, consider n disks. Looking back at Figure 7-9b, you can see that at one point in our algorithm the largest disk is isolated on one pole and $n - 1$ disks are on another. This configuration would have to be true of an optimal algorithm as well, for there is no other way to move the largest disk. Thus, the optimal algorithm must have moved these $n - 1$ disks from pole 1 to pole 2 in $M(n - 1) = m(n - 1)$ moves.

After moving the largest disk (Figure 7-9c), the optimal algorithm moves $n - 1$ disks from pole 2 to pole 3 in another $M(n - 1) = m(n - 1)$ moves. Altogether, the optimal algorithm makes at least $2\ M(n - 1) + 1$ moves. Thus,

$$M(n) \geq 2\ M(n - 1) + 1$$

Now apply the assumption that $M(n - 1) = m(n - 1)$ and then the recurrence for $m(n)$ given in Segment 7.32 to get

$$M(n) \geq 2\ m(n - 1) + 1 = m(n)$$

We have just shown that $M(n) \geq m(n)$. But since the optimal algorithm cannot require more moves than our algorithm, the expression $M(n) > m(n)$ cannot be true. Thus, we must have $M(n) = m(n)$ for all $n \geq 1$.

7.36 Finding an iterative algorithm to solve the Towers of Hanoi problem is not as easy as finding a recursive algorithm. We now know that any iterative algorithm will require at least as many moves as the recursive algorithm. An iterative algorithm will save the overhead—space and time—of tracking the recursive calls, but it will not really be more efficient than `solveTowers`. An algorithm that uses both iteration and recursion to solve the Towers of Hanoi problem is discussed in the section “Tail Recursion,” and an entirely iterative algorithm is the subject of Project 7 at the end of this chapter.

A Poor Solution to a Simple Problem

Some recursive solutions are so inefficient that you should avoid them. The problem that we will look at now is simple, occurs frequently in mathematical computations, and has a recursive solution that is so natural that you are likely to be tempted to use it. Don’t!

7.37 **Example: Fibonacci numbers.** Early in the 13th century, the mathematician Leonardo Fibonacci proposed a sequence of integers to model the number of descendants of a pair of rabbits. Later named the *Fibonacci sequence*, these numbers occur in surprisingly many applications.

The first two terms in the Fibonacci sequence are 1 and 1. Each subsequent term is the sum of the preceding two terms. Thus, the sequence begins as 1, 1, 2, 3, 5, 8, 13, . . . Typically, the sequence is defined by the equations

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \text{ when } n \geq 2$$

You can see why the following recursive algorithm would be a tempting way to generate the sequence:

```
Algorithm Fibonacci(n)
if (n <= 1)
   return 1
else
   return Fibonacci(n - 1) + Fibonacci(n - 2)
```

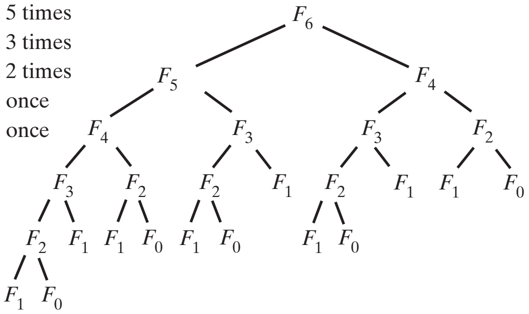
7.38 This algorithm makes two recursive calls. That fact in itself is not the difficulty. Earlier, you saw perfectly good algorithms—`displayArray` in Segment 7.18 and `solveTowers` in Segment 7.31—that make several recursive calls. The trouble here is that the same recursive calls are made repeatedly. A call to `Fibonacci(n)` invokes `Fibonacci(n - 1)` and then `Fibonacci(n - 2)`. But the call to `Fibonacci(n - 1)` has to compute `Fibonacci(n - 2)`, so the same Fibonacci number is computed twice.

Things get worse. The call to `Fibonacci(n - 1)` calls `Fibonacci(n - 3)` as well. The two previous calls to `Fibonacci(n - 2)` each invoke `Fibonacci(n - 3)`, so `Fibonacci(n - 3)` is computed three times. Figure 7-10a illustrates the dependency of F_6 on previous Fibonacci numbers and so indicates the number of times a particular number is computed repeatedly by the method `Fibonacci`. In contrast, Figure 7-10b shows that an iterative computation of F_6 computes each prior term once. The recursive solution is clearly less efficient. The next segments will show you just how inefficient it is.

FIGURE 7-10 The computation of the Fibonacci number F_6 using (a) recursion; (b) iteration

(a)

F_2 is computed 5 times
F_3 is computed 3 times
F_4 is computed 2 times
F_5 is computed once
F_6 is computed once



(b)

$F_0 = 1$
$F_1 = 1$
$F_2 = F_1 + F_0 = 2$
$F_3 = F_2 + F_1 = 3$
$F_4 = F_3 + F_2 = 5$
$F_5 = F_4 + F_3 = 8$
$F_6 = F_5 + F_4 = 13$

7.39 **The time efficiency of the algorithm `Fibonacci`.** We can investigate the efficiency of the `Fibonacci` algorithm by using a recurrence relation, as we did in Segments 7.22 through Segment 7.27. First, notice that F_n requires one add operation plus the operations that F_{n-1} and F_{n-2} require. So if $t(n)$ represents the time requirement of the algorithm in computing F_n, we have

$$t(n) = 1 + t(n - 1) + t(n - 2) \text{ for } n \geq 2$$
$$t(1) = 1$$
$$t(0) = 1$$

This recurrence relation looks like the recurrence for the Fibonacci numbers themselves. It should not surprise you then that $t(n)$ is related to the Fibonacci numbers. In fact, if you look at Figure 7-10a and count the occurrences of the Fibonacci numbers F_2 through F_6, you will discover a Fibonacci sequence.

To find a relationship between $t(n)$ and F_n, let's expand $t(n)$ for a few values of n:

$$t(2) = 1 + t(1) + t(0) = 1 + F_1 + F_0 = 1 + F_2 > F_2$$
$$t(3) = 1 + t(2) + t(1) > 1 + F_2 + F_1 = 1 + F_3 > F_3$$
$$t(4) = 1 + t(3) + t(2) > 1 + F_3 + F_2 = 1 + F_4 > F_4$$

We guess that $t(n) > F_n$ for $n \geq 2$. Notice that $t(0) = 1 = F_0$ and $t(1) = 1 = F_1$. These do not satisfy the strict inequality of our guess.

We now prove that our guess is indeed fact. (You can skip the proof on your first reading.)

7.40 **Proof by induction that $t(n) > F_n$ for $n \geq 2$.** Since the recurrence relation for $t(n)$ involves two recursive terms, we need two base cases. In the previous segment, we already showed that $t(2) > F_2$ and $t(3) > F_3$. Now if $t(n) > F_n$ for $n = 2, 3, \ldots, k$, we need to show that $t(k + 1) > F_{k+1}$. We can do this as follows:

$$t(k + 1) = 1 + t(k) + t(k - 1) > 1 + F_k + F_{k-1} = 1 + F_{k+1} > F_{k+1}$$

We can conclude that $t(n) > F_n$ for all $n \geq 2$.

Since we know that $t(n) > F_n$ for all $n \geq 2$, we can say that $t(n) = \Omega(F_n)$. Recall that the Big Omega notation means that $t(n)$ is at least as large as the Fibonacci number F_n. It turns out that we can compute F_n directly without using the recurrence relation given in Segment 7.37. It can be shown that

$$Fn = (a^n - b^n)/\sqrt{5}$$

where $a = (1 + \sqrt{5})/2$ and $b = (1 - \sqrt{5})/2$. Since $|1 - \sqrt{5}| < 2$, we have $|b| < 1$ and $|b^n| < 1$. Therefore, we have

$$Fn > (a^n - 1)/\sqrt{5}$$

Thus, $F_n = \Omega(a^n)$, and since we know that $t(n) = \Omega(F_n)$, we have $t(n) = \Omega(a^n)$. Some arithmetic shows that the previous expression for a equals approximately 1.6. We conclude that $t(n)$ grows exponentially with n. That is, the time required to compute F_n recursively increases exponentially as n increases.

7.41 At the beginning of this section, we observed that each Fibonacci number is the sum of the preceding two Fibonacci numbers in the sequence. This observation should lead us to an iterative solution that is O(n). (See Exercise 9 at the end of this chapter.) Although the clarity and simplicity of the recursive solution makes it a tempting choice, it is much too inefficient to use.

Programming Tip: Do not use a recursive solution that repeatedly solves the same problem in its recursive calls.

Question 11 If you compute the Fibonnaci number F_6 recursively, how many recursive calls are made, and how many additions are performed?

Question 12 If you compute the Fibonnaci number F_6 iteratively, how many additions are performed?

Tail Recursion

7.42 **Tail recursion** occurs when the last action performed by a recursive method is a recursive call. For example, the following method `countDown` from Segment 7.6 is tail recursive:

```
public static void countDown(int integer)
{
   if (integer >= 1)
   {
      System.out.println(integer);
      countDown(integer - 1);
   } // end if
} // end countDown
```

A method that implements the algorithm `Fibonacci` given in Segment 7.37 will not be tail recursive, even though a recursive call *appears* last in the method. A closer look reveals that the last *action* is an addition.

The tail recursion in a method simply repeats the method's logic with changes to parameters and variables. Thus, you can perform the same repetition by using iteration. Converting a tail-recursive method to an iterative one is usually a straightforward process. For example, let's see how to convert the recursive method `countDown` just given. First we replace the `if` statement with a `while` statement. Then, instead of the recursive call, we assign its argument `integer - 1` to the method's formal parameter `integer`. Doing so gives us the following iterative version of the method:

```
public static void countDown(int integer)
{
   while (integer >= 1)
   {
      System.out.println(integer);
      integer = integer - 1;
   } // end while
} // end countDown
```

This method is essentially the same as the iterative method given in Segment 7.7.

Because converting tail recursion to iteration is often uncomplicated, some languages other than Java automatically convert tail-recursive methods to iterative methods to save the overhead involved with recursion. Most of this overhead involves memory, not time. If you need to save space, you should consider replacing tail recursion with iteration.

7.43 **Example.** Let's replace the tail recursion in the second version of the algorithm `solveTowers` given in Segment 7.31:

```
Algorithm solveTowers(numberOfDisks, startPole, tempPole, endPole)
if (numberOfDisks > 0)
{
   solveTowers(numberOfDisks - 1, startPole, endPole, tempPole)
   Move disk from startPole to endPole
   solveTowers(numberOfDisks - 1, tempPole, startPole, endPole)
}
```

This algorithm contains two recursive calls. The second one is tail recursive, since it is the algorithm's last action. Thus, we could try replacing the second recursive call with appropriate assignment statements and use a loop to repeat the method's logic, including the first recursive call, as follows:

```
Algorithm solveTowers(numberOfDisks, startPole, tempPole, endPole)
while (numberOfDisks > 0)
{
   solveTowers(numberOfDisks - 1, startPole, endPole, tempPole)
   Move disk from startPole to endPole
   numberOfDisks = numberOfDisks - 1
   startPole = tempPole
   tempPole = startPole
   endPole = endPole
}
```

This isn't quite right, however. Obviously, assigning `endPole` to itself is superfluous. Assigning `tempPole` to `startPole` and then assigning `startPole` to `tempPole` destroys `startPole` but leaves `tempPole` unchanged. What we need to do is exchange `tempPole` and `startPole`. Let's look at what is really happening here.

The only instruction that actually moves disks is *Move disk from* `startPole` *to* `endPole`. This instruction moves the largest disk that is not already on `endPole`. The disk to be moved is at the bottom of a pole, so any disks that are on top of it need to be moved first. Those disks are moved by the first recursive call. If we want to omit the second recursive call, what would we need to do instead before repeating the first recursive call? We must make sure that `startPole` contains the disks that have not been moved to `endPole`. Those disks are on `tempPole` as a result of the first recursive call. Thus, we need to exchange the contents of `tempPole` and `startPole`.

Making these changes results in the following revised algorithm:

```
Algorithm solveTowers(numberOfDisks, startPole, tempPole, endPole)
while (numberOfDisks > 0)
{
   solveTowers(numberOfDisks - 1, startPole, endPole, tempPole)
   Move disk from startPole to endPole
   numberOfDisks--
   Exchange the contents of tempPole and startPole
}
```

This revised algorithm is unusual in that its loop contains a recursive call. The base case for this recursion occurs when numberOfDisks is zero. Even though the method does not contain an if statement, it does detect the base case, ending the recursive calls.

Note: In a tail-recursive method, the last action is a recursive call. This call performs a repetition that can be done by using iteration. Converting a tail-recursive method to an iterative one is usually a straightforward process.

Indirect Recursion

7.44 Some recursive algorithms make their recursive calls indirectly. For example, we might have the following chain of events: Method A calls Method B, Method B calls Method C, and Method C calls Method A. Such recursion—called **indirect recursion**—is more difficult to understand and trace, but it does arise naturally in certain applications.

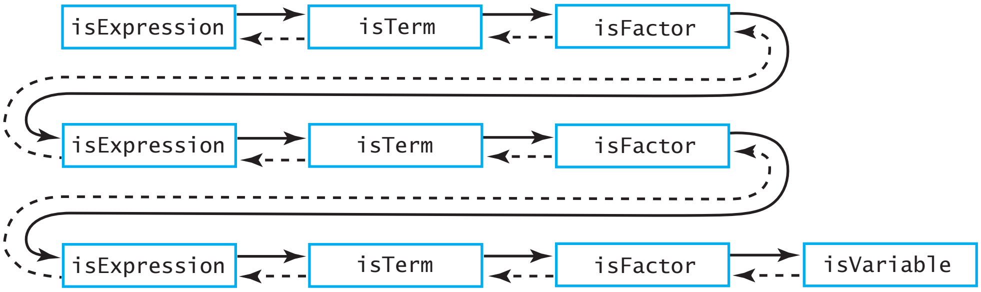
For example, the following rules describe strings that are valid algebraic expressions:

- An algebraic expression is either a term or two terms separated by a + or - operator.
- A term is either a factor or two factors separated by a * or / operator.
- A factor is either a variable or an algebraic expression enclosed in parentheses.
- A variable is a single letter.

Suppose that the methods isExpression, isTerm, isFactor, and isVariable detect whether a string is, respectively, an expression, a term, a factor, or a variable. The method isExpression calls isTerm, which in turn calls isFactor, which then calls isVariable and isExpression. Figure 7-11 illustrates these calls.

A special case of indirect recursion, where Method A calls Method B, and Method B calls Method A, is called **mutual recursion**. Project 10 at the end of this chapter describes an example of mutual recursion.

FIGURE 7-11 An example of indirect recursion



Using a Stack Instead of Recursion

7.45 One way to replace recursion with iteration is to simulate the program stack. In fact, we can implement a recursive algorithm by using a stack instead of recursion. As an example of converting a recursive method to an iterative one, we will consider the method `displayArray`, as given in Segment 7.18. For this demonstration, we will define `displayArray` as a nonstatic method in a class that has an array as a data field. With these changes, the method appears as follows:

```
public void displayArray(int first, int last)
{
   if (first == last)
      System.out.println(array[first] + " ");
   else
   {
      int mid = first + (last - first) / 2; // improved calculation of
                                            // midpoint
      displayArray(first, mid);
      displayArray(mid + 1, last);
   } // end if
} // end displayArray
```

7.46 We can replace the recursive method `displayArray` given in the previous segment with an iterative version by using a stack that mimics the program stack. To do so, we create a stack that is local to the method. We push objects onto this stack that are like the activation records described in Segment 7.10. An activation record in Java's program stack contains the method's arguments, local variables, and a reference to the current instruction. Since both recursive calls to `displayArray` are consecutive, there is no need for our activation records to distinguish between them by storing a representation of the program counter in the record. This simplification is not true in general, however.

To represent a record, we define a class that, in this case, has data fields for the method's arguments `first` and `last`. The following simple class is sufficient if we make it internal to our class containing `displayArray`:

```
private class Record
{
   private int first, last;

   private Record(int firstIndex, int lastIndex)
   {
      first = firstIndex;
      last = lastIndex;
   } // end constructor
} // end Record
```

7.47 In general, when a method begins execution, it pushes an activation record onto a program stack. At its return, a record is popped from this stack. We want an iterative `displayArray` to maintain its own stack. When the method begins execution, it should push a record onto this stack. Each recursive call should do likewise. As long as the stack is not empty, the method should remove a record from the stack and act according to the contents of the record. The method ends its execution when the stack becomes empty.

Here is an iterative version of `displayArray` that uses a stack as we just described:

```
private void displayArray(int first, int last)
{
   boolean done = false;
   StackInterface<Record> programStack = new LinkedStack<Record>();
   programStack.push(new Record(first, last));
   while (!done && !programStack.isEmpty())
   {
      Record topRecord = programStack.pop();
      first = topRecord.first;
      last = topRecord.last;
```

```
         if (first == last)
            System.out.println(array[first] + " ");
         else
         {
            int mid = first + (last - first) / 2;
            // Note the order of the records pushed onto the stack
            programStack.push(new Record(mid + 1, last));
            programStack.push(new Record(first, mid));
         } // end if
      } // end while
   } // end displayArray
```

This approach does not always produce an elegant solution. We certainly could write an iterative version of `displayArray` that was easier to understand than this version and did not require a stack. But sometimes a simple iterative solution is not apparent; in such cases, the stack approach offers a possible solution. You will see a more useful example of a stack-based iteration in Segment 24.14 of Chapter 24.

Chapter Summary

- Recursion is a problem-solving process that breaks a problem into identical but smaller problems.
- The definition of a recursive method must contain logic that involves an input—often a parameter—to the method and leads to different cases. One or more of these cases are base cases, or stopping cases, because they provide a solution that does not require recursion. One or more cases include a recursive invocation of the method that takes a step toward a base case by solving a "smaller" version of the task performed by the method.
- For each call to a method, Java records the values of the method's parameters and local variables in an activation record. The records are placed into an ADT called a stack that organizes them chronologically. The record most recently added to the stack is of the currently executing method. In this way, Java can suspend the execution of a recursive method and invoke it again with new argument values.
- A recursive method that processes an array often divides the array into portions. Recursive calls to the method work on each of these array portions.
- A recursive method that processes a chain of linked nodes needs a reference to the chain's first node as its parameter.
- A recursive method that is part of an implementation of an ADT often is private, because its use requires knowledge of the underlying data structure. Although such a method is unsuitable as an ADT operation, it can be called by a public method that implements the operation.
- A recurrence relation expresses a function in terms of itself. You can use a recurrence relation to describe the work done by a recursive method.
- Any solution to the Towers of Hanoi problem with n disks requires at least $2^n - 1$ moves. A recursive solution to this problem is clear and as efficient as possible. As an $O(2^n)$ algorithm, however, it is practical for small values of n.
- Each number in the Fibonacci sequence—after the first two—is the sum of the previous two numbers. Computing a Fibonacci number recursively is quite inefficient, as the required previous numbers are computed several times each.
- Tail recursion occurs when the last action of a recursive method is a recursive call. This recursive call performs a repetition that can be done by using iteration. Converting a tail-recursive method to an iterative one is usually a straightforward process.
- Indirect recursion results when a method calls a method that calls a method, and so on until the first method is called again.
- You can use a stack instead of recursion to implement a recursive algorithm. This stack mimics the behavior of the program stack.

Programming Tips

- An iterative method contains a loop. A recursive method calls itself. Although some recursive methods contain a loop *and* call themselves, if you have written a `while` statement within a recursive method, be sure that you did not mean to write an `if` statement.
- A recursive method that does not check for a base case, or that misses the base case, will not terminate normally. This situation is known as infinite recursion.
- Too many recursive calls can cause the error message "stack overflow." This means that the stack of activation records has become full. In essence, the method uses too much memory. Infinite recursion or large-size problems are the likely causes of this error.
- Do not use a recursive solution that repeatedly solves the same problem in its recursive calls.
- If a recursive method does not work, answer the following questions. Any "no" answers should guide you to the error.
  - Does the method have at least one parameter or input value?
  - Does the method contain a statement that tests a parameter or input value, leading to different cases?
  - Did you consider all possible cases?
  - Does at least one of these cases cause at least one recursive call?
  - Do these recursive calls involve smaller arguments, smaller tasks, or tasks that get closer to the solution?
  - If these recursive calls produce or return correct results, will the method produce or return a correct result?
  - Is at least one of the cases a base case that has no recursive call?
  - Are there enough base cases?
  - Does each base case produce a result that is correct for that case?
  - If the method returns a value, does each of the cases return a value?

Exercises

1. Consider the method `displayRowOfCharacters` that displays any given character the specified number of times on one line. For example, the call

```
displayRowOfCharacters('*', 5);
```

produces the line

```
*****
```

Implement this method in Java by using recursion.

2. Describe a recursive algorithm that draws concentric circles, given the diameter of the outermost circle. The diameter of each inner circle should be three-fourths the diameter of the circle that encloses it. The diameter of the innermost circle should exceed 1 inch.

3. Write a method that asks the user for integer input that is between 1 and 10, inclusive. If the input is out of range, the method should recursively ask the user to enter a new input value.

4. The factorial of a positive integer n—which we denote as $n!$—is the product of n and the factorial of $n - 1$. The factorial of 0 is 1. Write two different recursive methods that each return the factorial of n.

5. Write a recursive method that writes a given array backward. Consider the last element of the array first.

6. Repeat Exercise 5, but instead consider the first element of the array first.

7. Repeat Exercises 5 and 6, but write a string backward instead of an array.

8. A palindrome is a string that reads the same forward and backward. For example *deed* and *level* are palindromes. Write an algorithm in pseudocode that tests whether a string is a palindrome. Implement your algorithm as a static method in Java. Exercise 11 and Project 1 in Chapter 5 asked you to describe how to do this using a stack.

9. Segment 7.37 introduced the Fibonacci sequence. Computing this sequence recursively is inefficient and takes too much time. Write two methods that each compute the n^{th} term in the Fibonacci sequence by using iteration instead of recursion. One method should use an array to store the Fibonacci numbers. The other method should use three variables to contain the current term in the sequence and the two terms before it.

   What is the Big Oh for each of your iterative methods? Compare these results with the performance of the recursive algorithm.

10. For three disks, how many recursive calls are made by each of the two solveTowers algorithms given in Segment 7.31?

11. Write a recursive method that counts the number of nodes in a chain of linked nodes.

12. If n is a positive integer in Java, n % 10 is its rightmost digit and n / 10 is the integer obtained by dropping the rightmost digit from n. Using these facts, write a recursive method that displays an integer n in decimal. Now observe that you can display n in any base between 2 and 9 by replacing 10 with the new base. Revise your method to accommodate a given base.

13. Consider the method getFrequencyOf in the class ArrayBag, as given in Segment 2.17 of Chapter 2. Write a private recursive method that getFrequencyOf can call, and revise the definition of getFrequencyOf accordingly.

14. Repeat Exercise 13, but instead use the class LinkedBag and the method getFrequencyOf in Segment 3.16 of Chapter 3.

15. Write four different recursive methods that each compute the sum of integers in an array of integers. Model your methods after the displayArray methods given in Segments 7.15 through 7.18 and described in Question 5.

16. Write a recursive method that returns the smallest integer in an array of integers. If you divide the array into two pieces—halves, for example—and find the smallest integer in each of the two pieces, the smallest integer in the entire array will be the smaller of the these two integers. Since you will be searching a portion of the array—for example, the elements array[first] through array[last]—it will be convenient for your method to have three parameters: the array and two indices, first and last. You can refer to the method displayArray in Segment 7.18 for inspiration.

17. Trace the call f(16) to the following method by showing a stack of activation records:

```
public int f(int n)
{
   int result = 0;
   if (f <= 4)
      result = 1;

   else
      result = f(n / 2) + f(n / 4);
   return result;
} // end f
```

18. Write a recursive algorithm in pseudocode that finds the second smallest object in an array of Comparable objects. Implement your algorithm as a static method in Java.

19. Consider the class ArrayBag, as given in Chapter 2. Implement the method equals for ArrayBag so that it calls a private recursive method.

20. If

$t(1) = 2$
$t(n) = 1 + t(n - 1)$ for $n > 1$

find an expression for $t(n)$ that is not given in terms of itself. Prove that your result is correct by using induction.

21. If

$t(1) = 1$
$t(n) = 2 * t(n - 1)$ for $n > 1$

find an expression for $t(n)$ that is not given in terms of itself. Prove that your result is correct by using induction.

22. Consider a checkerboard that has a dollar amount printed on each of its squares. You can place a checker on the board anywhere you want and then move it across the board with standard diagonal moves. Once you reach the other side, you are finished. You will collect an amount of money equal to the sum of the values written on the squares that your checker traveled over.

   a. Give a recursive algorithm that will compute the maximum amount you can collect.
   b. Give an iterative algorithm that uses a stack to compute the maximum amount you can collect.

23. Consider the recursive method given in Segment 7.21 that displays the contents of a chain of linked nodes in backward order. Also consider the recursive method described in Exercise 5 that displays the contents of an array in backward order.

   a. What is the time complexity of each of these two methods, and how do they compare?
   b. Write an iterative method that displays the contents of a chain of linked nodes in backward order. What is this method's time complexity, and how does it compare to the complexities that you computed in Part *a*?

Projects

1. The following algorithm finds the square root of a positive number:

```
Algorithm squareRoot(number, lowGuess, highGuess, tolerance)
newGuess = (lowGuess + highGuess) / 2
if ((highGuess - newGuess) / newGuess < tolerance)
   return newGuess
else if (newGuess * newGuess > number)
   return squareRoot(number, lowGuess, newGuess, tolerance)
else if (newGuess * newGuess < number)
   return squareRoot(number, newGuess, highGuess, tolerance)
else
   return newGuess
```

To begin the computation, you need a value `lowGuess` less than the square root of the number and a value `highGuess` that is larger. You can use zero as `lowGuess` and the number itself as `highGuess`. The parameter `tolerance` controls the precision of the result independently of the magnitude of `number`. For example, computing the square root of 250 with `tolerance` equal to 0.00005 results in 15.81. This result has four digits of accuracy.

Implement this algorithm.

2. Implement the two versions of the `solveTower` algorithm given in Segment 7.31. Represent the towers by either single characters or strings. Each method should display directions that indicate the moves that must be made. Insert counters into each method to count the number of times it is called. These counters can be data fields of the class that contains these methods. Compare the number of recursive calls made by each method for various numbers of disks.

3. Repeat Project 5a of Chapter 5, but write a recursive implementation of the algorithm.

4. Implement the algorithms that Exercise 22 describes.

5. Imagine an array of *n* items from which you can choose. You will place the items you choose into a knapsack of size *k*. Each item has a size and a value. Of course, you cannot take more items than you have space for in the knapsack. Your goal is to maximize the total value of the items you take.

   a. Design a recursive algorithm `maxKnapsack` to solve this knapsack problem. The parameters to the algorithm are the knapsack, the array of items, and the position within the array of the next item to consider. The algorithm chooses the items for the knapsack and returns a knapsack containing the chosen items. A knapsack can report its size, its contents, the value of its contents, and the size of its contents.

   Hint: If any items in the array have not yet been considered, retrieve the next item in the array. You can either ignore the item or, if it fits, put it in the knapsack. To decide, make a recursive call for each of these two cases. Compare the knapsacks returned by these calls to see which one has the most valuable contents. Then return that knapsack.

   b. Write the classes `Knapsack` and `KnapsackItem`. Then write a program that defines the method `maxKnapsack`. The program should read the size of the knapsack and then the size, value, and name of each available item. Here is some sample input data for a knapsack of size 10:

```
Size  Value   Name
   1  50000   rare coin
   2   7000   small gold coin
   4  10000   packet of stamps
   4  11000   pearl necklace
   5  12000   silver bar
  10  60000   painting
```

   After displaying the items, call `maxKnapsack`. Then display the chosen items, their values, and their total value.

6. Suppose that you are scheduling a room. You are given a group of activities each of which has a start time and stop time. Two activities are compatible if they do not overlap. For example, in the following activities, activity A is compatible with activities B and D, but not activity C:

Activity	Start Time	Stop Time
A	1	2
B	2	5
C	1	3
D	5	6

Your goal is to schedule compatible activities that result in a maximum usage of the room.

   a. Design a recursive algorithm to solve this room-scheduling problem. The method whose signature is

```
maxRoomUse(int startTime, int stopTime, Activity[] activities)
```

   returns a pair consisting of the maximum usage in hours and an array of activities scheduled. Note that `startTime` is the first time that an activity can be scheduled, `stopTime` is the final time, and `activities` is an array of possible activities.

   b. Write the class `Activity` and a class `Schedule` that represents the pair that `maxRoomUse` returns. Then write a program that defines the method `maxRoomUse`. The program should read the start time and stop time for the room followed by the start and stop times for each activity (one activity per line). After displaying the given activities, display the maximum usage in hours of the room along with a list of the scheduled activities.

7. You can get a solution to the Towers of Hanoi problem by using the following iterative algorithm. Beginning with pole 1 and moving from pole to pole in the order pole 1, pole 3, pole 2, pole 1, and so on, make at most one move per pole according to the following rules:

- Move the topmost disk from a pole to the next possible pole in the specified order. Remember that you cannot place a disk on top of a smaller one.
- If the disk that you are about to move is the smallest of all the disks and you just moved it to the present pole, do not move it. Instead, consider the next pole.

This algorithm should make the same moves as the recursive algorithms given in Segment 7.31 and pictured in Figure 7-8. Thus, this iterative algorithm is O(2^n) as well.

Implement this algorithm.

8. Write an application or applet that animates the solution to the Towers of Hanoi problem. The problem asks you to move *n* disks from one pole to another, one at a time. You move only the top disk on a pole, and you place a disk only on top of larger disks on a pole. Since each disk has certain characteristics, such as its size, it is natural to define a class of disks.

Design and implement an ADT tower that includes the following operations:

- Add a disk to the top of the disks on the pole
- Remove the topmost disk

Also, design and implement a class that includes a recursive method to solve the problem.

9. Java's class `Graphics` has the following method to draw a line between two points:

```
/** Draws a line between the points (x1, y1) and (x2, y2).*/
public void drawLine(int x1, int y1, int x2, int y2)
```

`Graphics` uses a coordinate system that measures points from the top left corner of a window.

Write a recursive method that draws a picture of a 12-inch ruler. Mark inches, half inches, quarter inches, and eighth inches. Mark the half inches with marks that are smaller than those that mark the inches. Mark the quarter inches with marks that are smaller than those that mark the half inches, and so on. Your picture need not be full size. *Hint*: Draw a mark in the middle of the ruler and then draw rulers to the left and right of this mark.

10. Imagine a row of *n* lights that can be turned on or off only under certain conditions, as follows. The first light can be turned on or off anytime. Each of the other lights can be turned on or off only when the preceding light is on and all other lights before it are off. If all the lights are on initially, how can you turn them off? For three lights numbered 1 to 3, you can take the following steps, where 1 is a light that is on and 0 is a light that is off:

1 1 1	All on initially
0 1 1	Turn off light 1
0 1 0	Turn off light 3
1 1 0	Turn on light 1
1 0 0	Turn off light 2
0 0 0	Turn off light 1

You can solve this problem in general by using mutual recursion, as follows:

```
Algorithm turnOff(n)
// Turns off n lights that are initially on.
if (n == 1)
    Turn off light 1
```

```
else
{
   if (n > 2)
      turnOff(n - 2)
   Turn off light n
   if (n > 2)
      turnOn(n - 2)
   turnOff(n - 1)
}

Algorithm turnOn(n)
// Turns on n lights that are initially off.
if (n == 1)
   Turn on light 1
else
{
   turnOn(n - 1)
   if (n > 2)
      turnOff(n - 2)
   Turn on light n
   if (n > 2)
      turnOn(n - 2)
}
```

a. Implement these algorithms in Java. Use the results in a program to display directions to turn off *n* lights that initially are on.

b. What recurrence relation expresses the number of times that lights are switched on or off during the course of solving this problem for *n* lights?

11. Consider a maze made up of a rectangular array of squares, such as the following one:

```
X X X X X X X X X X X X X
      X       X X X X   X
X X       X         X   X
X X    X X  X   X   X   X
X X      X      X       X
X X    X X X X X
X X X X X X X X X X X X X
```

The Xs represent a blocked square and form the walls of the maze. Let's consider mazes that have only one entrance and one exit on opposite sides of the maze, as in our example. Beginning at the entrance at the top left side of the maze, find a path to the exit at the bottom right side. You can move only up, down, left, or right.

Each square in the maze can be in one of four states: clear, blocked, path, or visited. Initially, each square is either clear or blocked. If a square lies on a successful path, mark it with a period. If you enter a square but it does not lead to a successful path, mark the square as visited.

Let a two-dimensional array represent the maze. Use a recursive algorithm to find a path through the maze. Some mazes might have more than one successful path, while others have no path.

Answers to Self-Test Questions

1.

```
public static void skipLines(int givenNumber)
{
   if (givenNumber >= 1)
   {
      System.out.println();
      skipLines(givenNumber - 1);
   } // end if
} // end skipLines
```

2.
```
Algorithm drawConcentricCircles(givenNumber, givenDiameter, givenPoint)
if (givenNumber >= 1)
{
   Draw a circle whose diameter is givenDiameter and whose center is at givenPoint
   givenDiameter = 4 * givenDiameter / 3
   drawConcentricCircles(givenNumber - 1, givenDiameter, givenPoint)
}
```

3.
```
public static void countUp(int n)
{
   if (n >= 1)
   {
      countUp(n - 1);
      System.out.println(n);
   } // end if
} // end countUp
```

4.
```
public static int productOf(int n)
{
   int result = 1;
   if (n > 1)
      result = n * productOf(n - 1);
   return result;
} // end productOf
```

5.
```
public static void displayArray(int[] array, int first, int last)
{
   if (first == last)
      System.out.print(array[first] + " ");
   else
   {
      int mid = (first + last) / 2;
      displayArray(array, first, mid - 1);
      System.out.print(array[mid] + " ");
      displayArray(array, mid + 1, last);
   } // end if
} // end displayArray
```

6. The order of events is as follows:

```
displayBackward()
displayChainBackward(firstNode)
displayChainBackward(a reference to the second node)
displayChainBackward(a reference to the third node)
displayChainBackward(null)
Print the data in the third node
Print the data in the second node
Print the data in the first node
```

Activation records for the calls to `displayChainBackward` appear in a stack, as follows (`dCB` is an abbreviation for `displayChainBackward`; the stack is shown top to bottom):

```
dCB(firstNode)
dCB(reference to second node) dCB(firstNode)
dCB(reference to third node) dCB(reference to second node) dCB(firstNode)
dCB(null) dCB(reference to third node) dCB(reference to second node) dCB(firstNode)
dCB(reference to third node) dCB(reference to second node) dCB(firstNode)
Print the data in the third node
dCB(reference to second node) dCB(firstNode)
```

```
    Print the data in the second node
    dCB(firstNode)
    Print the data in the first node
```

7. O(n). You can use the same recurrence relation that was shown in Segments 7.22 and 7.23 for the method `countDown`.

8. **a.** $t(n) = 1 + t(n - 1)$ for $n > 0$, $t(0) = 1$.
b. Since $t(n) = n + 1$, the algorithm is O(n).

9. Move a disk from pole 1 to pole 2
Move a disk from pole 1 to pole 3
Move a disk from pole 2 to pole 3
Move a disk from pole 1 to pole 2
Move a disk from pole 3 to pole 1
Move a disk from pole 3 to pole 2
Move a disk from pole 1 to pole 2
Move a disk from pole 1 to pole 3
Move a disk from pole 2 to pole 3
Move a disk from pole 2 to pole 1
Move a disk from pole 3 to pole 1
Move a disk from pole 2 to pole 3
Move a disk from pole 1 to pole 2
Move a disk from pole 1 to pole 3
Move a disk from pole 2 to pole 3

10. 2 and 6, respectively.

11. 24 recursive calls and 12 additions.

12. 5 additions.

Chapter 8

An Introduction to Sorting

Contents

Prerequisites

Chapter	3	A Bag Implementation That Links Data
Chapter	4	The Efficiency of Algorithms
Chapter	7	Recursion

Objectives

After studying this chapter, you should be able to

- Sort an array into ascending order by using the following methods: selection sort, insertion sort, and Shell sort
- Sort a chain of linked nodes into ascending order by using an insertion sort
- Assess the efficiency of a sort and discuss the relative efficiencies of the various methods

We are all familiar with arranging objects in order from smallest to largest or from largest to smallest. Not only do we order numbers this way, but we also can arrange people by height, age, or name; music by title, artist, or album; and so on. Arranging things into either ascending or descending order is called **sorting**. You can sort any collection of items that can be compared with one another. Exactly how you compare two objects depends on the nature of the objects. For example, you can arrange a row of books on your bookshelf in several ways: by title, by author, by height, by color, and so on. The designer of a class of book objects would choose one of these ways when implementing the method `compareTo`.

Suppose you have a collection of items that need to be sorted in some way. For example, you might want to arrange a group of numbers from lowest to highest or from highest to lowest, or you might want to place some strings in alphabetical order. This chapter discusses and implements a few simple algorithms that sort items into ascending order. That is, our algorithms rearrange the first *n* entries in a collection so that

entry 1 ≤ entry 2 ≤ . . . ≤ entry *n*

With only small changes to our algorithms, you will be able to sort entries into descending order.

Sorting an array is usually easier than sorting a chain of linked nodes. For this reason, typical sorting algorithms sort an array. In particular, our algorithms will rearrange the first *n* values in an array `a` so that

```
a[0] ≤ a[1] ≤ a[2] ≤ . . . ≤ a[n - 1]
```

However, we also will use one of our algorithms to sort a chain of linked nodes.

Sorting is such a common and important task that many sorting algorithms exist. This chapter examines some basic algorithms for sorting data. Although most of our examples will sort integers, the Java implementations given will sort any `Comparable` objects—that is, objects of any class that implements the interface `Comparable` and, therefore, defines the method `compareTo`.

The efficiency of a sorting algorithm is significant, particularly when large amounts of data are involved. We will examine the performance of the algorithms in this chapter and find that they are relatively slow. The next chapter will present sorting algorithms that usually are much faster.

Organizing Java Methods That Sort an Array

8.1 One way to organize methods that sort an array is to create a class of static methods that perform the various sorts. The methods define a generic type `T` for the objects in the array. For example, we could write the header of such a method as follows:

```
public static <T> void sort(T[] a, int n)
```

Recall that the array passed to this method can contain objects of any one class.

For an array to be sortable, the objects in that array must be `Comparable`. Thus, the class that `T` represents must implement the interface `Comparable`. To ensure this requirement, we write

```
<T extends Comparable<T>>
```

instead of simply `<T>` before the return type in the headers of the sort methods. We then can use `T` as the data type of the parameters and local variables within the methods. For example, our class could begin as follows:

```
public class SortArray
{
   public static <T extends Comparable<T>> void sort(T[] a, int n)
   {  . . .
```

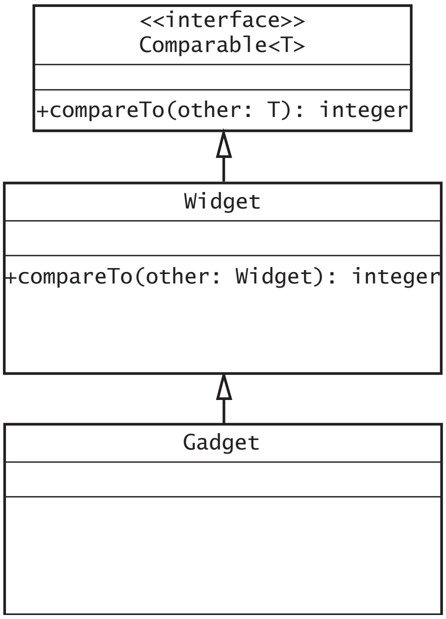
A client could sort an array of 50 objects, for example, by using the statement

```
SortArray.sort(myArray, 50);
```

If the objects in myArray are all of type Gadget, the compiler will discover that fact without our help. Gadget, however, must implement Comparable and have a compareTo method.

8.2 **An improvement.** By writing T extends Comparable<T>, we require that Gadget implement the interface Comparable<Gadget>. But insisting that a Gadget object be compared only to another Gadget object is more restrictive than we really need to be. What would happen if we had derived Gadget from Widget, where Widget implements Comparable<Widget>, as Figure 8-1 shows? If gadgets and widgets are similar enough to have the same basis for comparison, Gadget could use the method compareTo it inherits from Widget without defining its own. But then a call to the method sort with an array of gadgets and widgets as an argument would not compile.

FIGURE 8-1 The class Gadget is derived from the class Widget, which implements the interface Comparable



<<interface>>
Comparable<T>
+compareTo(other: T): integer
Widget
+compareTo(other: Widget): integer
Gadget

Instead of comparing an object of T only with other objects of T, we can allow comparisons of objects of a superclass of T. So instead of writing T extends Comparable<T>, we write

```
T extends Comparable<? super T>
```

The **wildcard** ? represents any type, but the notation ? super T means any superclass of T. Thus, the header for the method sort would be

```
public static <T extends Comparable<? super T>> void sort(T[] a, int n)
```

Programming Tip: To use Comparable with arbitrary types, write Comparable<? super T> instead of Comparable<T>.

Note: Bounded wildcards
When using generic types, the wild card `?` represents any class. You can **bound**, or limit, the wildcard in one of two ways. For example, `? super Gizmo` means any superclass of `Gizmo`. We say that `Gizmo` is the **lower bound** of the wildcard. Analogously, `? extends Gizmo` means any subclass of `Gizmo`. Here, `Gizmo` is the **upper bound** of the wildcard. In Chapter 4, Segments 4.13 and 4.17, respectively, provide other meanings for the terms upper bound and lower bound.

We now turn our attention to several ways of sorting an array.

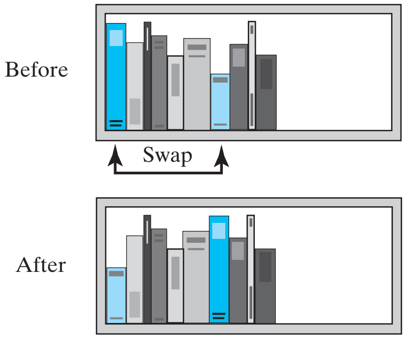
Selection Sort

8.3 Imagine that you want to rearrange the books on your bookshelf by height, with the shortest book on the left. You might begin by tossing all of the books onto the floor. You then could return them to the shelf one by one, in their proper order. If you first return the shortest book to the shelf, and then the next shortest, and so on, you would perform a kind of **selection sort**. But using the floor—or another shelf—to store your books temporarily uses extra space needlessly.

VideoNote
Selection sort

Instead, approach your intact bookshelf and *select* the shortest book. Since you want it to be first on the shelf, you remove the first book on the shelf and put the shortest book in its place. You still have a book in your hand, so you put it into the space formerly occupied by the shortest book. That is, the shortest book has traded places with the first book, as Figure 8-2 illustrates. You now ignore the shortest book and repeat the process for the rest of the bookshelf.

FIGURE 8-2 Before and after exchanging the shortest book and the first book



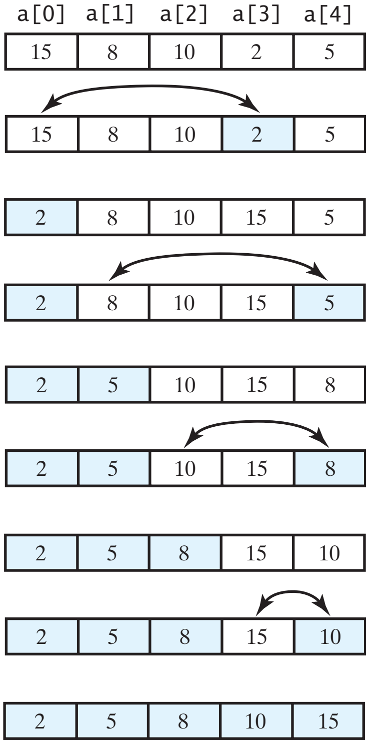
In terms of an array `a`, the selection sort finds the smallest entry in the array and exchanges it with `a[0]`. Then, ignoring `a[0]`, the sort finds the next smallest entry and swaps it with `a[1]`, and so on. Notice that we use only one array. We sort it by making entries trade places with other entries.

We could have copied the array into a second array and then moved the entries back to the original array in their proper order. But that would be like using the floor to store books temporarily. Fortunately, all of that extra space is unnecessary.

8.4 Figure 8-3 shows how a selection sort rearranges an array of integers by interchanging values. Beginning with the original array, the sort locates the smallest value in the array, that is, the 2 in `a[3]`. The value in `a[3]` is interchanged with the value in `a[0]`. After that interchange, the smallest value is in `a[0]` where it belongs.

The next smallest value is the 5 in a[4]. The sort then interchanges the value in a[4] with the value in a[1]. So far, the values in a[0] and a[1] are the smallest in the array and are in their correct position within the final sorted array. The algorithm then interchanges the next smallest entry—the 8—with a[2], and so on until the entire array is sorted.

FIGURE 8-3 A selection sort of an array of integers into ascending order



Iterative Selection Sort

8.5 The following pseudocode describes an iterative algorithm for the selection sort:

```
Algorithm selectionSort(a, n)
// Sorts the first n entries of an array a.

for (index = 0; index < n - 1; index++)
{
   indexOfNextSmallest = the index of the smallest value among
                           a[index], a[index + 1], . . . , a[n - 1]
   Interchange the values of a[index] and a[indexOfNextSmallest]
   // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[index], and these are the smallest
   // of the original array entries. The remaining array entries begin at a[index + 1].
}
```

Notice that during the last iteration of the for loop, the value of index is n - 2, even though the last array entry is a[n - 1]. Once the entries a[0] through a[n - 2] are in their correct places, only the one entry a[n - 1] remains to be positioned. But since the other entries are correctly positioned, it must already be in the correct place as well.

Note: Notation

In mathematics, one-letter variable names are common. Recognizing this, and seeking to save some space, we use a and n within the text and pseudocode to represent, respectively, an array and its number of entries. Within Java code elsewhere, we have tried to avoid one-letter identifiers, using them only sparingly. However, the code that you will see in this chapter and the next uses a and n simply to maintain consistency with the text.

8.6 The class in Listing 8-1 contains the public method `selectionSort` and two private methods that assist in sorting. We can add other sorting methods as we develop them.

It is easy to see that the definition of `selectionSort` is a direct translation of the previous pseudocode into Java code. The method `getIndexOfSmallest` searches the array entries `a[first]` through `a[last]` and returns the index of the smallest among them. The method uses two local variables, `min` and `indexOfMin`. At any point in the search, `min` references the smallest value found so far. That value occurs at `a[indexOfMin]`. At the end of the search, the method returns `indexOfMin`. Notice that for our purposes here, we could have assumed that `last` is always `n - 1` and omitted it as a parameter. However, this general version will be useful in other settings.

Since exchanging entries in an array does not involve the method `compareTo`, the method `swap` can simply use `Object` as the type of these entries.

LISTING 8-1 A class for sorting an array using selection sort

```java
/**
   Class for sorting an array of Comparable objects from smallest to
   largest.
*/
public class SortArray
{
   /** Sorts the first n objects in an array into ascending order.
       @param a  an array of Comparable objects
       @param n  an integer > 0 */
   public static <T extends Comparable<? super T>>
          void selectionSort(T[] a, int n)
   {
      for (int index = 0; index < n - 1; index++)
      {
         int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
         swap(a, index, indexOfNextSmallest);
         // Assertion: a[0] <= a[1] <= . . . <= a[index] <= all other
         // a[i]
      } // end for
   } // end selectionSort

   /** Finds the index of the smallest value in a portion of an array.
       @param a      an array of Comparable objects
       @param first  an integer >= 0 and < a.length that is the index of
                     the first array entry to consider
       @param last   an integer >= first and < a.length that is the
                     index of the last array entry to consider
       @return the index of the smallest value among
               a[first], a[first + 1], . . . , a[last] */
   private static <T extends Comparable<? super T>>
           int getIndexOfSmallest(T[] a, int first, int last)
   {
      T min = a[first];
      int indexOfMin = first;
      for (int index = first + 1; index <= last; index++)
      {
         if (a[index].compareTo(min) < 0)
         {
            min = a[index];
            indexOfMin = index;
         } // end if
         // Assertion: min is the smallest of a[first] through a[index].
      } // end for

      return indexOfMin;
   } // end getIndexOfSmallest

   /** Swaps the array entries a[i] and a[j].
       @param a  an array of objects
       @param i  an integer >= 0 and < a.length
       @param j  an integer >= 0 and < a.length */
   private static void swap(Object[] a, int i, int j)
   {
      Object temp = a[i];
      a[i] = a[j];
      a[j] = temp;
   } // end swap
} // end SortArray
```

? SELF-TEST

Question 1 Trace the steps that a selection sort takes when sorting the following array into ascending order: 9 6 2 4 8.

Recursive Selection Sort

8.7 Selection sort also has a natural recursive form. Often recursive algorithms that involve arrays operate on a portion of the array. Such algorithms use two parameters, `first` and `last`, to designate the portion of the array containing the entries `a[first]` through `a[last]`. The method `getIndexOfSmallest` in Listing 8-1 illustrates this technique. The recursive selection sort algorithm uses this notation as well:

```
Algorithm selectionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   indexOfNextSmallest = the index of the smallest value among
                         a[first], a[first + 1], . . . , a[last]
   Interchange the values of a[first] and a[indexOfNextSmallest]
   // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[first] and these are the smallest
   // of the original array entries. The remaining array entries begin at a[first + 1].
   selectionSort(a, first + 1, last)
}
```

After we place the smallest entry into the first position of the array, we ignore it and sort the rest of the array by using a selection sort. If the array has only one entry, sorting is unnecessary. In this case, `first` and `last` are equal, so the algorithm leaves the array unchanged.

8.8 When we implement the previous recursive algorithm in Java, the resulting method will have `first` and `last` as parameters. Thus, its header will differ from the header of the iterative method `selectionSort` given in Segment 8.6. We could, however, provide the following method to simply invoke the recursive method:

```
public static <T extends Comparable<? super T>>
       void selectionSort(T[] a, int n)
{
   selectionSort(a, 0, n - 1); // invoke recursive method
} // end selectionSort
```

Whether you make the recursive method `selectionSort` private or public is up to you, but making it public provides the client with a choice of two ways in which to invoke the sort. In a similar fashion, you could revise the iterative selection sort given in Segment 8.6 to use the parameters `first` and `last` (see Exercise 6) and then provide the method just given to invoke it.

With these observations in mind, we will make the subsequent sorting algorithms more general by giving them three parameters—`a`, `first`, and `last`—so that they sort the entries `a[first]` through `a[last]`.

The Efficiency of Selection Sort

8.9 In the iterative method `selectionSort`, the `for` loop executes $n - 1$ times, so it invokes the methods `getIndexOfSmallest` and `swap` $n - 1$ times each. In the $n - 1$ calls to `getIndexOfSmallest`, `last` is $n - 1$ and `first` ranges from 0 to $n - 2$. Each time `getIndexOfSmallest` is invoked, its loop executes `last - first` times. Thus, this loop executes a total of

$$(n - 1) + (n - 2) + \ldots + 1$$

times. This sum is $n(n - 1)/2$. Since the operations in the loop are O(1), the selection sort is $O(n^2)$. Notice that our discussion does not depend on the nature of the data in the array. It could be wildly out of order, nearly sorted, or completely sorted; in any case, selection sort would be $O(n^2)$.

The recursive selection sort performs the same operations as the iterative selection sort, and so it is also $O(n^2)$.

Note: The time efficiency of selection sort

Selection sort is $O(n^2)$ regardless of the initial order of the entries in an array. Although the sort requires $O(n^2)$ comparisons, it performs only $O(n)$ swaps. Thus, the selection sort requires little data movement.

Insertion Sort

8.10 Another intuitive sorting algorithm is the **insertion sort**. Suppose again that you want to rearrange the books on your bookshelf by height, with the shortest book on the left. If the leftmost book on the shelf were the only book, your shelf would be sorted. But you also have all the other books to sort. Consider the second book. If it is taller than the first book, you now have two sorted books. If not, you remove the second book, slide the first book to the right, and *insert* the book you just removed into the first position on the shelf. The first two books are now sorted.

VideoNote
Insertion sort

Now consider the third book. If it is taller than the second book, you now have three sorted books. If not, remove the third book and slide the second book to the right, as Parts *a* through *c* of Figure 8-4 illustrate. Now see whether the book in your hand is taller than the first book. If so, insert the book into the second position on the shelf, as shown in Figure 8-4d. If not, slide the first book to the right, and insert the book in your hand into the first position on the shelf. If you repeat this process for each of the remaining books, your bookshelf will be arranged by the heights of the books.

FIGURE 8-4 The placement of the third book during an insertion sort

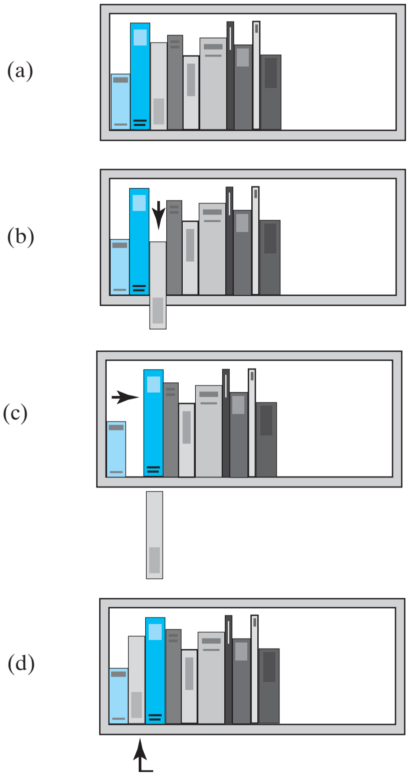
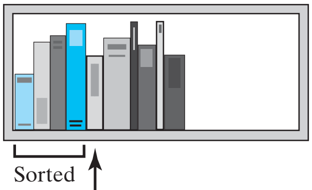


Figure 8-5 shows the bookshelf after several steps of the insertion sort. The books on the left side of the shelf are sorted. You remove the next unsorted book from the shelf and slide sorted books to the right, one at a time, until you find the right place for the book in your hand. You then insert this book into its new sorted location.

FIGURE 8-5 An insertion sort of books



1. Remove the next unsorted book.
2. Slide the sorted books to the right one by one until you find the right spot for the removed book.
3. Insert the book into its new position.

Iterative Insertion Sort

8.11 An insertion sort of an array **partitions**—that is, divides—the array into two parts. One part is sorted and initially contains just the first entry in the array. The second part contains the remaining entries. The algorithm removes the first entry from the unsorted part and inserts it into its proper sorted position within the sorted part. Just as you did with the bookshelf, you choose the proper position by comparing the unsorted entry with the sorted entries, beginning at the end of the sorted part and continuing toward its beginning. As you compare, you shift array entries in the sorted part to make room for the insertion.

Figure 8-6 illustrates these steps for a sort that has already positioned the first three entries of the array. The 3 is the next entry that must be placed into its proper position within the sorted region. Since 3 is less than 8 and 5 but greater than 2, the 8 and 5 are shifted to make room for the 3.

Figure 8-7 illustrates an entire insertion sort of an array of integers. At each pass of the algorithm, the sorted part expands by one e ntry as the unsorted part shrinks by one entry. Eventually, the unsorted part is empty and the array is sorted.

The following iterative algorithm describes an insertion sort of the entries at indices `first` through `last` of the array `a`. To sort the first `n` entries in the array, the call to the algorithm would be `insertionSort(a, 0, n - 1)`.

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] iteratively.

for (unsorted = first + 1 through last)
{
   nextToInsert = a[unsorted]
   insertInOrder(nextToInsert, a, first, unsorted - 1)
}
```

The sorted part contains one entry, `a[first]`, and so the loop in the algorithm begins at index `first` + 1 and processes the unsorted part. It then invokes another method—`insertInOrder`—to perform the insertions. In the pseudocode that follows for this method, `anEntry` is the value to be inserted into its proper position, and `begin` and `end` are array indices.

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted entries a[begin] through a[end].

index = end                     // index of last entry in the sorted portion
// make room, if needed, in sorted portion for another entry
```

```
while ( (index >= begin) and (anEntry < a[index]) )
{
   a[index + 1] = a[index]  // make room
   index--
}
// Assertion: a[index + 1] is available.
a[index + 1] = anEntry      // insert
```

FIGURE 8-6 Inserting the next unsorted entry into its proper location within the sorted portion of an array during an insertion sort

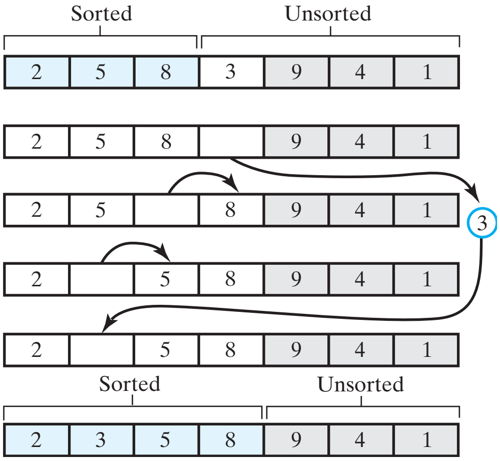
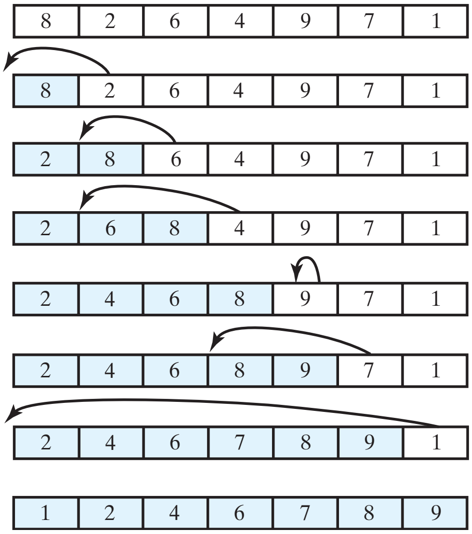


FIGURE 8-7 An insertion sort of an array of integers into ascending order



Question 2 Trace the steps that an insertion sort takes when sorting the following array into ascending order: 9 6 2 4 8.

Recursive Insertion Sort

8.12 You can describe an insertion sort recursively as follows. If you sort all but the last item in the array—a smaller problem than sorting the entire array—you then can insert the last item into its proper position within the rest of the array. The following pseudocode describes a recursive insertion sort:

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (the array contains more than one entry)
{
   Sort the array entries a[first] through a[last - 1]
   Insert the last entry a[last] into its correct sorted position within the rest of the array
}
```

We can implement this algorithm in Java as follows:

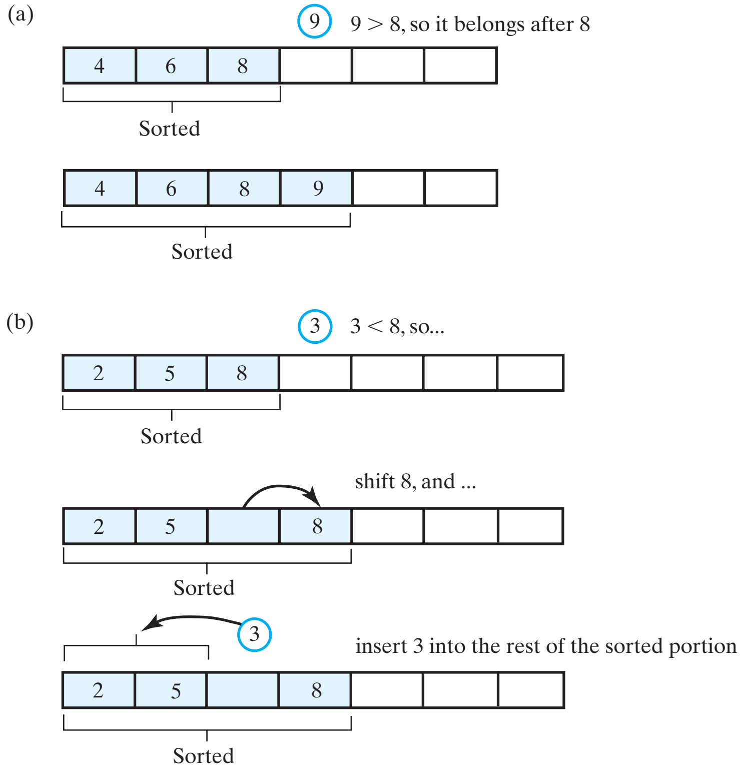
```
public static <T extends Comparable<? super T>>
       void insertionSort(T[] a, int first, int last)
{
   if (first < last)
   {
      // sort all but the last entry
      insertionSort(a, first, last - 1);

      // insert the last entry in sorted order
      insertInOrder(a[last], a, first, last - 1);
   } // end if
} // end insertionSort
```

8.13 **The algorithm `insertInOrder`: first draft.** The previous method can call the iterative version of `insertInOrder`, given earlier, or the recursive version that we now describe. If the entry to insert is greater than or equal to the last item in the sorted portion of the array, the entry belongs immediately after this last item, as Figure 8-8a illustrates. Otherwise, we move the last sorted item to the next higher position in the array and insert the entry into the remaining portion, as shown in Figure 8-8b.

We can describe these steps more carefully as follows:

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// First draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry
else
{
   a[end + 1] = a[end]
   insertInOrder(anEntry, a, begin, end - 1)
}
```

FIGURE 8-8 Inserting the first unsorted entry into the sorted portion of the array. (a) The entry is greater than or equal to the last sorted entry; (b) the entry is smaller than the last sorted entry



8.14 **The algorithm `insertInOrder`: final draft.** This algorithm is not quite right. The `else` clause will work only if we have more than one entry in the remaining portion of the array—that is, if `begin < end`. If `begin` and `end` were equal, for example, the recursive call would be equivalent to

```
insertInOrder(anEntry, a, begin, begin - 1);
```

which is incorrect.

Will `end` ever equal `begin`, if they were not equal initially? Yes. When `anEntry` is less than all entries `a[begin]`, . . . , `a[end]`, each recursive call decreases `end` by 1 until eventually `end` equals `begin`. What should we do when this happens? Since the sorted portion consists of one entry `a[end]`, we will move `a[end]` to the next higher position and place `anEntry` in `a[end]`.

The following revised algorithm reflects these changes:

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// Revised draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry
```

```
else if (begin < end)
{
   a[end + 1] = a[end]
   insertInOrder(anEntry, a, begin, end - 1)
}
else // begin == end and anEntry < a[end]
{
   a[end + 1] = a[end]
   a[end] = anEntry
}
```

The Efficiency of Insertion Sort

8.15 Look back at the iterative algorithm `insertionSort` given in Segment 8.11. For an array of n entries, `first` is 0 and `last` is $n - 1$. The `for` loop then executes $n - 1$ times, and so the method `insertInOrder` is invoked $n - 1$ times. Thus, within `insertInOrder`, `begin` is 0 and `end` ranges from 0 to $n - 2$. The loop within `insertInOrder` executes at most `end - begin` + 1 times each time the method is invoked. Thus, this loop executes at most a total of

$$1 + 2 + . . . + (n - 1)$$

times. This sum is $n\ (n - 1)/2$, so the insertion sort is $O(n^2)$. The recursive insertion sort performs the same operations as the iterative insertion sort, so it is also $O(n^2)$.

This analysis provides a worst-case scenario. In the best case, the loop in `insertInOrder` would exit immediately. Such is the case if the array is sorted already. In the best case, then, insertion sort is $O(n)$. In general, the more sorted an array is, the less work `insertInOrder` needs to do. This fact and its relatively simple implementation make the insertion sort popular for applications in which the array does not change much. For example, some customer databases add only a small percentage of new customers each day.

The next chapter will use the insertion sort when the array size is small.

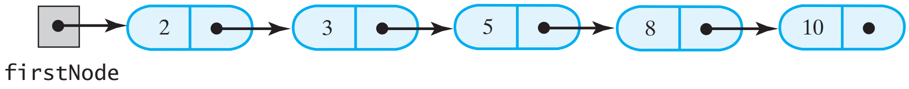
Note: The time efficiency of insertion sort
Insertion sort is at best $O(n)$ and at worst $O(n^2)$. The closer an array is to sorted order, the less work an insertion sort does.

Insertion Sort of a Chain of Linked Nodes

8.16 Usually you will sort arrays, but sometimes you might need to sort a chain of linked nodes. When you do, the insertion sort is one that is easy to understand.

Figure 8-9 shows a chain whose nodes contain integers that are sorted into ascending order. To begin to see how we can construct an insertion sort for a chain, imagine that we want to insert a node into this chain so that the integers in the nodes remain in sorted order.

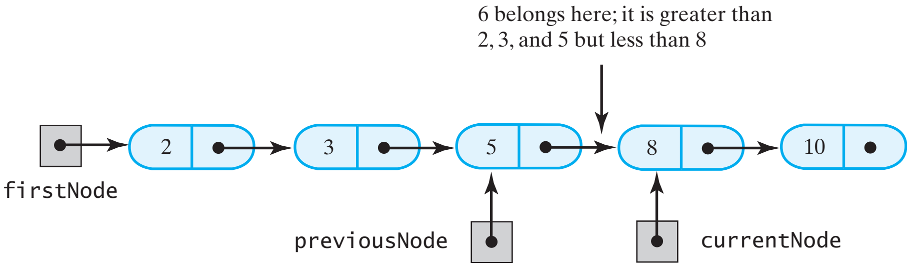
FIGURE 8-9 A chain of integers sorted into ascending order



Suppose that the node to be inserted into the chain contains the integer 6. We need to locate where in the chain the new node belongs. Since we have a reference, `firsNode`, to the first node in the chain, we can start there. We make comparisons as we move toward the end of the chain until we find the correct insertion point. Thus, we would compare 6 with 2, then with 3, with 5, and finally with 8 to see that 6 belongs between 5 and 8.

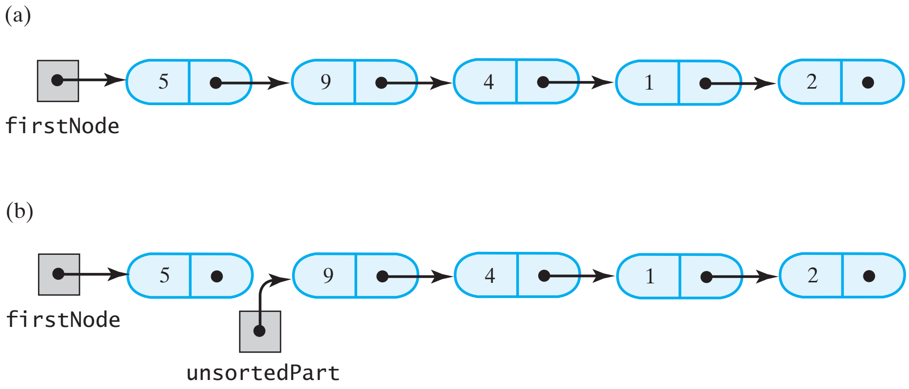
To insert a node into a chain, we need a reference to the node prior to the point of insertion. Thus, during the traversal of the chain, we save a reference to the node before the current one, as Figure 8-10 illustrates. Note that inserting at the beginning of the chain differs somewhat from inserting anywhere else in the chain.

FIGURE 8-10 During the traversal of a chain to locate the insertion point, save a reference to the node before the current one


6 belongs here; it is greater than
2, 3, and 5 but less than 8
2
3
5
8
10
firstNode
previousNode
currentNode

8.17 Now imagine that we have a method `insertInOrder(nodeToInsert)` that inserts a node into its correct sorted position within a chain, as just described. We can use this method to implement an insertion sort by adopting the same strategy that we used to sort an array: Divide the chain into two parts. The first part is sorted, and it initially contains only the first node. The second part is unsorted and initially is the rest of the chain. Figure 8-11 illustrates how to make this division. We first make the variable `unsortedPart` reference the second node and then set the link portion of the first node to `null`.

FIGURE 8-11 Breaking a chain of nodes into two pieces as the first step in an insertion sort: (a) the original chain; (b) the two pieces



To sort the nodes, we use the method `insertInOrder` to take each node from the unsorted part and insert it into the sorted part. Notice that our plan relinks existing nodes instead of creating new ones.

8.18 To give this discussion some context, suppose that we plan to add a sort method to a class LinkedGroup that uses a linked chain to represent a certain collection. As sorting requires us to compare the objects in the collection, they must belong to a class that implements the interface Comparable. Thus, the class definition begins as follows:

```
public class LinkedGroup<T extends Comparable<? super T>>
       implements GroupInterface<T>
{
   private Node firstNode;
   int length; // number of objects in the group
   . . .
```

assuming that we add the declaration of the sort method to GroupInterface.

This class has an inner class Node that has set and get methods for its private data fields. The following private method inserts the node that nodeToInsert references into the sorted chain that firstNode references.

```
private void insertInOrder(Node nodeToInsert)
{
   T item = nodeToInsert.getData();
   Node currentNode = firstNode;
   Node previousNode = null;

   // locate insertion point
   while ( (currentNode != null) &&
           (item.compareTo(currentNode.getData()) > 0) )
   {
      previousNode = currentNode;
      currentNode = currentNode.getNextNode();
   } // end while

   // make the insertion
   if (previousNode != null)
   { // insert between previousNode and currentNode
      previousNode.setNextNode(nodeToInsert);
      nodeToInsert.setNextNode(currentNode);
   }
   else // insert at beginning
   {
      nodeToInsert.setNextNode(firstNode);
      firstNode = nodeToInsert;
   } // end if
} // end insertInOrder
```

The local variable item is the data portion of the node to be inserted. The while loop compares item to the data in each node in the chain until either item is less than or equal to a data value or the end of the chain is reached. The references previousNode and currentNode are then used to insert the given node into its proper position.

8.19 The method to perform the insertion sort appears as follows. The local variable unsortedPart starts at the second node and then references each node in the rest of the chain as the loop executes. Each of these nodes is inserted in turn into the sorted part of the chain. Note that length is the number of nodes in the chain.

```
public void insertionSort()
{
   // if zero or one item is in the chain, there is nothing to do
   if (length > 1)
   {
      assert firstNode != null;
      // break chain into 2 pieces: sorted and unsorted
      Node unsortedPart = firstNode.getNextNode();
      assert unsortedPart != null;
      firstNode.setNextNode(null);

      while (unsortedPart != null)
      {
         Node nodeToInsert = unsortedPart;
         unsortedPart = unsortedPart.getNextNode();
         insertInOrder(nodeToInsert);
      } // end while
   } // end if
} // end insertionSort
```

Question 3 In the previous method `insertionSort`, if you move the line

```
unsortedPart = unsortedPart.getNextNode();
```

after the call to `insertInOrder`, will the method still work? Explain.

Question 4 The previous method `insertionSort` is not a static method. Why?

8.20 **The efficiency of an insertion sort of a chain.** For a chain of n nodes, the number of comparisons that the method `insertInOrder` makes is at most the number of nodes in the sorted portion of the chain. The method `insertionSort` calls `insertInOrder` $n - 1$ times. The first time it does so, the sorted portion contains one item, so one comparison is made. The second time, the sorted portion contains two items, so at most two comparisons are made. Continuing in this fashion, you can see that the maximum number of comparisons is

$$1 + 2 + \ldots + (n - 1)$$

This sum is $n\,(n - 1)/2$, so this insertion sort is $O(n^2)$.

Note: Sorting a chain of linked nodes can be difficult. The insertion sort, however, provides a reasonable way to perform this task.

Shell Sort

8.21 The sorting algorithms that we have discussed so far are simple and often useful, but they are too inefficient to use on large arrays. The Shell sort is a variation of the insertion sort that is faster than $O(n^2)$.

During an insertion sort, an array entry moves to an adjacent location. When an entry is far from its correct sorted position, it must make many such moves. So when an array is completely scrambled, an insertion sort takes a good deal of time. But when an array is almost sorted, an insertion sort is more efficient. In fact, Segment 8.15 showed that the more sorted an array is, the less work the method `insertInOrder` needs to do.

By capitalizing on these observations, Donald Shell devised in 1959 an improved insertion sort, now called the **Shell sort**. Shell wanted entries to move beyond their adjacent locations. To do so, he sorted subarrays of entries at equally spaced indices. Instead of moving to an adjacent location, an entry moves several locations away. The result is an array that is almost sorted—one that can be sorted efficiently by using an ordinary insertion sort.

8.22 For example, Figure 8-12 shows an array and the subarrays obtained by considering every sixth entry. The first subarray contains the integers 10, 9, and 7; the second subarray contains 16 and 6; and so on. There happen to be six of these subarrays.

Now we sort each of the six subarrays separately by using an insertion sort. Figure 8-13 shows the sorted subarrays and the state of the original array as a result. Notice that the array is "more sorted" than it was originally.

FIGURE 8-12 An array and the subarrays formed by grouping entries whose indices are 6 apart

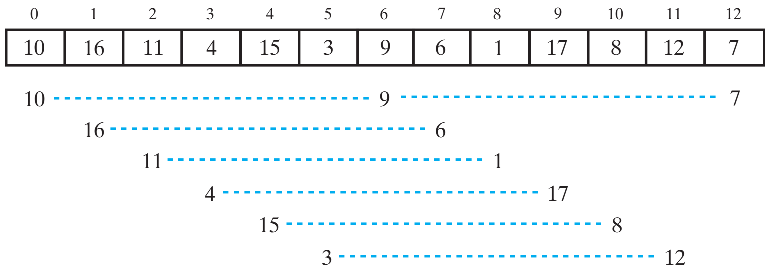
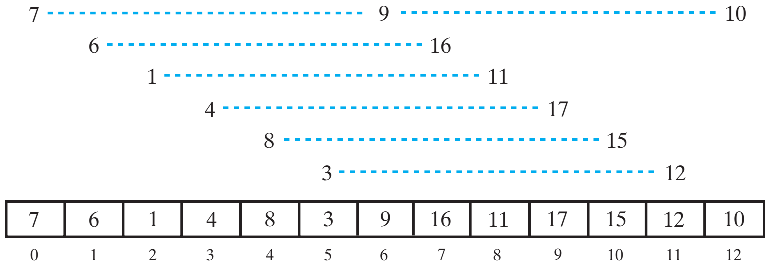


FIGURE 8-13 The subarrays of Figure 8-12 after each is sorted, and the array that contains them



8.23 Now we form new subarrays, but this time we reduce the separation between indices. Shell suggested that the initial separation between indices be $n/2$ and that you halve this value at each pass until it is 1. The array in our example has 13 entries, so we began with a separation of 6. We now reduce the separation to 3. Figure 8-14 shows the resulting subarrays, and Figure 8-15 shows the subarrays after they are sorted.

FIGURE 8-14 The subarrays of the array in Figure 8-13 formed by grouping entries whose indices are 3 apart

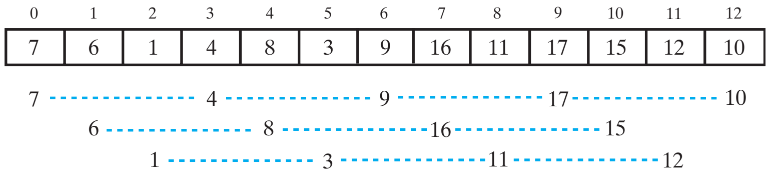
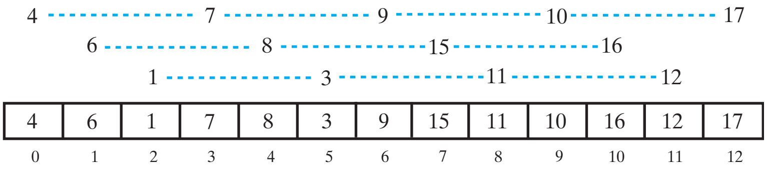


FIGURE 8-15 The subarrays of Figure 8-14 after each is sorted, and the array that contains them



Dividing the current separation, 3, by 2 results in 1. Therefore, the final step is simply an ordinary insertion sort of the entire array. This last step will sort the array regardless of what we have done to it beforehand. Thus, Shell sort will work if you use any sequence of index separations, as long as the last one is 1. But not just any sequence will make the Shell sort efficient, as you will see in Segment 8.25.

Question 5 Apply the Shell sort to the array 9 8 2 7 5 4 6 3 1, with index separations of 4, 2, and 1. What are the intermediate steps?

The Java Code

8.24 The heart of the Shell sort is the adaptation of the insertion sort to work on a subarray of equally spaced entries. By combining and modifying the two algorithms that describe the insertion sort, as given in Segment 8.11, we obtain the following method that sorts array entries whose indices are separated by an increment of `space`.

```
/** Sorts equally spaced entries of an array into ascending order.
    @param a      an array of Comparable objects
    @param first  the integer index of the first array entry to
                  consider; first >= 0 and < a.length
    @param last   the integer index of the last array entry to
                  consider; last >= first and < a.length
    @param space  the difference between the indices of the
                  entries to sort */
private static <T extends Comparable<? super T>>
        void incrementalInsertionSort(T[] a, int first, int last, int space)
{
   int unsorted, index;

   for (unsorted = first + space; unsorted <= last;
        unsorted = unsorted + space)
   {
      T nextToInsert = a[unsorted];

      for (index = unsorted - space; (index >= first) &&
                      (nextToInsert.compareTo(a[index]) < 0);
                      index = index - space)
      {
         a[index + space] = a[index];
      } // end for

      a[index + space] = nextToInsert;
   } // end for
} // end incrementalInsertionSort
```

A method to perform a Shell sort will invoke `incrementalInsertionSort` and supply any sequence of spacing factors. For example, we might write the following method, using the spacing that Segment 8.23 described:

```
public static <T extends Comparable<? super T>>
       void shellSort(T[] a, int first, int last)
{
   int n = last - first + 1; // number of array entries

   for (int space = n / 2; space > 0; space = space / 2)
   {
      for (int begin = first; begin < first + space; begin++)
         incrementalInsertionSort(a, begin, last, space);
   } // end for
} // end shellSort
```

SELF-TEST

Question 6 Trace the steps that a Shell sort takes when sorting the following array into ascending order: 9 6 2 4 8 7 5 3.

The Efficiency of Shell Sort

8.25 Since the Shell sort uses an insertion sort repeatedly, it certainly seems like much more work than using only one insertion sort. Actually, however, it is not. Although we used an insertion sort several times instead of just once, the initial sorts are of arrays that are much smaller than the original one, the later sorts are on arrays that are partially sorted, and the final sort is on an array that is almost entirely sorted. Intuitively, this seems good. But even though the Shell sort is not very complicated, its analysis is.

Since `incrementalInsertionSort` involves a loop and is called from within nested loops, the Shell sort uses three nested loops. Often such algorithms are O(n^3), but it turns out that the worst-case behavior of the Shell sort is still O(n^2). If n is a power of 2, the average-case behavior is O($n^{1.5}$). And if you tweak the spacing a bit, you can make the Shell sort even more efficient.

One improvement is to avoid even values of `space`. Figure 8-12 provided an example of the subarrays when `space` was 6. The first subarray contained 10, 9, and 7, for instance. Later, after we halved `space`, the first subarrray contained 7, 4, 9, 17, and 10, as Figure 8-14 shows. Notice that these two subarrays have entries in common, namely the 10, 9, and 7. Thus, the comparisons that you make when `space` is even will be repeated on the next pass when the increment is `space/2`.

To avoid this inefficiency, simply add 1 to `space` whenever it is even. This simple change results in consecutive increments that have no factor in common. The worst-case behavior of the Shell sort is then O($n^{1.5}$). Other sequences for `space` result in even greater efficiencies, although the proof that this is the case remains elusive. An improved Shell sort can be a reasonable choice for moderately large arrays.

Note: The time efficiency of Shell sort
The Shell sort, as implemented in this chapter, is O(n^2) in the worst case. By adding 1 to space anytime that it is even, you can improve the worst-case behavior to O($n^{1.5}$).

Comparing the Algorithms

8.26 Figure 8-16 summarizes the time efficiencies of the three sorting algorithms presented in this chapter. Generally, the selection sort is the slowest algorithm. The Shell sort, by capitalizing on the best-case behavior of the insertion sort, is the fastest.

FIGURE 8-16 The time efficiencies of three sorting algorithms, expressed in Big Oh notation

	Best Case	Average Case	Worst Case
Selection sort	O(n^2)	O(n^2)	O(n^2)
Insertion sort	O(n)	O(n^2)	O(n^2)
Shell sort	O(n)	O($n^{1.5}$)	O(n^2) or O($n^{1.5}$)

Chapter Summary

- A selection sort of an array selects the smallest entry and swaps it with the first one. Ignoring the new first entry, the sort then finds the smallest entry in the rest of the array and swaps it with the second entry, and so on.
- Typically, you perform a selection sort iteratively, although a simple recursive form is possible.
- A selection sort is O(n^2) in all cases.
- An insertion sort divides an array into two portions, sorted and unsorted. Initially, the array's first entry is in the sorted portion. The sort takes the next unsorted entry and compares it with entries in the sorted portion. As the comparisons continue, each sorted entry is shifted by one position toward the end of the array until the unsorted entry's correct position is located. The sort then inserts the entry into its correct position, which has been vacated by the shifts.
- You can perform an insertion sort either iteratively or recursively.
- An insertion sort is O(n^2) in the worst case but is O(n) in the best case. The more sorted an array is, the less work an insertion sort does.
- You can use an insertion sort to sort a chain of linked nodes, a task that typically is difficult.
- The Shell sort is a modification of the insertion sort that sorts subarrays of entries that are equally spaced within the array. The strategy efficiently arranges the array so that it is almost sorted, enabling an ordinary insertion sort to quickly finish the job.
- The worst-case behavior of Shell sort, as implemented in this chapter, is O(n^2). With a simple change, its worst-case behavior can be improved to at least O($n^{1.5}$).

Programming Tip

- To use `Comparable` with arbitrary types, write `Comparable<? super T>` instead of `Comparable<T>`

Exercises

1. Show the contents of the array of integers 5 7 4 9 8 5 6 3 each time a selection sort changes it while sorting the array into ascending order.
2. Repeat Exercise 1, but use an insertion sort instead.
3. Repeat Exercise 1, but use a Shell sort instead.

4. **a.** Write pseudocode for a selection sort algorithm that selects the largest, instead of the smallest, entry in the array and sorts the array into descending order.
b. Using your algorithm, repeat Exercise 1.
c. Revise the iterative method `selectionSort`, as given in Segment 8.6, so that it implements your algorithm.

5. Repeat Exercise 4, but this time sort the array into ascending order.

6. Revise the iterative method `selectionSort`, as given in Segment 8.6, so that it has `first` and `last` as parameters instead of `n`.

7. Consider a revised selection sort algorithm so that on each pass it finds both the largest and smallest values in the unsorted portion of the array. The sort then moves each of these values into its correct location by swapping array entries.

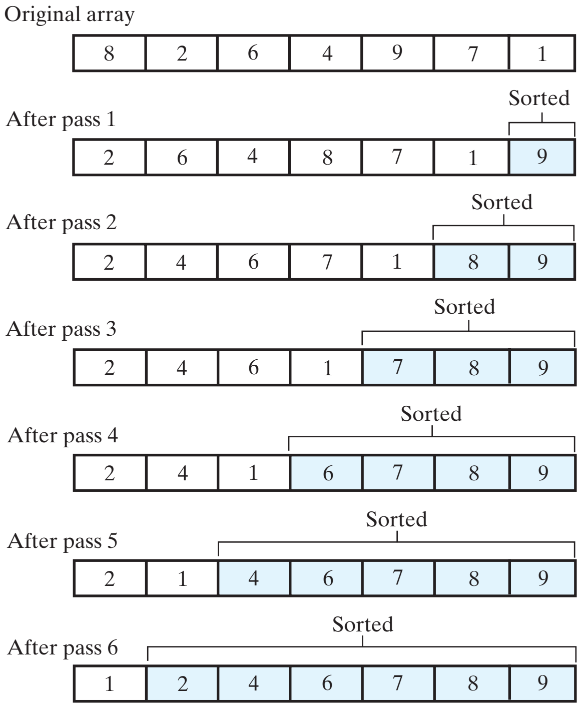
a. How many comparisons are necessary to sort *n* values?
b. Is the answer to Part *a* greater than, less than, or equal to the number of comparisons required by the original version of selection sort?

8. A **bubble sort** can sort an array of *n* entries into ascending order by making *n* – 1 passes through the array. On each pass, it compares adjacent entries and swaps them if they are out of order. For example, on the first pass, it compares the first and second entries, then the second and third entries, and so on. At the end of the first pass, the largest entry is in its proper position at the end of the array. We say that it has bubbled to its correct spot. Each subsequent pass ignores the entries at the end of the array, since they are sorted and are larger than any of the remaining entries. Thus, each pass makes one fewer comparison than the previous pass. Figure 8-17 gives an example of a bubble sort.

Implement the bubble sort

a. Iteratively
b. Recursively

FIGURE 8-17 A bubble sort of an array (see Exercise 8)



9. How does the efficiency of the bubble sort compare to the other sorting algorithms in this chapter?

10. The bubble sort in Exercise 8 always makes *n* passes. However, it is possible for the array to become sorted before all *n* passes are complete. For example, a bubble sort of the array

9 2 1 6 4 7 8

is sorted after only two passes:

2 1 6 4 7 8 9 (end of pass 1)
1 2 4 6 7 8 9 (end of pass 2)

But since a swap occurred during the second pass, the sort needs to make one more pass to check that the array is in order. Additional passes, such as the ones that the algorithm in Exercise 8 would make, are unnecessary.

You can skip these unnecessary passes and even do less work by remembering where the last swap occurred. During the first pass, the last swap is of the 9 and 8. The second pass checks up to the 8. But during the second pass, the last swap is of the 6 and 4. You now know that 6, 7, 8, and 9 are sorted. The third pass needs only to check up to the 4, instead of the 7, as an ordinary bubble sort would do. No swaps occur during the third pass, so the index of the last swap during this pass is taken as zero, indicating that no further passes are necessary. Implement this revised bubble sort.

11. Devise an algorithm that detects whether a given array is sorted into ascending order. Write a Java method that implements your algorithm. You can use your method to test whether a sort method has executed correctly.

12. Imagine wanting to perform a selection sort on a collection of `Comparable` objects. The collection is an instance of the class `Group`.

- **a.** What private methods would you need?
- **b.** Implement a method to perform a selection sort on the objects in an instance of `Group`.
- **c.** Using Big Oh notation, describe the time efficiency of your method.

13. Which recursive algorithms in this chapter are tail recursive?

14. As Segment 8.25 suggests, you can improve the efficiency of the Shell sort by adding 1 to `space` any time it is even.

- **a.** By looking at several examples, convince yourself that consecutive increments do not have a common factor.
- **b.** Subtracting 1 from `space` any time that it is even does not produce consecutive increments without common factors. Find an example of *n* that demonstrates this phenomenon.
- **c.** Revise the implementation of the Shell sort given in Segment 8.24 so that `space` is not even.

15. Suppose you want to find the largest entry in an unsorted array of *n* entries. Algorithm A searches the entire array sequentially and records the largest entry seen so far. Algorithm B sorts the array into descending order and then reports the first entry as the largest. Compare the time efficiency of the two approaches.

16. Consider the method `insertInOrder`, as given in Segment 8.11, that inserts an object into its correct position within a sorted portion of an array. If we were to use a similar algorithm to insert a node into a sorted chain of linked nodes, we would begin at the end of the chain. For example, to insert a node containing 6 into the chain shown in Figure 8-9, we first would compare 6 with the integer 10. Since 6 belongs before 10, we would then compare 6 with 8. Since 6 belongs before 8, we compare 6 with 5 and discover that 6 belongs between 5 and 8.

Describe how you could use this algorithm to define a method `insertInOrder` for a sorted chain of linked nodes.

17. Consider a class `Person` that has a string `phoneNumber` as a private data field. Phone numbers have an optional area code, but are written with dashes. For example, two possible phone numbers are 443-555-1232 and 555-0009. Write a method `compareTo` for `Person` that enables an array of `Person` objects to be sorted by phone number.

18. Consider a class `Student` that has private data fields for name, class rank, identification number, and grade point average. Imagine an array of `Student` objects that you want to sort according to any one of the previously given fields.

- **a.** What difficulty will you encounter in implementing such a sort?
- **b.** One solution to this problem defines a new class for each criterion for sorting. Each of these classes encapsulates a `Student` object. In this way, you can use the sorting methods given in this chapter. Provide the details necessary for another programmer to implement this solution.
- **c.** Another solution to this problem changes the signature and definition of the sorting method. One parameter of the method is an object that can compare two `Student` objects according to a certain criterion. This parameter belongs to one of several new classes that correspond to the sorting criteria. Provide the details necessary for another programmer to implement this solution.

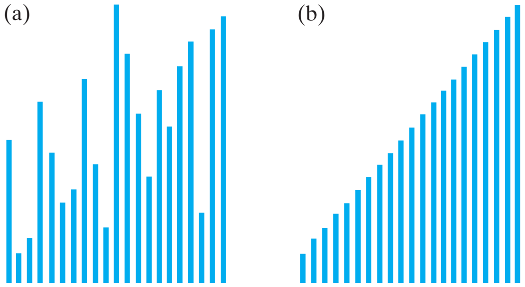
PROJECTS

1. Graphical demonstrations of various sorting algorithms are instructive, as they provide insight into how an algorithm behaves. Consider a collection of vertical lines of varying lengths, such as the ones in Figure 8-18a. Create a sorting demonstration that sorts the lines by length, as shown in Figure 8-18b. You should draw the configuration of lines after every swap or move that a given sorting algorithm makes. If you delay execution very briefly after each redraw, the result will be an animation of the sort.

You could begin by drawing 256 lines, each one pixel wide but of different lengths—and perhaps different colors—arranged from shortest to longest so that they appear as a triangle. The user then should exercise an option to scramble the lines. At a user signal, your sorting algorithm should sort the lines.

You can provide individual demonstrations, perhaps as applets, for each sort algorithm. Or you can include all the algorithms in one program that asks the user to choose an algorithm. Each sort should start with the same scrambled lines so the user can compare methods. You might also choose a sort algorithm at random and see whether the user can guess which one it is.

FIGURE 8-18 An animated sorting demonstration that sorts vertical lines (a) before its execution; (b) after its execution



2. Revise the implementations of the insertion sort and the Shell sort so that they count the number of comparisons made during a sort. Use your implementations to compare the two sorts on arrays of random `Integer` objects of various sizes. Also, compare the Shell sort as implemented in Segment 8.24 with a revised Shell sort that adds 1 to `space` any time it is even. For what array size does the difference in the number of comparisons become significant? Is the size consistent with the size predicted by the algorithm's Big Oh?

3. Complete the implementations of the sorting algorithms given in this chapter. Use your implementations to compare the run times of the sorts on various arrays of 50,000 random `Integer` objects. See the projects at the end of Chapter 4 for a description of how to time a block of Java code. Write a summary of which algorithm you find to be more efficient and why.

4. Consider an *n* by *n* array of integer values.

   a. Write an algorithm to sort the rows of the array by their first value.
   b. Using Big Oh notation, describe the efficiency of your algorithm.
   c. Implement your algorithm.

5. Suppose that you want to perform a Shell sort on a linked chain.

   a. Revise the method `incrementalInsertionSort` to work with a linked chain instead of an array.
   b. Compare the performance of `incrementalInsertionSort` on an array with its performance on a linked chain.
   c. Using the revised method, implement a Shell sort for a linked chain.
   d. Find the run time required to sort *n* values in a linked chain for different values of *n*. (See the projects at the end of Chapter 4 for a description of how to time a block of Java code.) Graph the run time versus *n*.
   e. Assuming that the performance of your sort is O(n^k), make an estimate for the value of *k*.

6. You can sort a large array of integers that are in the range 1 to *n* by using an array `count` of *n* entries to count the number of occurrences of each integer in the array. For example, consider the following array of 14 integers that range from 1 to 9:

   9 2 4 8 9 4 3 2 8 1 2 7 2 5

   Form an array `count` of 9 elements such that `count[i- 1]` contains the number of times that `i` occurs in the array to be sorted. Thus, `count` is

   1 4 1 2 1 0 1 2 2

   We now know that 1 occurs once in the original array, 2 occurs four times, and so on. Thus, the sorted array is

   1 2 2 2 2 3 4 4 5 7 8 8 9 9

   a. Implement this sorting algorithm.
   b. Using Big Oh notation, describe the efficiency of this algorithm.
   c. Is this algorithm useful as a general sorting algorithm? Explain.

Answers to Self-Test Questions

1.
```
9 6 2 4 8
2 6 9 4 8
2 4 9 6 8
2 4 6 9 8
2 4 6 8 9
```

2.
```
9 6 2 4 8
6 9 2 4 8
2 6 9 4 8
2 4 6 9 8
2 4 6 8 9
```

3. No; `insertInOrder` links the node to be inserted into the sorted part of the chain so that the node no longer references the rest of the unsorted part. Since `unsortedPart` still references the inserted node, executing the line in question next would make `unsortedPart` either reference a node in the sorted part or be `null`.

4. The public method `insertionSort` is to be invoked by using an object of `LinkedGroup`, which is the class that defines this method. Thus, the method should not be static.

5. First, you consider the subarray of equally spaced integers at the indices 0, 4, and 8 (they appear in bold):

9 8 2 7 **5** 4 6 3 **1**

Now sort them to get

1 8 2 7 **5** 4 6 3 **9**

The indices 0, 4, and 8 have a separation of 4. Next, consider the integers at indices 1 and 5:

1 **8** 2 7 5 **4** 6 3 9

Sort them to get

1 **4** 2 7 5 **8** 6 3 9

Then sort the integers at indices 2 and 6; they already are in order:

1 4 **2** 7 5 8 **6** 3 9

Next, consider the integers at indices 3 and 7. Sort them to get

1 4 2 **3** 5 8 6 **7** 9

Now decrease the separation between indices to 2. You consider the integers at the indices 0, 2, 4, 6, and 8:

1 4 **2** 3 **5** 8 **6** 7 **9**

You find that they are sorted. Then consider the integers at indices 1, 3, 5, and 7:

1 **4** 2 **3** 5 **8** 6 **7** 9

Sort them to get

1 **3** 2 **4** 5 **7** 6 **8** 9

Decreasing the separation to 1 results in an ordinary insertion sort of an array that is almost sorted.

6. **9** 6 2 4 **8** 7 5 3

8 6 2 4 **9** 7 5 3

8 **6** 2 4 9 **7** 5 3

8 6 **2** 4 9 7 **5** 3

8 6 2 **4** 9 7 5 **3**

8 6 2 **3** 9 7 5 **4**

8 6 **2** 3 **9** 7 **5** 4

2 6 **5** 3 **8** 7 **9** 4

2 **6** 5 **3** 8 **7** 9 **4**

2 **3** 5 **4** 8 **6** 9 **7**

Now apply a regular insertion sort.

Chapter 9

Faster Sorting Methods

Contents

Prerequisites

Chapter	4	The Efficiency of Algorithms
Chapter	7	Recursion
Chapter	8	An Introduction to Sorting

Objectives

After studying this chapter, you should be able to

- Sort an array into ascending order by using the following methods: merge sort, quick sort, and radix sort
- Assess the efficiency of a sort and discuss the relative efficiencies of the various methods

The sorting methods that you saw in the previous chapter often are sufficient when you want to sort small arrays. They even can be a reasonable choice if you need to sort a larger array once. Additionally, the insertion sort is a good way to sort a chain of linked nodes. However, when you need to sort very large arrays frequently, those methods take too much time. This chapter presents sorting algorithms that are much faster in general than the methods in Chapter 8.

Merge Sort

9.1 The **merge sort** divides an array into halves, sorts the two halves, and then merges them into one sorted array. The algorithm for merge sort is usually stated recursively. You know that a recursive algorithm expresses the solution to a problem in terms of a smaller version of the same problem. When you divide a problem into two or more smaller but *distinct* problems, solve *each* new problem, and then combine their solutions to solve the original problem, the strategy is said to be a **divide and conquer** algorithm. That is, you divide the problem into pieces and conquer each piece to reach a solution. Although divide and conquer algorithms often are expressed recursively, this is not a requirement.

VideoNote
Merge sort

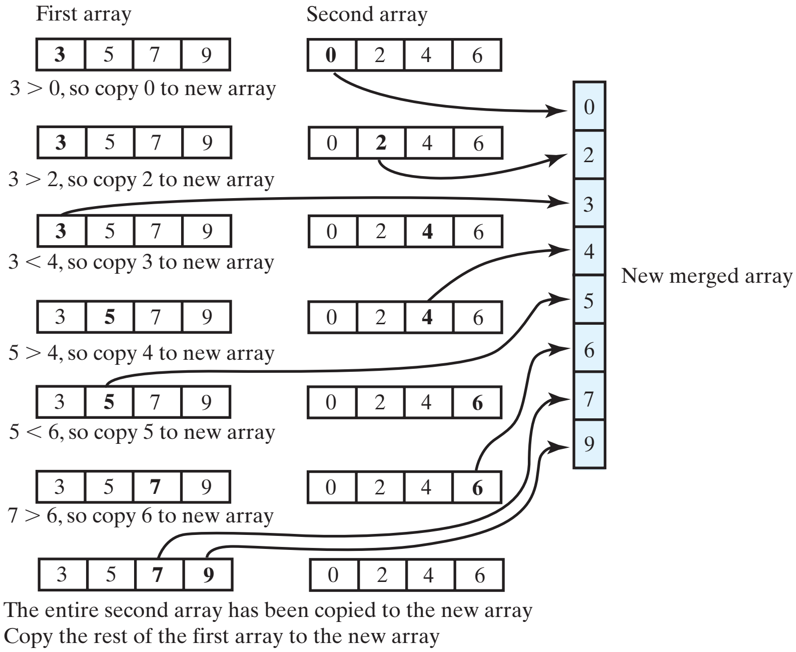
When expressed recursively, a divide and conquer algorithm contains two or more recursive calls. Most of the recursive solutions that you have seen so far do not use the divide and conquer strategy. For example, Segment 8.7 gave a recursive version of the selection sort. Even though that algorithm considers smaller and smaller arrays, it does not divide the problem into two sorting problems.

The real effort during the execution of a merge sort occurs during the merge step, and this is also the step that involves most of the programming effort, so we will begin there.

Merging Arrays

9.2 Imagine that you have two distinct arrays that are sorted. Merging two sorted arrays is not difficult, but it does require an additional array. Processing both arrays from beginning to end, you compare an entry in one array with an entry in the other and copy the smaller entry to a new third array, as Figure 9-1 shows. After reaching the end of one array, you simply copy the remaining entries from the other array to the new third array.

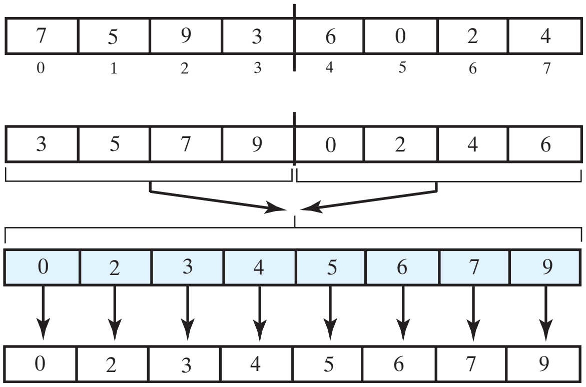
FIGURE 9-1 Merging two sorted arrays into one sorted array


First array
Second array
3 5 7 9
0 2 4 6
3 > 0, so copy 0 to new array
3 5 7 9
0 2 4 6
3 > 2, so copy 2 to new array
3 5 7 9
0 2 4 6
3 < 4, so copy 3 to new array
3 5 7 9
0 2 4 6
5 > 4, so copy 4 to new array
3 5 7 9
0 2 4 6
5 < 6, so copy 5 to new array
3 5 7 9
0 2 4 6
7 > 6, so copy 6 to new array
3 5 7 9
0 2 4 6
The entire second array has been copied to the new array
Copy the rest of the first array to the new array
0 2 3 4 5 6 7 9
New merged array

Recursive Merge Sort

9.3 **The algorithm.** In a merge sort, you merge two sorted arrays that are actually halves of the original array. That is, you divide the array into halves, sort each half, and merge the sorted halves into a second temporary array, as Figure 9-2 shows. You then copy the temporary array back to the original array.

FIGURE 9-2 The major steps in a merge sort



Divide the array into two halves

Sort the two halves

Merge the sorted halves into another array

Copy the merged array back into the original array

This sounds like a simple plan, but how did we sort the two halves of the array? By using a merge sort, of course! If `mid` is the index of the approximate midpoint of an array of *n* entries, we need to sort the entries indexed by `0` through `mid`, and then the entries indexed by `mid + 1` through `n - 1`. Since we perform these sorts by making recursive calls to the merge sort algorithm, the algorithm needs two parameters—`first` and `last`—to specify the first and last indices of the subrange of the array to be sorted. We will use the notation `a[first..last]` to mean the array entries `a[first]`, `a[first + 1]`, ..., `a[last]`.

Merge sort has the following recursive formulation:

```
Algorithm mergeSort(a, tempArray, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   mid = (first + last) / 2
   mergeSort(a, tempArray, first, mid)
   mergeSort(a, tempArray, mid + 1, last)
   Merge the sorted halves a[first..mid] and a[mid + 1..last] using the array tempArray
}
```

Notice that the algorithm ignores arrays of one or fewer entries.

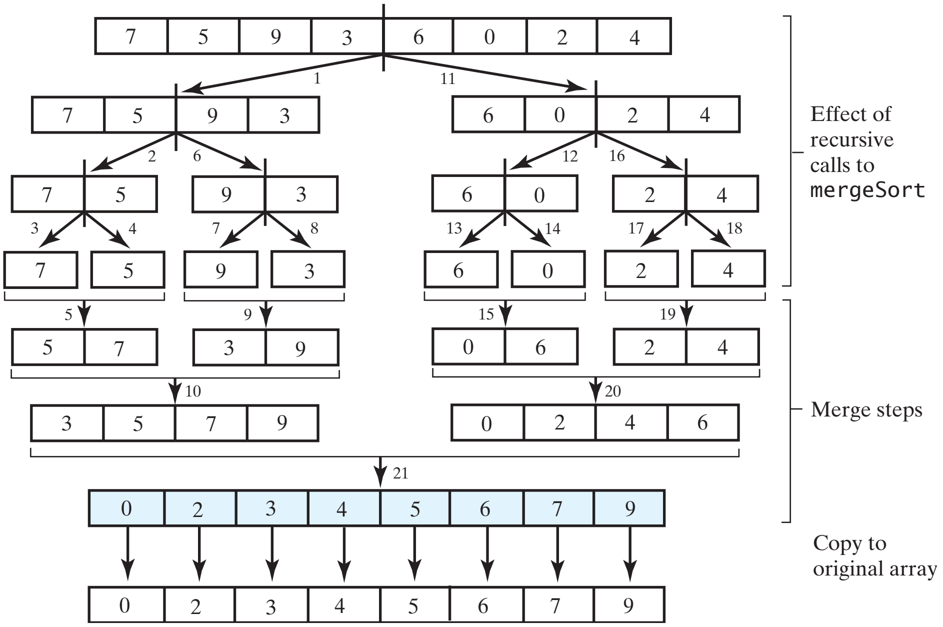
The following pseudocode describes the merge step:

```
Algorithm merge(a, tempArray, first, mid, last)
// Merges the adjacent subarrays a[first..mid] and a[mid + 1..last].

beginHalf1 = first
endHalf1 = mid
beginHalf2 = mid + 1
endHalf2 = last

// While both subarrays are not empty, compare an entry in one subarray with
// an entry in the other; then copy the smaller item into the temporary array
index = 0 // next available location in tempArray
```

```
while ( (beginHalf1 <= endHalf1) and (beginHalf2 <= endHalf2) )
{
   if (a[beginHalf1] <= a[beginHalf2])
   {
      tempArray[index] = a[beginHalf1]
      beginHalf1++
   }
   else
   {
      tempArray[index] = a[beginHalf2]
      beginHalf2++
   }
   index++
}
// Assertion: One subarray has been completely copied to tempArray.

Copy remaining entries from other subarray to tempArray
Copy entries from tempArray to array a
```

9.4 **Tracing the steps in the algorithm.** Let's examine what happens when we invoke `mergeSort` on the array halves. Figure 9-3 shows that `mergeSort` divides an array into two halves and then recursively divides each of those halves into two halves until each half contains only one entry. At this point in the algorithm, the merge steps begin. Pairs of one-entry subarrays are merged to form a two-entry subarray. Pairs of two-entry subarrays are merged to form a four-entry subarray, and so on.

FIGURE 9-3 The effect of the recursive calls and the merges during a merge sort



Numbers on the arrows in the figure indicate the order in which the recursive calls and the merges occur. Notice that the first merge occurs after four recursive calls to `mergeSort` and before other recursive calls to `mergeSort`. Thus, the recursive calls to `mergeSort` are interwoven with calls to `merge`. The actual sorting takes place during the merge steps and not during the recursive calls.

As you will see, we can use these observations in two ways. First, we can ascertain the algorithm's efficiency. Second, we can formulate the `mergeSort` algorithm iteratively.

Note: Merge sort rearranges the entries in an array during its merge steps.

Question 1 Trace the steps that a merge sort takes when sorting the following array into ascending order: 9 6 2 4 8 7 5 3.

9.5 **Implementation note.** Although the implementation of the recursive `mergesort` is straightforward, you should be careful to allocate the temporary array only once. Since the array is an implementation detail, you might be tempted to hide its allocation in the method `merge`. But since `merge` is called each time `mergesort` is called recursively, a temporary array would be allocated and initialized many times. Instead, we can allocate a temporary array in the following public version of `mergesort` and pass it to a private `mergesort` that implements the pseudocode given previously:

```
public static <T extends Comparable<? super T>>
       void mergeSort(T[] a, int first, int last)
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempArray = (T[])new Comparable<?>[a.length]; // unchecked cast
   mergeSort(a, tempArray, first, last);
} // end mergeSort
```

Segment 8.2 in the previous chapter introduced the notation `? super T` to mean any superclass of `T`. When we allocate an array of `Comparable` objects, we use a wildcard `?` to represent any object. We then cast the array to an array of type `T` objects.

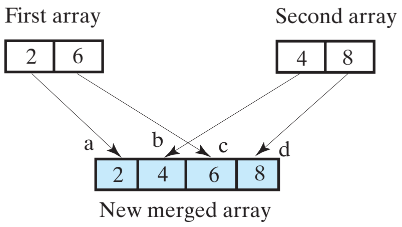
The Efficiency of Merge Sort

9.6 Assume for now that n is a power of 2, so that we can divide n by 2 evenly. The array in Figure 9-3 has $n = 8$ entries. The initial call to `mergeSort` makes two recursive calls to `mergeSort`, dividing the array into two subarrays of $n/2$, or 4, entries each. Each of the two recursive calls to `mergeSort` makes two recursive calls to `mergeSort`, dividing the two subarrays into four subarrays of $n/2^2$, or 2, entries each. Finally, recursive calls to `mergeSort` divide the four subarrays into eight subarrays of $n/2^3$, or 1, entry each. It takes three levels of recursive calls to obtain subarrays of one entry each. Notice that the original array contained 2^3 entries. The exponent 3 is the number of levels of recursive calls. In general, if n is 2^k, k levels of recursive calls occur.

Now consider the merge steps, because that is where the real work occurs. The merge step makes at most $n - 1$ comparisons among the n entries in the two subarrays. Figure 9-4 provides an example of a merge that requires $n - 1$ comparisons, while Figure 9-1 shows an example where fewer than $n - 1$ comparisons occur. Each merge also requires n moves to a temporary array and n moves back to the original array. In total, each merge requires at most $3n - 1$ operations.

Each call to `mergeSort` calls `merge` once. The merge operation as a result of the original call to `mergeSort` requires at most $3n - 1$ operations. It is O(n). An example of this merge appears as step 21 in Figure 9-3. The two recursive calls to `mergeSort` result in two calls to `merge`. Each call merges $n/2$ entries in at most $3n/2 - 1$ operations. The two merges then require at most $3n - 2$ operations. They are O(n). The next level of recursion involves 2^2 calls to `mergeSort` resulting in four calls to `merge`. Each call to `merge` merges $n/2^2$ entries in at most $3n/2^2 - 1$ operations. Together these four merges use at most $3n - 2^2$ operations, so are O(n).

FIGURE 9-4 A worst-case merge of two sorted arrays



a. 2 < 4, so copy 2 to new array

b. 6 > 4, so copy 4 to new array

c. 6 < 8, so copy 6 to new array

d. Copy 8 to new array

If n is 2^k, the k levels of recursive calls to `mergeSort` result in k levels of merges. The merges at each level are O(n). Since k is $\log_2 n$, `mergeSort` is O($n \log n$). When n is not a power of 2, we can find an integer k so that $2^{k-1} < n < 2^k$. For example, when n is 15, k is 4. Thus,

$$k - 1 < \log_2 n < k$$

so if we round $\log_2 n$ up, we will get k. Therefore, the merge sort is O($n \log n$) in this case as well. Notice that the merge steps are O(n) regardless of the initial order of the array. Merge sort is then O($n \log n$) in the worst, best, and average cases.

A disadvantage of merge sort is the need for the temporary array during the merge step. At the beginning of Chapter 8, we spoke of sorting the books on your bookshelf by height. We were able to do so without the extra space that another shelf or the floor would provide. You can see now that the merge sort would require this extra space. Later in this chapter, you will see another algorithm that sorts in O($n \log n$) time without a second array.

Note: The time efficiency of merge sort

Merge sort is O($n \log n$) in all cases. Its need for a temporary array is a disadvantage. The time required for copying entries, however, is less in Java than in other programming languages, since references are copied instead of the actual objects.

9.7 **Assessing efficiency another way.** In Chapter 7, we used a recurrence relation to estimate the time efficiency of recursive algorithms. We can use the same technique here. If $t(n)$ represents the time requirement of `mergeSort` in the worst case, the two recursive calls each require time $t(n/2)$. The merge step is O(n). Thus, we have the following:

$$\begin{aligned} t(n) &= t(n/2) + t(n/2) + n \\ &= 2\,t(n/2) + n \text{ when } n > 1 \\ t(1) &= 0 \end{aligned}$$

As a first step in solving this recurrence relation, we evaluate it for a specific value of n. Since $t(n)$ involves $n/2$, choosing n to be a power of 2—8, for example—is convenient. We then have

$$\begin{aligned} t(8) &= 2\,t(4) + 8 \\ t(4) &= 2\,t(2) + 4 \\ t(2) &= 2\,t(1) + 2 = 2 \end{aligned}$$

By substituting repeatedly, we get the following for $t(8)$:

$$\begin{aligned} t(8) &= 2\,t(4) + 8 \\ &= 2\,[2\,t(2) + 4] + 8 \\ &= 4\,t(2) + 8 + 8 \end{aligned}$$

$$= 4\,[2\,t(1) + 2] + 8 + 8$$
$$= 8 + 8 + 8$$
$$= 8 \times 3$$

Since $8 = 2^3$, $3 = \log_2 8$, so we guess that

$$t(n) = n \log_2 n$$

Just as we did in Chapter 7, we now need to prove that our guess is in fact true. We leave this proof as an exercise.

Iterative Merge Sort

9.8 Once we have the merge algorithm, developing the recursive merge sort is easy. Developing an iterative merge sort is not as simple. We begin by making some observations about the recursive solution.

The recursive calls simply divide the array into *n* one-entry subarrays, as you can see in Figure 9-3. Although we do not need recursion to isolate the entries in an array, the recursion controls the merging process. To replace the recursion with iteration, we will need to control the merges. Such an algorithm will be more efficient of both time and space than the recursive algorithm, since it will eliminate the recursive calls and, therefore, the stack of activation records. But an iterative merge sort will be trickier to code without error.

Basically, an iterative merge sort starts at the beginning of the array and merges pairs of individual entries to form two-entry subarrays. Then it returns to the beginning of the array and merges pairs of the two-entry subarrays to form four-entry subarrays, and so on. However, after merging all pairs of subarrays of a particular length, we might have entries left over. Merging these requires some care. Project 2 at the end of this chapter asks you to develop an iterative merge sort. You will see there that you can save much of the time necessary to copy the temporary array back to the original array during the merges.

Merge Sort in the Java Class Library

9.9 The class `Arrays` in the package `java.util` defines several versions of a static method `sort` to sort an array into ascending order. For an array of objects, `sort` uses a merge sort. The method

```
public static void sort(Object[] a)
```

sorts an entire array `a` of objects, while the method

```
public static void sort(Object[] a, int first, int after)
```

sorts the objects in `a[first]` through `a[after - 1]`. For both methods, objects in the array must define the `Comparable` interface.

The merge sort used by these methods skips the merge step if none of the entries in the left half of the array are greater than the entries in the right half. Since both halves are sorted already, the merge step is unnecessary in this case.

Note: **Stable sorts**
A sorting algorithm is **stable** if it does not change the relative order of objects that are equal. For example, if object `x` appears before object `y` in a collection of data, and `x.compareTo(y)` is zero, a stable sorting algorithm will leave object `x` before object `y` after sorting the data. Stability is important to certain applications. For example, suppose that you sort a group of people, first by name and then by age. A stable sorting algorithm will ensure that people of the same age will remain in alphabetical order.

The merge sorts in the Java Class Library are stable. Exercise 10 at the end of this chapter asks you to identify the stable sorting algorithms presented in this chapter and the previous one.

Question 2 Modify the merge sort algorithm given in Segment 9.3 so that it skips any unnecessary merges, as just described.

Quick Sort

9.10 We now look at another divide and conquer strategy for sorting an array. The **quick sort** divides an array into two pieces, but unlike merge sort, these pieces are not necessarily halves of the array. Instead, quick sort chooses one entry in the array—called the **pivot**—and rearranges the array entries so that

- The pivot is in the position that it will occupy in the final sorted array
- Entries in positions before the pivot are less than or equal to the pivot
- Entries in positions after the pivot are greater than or equal to the pivot

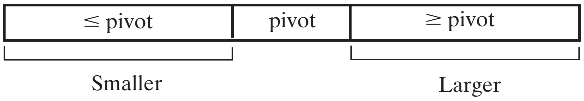
This arrangement is called a **partition** of the array.

VideoNote
Quick sort

Creating the partition divides the array into two pieces, which we will call *Smaller* and *Larger*, separated by the pivot, as Figure 9-5 illustrates. Since the entries in *Smaller* are less than or equal to the pivot, and the entries in *Larger* are greater than or equal to the pivot, the pivot is in its correct and final position within the sorted array. If we now sort the two subarrays *Smaller* and *Larger*—by using quick sort, of course—the original array will be sorted. The following algorithm describes our sorting strategy:

```
Algorithm quickSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   Choose a pivot
   Partition the array about the pivot
   pivotIndex = index of pivot
   quickSort(a, first, pivotIndex - 1)  // sort Smaller
   quickSort(a, pivotIndex + 1, last)   // sort Larger
}
```

FIGURE 9-5 A partition of an array during a quick sort



The Efficiency of Quick Sort

9.11 Notice that creating the partition—which accounts for most of `quickSort`'s work—occurs before the recursive calls to `quickSort`. Contrast this with merge sort, where most of the work occurs during the merge phase *after* the recursive calls to `mergeSort`. Partitioning will require no more than n comparisons, and so, like merging, it will be an O(n) task. Thus, we can assess the efficiency of quick sort, even though we have not yet developed a partitioning strategy.

The ideal situation occurs when the pivot moves to the center of the array, so the two subarrays that the partition forms are the same size. If every recursive call to `quickSort` forms a partition with equal-sized subarrays, the quick sort will be like merge sort in that the recursive calls halve the array. Thus, quick sort would be O($n \log n$), and this would be its best case.

This ideal situation might not always occur, however. In the worst case, each partition has one empty subarray. Although one recursive call will have nothing to do, the other call must sort $n - 1$ entries instead of $n/2$. The result is n levels of recursive calls instead of log n. Thus, in the worst case, quick sort is O(n^2).

The choice of pivots, then, affects quick sort's efficiency. Some pivot-selection schemes can lead to worst-case behavior if the array is already sorted or nearly sorted. In practice, nearly sorted arrays can occur more frequently than you might imagine. As you will see later, our pivot-selection scheme will avoid worst-case behavior for sorted arrays.

Although we will not prove it, quick sort is O(n log n) in the average case. While merge sort is always O(n log n), quick sort can be faster than merge sort in practice and does not require the additional memory that merge sort needs for merging.

Note: The time efficiency of quick sort
Quick sort is O(n log n) in the average case but O(n^2) in the worst case. The choice of pivots affects its behavior.

Creating the Partition

9.12 Various strategies are possible for choosing a pivot and for creating the partition in Figure 9-5. For now, we will assume that you have chosen a pivot, and so we will describe how to create a partition independently of your pivot-selection strategy. Later, our actual pivot-selection scheme will suggest minor changes to this partitioning process.

After choosing a pivot, swap it with the last entry in the array so that the pivot is not in your way while you create the partition. Figure 9-6a shows an array after this step. Starting at the beginning of the array and moving toward the end (left to right in the figure), look for the first entry that is greater than or equal to the pivot. In Figure 9-6b, that entry is 5 and occurs at the index `indexFromLeft`. In a similar fashion, starting at the next-to-last entry and moving toward the beginning of the array (right to left in the figure), look for the first entry that is less than or equal to the pivot. In Figure 9-6b, that entry is 2 and occurs at the index `indexFromRight`. Now, if `indexFromLeft` is less than `indexFromRight`, swap the two entries at those indices. Figure 9-6c shows the result of this step. The 2, which is less than the pivot, has moved toward the beginning of the array while the 5, which is greater than the pivot, has moved in the opposite direction.

Continue the searches from the left and from the right. Figure 9-6d shows that the search from the left stops at 4 and the search from the right stops at 1. Since `indexFromLeft` is less than `indexFromRight`, swap 4 and 1. The array now appears as in Figure 9-6e. Entries equal to the pivot are allowed in either piece of the partition.

Continue the searches again. Figure 9-6f shows that the search from the left stops at 6 while the search from the right goes beyond the 6 to stop at 1. Since `indexFromLeft` is not less than `indexFromRight`, no swap is necessary and the searches end. The only remaining step is to place the pivot between the subarrays *Smaller* and *Larger* by swapping `a[indexFromLeft]` and `a[last]`, as Figure 9-6g shows. The completed partition appears in Figure 9-6h.

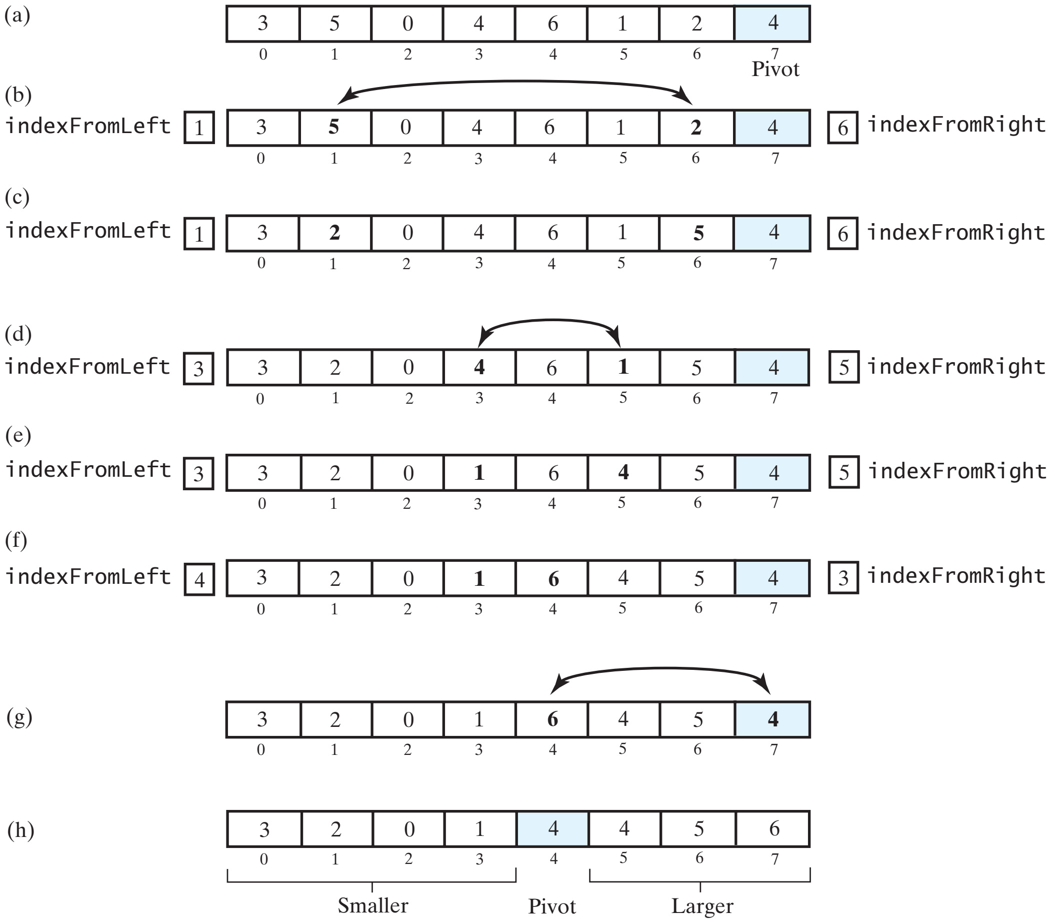
Note that the previous searches must not go beyond the ends of the array. Soon, in Segment 9.15, you will see a convenient way to implement this requirement.

9.13 **Entries equal to the pivot.** Notice that both of the subarrays *Smaller* and *Larger* can contain entries equal to the pivot. This might seem a bit strange to you. Why not always place any entries that equal the pivot into the same subarray? Such a strategy would tend to make one subarray larger than the other. However, to enhance quick sort's performance, we want the subarrays to be as nearly equal in size as possible.

Notice that both the search from the left and the search from the right stop when they encounter an entry that equals the pivot. This means that rather than leaving such entries in place, they are swapped. It also means that such an entry has a chance of landing in each of the subarrays.

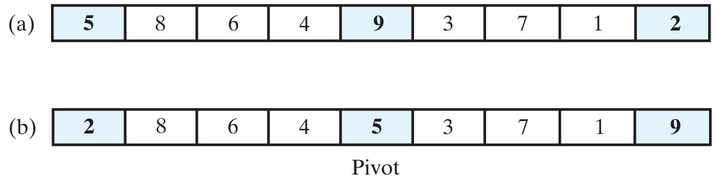
9.14 **Pivot selection.** Ideally, the pivot should be the median value in the array, so that the subarrays *Smaller* and *Larger* each have the same—or nearly the same—number of entries. One way to find the median value is to sort the array and then get the value in the middle. But sorting the array is the original problem, so this circular logic is doomed. Other ways to find the median are too slow to use.

FIGURE 9-6 A partitioning strategy for quick sort



Since choosing the best pivot takes too much time, we should at least try to avoid a bad pivot. So instead of finding the median of all values in the array, we will take as our pivot the median of three entries in the array: the first entry, the middle entry, and the last entry. One way to accomplish this task is to sort only those three entries and use the middle entry of the three as the pivot. Figure 9-7 shows an array both before and after its first, middle, and last entries are sorted. The pivot is the 5. This pivot selection strategy is called **median-of-three pivot selection**.

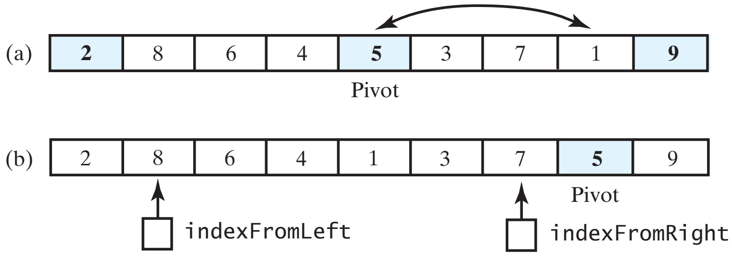
FIGURE 9-7 Median-of-three pivot selection: (a) The original array; (b) the array with its first, middle, and last entries sorted



Note: Median-of-three pivot selection avoids worst-case performance by quick sort when the given array is already sorted or nearly sorted. While it theoretically does not avoid worst-case performance for other arrays, such performance is unlikely in practice.

9.15 **Adjusting the partition algorithm.** Median-of-three pivot selection suggests some minor adjustments to our partitioning scheme. Previously, we swapped the pivot with the last entry in the array prior to partitioning. But here, the first, middle, and last entries in the array are sorted, so we know that the last entry is at least as large as the pivot. Thus, the last entry belongs in the subarray *Larger*. We can simply leave the last entry in place. To get the pivot out of the way, we can swap it with the next-to-last entry, `a[last - 1]`, as Figure 9-8 shows. Thus, the partition algorithm can begin its search from the right at index `last - 2`.

FIGURE 9-8 (a) The array with its first, middle, and last entries sorted; (b) the array after positioning the pivot and just before partitioning



Also notice that the first entry is at least as small as the pivot, and so it belongs in the subarray *Smaller*. Thus, we can leave the first entry in place and have the partition algorithm begin its search from the left at index `first + 1`. Figure 9-8b shows the status of the array at this point, just prior to partitioning.

This scheme provides a side benefit that simplifies the loops for the two searches. The search from the left looks for an entry that is greater than or equal to the pivot. That search will terminate because, at worst, it will stop at the pivot. The search from the right looks for an entry that is less than or equal to the pivot. That search will terminate because, at worst, it will stop at the first entry. Thus, the loops need not do anything special to prevent the searches from going beyond the ends of the array.

After the search loops end, we need to position the pivot between the subarrays *Smaller* and *Larger*. We do this by swapping the entries `a[indexFromLeft]` and `a[last - 1]`.

Note: Quick sort rearranges the entries in an array during the partitioning process. Each partition places one entry—the pivot—in its correct sorted position. The entries in each of the two subarrays that are before and after the pivot will remain in their respective subarrays.

Java Code for Quick Sort

9.16 **Pivot selection.** Median-of-three pivot selection requires us to sort three entries. We do this with simple comparisons and swaps, as follows:

```
/** Sorts the first, middle, and last entries of an array
    into ascending order.
    @param a      an array of Comparable objects
    @param first  the integer index of the first array entry;
                  first >= 0 and < a.length
    @param mid    the integer index of the middle array entry
    @param last   the integer index of the last array entry;
                  last - first >= 2, last < a.length */
private static <T extends Comparable<? super T>>
        void sortFirstMiddleLast(T[] a, int first, int mid, int last)
{
   order(a, first, mid); // make a[first] <= a[mid]
   order(a, mid, last);  // make a[mid] <= a[last]
   order(a, first, mid); // make a[first] <= a[mid]
} // end sortFirstMiddleLast

/** Orders two given array entries into ascending order
    so that a[i] <= a[j].
    @param a  an array of Comparable objects
    @param i  an integer >= 0 and < array.length
    @param j  an integer >= 0 and < array.length */
private static <T extends Comparable<? super T>>
        void order(T[] a, int i, int j)
{
   if (a[i].compareTo(a[j]) > 0)
      swap(a, i, j);
} // end order

/** Swaps the array entries array[i] and array[j]. */
private static void swap(Object[] array, int i, int j)
{
   Object temp = array[i];
   array[i] = array[j];
   array[j] = temp;
} // end swap
```

After passing the indices of the first, middle, and last entries of the array to the method `sortFirstMiddleLast`, we will have the pivot at the middle index.

9.17 **Partitioning.** Median-of-three pivot selection assumes that the array has at least three entries. If you have only three entries, the pivot selection sorts them, so there is no need for the partition method or for quick sort. Thus, the following partition method assumes that the array contains at least four entries:

```
/** Partitions an array as part of quick sort into two subarrays
    called Smaller and Larger that are separated by a single
    entry called the pivot.
    Entries in Smaller are <= pivot and appear before the
    pivot in the array.
    Entries in Larger are >= pivot and appear after the
    pivot in the array.
    @param a      an array of Comparable objects
    @param first  the integer index of the first array entry;
                  first >= 0 and < a.length
    @param last   the integer index of the last array entry;
                  last - first >= 3; last < a.length
    @return the index of the pivot */
private static <T extends Comparable<? super T>>
        int partition(T[] a, int first, int last)
{
   int mid = (first + last) / 2;
   sortFirstMiddleLast(a, first, mid, last);

   // Assertion: The pivot is a[mid]; a[first] <= pivot and
   // a[last] >= pivot, so do not compare these two array entries
   // with pivot.

   // move pivot to next-to-last position in array
   swap(a, mid, last - 1);
   int pivotIndex = last - 1;
   T pivot = a[pivotIndex];

   // determine subarrays Smaller = a[first..endSmaller]
   // and                 Larger  = a[endSmaller+1..last-1]
   // such that entries in Smaller are <= pivot and
   // entries in Larger are >= pivot; initially, these subarrays are empty

   int indexFromLeft = first + 1;
   int indexFromRight = last - 2;

   boolean done = false;
   while (!done)
   {
      // starting at beginning of array, leave entries that are < pivot;
      // locate first entry that is >= pivot; you will find one,
      // since last entry is >= pivot
      while (a[indexFromLeft].compareTo(pivot) < 0)
         indexFromLeft++;

      // starting at end of array, leave entries that are > pivot;
      // locate first entry that is <= pivot; you will find one,
      // since first entry is <= pivot

      while (a[indexFromRight].compareTo(pivot) > 0)
         indexFromRight--;

      assert a[indexFromLeft].compareTo(pivot) >= 0 &&
             a[indexFromRight].compareTo(pivot) <= 0;

      if (indexFromLeft < indexFromRight)
      {
         swap(a, indexFromLeft, indexFromRight);
         indexFromLeft++;
         indexFromRight--;
      }
      else
         done = true;
   } // end while

   // place pivot between Smaller and Larger subarrays
   swap(a, pivotIndex, indexFromLeft);
   pivotIndex = indexFromLeft;

   // Assertion:
   //   Smaller = a[first..pivotIndex-1]
   //   Pivot = a[pivotIndex]
   //   Larger = a[pivotIndex+1..last]

   return pivotIndex;
} // end partition
```

9.18 **The quick sort method.** Before completing the Java code for quick sort, we need to think about small arrays. You have seen that the array should contain at least four entries before you call the partition method. But simply agreeing to use quick sort only on large arrays is not enough. The pseudocode given for quick sort in Segment 9.10 shows that partitioning even a very large array will eventually lead to a recursive call that involves an array as small as two entries. The code for quick sort needs to screen out these small arrays and use another way to sort them. An insertion sort is a good choice for small arrays. In fact, using it instead of quick sort on arrays of as many as ten entries is reasonable. The following method implements quick sort with these observations in mind. The method assumes a constant MIN_SIZE that specifies the size of the smallest array on which we will use a quick sort.

```
/** Sorts an array into ascending order. Uses quick sort with
    median-of-three pivot selection for arrays of at least
    MIN_SIZE entries, and uses insertion sort for other arrays. */
public static <T extends Comparable<? super T>>
       void quickSort(T[] a, int first, int last)
{
   if (last - first + 1 < MIN_SIZE)
   {
      insertionSort(a, first, last);
   }
   else
   {
      // create the partition: Smaller | Pivot | Larger
      int pivotIndex = partition(a, first, last);

      // sort subarrays Smaller and Larger
      quickSort(a, first, pivotIndex - 1);
      quickSort(a, pivotIndex + 1, last);
   } // end if
} // end quickSort
```

Question 3 Trace the steps that the method quickSort takes when sorting the following array into ascending order: 9 6 2 4 8 7 5 3. Assume that MIN_SIZE is 4.

Quick Sort in the Java Class Library

9.19 The class Arrays in the package java.util uses a quick sort to sort arrays of primitive types into ascending order. The method

```
public static void sort(type[] a)
```

sorts an entire array a, while the method

```
public static void sort(type[] a, int first, int after)
```

sorts the entries in `a[first]` through `a[after - 1]`. Note that *type* is either `byte`, `char`, `double`, `float`, `int`, `long`, or `short`.

Radix Sort

9.20 The sorting algorithms that you have seen so far sort objects that can be compared. The **radix sort** does not use comparison, but to work, it must restrict the data that it sorts. For this restricted data, the radix sort is O(*n*), and so it is faster than any other sort in this chapter. However, it is not suitable as a general-purpose sorting algorithm, because it treats array entries as if they were strings that have the same length.

Let's look at an example of a radix sort of the following three-digit positive integers:

123 398 210 019 528 003 513 129 220 294

Notice that 19 and 3 are padded with zeros to make them three-digit integers. The radix sort begins by grouping the integers according to their rightmost digits. Since a digit can have one of 10 values, we need 10 groups, or **buckets**. If bucket *d* corresponds to the digit *d*, we place 123 into bucket 3, 398 into bucket 8, and so on. Figure 9-9a shows the result of this process. Notice that each bucket must retain the order in which it receives the integers.

Looking at the buckets sequentially, we see that the integers are now in the following order:

210 220 123 003 513 294 398 528 019 129

We move these integers from the buckets to the original array. We then group the integers by their middle digits, using the now empty buckets. Thus, 210 goes into bucket 1, 220 goes into bucket 2, 123 goes into bucket 2, and so on. Figure 9-9b shows the result of this pass.

The integers in the buckets are now in this order:

003 210 513 019 220 123 528 129 294 398

After moving these integers from the buckets back to the array, we group them by their leftmost digits. Thus, 003 goes into bucket 0, 210 goes into bucket 2, 513 goes into bucket 5, and so on. Figure 9-9c shows the result of this pass.

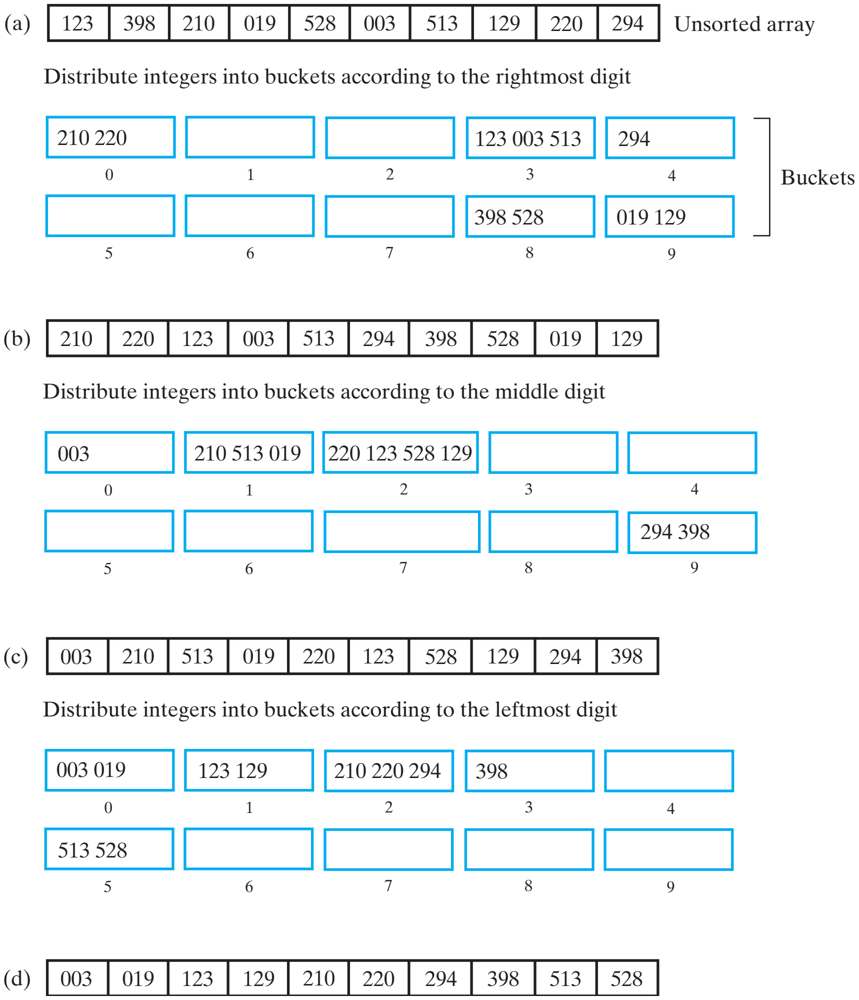
The integers in the buckets are now in their final sorted order:

003 019 123 129 210 220 294 398 513 528

Aside: Origin of the radix sort

During the early days of computing, data was stored on punched cards. Each card had 80 columns in which to store 80 characters. Each column had 12 rows that were the possible positions for holes. A machine called a card sorter distributed the cards among 12 bins according to the row punched in the column chosen by the machine's operator. These bins are analogous to the buckets in a radix sort. After running a stack of cards through the card sorter, the operator would gather the cards a bin at a time to create a new stack. The cards would be run through the sorter again to sort the next column of holes. By repeating this process, the operator could sort the cards.

FIGURE 9-9 Radix sort: (a) Original array and buckets after first distribution; (b) reordered array and buckets after second distribution; (c) reordered array and buckets after third distribution; (d) sorted array



Pseudocode for Radix Sort

9.21 Our previous description of radix sort assumed that the integers to be sorted each contain the same number of digits. Actually, this requirement is unnecessary as long as you get 0 when you ask for a digit that does not exist. For example, if you ask for the hundreds digit of a two-digit integer, you should get 0.

The following algorithm describes a radix sort of an array of positive decimal integers. We number the digits in each integer from the right beginning at zero. Thus, the units digit is digit 0, the tens digit is digit 1, and so on.

```
Algorithm radixSort(a, first, last, maxDigits)
// Sorts the array of positive decimal integers a[first..last] into ascending order;
// maxDigits is the number of digits in the longest integer.

for (i = 0 to maxDigits - 1)
{
   Clear bucket[0], bucket[1], . . . , bucket[9]
   for (index = first to last)
   {
      digit = digit i of a[index]
      Place a[index] at end of bucket[digit]
   }
   Place contents of bucket[0], bucket[1], . . . , bucket[9] into the array a
}
```

This algorithm uses an array of buckets. The nature of a bucket is unspecified, but after you read Chapter 10, you will see that a bucket can be an instance of the ADT queue.

Question 4 Trace the steps that the algorithm `radixSort` takes when sorting the following array into ascending order:

6340 1234 291 3 6325 68 5227 1638

The Efficiency of Radix Sort

9.22 If an array contains *n* integers, the inner loop in the previous algorithm iterates *n* times. If each integer contains *d* digits, the outer loop iterates *d* times. Thus, the radix sort is O(*d* x *n*). The *d* in this expression tells us that the actual running time for a radix sort depends on the size of the integers. But on a computer, the typical integer is restricted in size to about 10 decimal digits, or 32 bits. As long as *d* is fixed and is much smaller than *n,* radix sort is simply an O(*n*) algorithm.

Note: Although radix sort is an O(*n*) algorithm for certain data, it is not appropriate for all data.

Question 5 One of the difficulties with the radix sort is that the number of buckets depends on the kind of strings you are sorting. You saw that sorting integers requires 10 buckets; sorting words requires at least 26 buckets. If you use radix sort to alphabetize an array of words, what changes would be necessary to the given algorithm?

Comparing the Algorithms

9.23 Figure 9-10 summarizes the efficiencies of the sorting algorithms presented in this chapter and the previous chapter. Although a radix sort is fastest, it is not always applicable. The merge sort and quick sort are generally faster than any of the other algorithms.

FIGURE 9-10 The time efficiency of various sorting algorithms, expressed in Big Oh notation

	Average Case	Best Case	Worst Case
Radix sort	$O(n)$	$O(n)$	$O(n)$
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Shell sort	$O(n^{1.5})$	$O(n)$	$O(n^2)$ or $O(n^{1.5})$
Insertion sort	$O(n^2)$	$O(n)$	$O(n^2)$
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$

To give you an idea of how the problem size affects time efficiency, Figure 9-11 tabulates the four growth-rate functions that appear in Figure 9-10 for several values of n. You certainly could use an $O(n^2)$ sort algorithm when n is 10. When n is 100, a Shell sort is almost as fast as a quick sort in the average case. But when n is one million, an average-case quick sort is much faster than a Shell sort and much, much faster than an insertion sort.

If your array has relatively few entries, or if it is nearly sorted, the insertion sort is a good choice. Otherwise, the quick sort is generally preferable. Note that merge sort is useful when the data collection is in an external file because it is too large to reside in main memory all at once.

We will discuss another sorting algorithm, the heap sort, in Chapter 26. This technique is also $O(n \log n)$, but the quick sort is usually preferable.

FIGURE 9-11 A comparison of growth-rate functions as n increases

n	10	10^2	10^3	10^4	10^5	10^6
$n \log_2 n$	33	664	9966	132,877	1,660,964	19,931,569
$n^{1.5}$	32	10^3	31,623	10^6	31,622,777	10^9
n^2	10^2	10^4	10^6	10^8	10^{10}	10^{12}

Chapter Summary

- Merge sort is a divide and conquer algorithm that halves an array, recursively sorts the two halves, and then merges them into one sorted array.
- Merge sort is $O(n \log n)$. However, it does use additional memory to perform the merge step.
- Quick sort is another divide and conquer algorithm that partitions an array into two subarrays that are separated by one entry, the pivot. The pivot is in its correct sorted position. The entries in one subarray are less than or equal to the pivot, while the entries in the second subarray are greater than or equal to the pivot. Quick sort recursively sorts the two subarrays.
- Quick sort is $O(n \log n)$ most of the time. Although it is $O(n^2)$ in its worst case, you usually can avoid this case by choosing appropriate pivots.
- Even though merge sort and quick sort are $O(n \log n)$ algorithms, quick sort is usually faster in practice and does not require additional memory.

- Radix sort treats array entries as if they were strings that have the same length. Initially radix sort distributes the entries into buckets according to the character (digit) at one end of the strings. The sort then collects the strings and distributes them again among the buckets according to the character or digit in the next position. The sort continues this process until all character positions are considered.
- Radix sort does not compare array entries. Although it is O(n), it cannot sort all types of data. Thus, it is not appropriate as a general-purpose sorting algorithm.

Exercises

1. Suppose that 80 90 70 85 60 40 50 95 represents an array of `Integer` objects. Show the steps that a merge sort takes when sorting this array.

2. Consider the method `quickSort`, as given in Segment 9.18, that sorts an array of objects into ascending order by using a quick sort. Suppose that 80 90 70 85 60 40 50 95 represents an array of `Integer` objects.

   a. What does the array look like after `quickSort` partitions it for the first time? (Show all intermediate results.)
   b. How many comparisons did this partition process require?
   c. The pivot is now between two subarrays called *Smaller* and *Larger*. Will the position of this particular entry change during subsequent steps of the sort? Why or why not?
   d. What recursive call to `quickSort` occurs next?

3. Consider the merge step of the merge sort.

   a. What is the minimum number of comparisons needed to merge two subarrays each of size $n/2$?
   b. Give a recurrence relation that counts the number of comparisons made in the best case.
   c. Make an educated guess at the solution to the recurrence relation.

4. How many comparisons does quick sort require in the worst case when median-of-three partitioning is used? What is the Big Oh for the worst case?

5. Show the steps that a radix sort takes when sorting the following array of `Integer` objects:

   783 99 472 182 264 543 356 295 692 491 94

6. Show the steps that a radix sort takes when sorting the following array of strings into alphabetical order:

   joke book back dig desk word fish ward dish wit deed fast dog bend

7. Describe how a card player can use a radix sort to sort a hand of cards.

8. Consider a collection of `Comparable` objects that is represented by a chain of linked nodes. Suppose that you want to provide a sort operation for this collection.

   a. Implement a private method that merges two sorted chains into one new sorted chain.
   b. The method described in Part *a* could be part of a merge sort of a sorted chain. Describe how you could implement such a sort.

9. Recall that a sorting algorithm is stable if it does not change the relative order of objects that are equal. What sorting algorithms in this chapter and the previous one are stable?

10. Segment 9.7 showed that you can compute the efficiency of merge sort by solving the recurrence relation

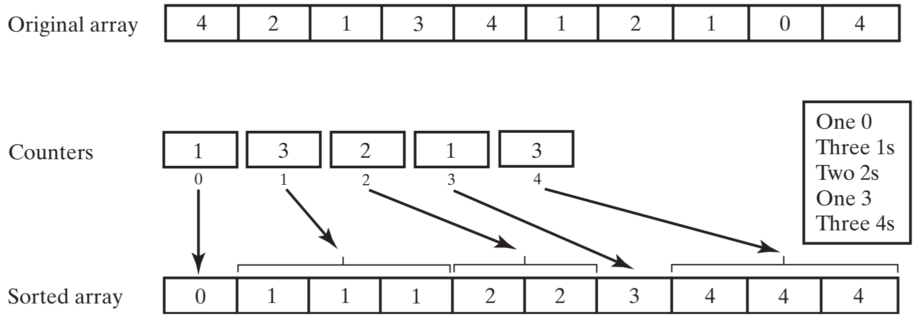
    $t(n) = 2\ t(n/2) + n$ when $n > 1$
    $t(1) = 0$

    Prove by induction that $t(n) = n \log_2 n$.

11. A **counting sort** is a simple way to sort an array of *n* positive integers that lie between 0 and *m*, inclusive. You need *m* + 1 counters. Then, making only one pass through the array, you count the number of times each integer occurs in the array. For example, Figure 9-12 shows an array of integers that lie between 0 and 4 and the five counters after a counting sort has made its pass through the array. From the counters, you can see that the array contains one 0, three 1s, two 2s, one 3, and three 4s. These counts enable you to determine that the sorted array should contain 0 1 1 1 2 2 3 4 4 4.

   a. Write a method that performs a counting sort.
   b. How does the efficiency of a counting sort compare to that of an insertion sort or a quick sort?

FIGURE 9-12 A counting sort of an array (see Exercise 11)



Projects

1. Implement the recursive algorithm for merge sort.

2. Segment 9.8 introduced you to an iterative merge sort. This project continues that discussion by providing more details about the merge steps.

   a. If *n* is a power of 2, as it is in Figure 9-3, you would merge pairs of individual entries, starting at the beginning of the array. Then you would return to the beginning of the array and merge pairs of two-entry subarrays. Finally, you would merge one pair of four-entry subarrays. Notice that the subarrays in each pair of subarrays contain the same number of entries.

   In general, *n* might not be a power of 2. After merging a certain number of pairs of subarrays, you might have too few entries left to make up a complete pair of subarrays. In Figure 9-13a, after merging pairs of single entries, one entry is left over. You then merge one pair of two-entry subarrays, and merge the leftover two-entry subarray with the leftover single entry. Parts *b* and *c* of Figure 9-13 show two other possibilities.

   Implement an iterative merge sort. Use the algorithm `merge` that was given in Segment 9.3. A private method that uses `merge` to merge the pairs of subarrays is useful. After the method completes its task, you can handle the leftovers that we just described.

   b. Merging two subarrays requires an additional temporary array. Although you need to use this extra space, you can save much of the time that our earlier merge algorithm spends in copying entries from the temporary array back to the original array. If *a* is the original array and *t* is the temporary array, you first merge subarrays of *a* into the array *t*. Instead of copying *t* back to *a* and continuing the merge, you determine subarrays of *t* and merge them into *a*. If you can do this an even number of times, no additional copying is necessary. Make these changes to the iterative merge sort that you wrote in Part *a*.

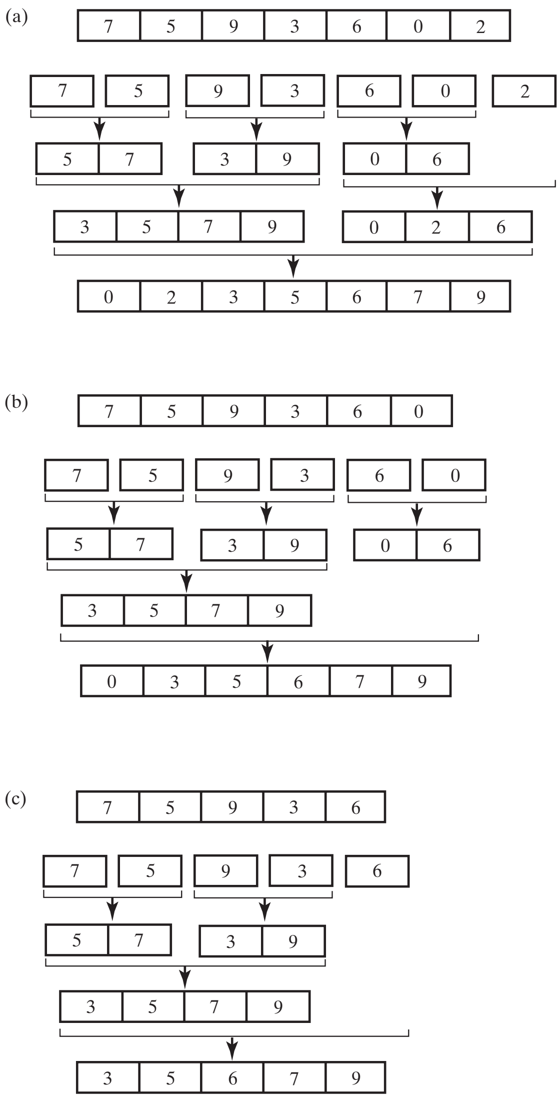
3. Consider the following implementation of an iterative merge sort. Scan the array from its beginning and partition it into segments that are each sorted. As you find each segment, represent it as a pair of indices and place the pair at the end of an initially empty vector.

Next, remove the first two pairs from the vector and merge the array segments they represent. Notice that these segments are adjacent in the array. The merge results in a larger segment that is sorted. Place the pair of indices that represents the resulting segment at the end of the vector. Repeat the steps in this paragraph until only one entry remains in the vector.

Sometimes during the process, the two pairs at the beginning of the vector will represent segments that are not adjacent. In this case, move the first pair to the end of the vector and continue.

a. What is the best-case performance of this algorithm?
b. What is the worst-case performance of this algorithm?
c. Implement the algorithm.

FIGURE 9-13 Special cases in an iterative merge sort after merging one-entry subarrays



4. Revise the implementation of quick sort as follows. If the array has 7 entries, choose the middle entry as the pivot. For arrays of between 8 and 40 entries, use the median-of-three pivot-selection scheme described in Segments 9.14 and 9.16. For larger arrays, the pivot is the median of 9 entries that are about equally spaced, including the first, last, and middle entries. For arrays of fewer than 7 entries, use insertion sort instead of quick sort.

5. Extend Project 1 of the previous chapter to provide graphical demonstrations of the merge sort and quick sort algorithms introduced in this chapter.

6. The *median* of a collection of data is the middle value. One way to find the median is to sort the data and take the value that is at—or nearly at—the center of the collection. But sorting does more than necessary to find the median. You need to find only the k^{th} smallest entry in the collection for an appropriate value of k. To find the median of n items, you would take k as $n/2$ rounded up—that is, $\lceil n/2 \rceil$.

You can use the partitioning strategy of quick sort to find the k^{th} smallest entry in an array. After choosing a pivot and forming the subarrays *Smaller* and *Larger*, as described in Segment 9.10, you can draw one of the following conclusions:

- If *Smaller* contains k or more entries, it must contain the k^{th} smallest entry.
- If *Smaller* contains $k - 1$ entries, the k^{th} smallest entry is the pivot.
- If *Smaller* contains fewer than $k - 1$ entries, the k^{th} smallest entry is in *Larger*.

You now can develop a recursive solution to finding the k^{th} smallest entry. The first and last conclusions correspond to the recursive calls. The remaining one is the base case.

Implement a recursive method that finds the k^{th} smallest entry in an unsorted array. Use your method to find the median in the array.

7. A binary radix sort will sort an array a of n integer values based on their binary bits instead of their decimal digits. This sort will need only two buckets. Represent the buckets as a 2 by n array. You can avoid some work by not copying the contents of the buckets back into the array a at the end of each pass. Instead just add the values from the second bucket to the end of the first bucket.

Implement this algorithm.

8. Repeat Project 3 of the previous chapter, but instead compare the merge sort and the quick sort.

9. Implement a merge sort of the objects in a chain of linked nodes. Compare the run times of this version of merge sort and a quick sort of the same data in an array. See the projects at the end of Chapter 4 for a description of how to time a block of Java code.

10. Implement a radix sort of the strings in a chain of linked nodes.

Answers to Self-Test Questions

1.

```
      9 6 2 4 8 7 5 3
  9 6 2 4          8 7 5 3
 9 6    2 4       8 7    5 3
9  6   2  4      8  7   5  3
 6 9    2 4       7 8    3 5
  2 4 6 9          3 5 7 8
      2 3 4 5 6 7 8 9
```

2.

```
Algorithm mergeSort(a, tempArray, first, last)
if (first < last)
{
   mid = (first + last) / 2
   mergeSort(a, first, mid)
   mergeSort(a, mid + 1, last)
   if (array[mid] > array[mid + 1]))
      Merge the sorted halves a[first..mid] and a[mid+1..last] using the array tempArray
}
```

3.

```
quickSort(array, 0, 7)
partition(array, 0, 7)
 9 6 2 4 8 7 5 3
 3 6 2 4 8 7 5 9
 3 6 2 5 8 7 4 9
 3 2 6 5 8 7 4 9
 3 2 4 5 8 7 6 9
quickSort(array, 0, 1)
insertionSort(array, 0, 1)
 2 3 4 5 8 7 6 9
quickSort(array, 3, 7)
partition(array, 3, 7)
 2 3 4 5 8 7 6 9
 2 3 4 5 8 6 7 9
 2 3 4 5 6 8 7 9
 2 3 4 5 6 7 8 9
quickSort(array, 3, 4)
insertionSort(array, 3, 4)
 2 3 4 5 6 7 8 9
quickSort(array, 6, 7)
insertionSort(array, 6, 7)
 2 3 4 5 6 7 8 9
```

4.

6340 1234 0291 0003 6325 0068 5227 1638
634**0** 029**1** 000**3** 123**4** 632**5** 522**7** 006**8** 163**8**
00**0**3 63**2**5 52**2**7 12**3**4 16**3**8 63**4**0 00**6**8 02**9**1
0**0**03 0**0**68 5**2**27 1**2**34 0**2**91 6**3**25 6**3**40 1**6**38
0003 **0**068 **0**291 **1**234 **1**638 **5**227 **6**325 **6**340
0003 0068 0291 1234 1638 5227 6325 6340

5.

```
Algorithm radixSort(a, first, last, wordLength)
// Sorts the array of lowercase words a[first..last] into ascending order;
// treats each word as if it was padded on the right with blanks to make all words have
// the same length, wordLength.

for (i = 1 to wordlength)
{
   Clear bucket['a'], bucket['b'], . . . , bucket['z'], bucket[' ']
   for (index = first to last)
   {
      letter = $i^{th}$ letter from the right of a[index]
      Place a[index] at end of bucket[letter]
   }
   Place contents of bucket['a'], bucket['b'], . . . , bucket['z'], bucket[' ']
      into the array a
}
```

This page intentionally left blank

Queues, Deques, and Priority Queues

Chapter

10

Contents

Prerequisites

Appendix	D	Designing Classes
Chapter	5	Stacks
Chapter	8	An Introduction to Sorting

Objectives

After studying this chapter, you should be able to

- Describe the operations of the ADT queue
- Use a queue to simulate a waiting line
- Use a queue in a program that organizes data in a first-in, first-out manner
- Describe the operations of the ADT deque
- Use a deque in a program that organizes data chronologically and can operate on both the oldest and newest entries
- Describe the operations of the ADT priority queue
- Use a priority queue in a program that organizes data objects according to their priorities

Waiting for your turn is a fact of life. Most people have spent much time standing in lines at stores, banks, or movie theaters. You have probably waited on the telephone for an airline representative or a technical support person, and you may have waited for your printed output to finally reach the printer in the computer lab. In each of these examples, people wait with the expectation that they will be served before everyone who has come after them. That is, first come, first served.

A queue is another name for a waiting line, and it is the name of one of the ADTs that we will investigate in this chapter. Queues are used within operating systems and to simulate real-world events—that is, they come into play whenever processes or events must wait.

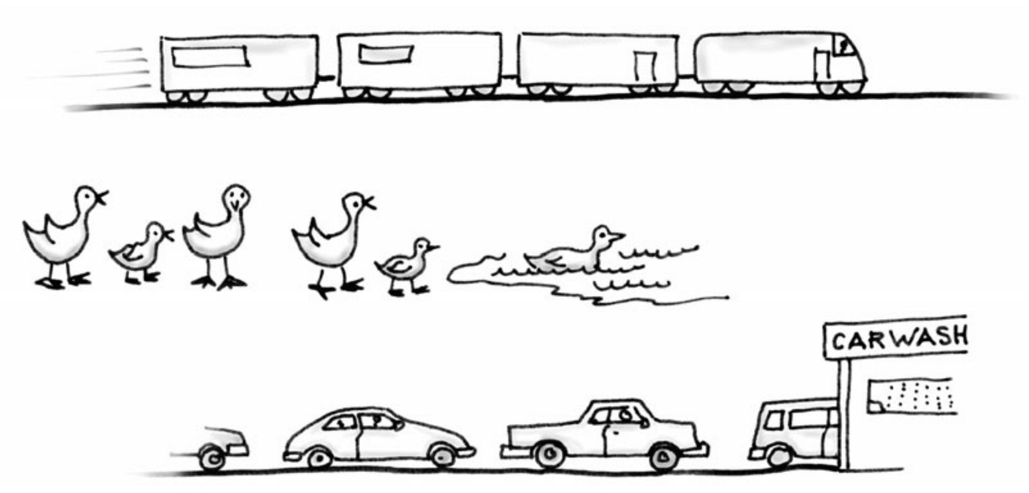
Sometimes you need more flexibility than a queue permits. A double-ended queue, or deque, organizes data like a queue but enables you to operate on both its oldest and newest entries. And when the importance of an object depends on criteria other than its arrival time, you can assign it a priority. You can organize such objects within a priority queue according to their priorities instead of chronologically.

The queue, deque, and priority queue are three ADTs that this chapter will explore.

The ADT Queue

10.1 Like a stack, the ADT **queue** organizes its entries according to the order in which they were added. But while a stack has a last-in, first-out behavior, a queue exhibits a **first-in**, **first-out**, or **FIFO**, behavior. To achieve this behavior, all additions to a queue are at its **back**. The item added most recently, then, is at the back of a queue. The item that was added earliest is at the **front** of a queue. Figure 10-1 provides examples of some common queues.

FIGURE 10-1 Some everyday queues



Note: Among the items in a queue, the one added first, or earliest, is at the **front** of the queue, and the one added most recently is at the **back** of the queue.

VideoNote
The ADT queue

A queue, like a stack, restricts access to its entries. Although someone might cut into a line of people, additions to a software queue must occur at its back. A client can look at or remove only the entry at the front of the queue. The only way to look at an entry that is not at the front of a queue is to repeatedly remove items from the queue until the desired item reaches the front. If you were to remove all of a queue's entries one by one, you would get them in chronological order, beginning with the first item added to the queue.

The queue has no search operation. An entry's value is not relevant to the queue or to the entry's position within the queue.

10.2 The operation that adds an entry to a queue is traditionally called `enqueue` (pronounced "N-Q"). The operation to remove an entry is `dequeue` (pronounced "D-Q"). The operation that retrieves the queue's front entry is called `getFront`. The following specifications define a set of operations for the ADT queue:

Abstract Data Type: Queue

Data

- A collection of objects in chronological order and having the same data type

Operations

Pseudocode	UML	Description
`enqueue(newEntry)`	`+enqueue(newEntry: integer): void`	Task: Adds a new entry to the back of the queue. Input: `newEntry` is the new entry. Output: None.
`dequeue()`	`+deque(): T`	Task: Removes and returns the entry at the front of the queue. Input: None. Output: Returns either the queue's front entry or, if the queue is empty before the operation, `null`.
`getFront()`	`+getFront(): T`	Task: Retrieves the queue's front entry without changing the queue in any way. Input: None. Output: Returns either the queue's front entry or, if the queue is empty, `null`.
`isEmpty()`	`+isEmpty(): boolean`	Task: Detects whether the queue is empty. Input: None. Output: Returns `true` if the queue is empty.
`clear()`	`+clear(): void`	Task: Removes all entries from the queue. Input: None. Output: None.

Note: Alternate names for methods
As we mentioned in Chapter 5, class designers often include aliases for certain methods. For a queue, you could include the additional methods `put` and `get` to mean `enqueue` and `dequeue`. The names `add`, `insert`, `remove`, and `delete` are also reasonable aliases. Likewise, you could provide a method `peek` to mean `getFront`.

10.3 The Java interface in Listing 10-1 specifies a queue of objects. The generic type `T`—which can be any class type—represents the data type of the items in the queue.

LISTING 10-1 An interface for the ADT queue

```
public interface QueueInterface<T>
{
   /** Adds a new entry to the back of the queue.
       @param newEntry  an object to be added */
   public void enqueue(T newEntry);

   /** Removes and returns the entry at the front of this queue.
       @return either the object at the front of the queue or, if the
               queue is empty before the operation, null */
   public T dequeue();

   /** Retrieves the entry at the front of this queue.
       @return either the object at the front of the queue or, if the
               queue is empty, null */
   public T getFront();

   /** Detects whether this queue is empty.
       @return true if the queue is empty, or false otherwise */
   public boolean isEmpty();

   /** Removes all entries from this queue. */
   public void clear();
} // end QueueInterface
```

10.4 **Example: Demonstrating the queue methods.** The following statements add, retrieve, and remove strings from a queue. We assume that the class `LinkedQueue` implements `QueueInterface` and is available.

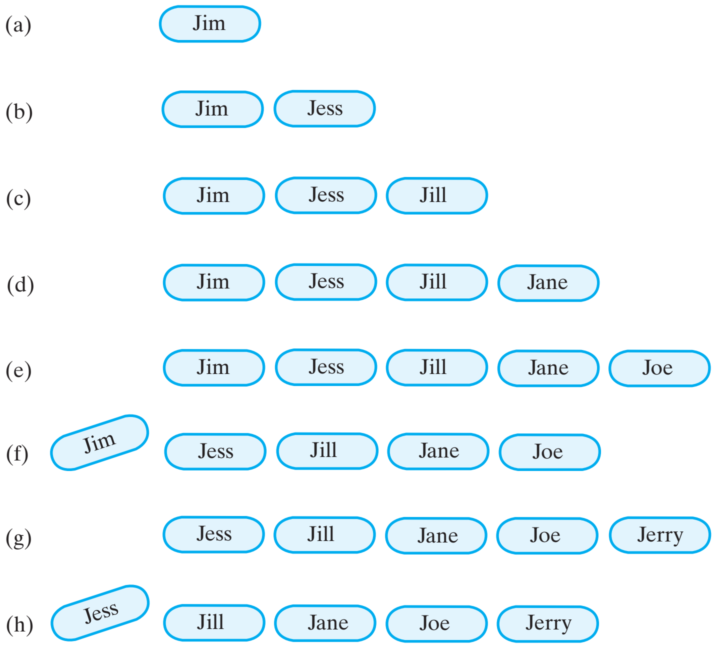
```
QueueInterface<String> myQueue = new LinkedQueue<String>();
myQueue.enqueue("Jim");
myQueue.enqueue("Jess");
myQueue.enqueue("Jill");
myQueue.enqueue("Jane");
myQueue.enqueue("Joe");

String front = myQueue.getFront(); // returns "Jim"
System.out.println(front + " is at the front of the queue.");

front = myQueue.dequeue();         // removes and returns "Jim"
System.out.println(front + " is removed from the queue.");

myQueue.enqueue("Jerry");
```

```
front = myQueue.getFront();         // returns "Jess"
System.out.println(front + " is at the front of the queue.");

front = myQueue.dequeue();          // removes and returns "Jess"
System.out.println(front + " is removed from the queue.");
```

Parts *a* through *e* of Figure 10-2 illustrate the five additions to the queue. Following these additions, the queue contains—from front to back—the strings *Jim*, *Jess*, *Jill*, *Jane*, and *Joe*. The string at the front of the queue is *Jim*; `getFront` retrieves it. The method `dequeue` retrieves *Jim* again and then removes it from the queue (Figure 10-2f). A subsequent call to `enqueue` adds *Jerry* to the back of the queue but does not affect the front (Figure 10-2g). Thus, `getFront` retrieves *Jess*, and `dequeue` retrieves *Jess* and then removes it (Figure 10-2h).

If we now were to execute `dequeue` repeatedly until the queue was empty, an additional call to either `dequeue` or `getFront` would return `null`.

FIGURE 10-2 A queue of strings after (a) `enqueue` adds *Jim*; (b) `enqueue` adds *Jess*; (c) `enqueue` adds *Jill*; (d) `enqueue` adds *Jane*; (e) `enqueue` adds *Joe*; (f) `dequeue` retrieves and removes *Jim*; (g) `enqueue` adds *Jerry*; (h) `dequeue` retrieves and removes *Jess*


(a) Jim
(b) Jim Jess
(c) Jim Jess Jill
(d) Jim Jess Jill Jane
(e) Jim Jess Jill Jane Joe
(f) Jim Jess Jill Jane Joe
(g) Jess Jill Jane Joe Jerry
(h) Jess Jill Jane Joe Jerry

Question 1 After the following nine statements execute, what string is at the front of the queue and what string is at the back?

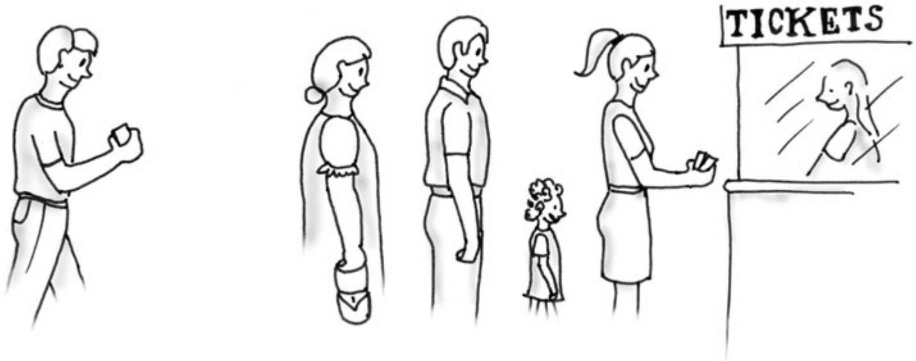
```
QueueInterface<String> myQueue = new LinkedQueue<String>();
myQueue.enqueue("Jim");
myQueue.enqueue("Jess");
myQueue.enqueue("Jill");
myQueue.enqueue("Jane");
String name = myQueue.dequeue();
myQueue.enqueue(name);
myQueue.enqueue(myQueue.getFront());
name = myQueue.dequeue();
```

Programming Tip: Methods such as `getFront` and `dequeue` must behave reasonably when the queue is empty. Here, we specify that they return `null`. Another possibility is to have them throw an exception.

A Problem Solved: Simulating a Waiting Line

In many everyday situations, you will wait in a line. Whether the line is at a store, a ticket window, or a car wash, a line behaves like the ADT queue. The person at the front of the line is served first; newcomers go to the back of the line, as Figure 10-3 shows. In this problem, we will perform a computer simulation of a waiting line.

FIGURE 10-3 A line, or queue, of people



10.5 Most businesses are concerned with the time that their customers must wait for service. A short wait time enables an organization to increase customer satisfaction, serve more people, and make more money. If two agents serve one line, you will wait less time than if only one agent is on duty. A business, however, does not want to employ more people than necessary. And a car wash certainly would not build an additional service bay to test its effect on the time its customers wait in a single line.

Computer simulation of a real-world situation is a common way to test various business scenarios. In this example, we will simulate one line of people waiting for service from one agent. Customers arrive at different intervals and require various times to complete their transactions. One way to achieve this variety is to assume that the events are random.

In a **time-driven simulation**, a counter enumerates simulated units of time—minutes, for example. Customers arrive at random times during the simulation and enter the queue. Each customer is assigned a random transaction time—that is, the amount of time required for the customer's transaction—that does not exceed some arbitrary upper bound. During the simulation, the time that each customer waits in the queue is recorded. At the conclusion of the simulation, summary statistics are generated, including the number of customers served and the average time that each waited.

10.6 **Solution design.** Two kinds of objects occur in the description of this problem: the waiting line and the customers. We can design a class for each of these.

The class WaitLine simulates the waiting line for a given period of time. During this time, customers enter the line at random intervals and leave it after being served. At the conclusion of the simulation, the class computes the summary statistics. Figure 10-4 shows a CRC card for this class.

The class Customer records and makes available the customer's arrival time, transaction time, and customer number. Figure 10-5 contains a class diagram for WaitLine and Customer.

FIGURE 10-4 A CRC card for the class WaitLine

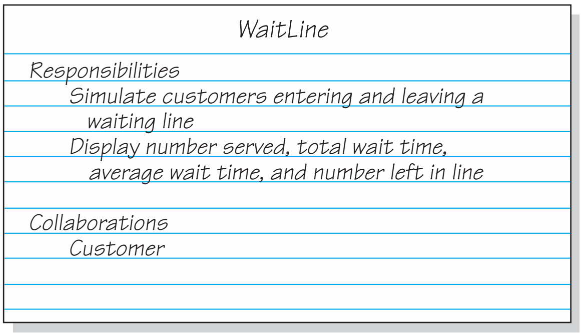
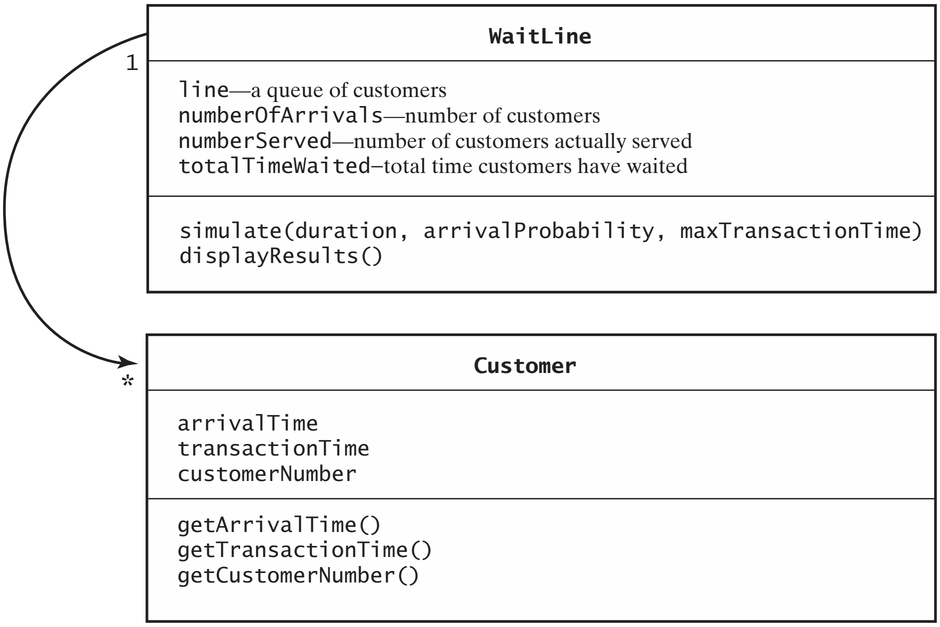

WaitLine
Responsibilities
Simulate customers entering and leaving a waiting line
Display number served, total wait time, average wait time, and number left in line
Collaborations
Customer

FIGURE 10-5 A diagram of the classes WaitLine and Customer


WaitLine
1
line—a queue of customers
numberOfArrivals—number of customers
numberServed—number of customers actually served
totalTimeWaited–total time customers have waited
simulate(duration, arrivalProbability, maxTransactionTime)
displayResults()
*
Customer
arrivalTime
transactionTime
customerNumber
getArrivalTime()
getTransactionTime()
getCustomerNumber()

10.7 **The method simulate.** The method simulate is the heart of this example and of the class WaitLine. To maintain the clock for this time-driven simulation, simulate contains a loop that counts up to a given duration. For example, the clock could simulate one hour by counting minutes, beginning at 0 and continuing until 60.

At each value of the clock, the method sees whether the current customer is still being served and whether a new customer has arrived. If a new customer arrives, the method creates a new customer object, assigns it a random transaction time, and places the customer into the queue. If a customer is still being served, the clock advances; if not, a customer leaves the front of the queue and begins service. At this point, the time the customer waited is noted. Figure 10-6 provides an example of the queue for a portion of the simulation.

The following pseudocode describes the method `simulate`. It assumes that the class `WaitLine` has initialized its data fields as follows: `line` is an empty queue, and `numberOfArrivals`, `numberServed`, and `totalTimeWaited` are each zero.

```
Algorithm simulate(duration, arrivalProbability, maxTransactionTime)
transactionTimeLeft = 0
for (clock = 0; clock < duration; clock++)
{
   if (a new customer arrives)
   {
      numberOfArrivals++
      transactionTime = a random time that does not exceed maxTransactionTime
      nextArrival = a new customer containing clock, transactionTime, and
                    a customer number that is numberOfArrivals
      line.enqueue(nextArrival)
   }

   if (transactionTimeLeft > 0) // if present customer is still being served
      transactionTimeLeft--
   else if (!line.isEmpty())
   {
      nextCustomer = line.dequeue()
      transactionTimeLeft = nextCustomer.getTransactionTime() - 1
      timeWaited = clock - nextCustomer.getArrivalTime()
      totalTimeWaited = totalTimeWaited + timeWaited
      numberServed++
   }
}
```

Question 2 Consider the simulation begun in Figure 10-6.

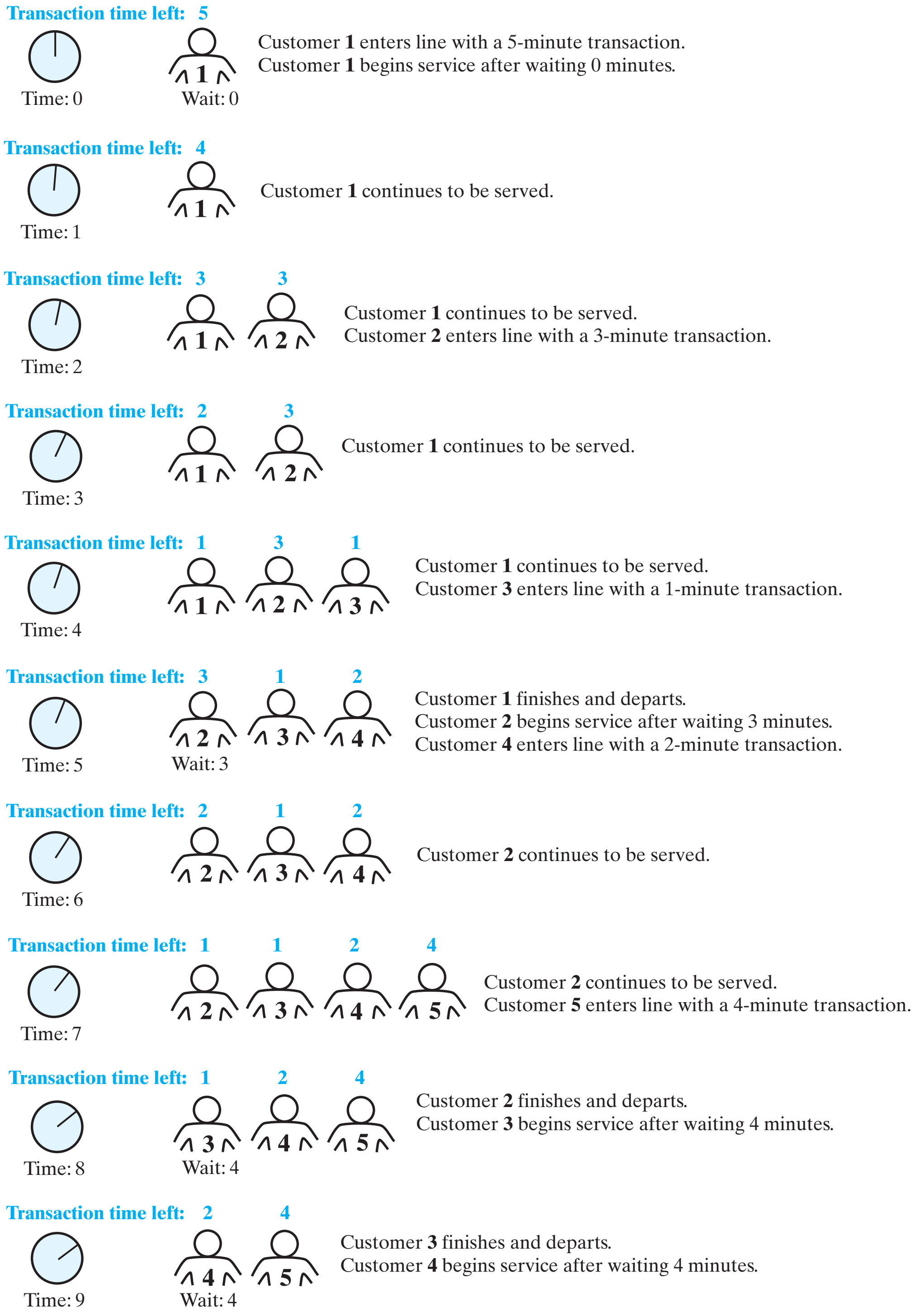
a. At what time does Customer 4 finish and depart?

b. How long does Customer 5 wait before beginning the transaction?

10.8 **Implementation details for `simulate`.** At each value of the clock, `simulate` must determine whether a new customer has arrived. To do so, it needs the probability that a customer will arrive. This arrival probability is a parameter of the method and has a value between 0 and 1. For example, if there is a 65 percent chance that a customer will arrive at any given time, the arrival probability is 0.65. We then generate a random number between 0 and 1 by using the method `random` in Java's class `Math`. If the value returned by `Math.random()` is less than the given arrival probability, `simulate` creates a new customer and places it into the queue.

The method assigns to each new customer a random transaction time. Given a maximum value for this time, we can multiply it by `Math.random()` to get a random transaction time. Adding 1 to the result ensures that the transaction time is never 0 but allows a small chance that the transaction time will exceed the given maximum value by 1. For simplicity, we will tolerate this small imprecision.

FIGURE 10-6 A simulated waiting line


Transaction time left: 5
1
Time: 0
Wait: 0
Transaction time left: 4
1
Time: 1
Transaction time left: 3 3
1 2
Time: 2
Transaction time left: 2 3
1 2
Time: 3
Transaction time left: 1 3 1
1 2 3
Time: 4
Transaction time left: 3 1 2
2 3 4
Time: 5
Wait: 3
Transaction time left: 2 1 2
2 3 4
Time: 6
Transaction time left: 1 1 2 4
2 3 4 5
Time: 7
Transaction time left: 1 2 4
3 4 5
Time: 8
Wait: 4
Transaction time left: 2 4
4 5
Time: 9
Wait: 4

Customer **1** enters line with a 5-minute transaction.
Customer **1** begins service after waiting 0 minutes.

Customer **1** continues to be served.

Customer **1** continues to be served.
Customer **2** enters line with a 3-minute transaction.

Customer **1** continues to be served.

Customer **1** continues to be served.
Customer **3** enters line with a 1-minute transaction.

Customer **1** finishes and departs.
Customer **2** begins service after waiting 3 minutes.
Customer **4** enters line with a 2-minute transaction.

Customer **2** continues to be served.

Customer **2** continues to be served.
Customer **5** enters line with a 4-minute transaction.

Customer **2** finishes and departs.
Customer **3** begins service after waiting 4 minutes.

Customer **3** finishes and departs.
Customer **4** begins service after waiting 4 minutes.

An implementation of the class `WaitList` appears in Listing 10-2. The definition of the method `simulate` contains print statements to help you follow the simulation. The other methods in the class are straightforward.

LISTING 10-2 The class `WaitLine`

```
/** Simulates a waiting line. */
public class WaitLine
{
   private QueueInterface<Customer> line;
   private int numberOfArrivals;
   private int numberServed;
   private int totalTimeWaited;

   public WaitLine()
   {
      line = new LinkedQueue<Customer>();
      reset();
   } // end default constructor

   /** Simulates a waiting line with one serving agent.
       @param duration  the number of simulated minutes
       @param arrivalProbability  a real number between 0 and 1, and the
                                  probability that a customer arrives at
                                  a given time
       @param maxTransactionTime  the longest transaction time for a
                                  customer */
   public void simulate(int duration, double arrivalProbability,
                        int maxTransactionTime)
   {
      int transactionTimeLeft = 0;

      for (int clock = 0; clock < duration; clock++)
      {
         if (Math.random() < arrivalProbability)
         {
            numberOfArrivals++;
            int transactionTime = (int)(Math.random()
                                        * maxTransactionTime + 1);
            Customer nextArrival = new Customer(clock, transactionTime,
                                                numberOfArrivals);
            line.enqueue(nextArrival);
            System.out.println("Customer " + numberOfArrivals
                               + " enters line at time " + clock
                               + ". Transaction time is "
                               + transactionTime);
         } // end if

         if (transactionTimeLeft > 0)
```

```
               transactionTimeLeft--;
            else if (!line.isEmpty())
            {
               Customer nextCustomer = line.dequeue();
               transactionTimeLeft = nextCustomer.getTransactionTime() - 1;
               int timeWaited = clock - nextCustomer.getArrivalTime();
               totalTimeWaited = totalTimeWaited + timeWaited;
               numberServed++;
               System.out.println("Customer "
                                + nextCustomer.getCustomerNumber()
                                + " begins service at time " + clock
                                + ". Time waited is " + timeWaited);
            } // end if
         } // end for
      } // end simulate

      /** Displays summary results of the simulation. */
      public void displayResults()
      {
         System.out.println();
         System.out.println("Number served = " + numberServed);
         System.out.println("Total time waited = " + totalTimeWaited);
         double averageTimeWaited = ((double)totalTimeWaited) /
                                      numberServed;
         System.out.println("Average time waited = " + averageTimeWaited);
         int leftInLine = numberOfArrivals - numberServed;
         System.out.println("Number left in line = " + leftInLine);
      } // end displayResults

      /** Initializes the simulation. */
      public final void reset()
      {
         line.clear();
         numberOfArrivals = 0;
         numberServed = 0;
         totalTimeWaited = 0;
      } // end reset
   } // end WaitLine
```

10.9 **Sample output.** The Java statements

```
WaitLine customerLine = new WaitLine();
customerLine.simulate(20, 0.5, 5);
customerLine.displayResults();
```

simulate the line for 20 minutes with a 50 percent arrival probability and a 5-minute maximum transaction time. They produce the following results:

```
Customer 1 enters line at time 0. Transaction time is 4
Customer 1 begins service at time 0. Time waited is 0
Customer 2 enters line at time 2. Transaction time is 2
Customer 3 enters line at time 4. Transaction time is 1
Customer 2 begins service at time 4. Time waited is 2
Customer 4 enters line at time 6. Transaction time is 4
Customer 3 begins service at time 6. Time waited is 2
Customer 4 begins service at time 7. Time waited is 1
Customer 5 enters line at time 9. Transaction time is 1
Customer 6 enters line at time 10. Transaction time is 3
Customer 5 begins service at time 11. Time waited is 2
Customer 7 enters line at time 12. Transaction time is 4
Customer 6 begins service at time 12. Time waited is 2
Customer 8 enters line at time 15. Transaction time is 3
Customer 7 begins service at time 15. Time waited is 3
Customer 9 enters line at time 16. Transaction time is 3
Customer 10 enters line at time 19. Transaction time is 5
Customer 8 begins service at time 19. Time waited is 4

Number served = 8
Total time waited = 16
Average time waited = 2.0
Number left in line = 2
```

Since this example uses random numbers, another execution of the Java statements likely will have different results.

Note: Pseudo-random numbers

Java's method `Math.random` generates numbers that are uniformly distributed over the interval from 0 to 1. Actual times for processing customer transactions, however, are not uniformly distributed. They are close together, and few times are far from the average transaction time. One such distribution is called a *Poisson distribution*. Ideally, this simulation should use a different pseudo-random number generator. Since our maximum transaction time is small, however, using `Math.random` probably has little effect on the average wait time.

A Problem Solved: Computing the Capital Gain in a Sale of Stock

Suppose that you buy *n* shares of a stock or mutual fund for *d* dollars each. Later you sell some of these shares. If the sale price exceeds the purchase price, you have made a profit—a *capital gain*. On the other hand, if the sale price is lower than the purchase price, you experience a loss. We will designate a loss as a negative capital gain.

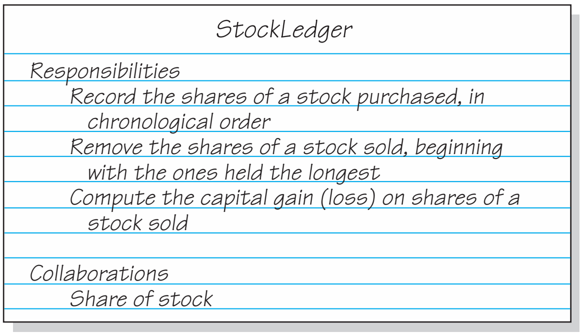
Typically, investors buy shares in a particular company or fund over a period of time. For example, suppose that last year you bought 20 shares of Presto Pizza at $45 per share. Last month, you bought 20 additional shares at $75 per share, and today you sold 30 shares at $65 per share. What is your capital gain? Well, which of your 40 shares did you actually sell? Unfortunately, you cannot pick and choose. When computing capital gains, you must assume that you sell shares in the order in which you purchased them (meaning that stock sales are a first-in, first-out application). So in our example, you sold the 20 shares that you bought at $45 each and 10 of the shares that you bought at $75 each. Your cost for the 30 shares is $1650. You sold them for $1950, a profit of $300.

Design a way to record your investment transactions chronologically and to compute the capital gain of any stock sale.

10.10 **Solution design.** To simplify the example, we assume that all transactions are for stocks of a single company and that there is no commission charge for the transactions. The class `StockPurchase` records the cost of a single share of stock.

Figure 10-7 shows a CRC card for the class `StockLedger`. The class enables us to record stock purchases in chronological order. At the time of sale, the class computes the capital gain and updates the record of stocks owned. These last two steps are related, so we combine them into one method. Thus, the class has two methods, `buy` and `sell`, as Figure 10-8 illustrates.

FIGURE 10-7 A CRC card for the class `StockLedger`

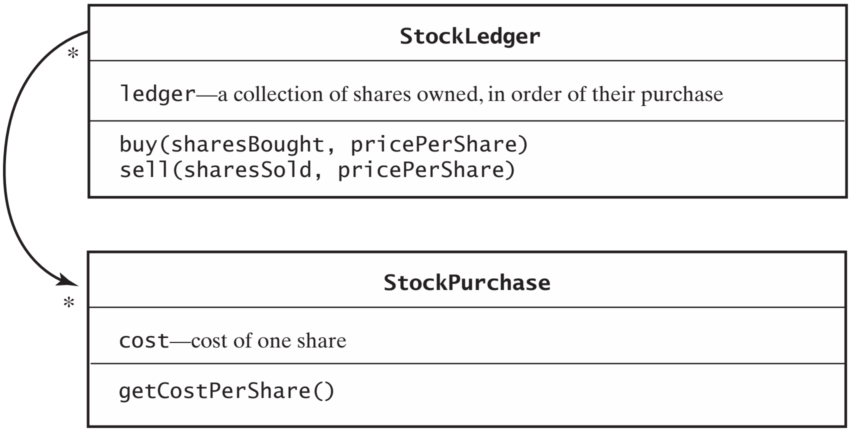


StockLedger

Responsibilities
    Record the shares of a stock purchased, in
      chronological order
    Remove the shares of a stock sold, beginning
      with the ones held the longest
    Compute the capital gain (loss) on shares of a
      stock sold

Collaborations
    Share of stock

FIGURE 10-8 A diagram of the classes `StockLedger` and `StockPurchase`


*
StockLedger
ledger—a collection of shares owned, in order of their purchase
buy(sharesBought, pricePerShare)
sell(sharesSold, pricePerShare)
*
StockPurchase
cost—cost of one share
getCostPerShare()

The following statements demonstrate how we could use `StockLedger` to record the transactions given in the problem description:

```
StockLedger myStocks = new StockLedger();
myStocks.buy(20, 45);                  // buy  20 shares at $45
myStocks.buy(20, 75);                  // buy  20 shares at $75
double capGain = myStocks.sell(30, 65); // sell 30 shares at $65
```

10.11 **Implementation.** In this example, `StockLedger` records instances of `StockPurchase`—which represent the shares we own—in a queue. A queue orders the shares chronologically, so we can sell them in the order in which we purchased them. The method `buy` then just enqueues each share that is bought.

The method `sell` removes from the queue as many shares as are sold. As it does this, it computes the total capital gain from the sale and returns it. The class `StockLedger` is given in Listing 10-3.

LISTING 10-3 The class StockLedger

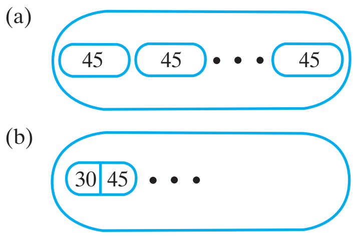
```
/** Records the purchase and sale of stocks, and provides the capital
    gain or loss. */
public class StockLedger
{
   private QueueInterface<StockPurchase> ledger;

   public StockLedger()
   {
      ledger = new LinkedQueue<StockPurchase>();
   } // end default constructor

   /** Records a stock purchase in this ledger.
       @param sharesBought   the number of shares purchased
       @param pricePerShare  the price per share */
   public void buy(int sharesBought, double pricePerShare)
   {
      for (; sharesBought > 0; sharesBought--)
      {
         StockPurchase purchase = new StockPurchase(pricePerShare);
         ledger.enqueue(purchase);
      } // end for
   } // end buy

   /** Removes from this ledger any shares that were sold
       and computes the capital gain or loss.
       @param sharesSold     the number of shares sold
       @param pricePerShare  the price per share
       @return the capital gain (loss) */
   public double sell(int sharesSold, double pricePerShare)
   {
      double saleAmount = sharesSold * pricePerShare;
      double totalCost = 0;

      while (sharesSold > 0)
      {
         StockPurchase share = ledger.dequeue();
         double shareCost = share.getCostPerShare();
         totalCost = totalCost + shareCost;
         sharesSold--;
      } // end while

      return saleAmount - totalCost; // gain or loss
   } // end sell
} // end StockLedger
```

10.12 **An observation about this solution.** A typical stock transaction involves multiple shares, and the two methods buy and sell reflect this reality in their parameters. For example, the invocation myStocks.buy(30, 45) indicates a purchase of 30 shares at $45 per share. However, notice that the implementation of buy adds each of the 30 shares to a queue. Figure 10-9a shows such a queue. The advantage of this approach is that sell can remove as many or as few shares as necessary.

Suppose that we instead encapsulate the purchase of 30 shares into one object and add it to the queue, as Figure 10-9b illustrates. If we then sell 20 of those shares, we would remove the object from the queue and learn the shares' purchase price. But we would have 10 shares that must remain in the queue. Since these are the oldest shares, we could not simply add them to the back of the queue; they must remain at the front. The ADT queue has no operation that modifies its front entry, nor does it have one to add an object to its front. If each entry has set methods, however, Java will allow the client to modify the entry at the front by using the reference that getFront returns. In this case, you would not remove the front entry until you have sold all of the shares it represents. Exercise 10 at the end of this chapter asks you to explore this approach.

On the other hand, if each entry in the queue does not have set methods, you would not be able to modify it. The queue would not be the right ADT to use when each entry represents more than one share of stock. Segment 10.14 explores another ADT that you can use instead.

FIGURE 10-9 A queue of (a) individual shares of stock; (b) grouped shares



Note: A class that has set methods is a class of **mutable objects**. A class without set methods is a class of **immutable objects**. Chapter 30 talks about such classes in more detail.

Java Class Library: The Interface Queue

10.13 The standard package java.util in the Java Class Library contains an interface Queue that is similar to our QueueInterface but specifies more methods. We list here a selection of method headers similar to the ones you have seen in this chapter. We have highlighted where they differ from our methods. Once again, T is the generic type.

```
public boolean add(T newEntry)
```

Adds a new entry to the back of this queue, returning true if successful and throwing an exception if not.

```
public boolean offer(T newEntry)
```

Adds a new entry to the back of this queue, returning true or false according to the success of the operation.

```
public T remove()
```

Retrieves and removes the entry at the front of this queue, but throws NoSuchElementException if the queue is empty prior to the operation.

```
public T poll()
```

Retrieves and removes the entry at the front of this queue, but returns null if the queue is empty prior to the operation.

```
public T element()
```

Retrieves the entry at the front of this queue, but throws `NoSuchElementException` if the queue is empty.

`public T peek()`
Retrieves the entry at the front of this queue, but returns `null` if the queue is empty.

`public boolean isEmpty()`
Detects whether this queue is empty.

`public void clear()`
Removes all entries from this queue.

`public int size()`
Gets the number of elements currently in this queue.

Some of these methods occur in pairs. Both `add` and `offer` add a new entry to the queue. If the operation is unsuccessful, `add` throws an exception but `offer` returns false. Likewise, each of the methods `remove` and `poll` removes and returns the entry at the front of the queue. If the queue is empty before the operation, `remove` throws an exception but `poll` returns `null`. Finally, `peek` and `element` each retrieve the entry at the front of the queue. If the queue is empty, `element` throws an exception but `peek` returns `null`.

You can learn more about `Queue` and the other components of the Java Class Library at `download.oracle.com/javase/7/docs/api/`.

The ADT Deque

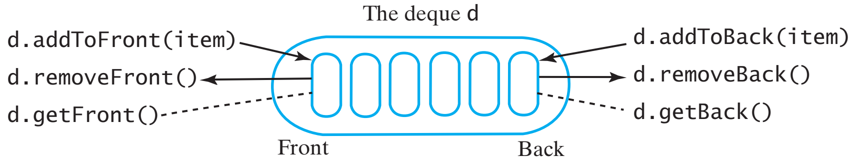
10.14 Imagine that you are in a line at the post office. When it is finally your turn, the postal agent asks you to fill out a form. You step aside to do so and let the agent serve the next person in the line. After you complete the form, the agent will serve you next. Essentially, you go to the front of the line, rather than waiting in line twice.

Similarly, suppose that you join a line at its end but then immediately decide it is too long, so you leave it. To simulate both of these examples, you want an ADT whose operations enable you to add, remove, or retrieve entries at both the front and back of a queue. Such an ADT is called a **double-ended queue**, or **deque** (pronounced “deck”).

A deque has both queuelike operations and stacklike operations. For example, the deque operations `addToBack` and `removeFront` resemble the queue operations `enqueue` and `dequeue`, respectively. And `addToFront` and `removeFront` are like the stack operations `push` and `pop`, respectively. In addition, a deque has the operations `getFront`, `getBack`, and `removeBack`. Figure 10-10 illustrates a deque and these methods.

Note: Although the ADT deque is called a double-ended queue, it actually behaves like a double-ended stack. As Figure 10-10 shows, you can push, pop, or get items at either of its ends.

FIGURE 10-10 An instance `d` of a deque



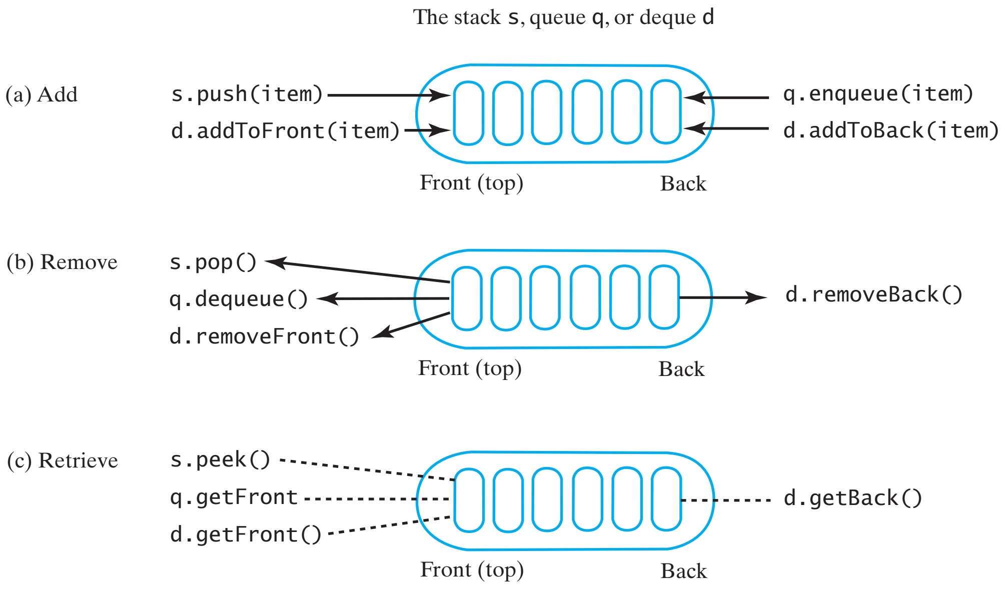
Since the specifications for the deque operations are like those you have already seen for a queue and a stack, we provide the brief Java interface in Listing 10-4 without comments.

LISTING 10-4 An interface for the ADT deque

```
public interface DequeInterface<T>
{
   public void addToFront(T newEntry);
   public void addToBack(T newEntry);
   public T removeFront();
   public T removeBack();
   public T getFront();
   public T getBack();
   public boolean isEmpty();
   public void clear();
} // end DequeInterface
```

A comparison of the operations that add, remove, and retrieve the entries of a stack, queue, and deque is provided in Figure 10-11.

FIGURE 10-11 A comparison of operations for a stack `s`, a queue `q`, and a deque `d`: (a) add; (b) remove; (c) retrieve



Question 3 After the following nine statements execute, what string is at the front of the deque and what string is at the back?

```
DequeInterface<String> myDeque = new LinkedDeque<String>();
myDeque.addToFront("Jim");
myDeque.addToBack("Jess");
myDeque.addToFront("Jill");
myDeque.addToBack("Jane");
String name = myDeque.getFront();
myDeque.addToBack(name);
myDeque.removeFront();
myDeque.addToFront(myDeque.removeBack());
```

10.15 **Example.** When typing at your keyboard, you might make a mistake. If you backspace to correct your mistake, what logic is used to decipher your intention? For example, if the symbol ← represents a backspace, and you type

cm←ompte←←utr←er

the result should be

computer

Each backspace erases the previous character entered.

To replicate this process, as characters are entered, we retain them in an ADT. We want this ADT to be stacklike so we can access the most recently entered character. But since we ultimately want the corrected characters to be in the order in which they were entered, we want the ADT to also behave like a queue. The ADT deque can satisfy these requirements.

The following pseudocode uses a deque to read and display a line of keyboard input:

```
// read a line
d = a new empty dequeue
while (not end of line)
{
   character = next character read
   if (character == ←)
      d.removeBack()
   else
      d.addToBack(character)
}
// display the corrected line
while (!d.isEmpty())
   System.out.print(d.removeFront())
System.out.println()
```

A Problem Solved: Computing the Capital Gain in a Sale of Stock

When we concluded the capital gain example, Segment 10.12 noted that our queue contained individual shares of stock. Since a typical stock transaction involves more than one share, representing a transaction as one object is more natural. But you saw that the transaction object must have set methods if we use a queue. That would not be the case if we used a deque instead.

10.16 In this section, we revise the implementation, but not the design, of the class `StockLedger` that was introduced in Segment 10.10. We also revise the class `StockPurchase` so that it represents the purchase of *n* shares of stock at *d* dollars per share, as Segment 10.12 suggests. The revised class has the data fields `shares` and `cost`, a constructor, and the accessor methods `getNumberOfShares` and `getCostPerShare`.

We can revise the implementation of the class `StockLedger` given in Segment 10.11 as follows. The data field `ledger` is now an instance of a deque instead of a queue. The method `buy` creates an instance of `StockPurchase` and places it at the back of the deque, as follows:

```java
public void buy(int sharesBought, double pricePerShare)
{
   StockPurchase purchase = new StockPurchase(sharesBought, pricePerShare);
   ledger.addToBack(purchase);
} // end buy
```

The method `sell` is more involved. It must remove a `StockPurchase` object from the front of the deque and decide whether that object represents more shares than the number sold. If it does, the method creates a new instance of `StockPurchase` to represent the shares that remain in the portfolio.

It then adds that instance to the front of the deque, since it is these shares that would be sold next.

```
public double sell(int sharesSold, double pricePerShare)
{
   double saleAmount = sharesSold * pricePerShare;
   double totalCost = 0;

   while (sharesSold > 0)
   {
      StockPurchase transaction = ledger.removeFront();
      double shareCost = transaction.getCostPerShare();
      int numberOfShares = transaction.getNumberOfShares();

      if (numberOfShares > sharesSold)
      {
         totalCost = totalCost + sharesSold * shareCost;
         int numberToPutBack = numberOfShares - sharesSold;
         StockPurchase leftOver = new StockPurchase(numberToPutBack, shareCost);
         ledger.addToFront(leftOver); // return leftover shares
         // Note: loop will exit since sharesSold will be <= 0 later
      }
      else
         totalCost = totalCost + numberOfShares * shareCost;

      sharesSold = sharesSold - numberOfShares;
   } // end while

   return saleAmount - totalCost; // gain or loss
} // end sell
```

Java Class Library: The Interface Deque

10.17 The standard package `java.util` in the Java Class Library contains an interface `Deque` that is similar to our `DequeInterface` but specifies more methods. Here is a selection of the method headers that this interface declares. The methods that either add, remove, or retrieve entries occur in pairs. One method in a pair throws an exception if the operation is unsuccessful, while the other method returns either `null` or false. `T` is the generic type of the entries in a deque.

`public void addFirst(T newEntry)`
Adds a new entry to the front of this deque, but throws one of several exceptions if it cannot.

`public boolean offerFirst(T newEntry)`
Adds a new entry to the front of this deque, returning true or false according to the success of the operation.

`public void addLast(T newEntry)`
Adds a new entry to the back of this deque, but throws one of several exceptions if it cannot.

`public boolean offerLast(T newEntry)`
Adds a new entry to the back of this deque, returning true or false according to the success of the operation.

`public T removeFirst()`
Retrieves and removes the entry at the front of this deque, but throws `NoSuchElementException` if the deque is empty prior to the operation.

`public T pollFirst()`
Retrieves and removes the entry at the front of this deque, but returns `null` if the deque is empty prior to the operation.

`public T removeLast()`
Retrieves and removes the entry at the back of this deque, but throws `NoSuchElementException` if the deque is empty prior to the operation.

`public T pollLast()`
Retrieves and removes the entry at the back of this deque, but returns `null` if the deque is empty prior to the operation.

`public T getFirst()`
Retrieves the entry at the front of this deque, but throws `NoSuchElementException` if the deque is empty.

`public T peekFirst()`
Retrieves the entry at the front of this deque, but returns `null` if the deque is empty.

`public T getLast()`
Retrieves the entry at the back of this deque, but throws `NoSuchElementException` if the deque is empty.

`public T peekLast()`
Retrieves the entry at the back of this deque, but returns `null` if the deque is empty.

`public boolean isEmpty()`
Detects whether this deque is empty.

`public void clear()`
Removes all entries from this deque.

`public int size()`
Gets the number of entries currently in this deque.

The interface `Deque` extends the interface `Queue`, so it also has the methods `add`, `offer`, `remove`, `poll`, `element`, and `peek` that were described earlier in Segment 10.13. In addition, `Deque` declares the following two stack methods:

```
public void push(T newEntry)
public T pop()
```

These methods are like the ones defined in the class `java.util.Stack`, which we encountered earlier in Segment 5.23 of Chapter 5, except that `push` is a `void` method in `Deque`. As we mentioned in Chapter 5, you should no longer use the standard class `Stack`. The following segment describes an alternate class for you to use.

The online documentation for the interface `Deque`, as given at `download.oracle.com/javase/7/docs/api`, tabulates the correspondence between deque methods and both queue and stack methods.

Java Class Library: The Class ArrayDeque

10.18 The standard package `java.util` in the Java Class Library contains the class `ArrayDeque`, which implements the interface `Deque` that we just described. Since `Deque` declares methods appropriate for a deque, a queue, and a stack, you can use `ArrayDeque` to create instances of any of these data collections.

The following two constructors are defined by this class:

`public ArrayDeque()`
Creates an empty deque whose initial capacity is 16 entries.

`public ArrayDeque(int initialCapacity)`
Creates an empty deque having a given initial capacity.

Instances of `ArrayDeque` grow in size as needed by a client.

Note: If you want to use a standard class instead of your own to create a stack, you should use an instance of the standard class `ArrayDeque`, but not the standard class `Stack`. `Stack` is now considered a legacy class. That is, it is retained in the Java Class Library only to support previously written Java programs.

The ADT Priority Queue

10.19 Although a bank serves its customers in the order in which they arrive, an emergency room treats patients according to the urgency of their malady. The bank organizes its customers into chronological order by using a queue. A hospital assigns a **priority** to each patient that overrides the time at which the patient arrived.

VideoNote
The ADTs deque and priority queue

The ADT **priority queue** organizes objects according to their priorities. Exactly what form a priority takes depends on the nature of the object. Priorities can be integers, for example. A priority of 1 can be the highest priority, or it can be the lowest. By making the objects `Comparable`, we can hide this detail in the objects' method `compareTo`. The priority queue then can use `compareTo` to compare objects by their priorities. Thus, the priority queue can have the Java interface given in Listing 10-5. We use the notation `? super T`, which Segment 8.2 of Chapter 8 introduced, to mean any super class of the generic type `T`.

LISTING 10-5 An interface for the ADT priority queue

```
public interface PriorityQueueInterface<T extends Comparable<? super T>>
{
   /** Adds a new entry to this priority queue.
       @param newEntry  an object */
   public void add(T newEntry);

   /** Removes and returns the item with the highest priority.
       @return either the object with the highest priority or, if the
               priority queue is empty before the operation, null */
   public T remove();

   /** Retrieves the item with the highest priority.
       @return either the object with the highest priority or, if the
               priority queue is empty, null */
   public T peek();

   /** Detects whether this priority queue is empty.
       @return true if the priority queue is empty, or false otherwise */
   public boolean isEmpty();
   /** Gets the size of this priority queue.
       @return the number of entries currently in the priority queue */
   public int getSize();

   /** Removes all entries from this priority queue */
   public void clear();
} // end PriorityQueueInterface
```

Question 4 After the following statements execute, what string is at the front of the priority queue and what string is at the back?

```
PriorityQueueInterface<String> myPriorityQueue =
                               new LinkedPriorityQueue<String>();
myPriorityQueue.add("Jane");
myPriorityQueue.add("Jim");
myPriorityQueue.add("Jill");
String name = myPriorityQueue.remove();
myPriorityQueue.add(name);
myPriorityQueue.add("Jess");
```

A Problem Solved: Tracking Your Assignments

> Professors and bosses like to assign tasks for us to do by certain dates. Using a priority queue, organize these assignments in the order in which we should complete them.

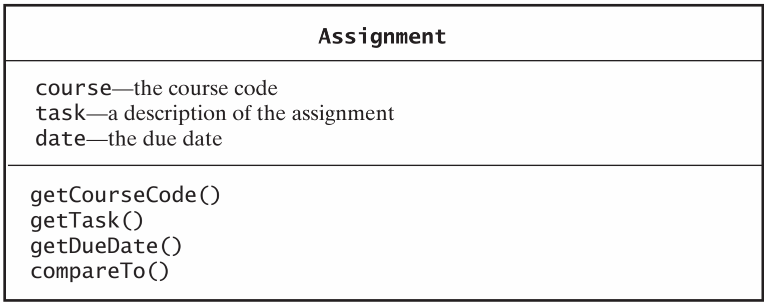
10.20 To keep our example simple, we will order the assignments by their due dates. A task with the earliest due date will have the highest priority.

We can define a class Assignment of tasks that includes a data field date representing a task's due date. Figure 10-12 shows a diagram of such a class. We assume that date is an instance of a Comparable class such as java.sql.Date in the Java Class Library. Thus, date.compareTo(otherDate) is negative, for example, if date occurs before otherDate. The compareTo method for Assignment is then

```
public int compareTo(Assignment other)
{
   return -date.compareTo(other.date);
} // end compareTo
```

A more sophisticated version of Assignment could include other criteria in compareTo to assess priority.

FIGURE 10-12 A diagram of the class Assignment



Note: The class `java.sql.Date`

The class `Date` in the package `java.sql` of the Java Class Library has a constructor whose parameter specifies the date as the number of milliseconds since midnight GMT on January 1, 1970. A more convenient way for us to construct a `Date` object is to use the following static method `valueOf`:

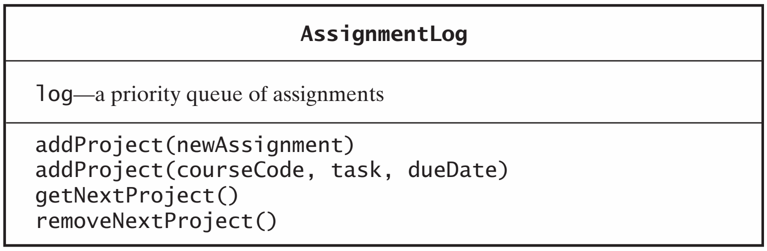
```
public static Date valueOf(String s)
```
Returns a `Date` object whose value is given by a string `s` in the form *yyyy-mm-dd.*

For example, the expression `Date.valueOf("2012-02-29")` returns a `Date` object representing February 29, 2012.

`Date` implements the interface `Comparable<Date>` and overrides `toString`.

10.21 We can either add instances of `Assignment` directly to a priority queue or write a simple wrapper class `AssignmentLog` to organize our assignments. As Figure 10-13 shows, `AssignmentLog` has a data field `log`, which is an instance of a priority queue that contains the assignments in priority order. The methods `addProject`, `getNextProject`, and `removeNextProject` manipulate the priority queue indirectly.

FIGURE 10-13 A diagram of the class `AssignmentLog`


AssignmentLog
log—a priority queue of assignments
addProject(newAssignment)
addProject(courseCode, task, dueDate)
getNextProject()
removeNextProject()

An implementation of `AssignmentLog` appears in Listing 10-6.

LISTING 10-6 The class `AssignmentLog`

```
import java.sql.Date;
public class AssignmentLog
{
   private PriorityQueueInterface<Assignment> log;

   public AssignmentLog()
   {
      log = new PriorityQueue<Assignment>();
   } // end constructor

   public void addProject(Assignment newAssignment)
   {
      log.add(newAssignment);
   } // end addProject
   public void addProject(String courseCode, String task, Date dueDate)
   {
      Assignment newAssignment = new Assignment(courseCode, task,
                                                dueDate);
      addProject(newAssignment);
   } // end addProject

   public Assignment getNextProject()
   {
      return log.peek();
   } // end getNextProject

   public Assignment removeNextProject()
   {
      return log.remove();
   } // end removeNextProject
} // end AssignmentLog
```

10.22 The following statements could appear in a client of `AssignmentLog`:

```
AssignmentLog myHomework = new AssignmentLog();
myHomework.addProject("CSC211", "Pg 50, Ex 2",
                      Date.valueOf("2012-10-21"));
Assignment pg75Ex8 = new Assignment("CSC215", "Pg 75, Ex 8",
                                    Date.valueOf("2012-10-14"));
myHomework.addProject(pg75Ex8);
. . .
System.out.println("The following assignment is due next:");
System.out.println(myHomework.getNextProject());
```

The assignment with the earliest due date is displayed but is not removed from the assignment log.

Java Class Library: The Class `PriorityQueue`

10.23 The standard package `java.util` in the Java Class Library contains the class `PriorityQueue`. This class implements the interface `Queue` that we described earlier in this chapter. An instance of `PriorityQueue` behaves like a priority queue, not a queue, in that its entries are ordered, with the entry having the smallest value, and therefore the highest priority, at the front of the priority queue. Since `PriorityQueue` uses the method `compareTo` to order its entries, the entries must belong to a class that implements the interface `Comparable`. In addition, the entries cannot be `null`.

Here are the basic constructors and methods of the class `PriorityQueue`:

`public PriorityQueue()`
Creates an empty priority queue whose initial capacity is 11 entries.

`public PriorityQueue(int initialCapacity)`
Creates an empty priority queue having a given initial capacity.

`public boolean add(T newEntry)`
Adds a new entry to this priority queue, returning true if successful and throwing an exception if not.

`public boolean offer(T newEntry)`
Adds a new entry to this priority queue, returning true or false according to the success of the operation.

`public T remove()`
Retrieves and removes the entry at the front of this priority queue, but throws `NoSuchElementException` if the priority queue is empty prior to the operation.

`public T poll()`
Retrieves and removes the entry at the front of this priority queue, but returns `null` if the priority queue is empty prior to the operation.

`public T element()`
Retrieves the entry at the front of this priority queue, but throws `NoSuchElementException` if the priority queue is empty.

`public T peek()`
Retrieves the entry at the front of this priority queue, but returns `null` if the priority queue is empty.

`public boolean isEmpty()`
Detects whether this priority queue is empty.

`public void clear()`
Removes all entries from this priority queue.

`public int size()`
Gets the number of elements currently in this priority queue.

Instances of `PriorityQueue` grow in size as needed by a client.

Chapter Summary

- The ADT queue organizes its entries on a first-in, first-out basis. Among its items, the one added first, or earliest, is at the front of the queue, and the one added most recently is at the back of the queue.
- A queue's major operations—`enqueue`, `dequeue`, and `getFront`—deal only with the ends of the queue. The method `enqueue` adds an entry to the back of the queue; `dequeue` removes and returns the entry at the front of the queue, and `getFront` just returns it.
- You can use a queue to simulate a waiting line. A time-driven simulation counts simulated units of time. Customers arrive at random times, are assigned a random transaction time, and enter a queue.
- When computing the capital gain from a sale of stock, you must sell shares in the order in which you purchased them. If you record your purchases of individual shares in a queue, they will be in the order in which they must be sold.
- A double-ended queue, or deque, has operations that add, remove, or retrieve entries at both its front and back. As such, it combines and expands the operations of a queue and a stack. The deque's major operations are `addToFront`, `removeFront`, `getFront`, `addToBack`, `removeBack`, and `getBack`.
- A priority queue organizes its entries according to their priorities, as determined by the entries' `compareTo` method. Besides adding entries to a priority queue, you can retrieve and remove the entry with the highest priority.
- A mutable object has set methods; an immutable object does not.

Programming Tip

- Methods such as `getFront` and `dequeue` must behave reasonably when a queue is empty. Here, we specify that they return `null`. Another possibility is to have them throw an exception.

Exercises

1. If you add the objects x, y, and z to an initially empty queue, in what order will three dequeue operations remove them from the queue?

2. If you add the objects x, y, and z to an initially empty deque, in what order will three removeBack operations remove them from the deque?

3. After the following statements execute, what are the contents of the queue?

```
QueueInterface<String> myQueue = new LinkedQueue<String>();
myQueue.enqueue("Jane");
myQueue.enqueue("Jess");
myQueue.enqueue("Jill");
myQueue.enqueue(myQueue.dequeue());
myQueue.enqueue(myQueue.getFront());
myQueue.enqueue("Jim");
String name = myQueue.dequeue();
myQueue.enqueue(myQueue.getFront());
```

4. After the following statements execute, what are the contents of the deque?

```
DequeInterface<String> myDeque = new LinkedDeque<String>();
myDeque.addToFront("Jim");
myDeque.addToFront("Jess");
myDeque.addToBack("Jill");
myDeque.addToBack("Jane");
String name = myDeque.removeFront();
myDeque.addToBack(name);
myDeque.addToBack(myDeque.getFront());
myDeque.addToFront(myDeque.removeBack());
myDeque.addToFront(myDeque.getBack());
```

5. After the following statements execute, what are the contents of the priority queue?

```
PriorityQueueInterface<String> myPriorityQueue = new LinkedPriorityQueue<String>();
myPriorityQueue.add("Jim");
myPriorityQueue.add("Jess");
myPriorityQueue.add("Jill");
myPriorityQueue.add("Jane");
String name = myPriorityQueue.remove();
myPriorityQueue.add(name);
myPriorityQueue.add(myPriorityQueue.peek());
myPriorityQueue.add("Jim");
myPriorityQueue.remove();
```

6. Consider strings that can be split so that their first half is the same as their second half (ignoring blanks, punctuation, and case). For example, the string "booboo" can be split into "boo" and "boo". Another example is "hello, hello". After ignoring blanks and the comma, the two halves of the string are the same. However, the string "rattan" has unequal halves, as does the string "abcab". Describe how you could use a queue to test whether a string has this property.

7. Complete the simulation begun in Figure 10-6. Let Customer 6 enter the line at time 10 with a transaction time of 2.

8. Assume that customerLine is an instance of the class WaitLine, as given in Segment 10.8. The invocation customerLine.simulate(15, 0.5, 5) produces the following random events:

   Customer 1 enters the line at time 6 with a transaction time of 3.
   Customer 2 enters the line at time 8 with a transaction time of 3.
   Customer 3 enters the line at time 10 with a transaction time of 1.
   Customer 4 enters the line at time 11 with a transaction time of 5.

   During the simulation, how many customers are served, and what is their average waiting time?

9. Repeat Exercise 8, but instead use the following random events:

   Customer 1 enters the line at time 0 with a transaction time of 4.
   Customer 2 enters the line at time 1 with a transaction time of 4.
   Customer 3 enters the line at time 3 with a transaction time of 1.
   Customer 4 enters the line at time 4 with a transaction time of 4.
   Customer 5 enters the line at time 9 with a transaction time of 3.
   Customer 6 enters the line at time 12 with a transaction time of 2.
   Customer 7 enters the line at time 13 with a transaction time of 1.

10. When using a queue to compute capital gains, we observed in Segment 10.12 that the queue's entries could represent more than one share of stock if they have set methods. Revise the class `StockPurchase` so that each of its instances has set methods and represents the purchase of multiple shares of one company's stock. Then revise the class `StockLedger`, using a queue to contain the `StockPurchase` objects.

11. Stacks, queues, and deques are similar in operation in many ways. Suppose that we wanted to create an abstract base class `QueueBase` and then use it and inheritance to implement each of the three ADTs. Design the class `QueueBase`. Indicate whether each field and method should be public, protected, or private, and explain why.

12. Segment 9.21 in Chapter 9 provided the pseudocode for a radix sort of an array. Each bucket in that algorithm is actually a queue. Describe why you can use a queue but not a stack for a radix sort.

13. Exercise 11 of Chapter 5 describes a palindrome. Can you use one of the ADTs described in this chapter instead of a stack to see whether a string is a palindrome? If so, develop an algorithm to do so for each applicable ADT.

14. Consider a special kind of stack that has a finite size but allows an unlimited number of `push` operations. If the stack is full when a `push` occurs, the stack makes room for the new entry by deleting the entry at its bottom. A browser that maintains a limited history could use this kind of stack. Implement this stack by using a deque.

Projects

1. Project 3 of Chapter 9 used a vector in the implementation of an iterative merge sort. In that project, the vector was used as if it were a queue. Repeat the project, but use a queue instead of a vector.

2. Implement the radix sort, as given in Segment 9.21 of Chapter 9, by using a queue for each bucket.

3. Expand the capital gains example described in this chapter to allow more than one type of stock in the portfolio. Identify different stocks by using a string for the stock's symbol. Record the shares of each company in a separate queue, deque, or priority queue. Maintain the collection of these ADTs in a bag or stack.

4. Simulate a small airport with one runway. Airplanes waiting to take off join a queue on the ground. Planes waiting to land join a queue in the air. Only one plane can use the runway at any given time. All planes in the air must land before any plane can take off.

5. Repeat Project 4, but use a priority queue for the planes waiting to land. Develop a priority schedule for situations such as low fuel or mechanical problems.

6. When each object in a collection has a priority, how should you organize several objects that have the same priority? One way is to order the objects with the same priority in chronological order. Thus, you can create a priority queue of queues. Design such an ADT.

7. Write a program to simulate a train route. A train route consists of a number of stations, starting and ending with a terminal station. The time that the train needs to travel between a pair of consecutive stations on the route is given. Associated with each station is a queue of passengers. Passengers are generated at random times, assigned to entry stations randomly, and given random destination stations. Trains leave a terminal at regular intervals and visit the stations on the route. When a train stops at a station, all passengers for that station exit first. Then any passengers waiting in the queue at the station board the train until either the queue is empty or the train is full.

8. Write a program to simulate job scheduling in an operating system. Jobs are generated at random times. Each job is given both a random priority from 1 to 4—where 1 is the highest priority—and a random amount of time to complete its execution.

Jobs do not begin execution and run to completion, but instead share the processor. The operating system executes a job for a fixed unit of time called a *time slice*. At the end of the time slice, the current job's execution is suspended. The job is then placed on a priority queue, where it waits for its next share of processor time. The job having the highest priority is then removed from the priority queue and executed for a time slice.

When a job is first generated, it will begin executing immediately if the processor is free. Otherwise it will be placed on the priority queue.

9. Repeat the online Project 6 for Appendix B to create the class `Huge` of large integers. Use a deque instead of an array to represent the value of an integer.

10. One way to shuffle playing cards is to use a *perfect shuffle*. First, you divide a deck of 52 cards into two halves of 26 cards each. Next, you merge the halves by interleaving the cards as follows. Beginning with the top half and alternating halves, you take the bottom card from a half and place it on top of a new deck.

For example, if our deck contains the six cards 1 2 3 4 5 6, the top half is 1 2 3, and the bottom half is 4 5 6. The 3 at the bottom of the top half becomes the bottom card in the shuffled deck. We then place the 6, which is at the bottom of the bottom half, on top of the shuffled deck. Next, we place 2 on top, then 5, 1, and finally 4. The shuffled deck is then 4 1 5 2 6 3. Notice that the card that was on top of the original deck is now second in the shuffled result, and the bottom card in the original deck is now second from the bottom in the shuffled deck. This shuffle is called an *in-shuffle* and is achieved by beginning with the top half when you move cards into the shuffled result. If you begin with the bottom half, you get an *out-shuffle*, whereby the original top card and bottom card remain in their positions in the shuffled deck.

Define a class of playing-card decks by using a deque to contain the cards. Your class should define methods to perform perfect in-shuffles and perfect out-shuffles. Using your class,

a. Determine the number of perfect out-shuffles needed to return a deck of *n* cards to its original order.
b. Determine the number of perfect in-shuffles needed to return a deck of *n* cards to its original order.
c. You can move a deck's top card, which is at position 0, to any desired position *m* by performing a sequence of in-shuffles and out-shuffles, as follows. You write *m* in binary. Beginning with the leftmost 1 and proceeding to the right, you perform an in-shuffle for each 1 encountered and an out-shuffle for each 0. For example, if *m* is 8, we have 1000 for its binary equivalent. We would perform one in-shuffle followed by three out-shuffles to move the original top card to position 8, that is, so it is the ninth card from the top of the deck. Define a method to perform this card trick.

Answers to Self-Test Questions

1. *Jill* is at the front, *Jess* is at the back.

2. **a.** 11.
b. 4.

3. *Jill* is at the front, *Jane* is at the back.

4. *Jim* is at the front, *Jane* is at the back.

Queue, Deque, and Priority Queue Implementations

Chapter 11

Contents

Prerequisites

Chapter	2	Bag Implementations That Use Arrays
Chapter	3	A Bag Implementation That Links Data
Chapter	6	Stack Implementations
Chapter	10	Queues, Deques, and Priority Queues

Objectives

After studying this chapter, you should be able to

- Implement the ADT queue by using either a chain of linked nodes, an array, or a vector
- Add or delete nodes at either end of a chain of doubly linked nodes
- Implement the ADT deque by using a chain of doubly linked nodes
- Implement the ADT priority queue by using either an array or a chain of linked nodes

The implementations of the ADT queue that are in this chapter use techniques like the ones we used to implement the ADT bag and the ADT stack. We will use either a chain of linked nodes, an array, or an instance of `Vector` to store the queue's entries. Although the stack implementations we saw in Chapter 6 were quite simple, the implementations of a queue are a bit more involved.

We also present a linked implementation of the double-ended queue, or deque. Since the deque allows access to both its front and its back, an ordinary chain of linked nodes is not sufficient. For example, deleting the last node in a chain is not possible without a reference to the preceding node. Thus, we use a new kind of chain, one that links its nodes in both directions. That is, a node in this chain references both the next node and the one that precedes it. Such a chain provides an efficient implementation of the deque.

Finally, we suggest some implementations of the ADT priority queue. We note, however, that a more efficient implementation will be possible when we encounter the ADT heap in Chapters 23 and 26.

A Linked Implementation of a Queue

11.1 If we use a chain of linked nodes to implement a queue, the two ends of the queue will be at opposite ends of the chain. If we have only a head reference to the chain, accessing the chain's last node will be inefficient. Adding a **tail reference**—an external reference to the last node in the chain—is one approach to this problem and is the one we will take here.

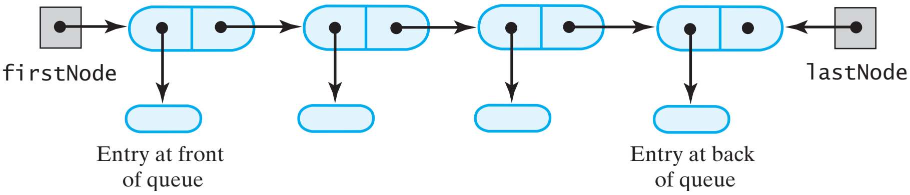
VideoNote
The class `LinkedQueue`

With both head and tail references, which node should be the front of the queue and which node should be the back? We must be able to remove the entry at the front of the queue. If it is at the beginning of the chain, we will be able to remove it easily. If it is at the end of the chain, however, removing it requires a reference to the preceding node. To get such a reference, we must traverse the chain. Thus, we reject this option and make the chain's first node contain the queue's front entry.

Placing the front of the queue at the beginning of the chain obviously forces the back of the queue to the chain's end. Since we add entries only to the back of the queue, and since we have a tail reference for the chain, this arrangement will work.

Figure 11-1 illustrates a chain of linked nodes with both head and tail references. The chain contains one node for each entry in the queue. Nodes are allocated only when needed for a new entry and are deallocated when an entry is removed.

FIGURE 11-1 A chain of linked nodes that implements a queue



11.2 **An outline of the class.** The linked implementation of the queue has two data fields. The field `firstNode` references the chain's first node, which contains the queue's front entry. And `lastNode` references the chain's last node, which contains the entry at the back of the queue. Since both of these fields are `null` when the queue is empty, the default constructor sets them to `null`. An outline of our class appears in Listing 11-1.

The class also contains the private class `Node`, like the one you saw in Listing 3-4 of Chapter 3. We also used this class in Chapter 6 for an implementation of the ADT stack.

LISTING 11-1 An outline of a linked implementation of the ADT queue

```
/**
   A class that implements a queue of objects by using
   a chain of linked nodes.
   @author Frank M. Carrano
*/
public class LinkedQueue<T> implements QueueInterface<T>
{
   private Node firstNode; // references node at front of queue
   private Node lastNode;  // references node at back of queue

   public LinkedQueue()
   {
      firstNode = null;
      lastNode = null;
   } // end default constructor

   < Implementations of the queue operations go here. >
   . . .

   private class Node
   {
      private T    data; // entry in queue
      private Node next; // link to next node

      < Constructors and the methods getData, setData, getNextNode, and setNextNode
        are here. >
      . . .
   } // end Node
} // end LinkedQueue
```

11.3 **Adding to the back.** To add an entry to the back of the queue, we allocate a new node and add it to the end of the chain. If the queue—and therefore the chain—is empty, we make both data fields, `firstNode` and `lastNode`, reference the new node, as Figure 11-2 illustrates. Otherwise, both the last node in the chain and the data field `lastNode` must reference the new node, as shown in Figure 11-3. Thus, the definition of `enqueue` appears as follows:

```
public void enqueue(T newEntry)
{
   Node newNode = new Node(newEntry, null);

   if (isEmpty())
      firstNode = newNode;
   else
      lastNode.setNextNode(newNode);

   lastNode = newNode;
} // end enqueue
```

This operation requires no search and is independent of the other entries in the queue. Its performance is thus O(1).

FIGURE 11-2 (a) Before adding a new node to an empty chain; (b) after adding it

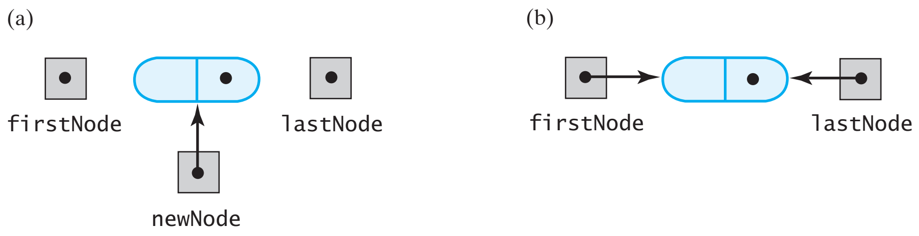
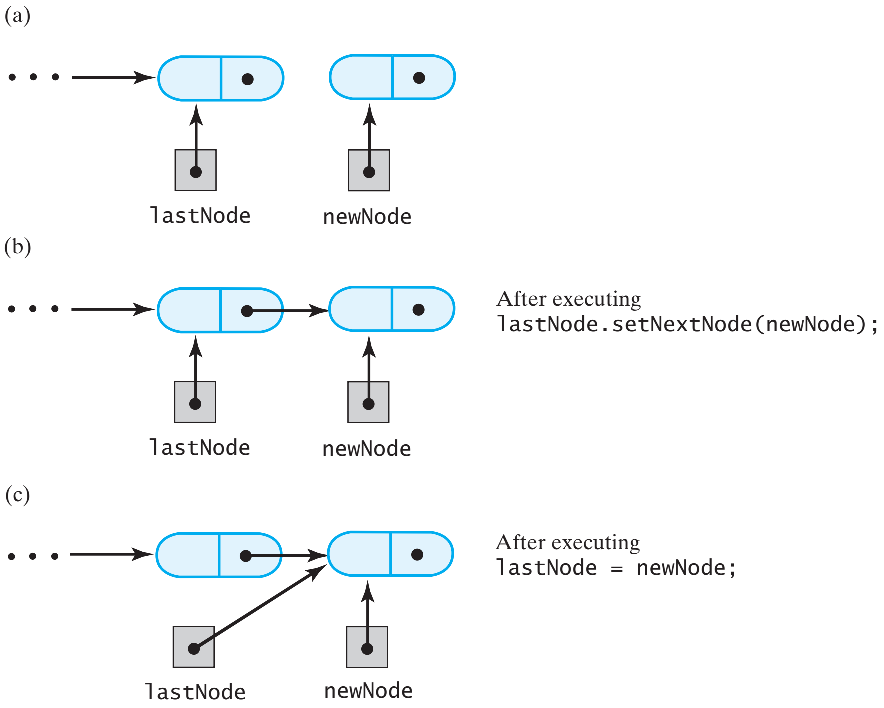


FIGURE 11-3 (a) Before, (b) during, and (c) after adding a new node to the end of a nonempty chain that has a tail reference



11.4 **Retrieving the front entry.** We get the entry at the front of the queue by accessing the data portion of the first node in the chain. Like `enqueue`, `getFront` is an O(1) operation.

```
public T getFront()
{
   T front = null;

   if (!isEmpty())
      front = firstNode.getData();
   return front;
} // end getFront
```

11.5 **Removing the front entry.** The method `dequeue` retrieves the entry at the front of the queue and then removes the chain's first node by making `firstNode` reference the second node in the chain, as shown in Figure 11-4. If the chain had only one node, `dequeue` would make the chain empty by setting both `firstNode` and `lastNode` to `null`, as Figure 11-5 illustrates.

```
public T dequeue()
{
   T front = null;

   if (!isEmpty())
   {
      front = firstNode.getData();
      firstNode = firstNode.getNextNode();

      if (firstNode == null)
         lastNode = null;
   } // end if

   return front;
} // end dequeue
```

Like `enqueue`, `dequeue` requires no search and is independent of the other entries in the queue. Its performance is thus O(1).

FIGURE 11-4 (a) A queue of more than one entry; (b) after removing the entry at the front of the queue

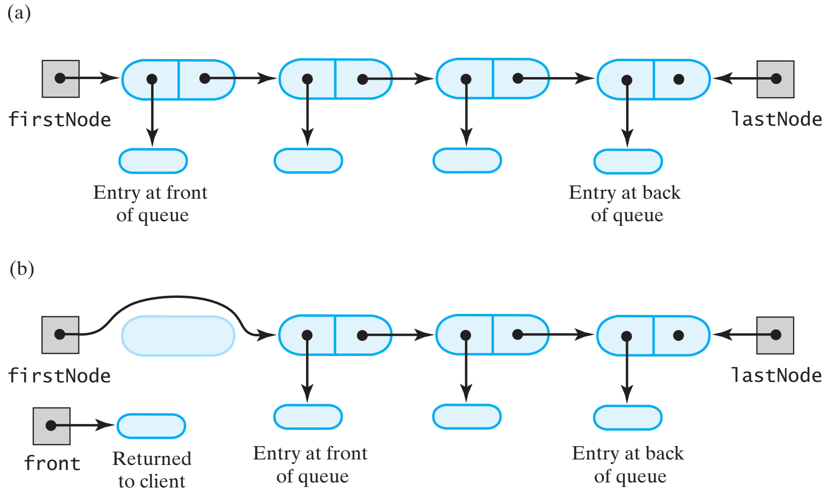
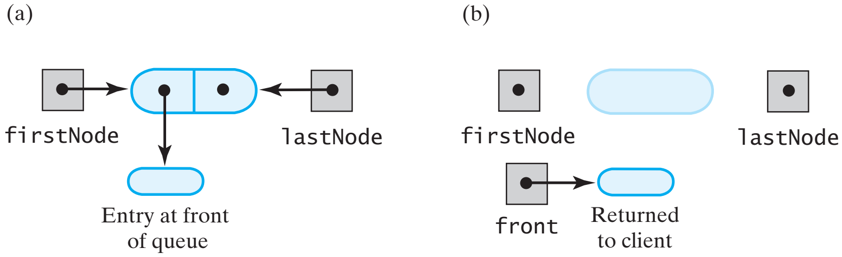


FIGURE 11-5 (a) A queue of one entry; (b) after removing the entry at the front of the queue



11.6 **The rest of the class.** The remaining public methods `isEmpty` and `clear` are straightforward:

```
public boolean isEmpty()
{
   return (firstNode == null) && (lastNode == null);
} // end isEmpty

public void clear()
{
   firstNode = null;
   lastNode = null;
} // end clear
```

? SELF-TEST

Question 1 Why is a tail reference desirable when you use a chain of linked nodes to implement a queue?

An Array-Based Implementation of a Queue

11.7 If we use an array `queue` to contain the entries in a queue, we could let `queue[0]` be the queue's front, as Figure 11-6a shows. Here, `frontIndex` and `backIndex` are the indices of the entries at the queue's front and back, respectively. But what happens when we remove the front entry? If we insist that the new front entry be in `queue[0]`, we would need to shift each array element by one position toward the beginning of the array. This arrangement would make the operation `dequeue` inefficient.

Instead, we can leave other array entries in their current positions when we remove the queue's front entry. For example, if we begin with the array in Figure 11-6a and execute `dequeue` twice, the array will be as shown in Figure 11-6b. Not moving array elements is attractive, but after several additions and removals, the array can look like the one pictured in Figure 11-6c. The queue entries have migrated to the end of the array. The last available array location is allocated to the last entry added to the queue. We could expand the array, but the queue has only three entries. Since most of the array is unoccupied, why not use this space for future additions? In fact, that is just what we will do next.

A Circular Array

11.8 Once the queue reaches the end of the array, as in Figure 11-6c, we can add subsequent entries to the queue at the beginning of the array. Figure 11-6d shows the array after two such additions to the queue. We make the array behave as though it were **circular**, so that its first location follows its last one. To do this, we use modulo arithmetic on the indices. Specifically, when we add an entry to the queue, we first increment `backIndex` modulo the size of the array. For example, if `queue` is the name of the array, we increment `backIndex` with the statement

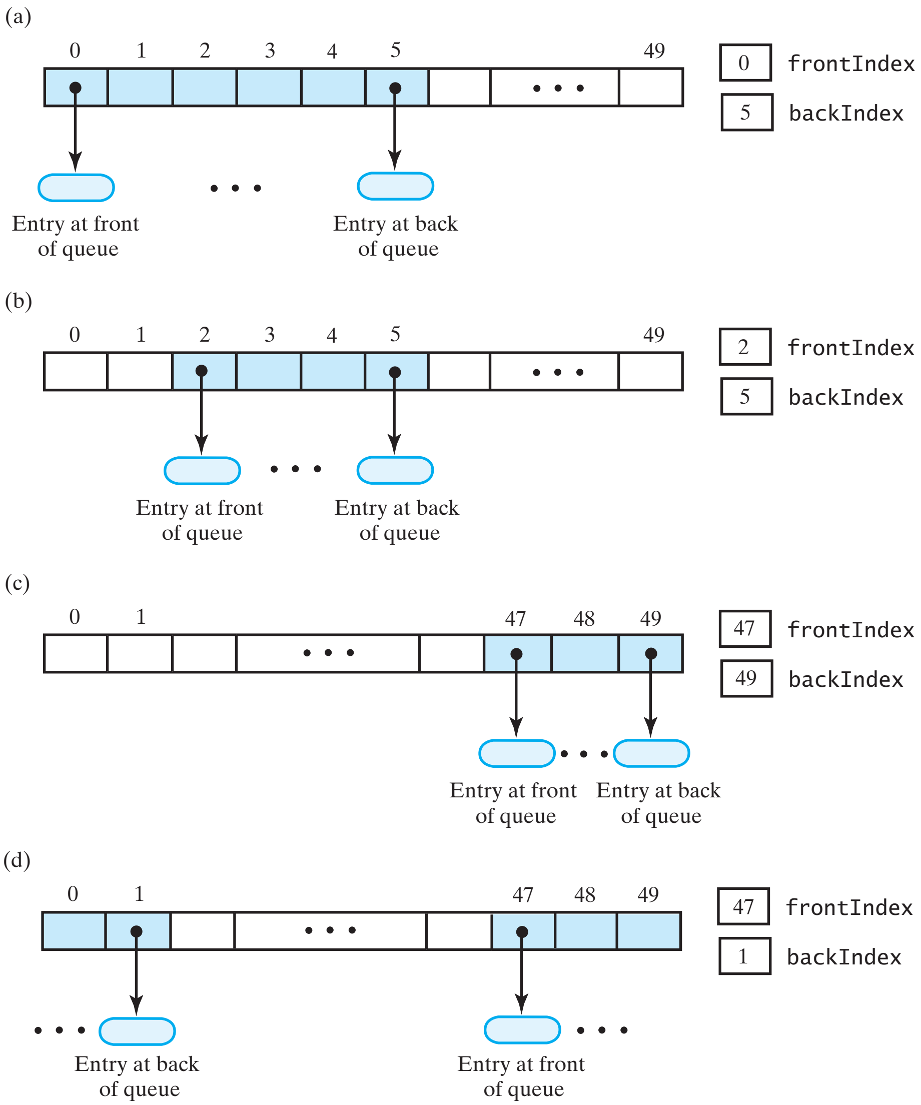
```
backIndex = (backIndex + 1) % queue.length;
```

To remove an entry, we increment `frontIndex` modulo the size of the array in a similar fashion.

? SELF-TEST

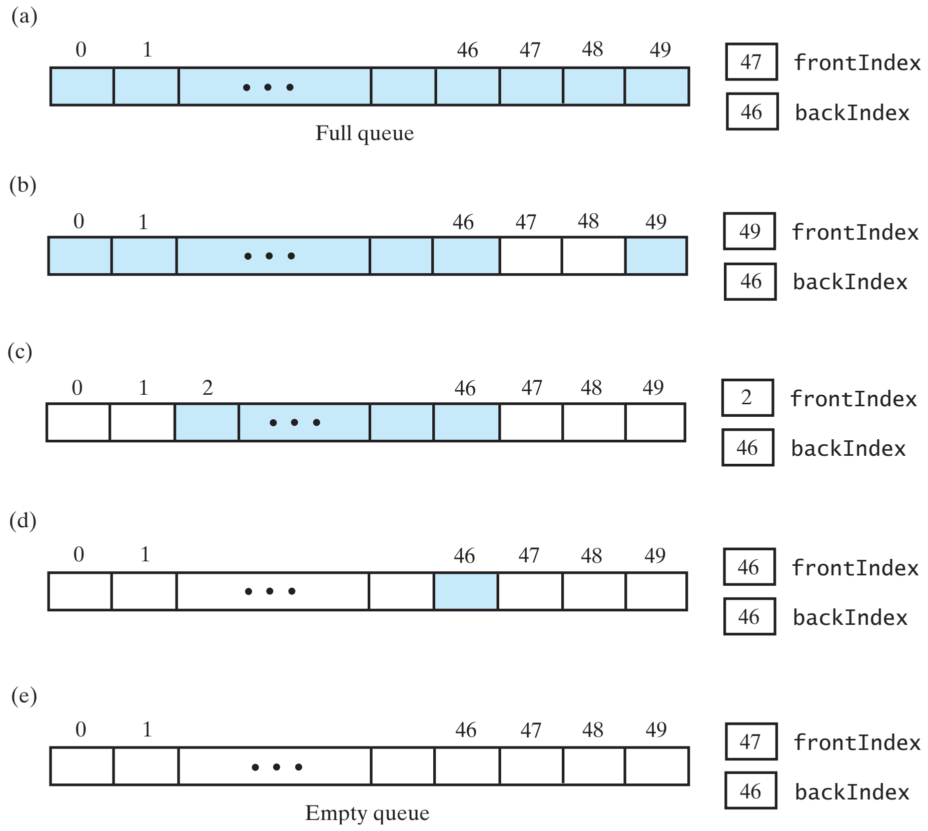
Question 2 When we removed an entry from an array-based bag, as we did in Chapter 2, we replaced the removed entry with the last one in the array. Yet the implementation of the queue just described does not do so. Explain this difference in implementations.

FIGURE 11-6 An array that represents a queue without moving any entries: (a) initially; (b) after removing the entry at the front twice; (c) after several more additions and removals; (d) after two additions that wrap around to the beginning of the array



11.9 **Complications.** Using a circular array complicates the implementation somewhat. For example, how can we detect when the array is full? Clearly the array in Figure 11-7a is full. This array is the result of several additions to the queue pictured in Figure 11-6d. So it appears that the queue is full when `frontIndex` is `backIndex` + 1.

FIGURE 11-7 A circular array that represents a queue: (a) when full; (b) after removing two entries; (c) after removing three more entries; (d) after removing all but one entry; (e) after removing the remaining entry



Now remove some entries from the queue. Figure 11-7b shows the array after executing `dequeue` twice. Notice that `frontIndex` advances to 49. If we continue to remove items from the queue, `frontIndex` will wrap around to zero and beyond. Figure 11-7c shows the array after three more items are removed. As we remove more items from the queue, `frontIndex` advances. Figure 11-7d shows the array after all but one item is removed from the queue. Now let's remove that one item. In Figure 11-7e, we see that this last removal has caused `frontIndex` to advance so that it is 1 more than `backIndex`. Although the queue is empty, `frontIndex` is `backIndex` + 1. This is exactly the same condition we encountered in Figure 11-7a when the queue was full.

Note: With a circular array, `frontIndex` equals `backIndex` + 1 both when the queue is empty and when it is full.

As you can see, we cannot test whether the queue is empty or full by using `frontIndex` and `backIndex`. One solution is to maintain a count of queue items. If the count is zero, the queue is empty; if the count equals the array's capacity, the queue is full. When the queue is full, the next `enqueue` operation can double the array's size before adding a new entry.

Having a counter as a data field leads to a reasonable implementation, but each `enqueue` and `dequeue` must update the count. We can avoid this extra work by leaving one array location unused. We develop this approach next.

A Circular Array with One Unused Location

11.10 Not using one array location allows us to distinguish between an empty queue and a full queue by examining only `frontIndex` and `backIndex`. In Java, each array location contains only a reference, so we waste little memory by having an unused location. Here we will leave unused the array location that follows the back of the queue. Project 2 at the end of this chapter considers a different location.

VideoNote
The class `ArrayQueue`

Figure 11-8 illustrates a seven-element circular array that represents a queue of at most six entries. As we add and remove entries, you should observe the effect on the indices `frontIndex` and `backIndex`. Part *a* of the figure shows the array initially, when the queue is empty. Notice that `frontIndex` is zero and `backIndex` contains the index of the array's last location. Adding an entry to this queue increments the initial value of `backIndex` so that it becomes zero, as shown in Part *b*. Part *c* illustrates the queue after five more additions, making it full. Now remove the front entry and add an entry to the back, as Parts *d* and *e* show. The queue is full once again. Repeating this pair of operations leads to the queues shown in Parts *f* and *g*. Now repeatedly remove the entry at the front until the queue is empty. Part *h* shows the queue after the first of these `dequeue` operations, Part *i* shows it after all but one entry is removed, and Part *j* shows the empty queue.

To summarize, the queue is full in Parts *c*, *e*, and *g* of this figure. In each of these examples, the index of the unused location is 1 more than `backIndex` and 1 less than `frontIndex`, if we treat the array as circular. That is, `frontIndex` is 2 more than `backIndex`. Thus, the queue is full when

`frontIndex` equals `(backIndex + 2) % queue.length`

The queue is empty in Parts *a* and *j*. In those cases, `frontIndex` is 1 more than `backIndex`. Thus, the queue is empty when

`frontIndex` equals `(backIndex + 1) % queue.length`

Admittedly, these criteria are more involved than checking a counter of the number of entries in the queue. However, once we have them, the rest of the implementation is simpler and more efficient because there is no counter to maintain.

11.11 **An outline of the class.** This array-based implementation of a queue begins with four data fields and two constructors. The fields are the array of queue entries, indices to the front and back of the queue, and an initial capacity for the queue that the default constructor creates. Another constructor lets the client choose the initial queue capacity. The initial size of the array is one more than the queue's initial capacity. Listing 11-2 outlines the class.

LISTING 11-2 An outline of an array-based implementation of the ADT queue

```java
/**
   A class that implements a queue of objects by using an array.
   @author Frank M. Carrano
*/

public class ArrayQueue<T> implements QueueInterface<T>
{
   private T[] queue; // circular array of queue entries and one unused
                      // location
   private int frontIndex;
   private int backIndex;
   private static final int DEFAULT_INITIAL_CAPACITY = 50;

   public ArrayQueue()
   {
      this(DEFAULT_INITIAL_CAPACITY);
   } // end default constructor

   public ArrayQueue(int initialCapacity)
   {
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempQueue = (T[]) new Object[initialCapacity + 1];
      queue = tempQueue;
      frontIndex = 0;
      backIndex = initialCapacity;
   } // end constructor

   < Implementations of the queue operations go here. >
   . . .
} // end ArrayQueue
```

11.12 **Adding to the back.** The method enqueue first calls the private method ensureCapacity, which doubles the size of the array if it is full, and then places the new entry immediately after the last occupied location in the array. To determine the index of this location, we increment backIndex. But since the array is circular, we use the operator % to make backIndex zero after reaching its maximum value.

```
public void enqueue(T newEntry)
{
   ensureCapacity();
   backIndex = (backIndex + 1) % queue.length;
   queue[backIndex] = newEntry;
} // end enqueue
```

The implementation of ensureCapacity differs from the one given in Chapter 6 because the array here is circular. We will see how to implement it shortly.

The performance of enqueue when it does not resize the array is independent of the number of entries in the queue. Thus, it is O(1) in this case. However, its performance degrades to O(n) when the array is full, because resizing the array is an O(n) operation. If this happens, however, the very next enqueue is O(1) again. As we mentioned in Segment 6.9, we could amortize the cost of doubling the array over all additions to the queue. That is, we let all enqueue operations share the cost of resizing the array. Unless the array is resized many times, each enqueue is almost O(1).

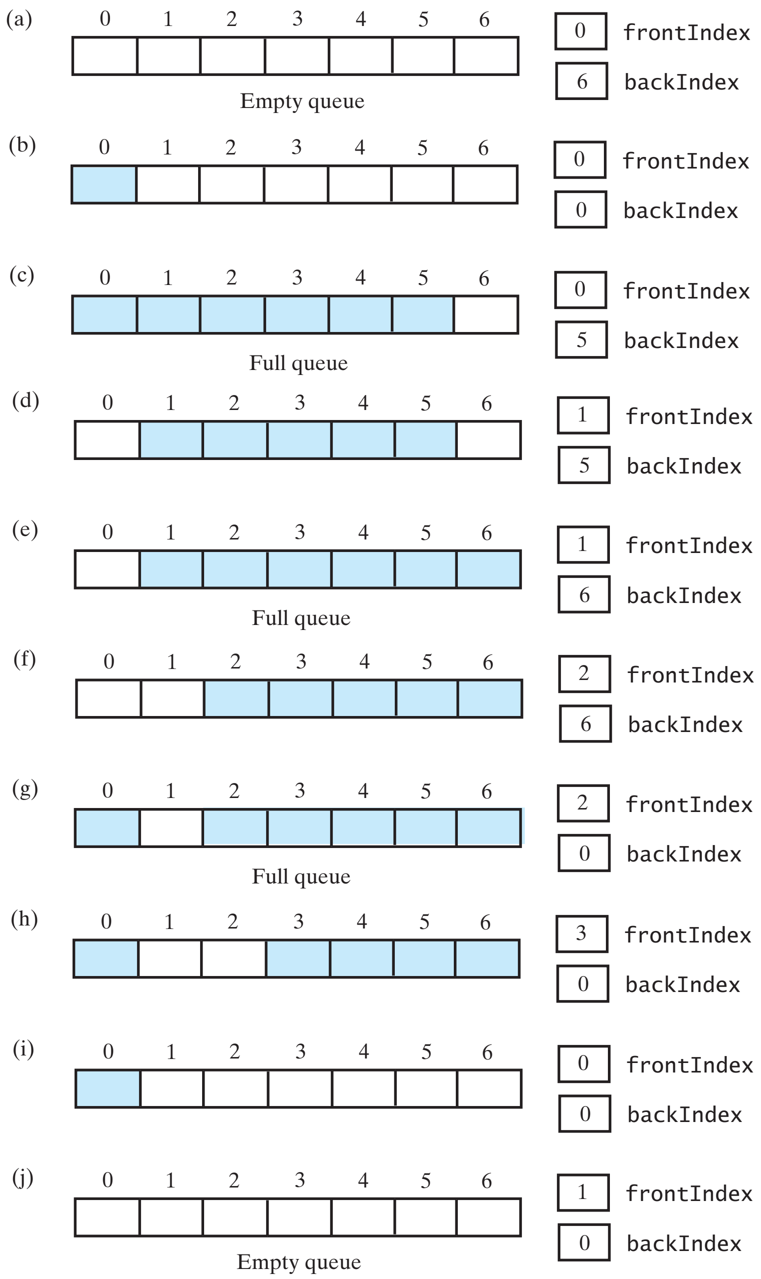
11.13 **Retrieving the front entry.** The method getFront returns either the array element at frontIndex or null if the queue is empty:

```
public T getFront()
{
   T front = null;
```

```
      if (!isEmpty())
         front = queue[frontIndex];

      return front;
   } // end getFront
```

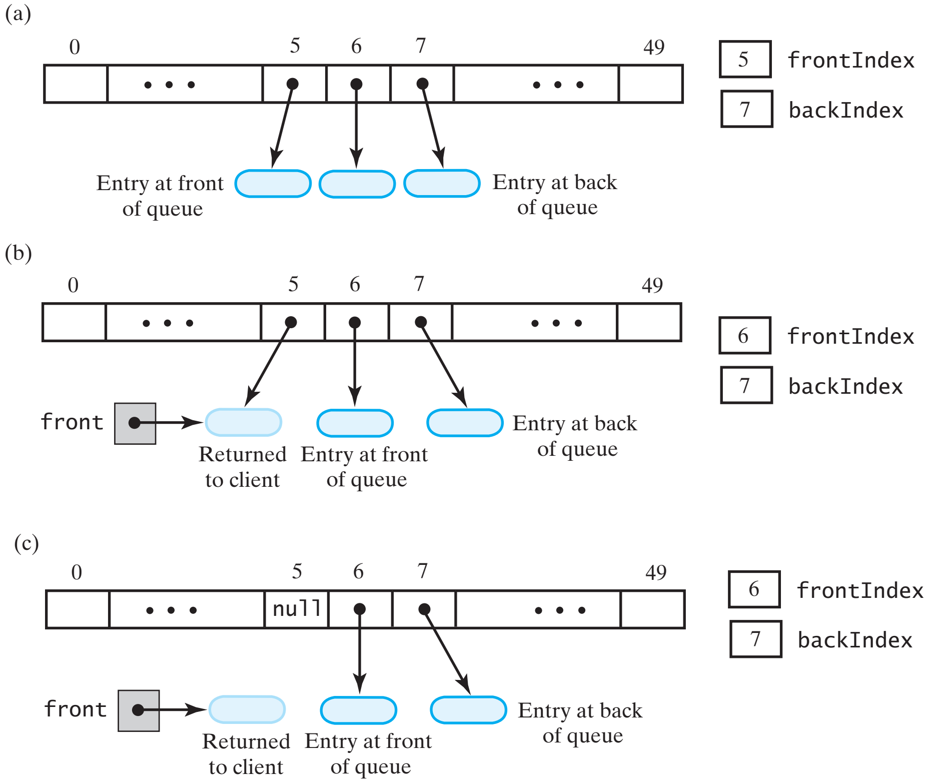
This operation is O(1).

FIGURE 11-8 A seven-location circular array that contains at most six entries of a queue



11.14 **Removing the front entry.** The method `dequeue`, like `getFront`, retrieves the entry at the front of the queue, but then it removes it. To remove the front entry of the queue shown in Figure 11-9a, we could simply increment `frontIndex`, as Figure 11-9b illustrates. This step would suffice because the other methods would behave correctly. For example, `getFront` would return the item that `queue[6]` references. However, the object that previously was the front of the queue and is returned to the client would still be referenced by the array. This fact is of no real concern if our implementation is correct. To be safe, `dequeue` can set `queue[frontIndex]` to `null` before incrementing `frontIndex`. Figure 11-9c illustrates the queue in this case.

FIGURE 11-9 An array-based queue: (a) initially; (b) after removing its front entry by incrementing `frontIndex`; (c) after removing its front entry by setting `queue[frontIndex]` to `null` and then incrementing `frontIndex`



The following implementation of `dequeue` reflects these comments:

```
public T dequeue()
{
   T front = null;

   if (!isEmpty())
   {
      front = queue[frontIndex];
      queue[frontIndex] = null;
      frontIndex = (frontIndex + 1) % queue.length;
   } // end if
```

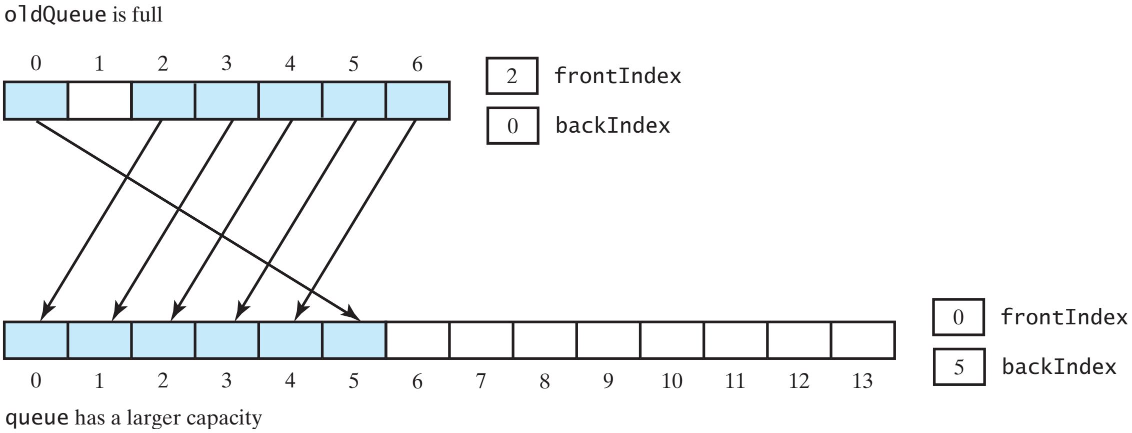
```
      return front;
   } // end dequeue
```

Like getFront, dequeue is an O(1) operation.

11.15 **The private method ensureCapacity.** As you saw in Segment 2.32 of Chapter 2, when we increase the size of an array, we must copy its entries into the newly allocated space. We need to be careful, though, because here the array is circular. We must copy entries in the order in which they appear in the queue.

For example, the seven-element array in Figure 11-8g is full and appears again in Figure 11-10. Call this array oldQueue. After allocating a new array queue of 14 locations, we copy the front of the queue from oldQueue[frontIndex] to queue[0]. We continue copying elements from the old array to the new array, proceeding to the end of the old array and wrapping around to its beginning, as the figure shows. In addition, we must set frontIndex and backIndex to reflect the reorganized array.

FIGURE 11-10 Doubling the size of an array-based queue



The following definition of ensureCapacity detects when the array is full by using the criterion given in Segment 11.10:

```
// Doubles the size of the array queue if it is full
private void ensureCapacity()
{
   if (frontIndex == ((backIndex + 2) % queue.length)) // if array is full,
   {                                                   // double size of array
      T[] oldQueue = queue;
      int oldSize = oldQueue.length;

      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempQueue = (T[]) new Object[2 * oldSize];
      queue = tempQueue;
      for (int index = 0; index < oldSize - 1; index++)
      {
         queue[index] = oldQueue[frontIndex];
         frontIndex = (frontIndex + 1) % oldSize;
      } // end for

      frontIndex = 0;
      backIndex = oldSize - 2;
   } // end if
} // end ensureCapacity
```

You can use the method `System.arraycopy` to copy the array. However, since the array is circular, you will need two calls to this method. Exercise 1 at the end of this chapter asks you to revise `ensureCapacity` in this way.

11.16 **The rest of the class.** The public method `isEmpty` has the following implementation, based on our comments at the end of Segment 11.10:

```
public boolean isEmpty()
{
   return frontIndex == ((backIndex + 1) % queue.length);
} // end isEmpty
```

The method `clear` could simply set `frontIndex` to 0 and `backIndex` to `queue.length - 1`. The other queue methods would behave as expected for an empty queue. However, the objects that were in the queue would then remain allocated. To deallocate them, `clear` should set to `null` each array location that was used for the queue. Alternatively, `clear` could call `dequeue` repeatedly until the queue is empty, if `dequeue` sets `queue[frontIndex]` to `null`. We leave the implementation of `clear` as an exercise.

? SELF-TEST

Question 3 Write an implementation of `clear` that sets to `null` each array location that was used for the queue.

Question 4 Write an implementation of `clear` that repeatedly calls `dequeue` until the queue is empty. How does this implementation compare to the one you wrote for Question 3?

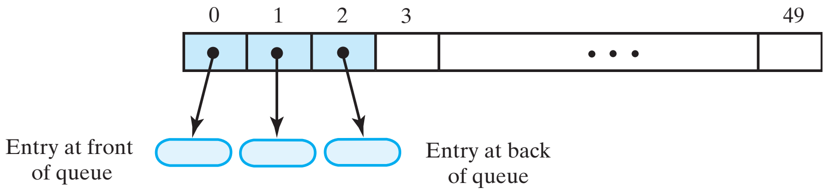
Question 5 If `queue` is an array that contains the entries in a queue, and `queue` is not treated as a circular array, what is a disadvantage of maintaining the back of the queue at `queue[0]`?

Note: In some languages other than Java, leaving an array location empty wastes memory because the location contains an object instead of a reference to an object. Project 3 at the end of this chapter considers an array-based implementation of a queue that does not have an unused location and does not maintain a counter.

A Vector-Based Implementation of a Queue

11.17 Using a vector to represent a queue's entries is relatively easy. We maintain the front of the queue at the beginning of the vector, as Figure 11-11 illustrates. We can use `Vector`'s method `add` to add an entry to the back of the queue. When we remove the queue's front entry from the vector, the vector's elements move so that the new front entry in the queue will be at the beginning of the vector. Thus, we do not need to maintain indices to the front and back of the queue. Also, the vector expands as necessary, so we do not have to worry about this detail.

FIGURE 11-11 A vector that represents a queue



11.18 **An outline of the class.** The class that implements the queue begins by declaring a vector as a data field and allocating the vector in its constructors, as Listing 11-3 shows.

LISTING 11-3 An outline of a vector-based implementation of the ADT queue

```
import java.util.Vector;
/**
   A class that implements a queue of objects by using a vector.
   @author Frank Carrano
*/
public class VectorQueue<T> implements QueueInterface<T>
{
   private Vector<T> queue; // queue's front entry is first in the vector

   public VectorQueue()
   {
      queue = new Vector<T>(); // vector doubles in size if necessary
   } // end default constructor

   public VectorQueue(int initialCapacity)
   {
      queue = new Vector<T>(initialCapacity);
   } // end constructor

   < Implementations of the queue operations go here. >
   . . .
} // end VectorQueue
```

11.19 **Adding to the back.** We use `Vector`'s method `add` to add an entry to the end of the vector—that is, to the back of the queue.

```
public void enqueue(T newEntry)
{
   queue.add(newEntry);
} // end enqueue
```

11.20 **Retrieving the front entry.** We retrieve the entry at the front of the queue by using `Vector`'s method `get` with an argument of zero:

```
public T getFront()
{
   T front = null;

   if (!isEmpty())
      front = queue.get(0);

   return front;
} // end getFront
```

11.21 **Removing the front entry.** We can retrieve and remove the entry at the front of the queue by using `Vector`'s method `remove` with an argument of zero. This method returns the removed entry, which is just what `dequeue` needs to do:

```
public T dequeue()
{
   T front = null;

   if (!isEmpty())
      front = queue.remove(0);

   return front;
} // end dequeue
```

11.22 **The rest of the class.** The remaining public methods `isEmpty` and `clear` invoke analogous `Vector` methods:

```
public boolean isEmpty()
{
   return queue.isEmpty();
} // end isEmpty

public void clear()
{
   queue.clear();
} // end clear
```

11.23 **Efficiency.** The implementation of `Vector` is based on an array that expands dynamically, but not one that is circular. Since we add entries to one end of a queue and remove them from the other end, the vector implementation inherently moves its entries after each removal. Thus, `dequeue` is O(n), while the other operations are O(1). However, expanding the vector degrades the performance of `enqueue`. The efficiency of this implementation is not as good as that of the one that uses a circular array.

Circular Linked Implementations of a Queue

11.24 Figure 11-1 in Segment 11.1 shows a chain of linked nodes that implements the ADT queue. This chain has two external references—one to the first node and one to the last node in the chain. Recall that these references are particularly useful for a queue implementation, since a queue's operations affect both of its ends. Like the chains you have seen before, the last node in this chain contains `null`. Such chains are sometimes called **linear linked chains**, regardless of whether they have a tail reference in addition to a head reference.

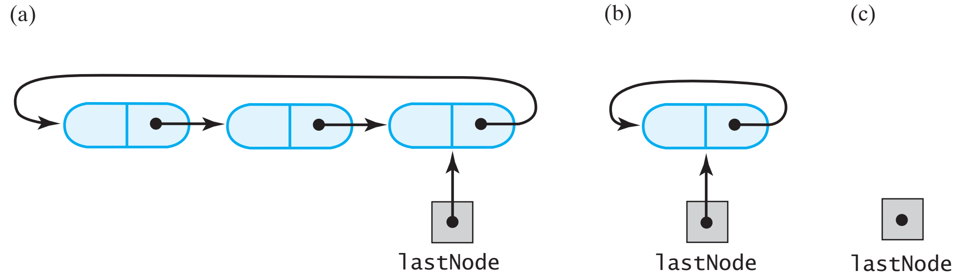
VideoNote
Other queue implementations

In a **circular linked chain**, the last node references the first node, so no node contains `null` in its `next` field. Despite the fact that each node references the next node, a circular linked chain has a beginning and an end. We could have an external reference to the chain's first node, but then a traversal of the chain would be necessary to locate the last node. Having both a reference to the first node and a reference to the last node is usually more than is necessary. Since the chain's last node references its first node, we can have a solitary reference to the last node and still locate the first node quickly. Figure 11-12 illustrates such a chain.

When a class uses a circular linked chain to represent a queue, its only data field is the reference `lastNode` to the chain's last node. The implementation therefore does not have the overhead of maintaining a data field that references the first node. Any time such a reference is needed, the expression `lastNode.getNextNode()` provides it. Despite this simplification, this approach is not necessarily better than the one used in the first section of this chapter. It is mostly just different, as you will see if you complete Project 4 at the end of this chapter.

We now investigate another way to use a circular linked chain to represent a queue.

FIGURE 11-12 A circular linked chain with an external reference to its last node that (a) has more than one node; (b) has one node; (c) is empty



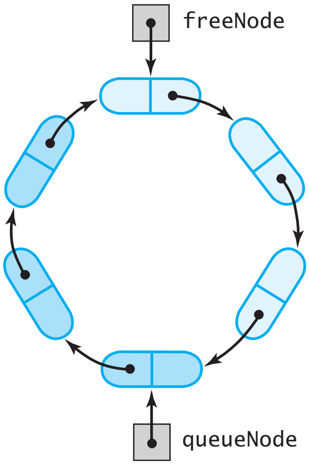
A Two-Part Circular Linked Chain

11.25 When a linked chain—whether it is linear or circular—represents a queue, it has one node for each entry in the queue. When we add an entry to the queue, we allocate a new node for the chain. When we remove an entry from the queue, a node is deallocated.

In the circular array implementation, the queue uses a subset of the fixed number of array locations available. When we add an entry to the queue, we use the next unoccupied location in the array. When we remove an entry from the queue, we make its array location available for the queue's later use. Since additions and removals are at the ends of a queue, the queue occupies contiguous locations in the circular array. The available locations also are contiguous, again because the array is circular. Thus, the circular array has two parts: One part contains the queue and the other part is available for the queue.

Suppose that we had two parts in a circular linked chain. The linked nodes that form the queue are followed by linked nodes that are available for use in the queue, as Figure 11-13 illustrates. Here `queueNode` references the node assigned to the front of the queue; `freeNode` references the first available node that follows the end of the queue. You could think of this configuration as two chains—one for the queue and one for the available nodes—that are joined at their ends to form a circle.

FIGURE 11-13 A two-part circular linked chain that represents both a queue and the nodes available to the queue



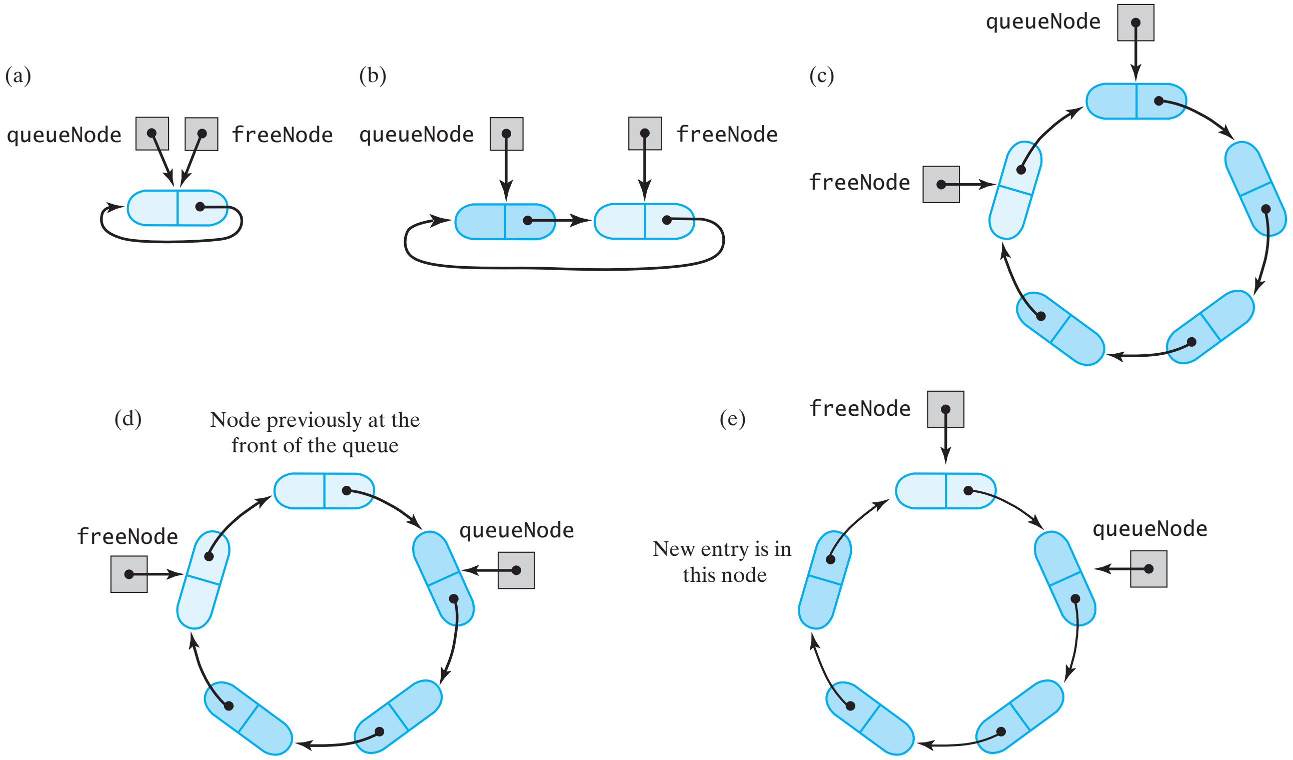
The available nodes are not allocated all at once the way locations are allocated for an array. Initially there are no available nodes; we allocate a node each time we add a new entry to the queue. However, when we remove an entry from the queue, we keep its node in the circle of nodes rather than deallocating it. Thus, a subsequent addition to the queue uses a node from the chain of available nodes. But if no such node is available, we allocate a new one and link it into the chain.

11.26 Detecting an empty queue or an absence of available nodes is easier if one node in the circular linked chain is unused. The situation is analogous to the circular array that we used in Segment 11.10. Figure 11-14a shows the queue when it is empty. Both `queueNode` and `freeNode` reference the same unused node. Notice that the node references itself. We can tell that the queue is empty because `queueNode` equals `freeNode`.

To add an entry to this empty queue, we allocate a new node and link it into the circular chain. Figure 11-14b shows the resulting chain for a queue of one entry. To simplify the figure, we have not illustrated the actual object in the queue. Although a node in the chain references an object in the queue, we will sometimes say that the node is in the queue.

While `queueNode` references the node assigned to the queue, `freeNode` still references the unused node. After three more additions to the queue, three more nodes are allocated and linked into the chain. Segment 11.28 will describe exactly how to accomplish this. The chain now appears as in Figure 11-14c. Again, `freeNode` references the unused node. Since `queueNode` references the node at the front of the queue, retrieving the front entry is easy.

FIGURE 11-14 A two-part circular linked chain that represents a queue: (a) when it is empty; (b) after adding one entry; (c) after adding three more entries; (d) after removing the front entry; (e) after adding one more entry



Now if we remove the entry at the front of the queue, we advance queueNode so the chain is as pictured in Figure 11-14d. The node that was at the front of the queue is not deallocated. A subsequent addition—since it is at the back of the queue—uses the node that freeNode references. We then advance freeNode. Figure 11-14e shows the chain at this point. Notice that we did not allocate a new node for the additional entry in this case.

How can we tell whether we must allocate a new node when we add to the queue? We must do so if we want to add to the queue shown in Figure 11-14e. At this point, queueNode equals freeNode.getNextNode(). That was not the case when we added an entry to the queue in Figure 11-14d; a node was available without allocating a new one. But notice in Figure 11-14a that queueNode also equals freeNode.getNextNode() when the queue is empty. This makes sense, because to add to an empty queue, we need to allocate a new node.

Note: In a two-part circular linked implementation of a queue, one node is unused. Two external references partition the chain into two parts: queueNode references the front node of the queue and freeNode references the node that follows the queue. The queue is empty if queueNode equals freeNode. You use the node at freeNode for a new entry. This node is either the first available node or the unused node. You must allocate a new unused node if queueNode equals freeNode.getNextNode().

11.27 **An outline of the class.** The class that implements the queue by using a two-part circular linked chain has the references queueNode and freeNode as data fields. Since the chain must always contain at least one node, the default constructor allocates a node, makes the node reference itself, and sets queueNode and freeNode to reference this new node. Thus, the class appears as outlined in Listing 11-4.

LISTING 11-4 An outline of a two-part circular linked implementation of the ADT queue

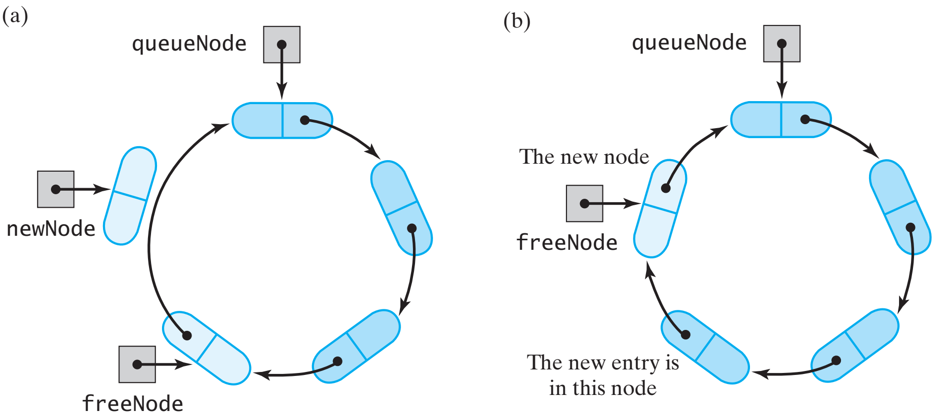
```
/**
   A class that implements a queue of objects by using
   a two-part circular chain of linked nodes
   @author Frank M. Carrano
*/
public class TwoPartCircularLinkedQueue<T> implements QueueInterface<T>
{
   private Node queueNode; // references first node in queue
   private Node freeNode;  // references node after back of queue

   public TwoPartCircularLinkedQueue()
   {
      freeNode = new Node(null, null);
      freeNode.setNextNode(freeNode);
      queueNode = freeNode;
   } // end default constructor

   < Implementations of the queue operations go here. >
   . . .
```

```
      private class Node
      {
         private T    data; // queue entry
         private Node next; // link to next node

         < Constructors and the methods getData, setData, getNextNode, and setNextNode
            are here. >
         . . .
      } // end Node
   } // end TwoPartCircularLinkedQueue
```

Programming Tip: When a circular linked chain has one node, the node must reference itself. Forgetting this step is easy to do and leads to an error during execution.

11.28 **Adding to the back.** Before adding an entry to the queue, we see whether a node is available in the chain. If one is not, we must allocate a new one and link it into the chain. We insert a new node into the chain *after* the node that `freeNode` references, as we are about to do in Figure 11-15a. We do not insert it before this node, because we would need a reference to the previous node to do so. Getting such a reference would take time. The node that `freeNode` references joins the queue and will contain the new entry. The new node becomes the unused node, and we make `freeNode` reference it, as Figure 11-15b shows.

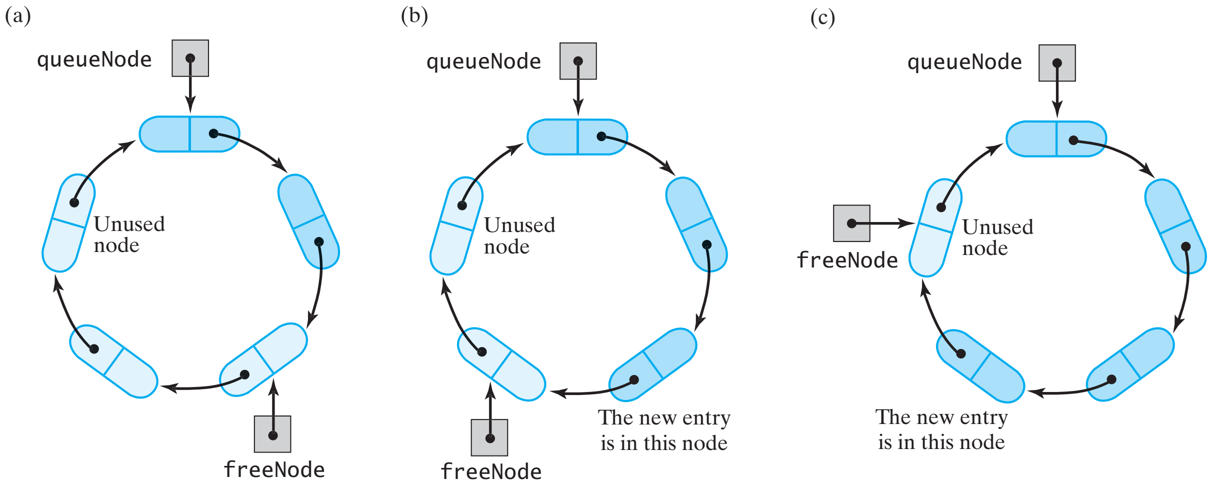
FIGURE 11-15 A chain that requires a new node for an addition to a queue: (a) before the addition; (b) after the addition



If a node is available in the chain, we use the node that `freeNode` references for the new entry. Figure 11-16 shows the chain before and after two existing nodes become part of the queue. After each addition, `freeNode` references the node that follows the back of the queue. In Figure 11-16b, this node is available for another addition, but in Figure 11-16c, it is unused.

The method `enqueue` is easier to write and to understand if we hide the detail of seeing whether to allocate a new node within the private method `isChainFull`. It returns `true` if the chain has no nodes available for use in the queue. The implementation of `isChainFull` is not difficult and appears later in Segment 11.31.

FIGURE 11-16 (a) A chain with nodes available for additions to a queue; (b) the chain after one addition; (c) the chain after another addition



The following implementation of enqueue is an O(1) operation:

```
public void enqueue(T newEntry)
{
   freeNode.setData(newEntry);

   if (isChainFull())
   {
      // allocate a new node and insert it after the node that
      // freeNode references
      Node newNode = new Node(null, freeNode.getNextNode());
      freeNode.setNextNode(newNode);
   } // end if

   freeNode = freeNode.getNextNode();
} // end enqueue
```

Question 6 Adding an entry to the queue pictured in Figure 11-16c requires the creation of a new node. Where in the chain would you insert the new node? Which node would contain the new entry?

11.29 **Retrieving the front.** If the queue is not empty, queueNode references its front node. The method getFront is therefore straightforward:

```
public T getFront()
{
   T front = null;

   if (!isEmpty())
     front = queueNode.getData();

   return front;
} // end getFront
```

This method is O(1).

11.30 **Removing the front.** The method dequeue returns the entry at the front of the queue. It then moves the node at the front from the queue's part of the chain to the available part simply by advancing `queueNode`. Parts *c* and *d* of Figure 11-14 show a chain before and after this step. Since the node that contained the removed entry is not deallocated, it still references the removed entry. Thus, we set the node's data portion to `null`.

Like `getFront`, dequeue is an O(1) operation:

```
public T dequeue()
{
   T front = null;

   if (!isEmpty())
   {
      front = queueNode.getData();
      queueNode.setData(null);
      queueNode = queueNode.getNextNode();
   } // end if

   return front;
} // end dequeue
```

11.31 **The rest of the class.** The methods `isEmpty` and `isChainFull` follow from the discussion in Segment 11.26:

```
public boolean isEmpty()
{
   return queueNode == freeNode;
} // end isEmpty

private boolean isChainFull()
{
   return queueNode == freeNode.getNextNode();
} // end isChainFull
```

The method `clear` sets `queueNode` equal to `freeNode` to make the queue appear empty. It retains all nodes currently in the chain. However, unless you set the data portions of these nodes to `null`, the objects in the queue are not deallocated. We leave the implementation of `clear` as an exercise.

? SELF-TEST

Question 7 Describe two different ways in which you could implement the method `clear`.

11.32 **Choosing a linked implementation.** So far, we have discussed several possible linked implementations of the ADT queue. You can use a linear chain with both head and tail references, as shown in Figure 11-1, or an equivalent circular chain with one external reference, as shown in Figure 11-12. In both of these implementations, removing an entry from the queue disconnects and deallocates a node in the chain. If, after removing entries from the queue, you seldom add entries, these implementations are fine. But if you frequently add an entry after removing one, the two-part circular chain saves the time of deallocating and reallocating nodes.

Java Class Library: The Class `AbstractQueue`

11.33 The standard package `java.util` in the Java Class Library contains the abstract class `AbstractQueue`. This class implements the interface `java.util.Queue` and does not allow `null` entries in the queue. Recall from Segment 10.13 of the previous chapter the following methods in this interface:

```
public boolean add(T newEntry)
public boolean offer(T newEntry)
public T remove()
public T poll()
public T element()
public T peek()
public boolean isEmpty()
public void clear()
public int size()
```

AbstractQueue provides implementations of the methods add, remove, and element that invoke offer, poll, and peek, respectively.

You can define a class of queues by using inheritance to extend AbstractQueue. Your class must override at least the following methods: offer, poll, peek, and size. Note that the class java.util.PriorityQueue, which we mentioned in the previous chapter, extends AbstractQueue and, thereby, implements the methods declared in the interface java.util.Queue.

To learn more about AbstractQueue, consult the online documentation for the Java Class Library.

A Doubly Linked Implementation of a Deque

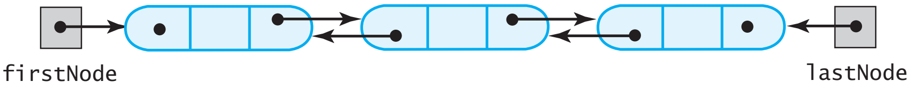
11.34 Earlier, in Segment 11.1, we planned the linked implementation of the queue and noticed that the front of the queue should not be at the tail of the chain of linked nodes. If it were, we would have to traverse the chain to get a reference to the preceding node so that we could remove the queue's front entry.

Although placing the front of the queue at the head of the chain solved our problem, such is not the case for a deque. We must be able to remove both the front *and* the back of a deque. So even if the deque's front is at the head of the chain, the deque's back will be at the chain's tail—and therein lies the problem.

Each node in a chain references only the next node. Thus, a chain, with its head reference, permits us to begin at the first node and move ahead from node to node. Having a tail reference lets us access the last node in the chain, but not the next-to-last node. That is, we cannot move backward from a node, and this is just what we need to do to remove the back of a deque.

11.35 What we need is a node that can reference the previous node as well as the next node in a chain. We call a chain of such nodes a **doubly linked chain**. We sometimes will call an ordinary chain a **singly linked chain** when a distinction is necessary. Figure 11-17 illustrates a doubly linked chain with its head and tail references. While an interior node references both the next node and the previous node, the first and last nodes each contain one null reference. Thus, when traversing the chain from the first node to the last, we will encounter null when we reach the last node. Likewise, when traversing the chain from the last node to the first, we will encounter null when we reach the first node.

FIGURE 11-17 A doubly linked chain with head and tail references



The node in a doubly linked chain is an instance of an inner class similar to the class Node. We will call this inner class DLNode and give it three data fields: next and previous are references to two other nodes, and data is a reference to the node's data. DLNode also has the methods getData, setData, getNextNode, setNextNode, getPreviousNode, and setPreviousNode.

11.36 **An outline of the class.** The doubly linked implementation of the deque begins much like the linked implementation of the queue given in Segment 11.2. The class has two data fields—`firstNode` and `lastNode`—that the default constructor sets to `null`, as you can see in Listing 11-5.

LISTING 11-5 An outline of a linked implementation of the ADT deque

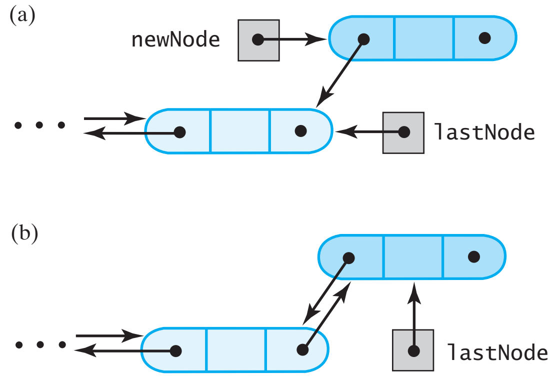
```
/**
   A class that implements a deque of objects by using
   a chain of doubly linked nodes.
   @author Frank M. Carrano
*/
public class LinkedDeque<T> implements DequeInterface<T>
{
   private DLNode firstNode; // references node for front of deque
   private DLNode lastNode;  // references node for back of deque

   public LinkedDeque()
   {
      firstNode = null;
      lastNode = null;
   } // end default constructor

   < Implementations of the deque operations go here. >
   . . .

   private class DLNode
   {
      private T      data;     // deque entry
      private DLNode next;     // link to next node
      private DLNode previous; // link to previous node

      < Constructors and the methods getData, setData, getNextNode, setNextNode,
        getPreviousNode, and setPreviousNode are here. >
      . . .
   } // end DLNode
} // end LinkedDeque
```

11.37 **Adding an entry.** The implementation of the method `addToBack` is like the implementation of `enqueue` given in Segment 11.3. Both methods add a node to the end of a chain so that the chain's current last node references the new node. Here, we also make the new node reference the current last node by passing the deque's data field `lastNode` to the node's constructor. The addition to the back of a chain that is not empty is illustrated in Figure 11-18. An implementation of the method follows:

```
public void addToBack(T newEntry)
{
   DLNode newNode = new DLNode(lastNode, newEntry, null);

   if (isEmpty())
      firstNode = newNode;
   else
      lastNode.setNextNode(newNode);
   lastNode = newNode;
} // end addToBack
```

Aside from its name, this method differs from enqueue only in the statement that allocates a new node.

FIGURE 11-18 Adding to the back of a nonempty deque: (a) after the new node is allocated; (b) after the addition is complete



The method addToFront has an analogous implementation. When adding a node to the beginning of a doubly linked chain, we must make the chain's current first node reference the new node by passing the deque's data field firstNode to the node's constructor. Compare the following definition for addToFront with the one just given for addToBack:

```
public void addToFront(T newEntry)
{
   DLNode newNode = new DLNode(null, newEntry, firstNode);

   if (isEmpty())
      lastNode = newNode;
   else
      firstNode.setPreviousNode(newNode);

   firstNode = newNode;
} // end addToFront
```

As given here, both addToFront and addToBack are O(1) operations.

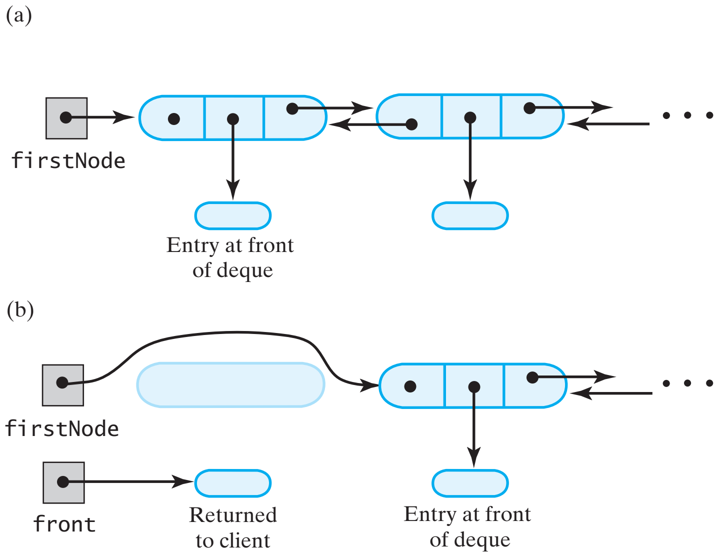
11.38 **Removing an entry.** The method removeFront has an implementation much like that of dequeue given in Segment 11.5, but it has one other concern. After detaching the first node, if the deque is not empty, removeFront must set the field previous in the new first node to null. This step occurs in the else clause of the following definition:

```
public T removeFront()
{
   T front = null;

   if (!isEmpty())
   {
      front = firstNode.getData();
      firstNode = firstNode.getNextNode();

      if (firstNode == null)
         lastNode = null;
      else
         firstNode.setPreviousNode(null);
   } // end if
```

```
      return front;
   } // end removeFront
```

Aside from its name, this method differs from dequeue only in the addition of the else clause. Figure 11-19 illustrates the effect of removeFront for a deque of at least two entries.

FIGURE 11-19 (a) A deque containing at least two entries; (b) after removing the first node and obtaining a reference to the deque's new first entry



The method removeBack has an analogous definition:

```
public T removeBack()
{
   T back = null;

   if (!isEmpty())
   {
      back = lastNode.getData();
      lastNode = lastNode.getPreviousNode();

      if (lastNode == null)
         firstNode = null;
      else
         lastNode.setNextNode(null);
   } // end if

   return back;
} // end removeBack
```

The implementations of removeFront and removeBack are each O(1).

11.39 **Retrieving an entry.** The method getFront has the same implementation as given in Segment 11.4 for a queue. The method getBack is analogous to getFront and is left as an exercise. Both getFront and getBack are O(1) operations.

SELF-TEST

Question 8 Implement the method getBack for the ADT deque when a doubly linked chain contains the deque's entries.

11.40 **Reusing this implementation.** Once you have implemented the ADT deque, you can use it to implement other ADTs such as the queue and the stack. These implementations are straightforward and are left as exercises.

Note: In a doubly linked chain, the first and last nodes each contain one `null` reference, since the first node has no previous node and the last node has no node after it. In a **circular doubly linked chain**, the first node references the last node, and the last node references the first. Only one external reference is necessary—a reference to the first node—since you can quickly get to the last node from the first node. You can use a circular doubly linked chain in an implementation of the ADT deque. Project 8 asks you to do this.

Possible Implementations of a Priority Queue

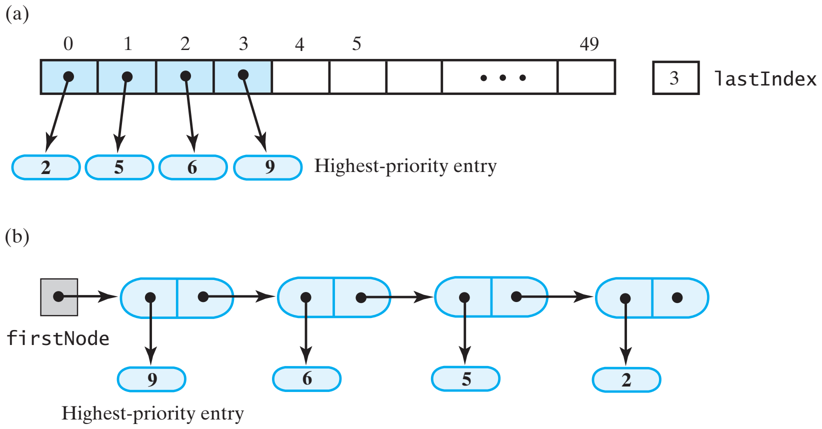
11.41 We can use an array, a linked chain, or a vector to implement the ADT priority queue. In each of these cases, we would maintain the entries in sorted order by their priorities. With an array, the entry with the highest priority should occur at the end of the array, so removing it would leave the other entries in their present places. Figure 11-20a illustrates this implementation.

If a linked chain contains the entries in a priority queue, the entry with the highest priority should occur at the beginning of the chain, where it is easy to remove. Figure 11-20b shows such a chain.

The next chapter will introduce the ADT list, and Chapter 16 will discuss a kind of list called the sorted list. A sorted list can maintain a priority queue's entries in priority order, doing much of the work for us. Project 10 at the end of Chapter 16 asks you to complete such an implementation.

Chapter 23 describes a more efficient implementation of a priority queue that uses an ADT called a heap.

FIGURE 11-20 Two possible implementations of a priority queue using (a) an array; (b) a chain of linked nodes



Chapter Summary

- You can implement a queue by using a chain of linked nodes that has both a head reference and a tail reference. The first node in the chain represents the front of the queue, because you can remove or access a chain's first node faster than any other node. The tail reference allows you to quickly add a node to the end of the chain, which is the queue's back.
- The queue operations are O(1) for a linked implementation.

- You can implement a queue by using an array. Once a queue entry is added to the array, it does not move. After many additions, the last array location will be used. Removals, however, will free locations at the beginning of the array. Thus, the array can appear full even when it is not. To solve this problem, you treat the array as if it were circular.
- The queue operations are O(1) for an array-based implementation. However, when the array is full, `enqueue` doubles the size of the array. In that case, `enqueue` is O(*n*). Typically, we amortize the cost of resizing the array over all additions to the queue. If the array is resized occasionally, each `enqueue` is almost O(1).
- You can implement a queue by using a vector. Since the entry at the front of the queue is always first in the vector, and since the implementation of `Vector` is based on an array that is expanded as necessary, entries in the vector move. Thus, the vector-based implementation is less efficient than our array-based implementation. However, it is easier to write.
- In a circular linked chain, every node references the next node in the chain. No node contains `null` in its `next` field. A circular linked chain can have a beginning and an end. Since the last node references the first node in the chain, one external reference to the last node provides convenient access to both the chain's last node and its first node.
- You can use a circular linked chain to implement a queue in much the same way that you use a linear linked chain that has both head and tail references. With both kinds of chain, `dequeue` removes a node and deallocates it.
- Another implementation of a queue uses a circular linked chain that has two parts. One part is used for the queue and the other part contains one unused node and any nodes that are available for use. In this implementation, `dequeue` removes an entry from the queue but does not remove a node from the chain. Instead, the node joins the available part of the chain.
- Since a deque has operations that add and remove entries at both ends, you can use a doubly linked chain whose nodes reference both the next node and the previous node. A doubly linked chain with head and tail references provides O(1) implementations of the deque operations.
- In a circular doubly linked chain, every node references the next node as well as the previous node in the chain. No node contains `null` in its `next` and `previous` fields. One external reference to the first node provides fast access to both the chain's last node and its first node. You can use a circular doubly linked chain in the implementation of a deque.
- You can use an array or a chain to implement a priority queue, but a more efficient implementation is possible by using a heap.

Programming Tip

- When a circular linked chain has one node, the node must reference itself. Forgetting this step is easy to do and leads to an error during execution.

Exercises

1. Segment 11.15 defines the private method `ensureCapacity` for an array-based implementation of the ADT queue. Revise that method to use the method `System.arraycopy` to copy the old array to a new expanded array.
2. Segment 11.31 describes an implementation of the queue's method `clear` when a two-part circular linked chain represents the queue. Write two different implementations of `clear`. One version should repeatedly invoke `dequeue`. The other version should set the data portion of each node in the queue to `null`.
3. Suppose that we want to add a method to a class of queues that will splice two queues together. This method adds to the end of a queue all items that are in a second queue. The header of the method could be as follows:

```
public void splice(QueueInterface<T> anotherQueue)
```

Write this method in such a way that it will work in any class that implements `QueueInterface<T>`.

4. Consider the method `splice` that Exercise 3 describes. Implement this method specifically for the class `ArrayQueue`. Take advantage of your ability to manipulate the array representation of the queue.

5. Consider the method `splice` that Exercise 3 describes. Implement this method specifically for the class `LinkedQueue`. Take advantage of your ability to manipulate the chain that represents the queue.

6. Using Big Oh notation, what is the time complexity of each queue operation in the class `ArrayQueue`? Briefly explain your answers.

7. Using Big Oh notation, what is the time complexity of each deque operation in the class `LinkedDeque`? Briefly explain your answers.

8. Implement the ADT queue by using an ADT deque to contain its entries.

9. Implement the ADT stack by using an ADT deque to contain its entries.

10. Describe an implementation of a queue that uses two stacks, and comment on its efficiency.

11. Implement the ADT deque by using a vector to contain its entries.

12. Consider an application that uses a priority queue. You have two implementations available. One implementation uses an array to maintain the entries in the priority queue, while the other uses a linked chain. Compare the performances of these implementations for each of the following sequences of operations on a priority queue.

    a. Insert 100 objects having the priorities 1, 2, 3,..., 99, 100.
    b. Insert 100 objects having priorities 100, 99, 98,...., 2, 1.
    c. Add 100 objects having random priorities within the range 1 to 100.
    d. Starting with 100 objects in the priority queue having priorities 1 through 100, remove them all.
    e. Starting with 100 objects in the priority queue having priorities 1 through 100, repeat the following pair of operations 1000 times:

    - Add an item having a random priority within the range 1 to 100.
    - Remove an item.

Projects

1. Use a circular array, as described in Segments 11.8 and 11.9, to implement the queue. Count entries to ascertain whether the queue is empty or full.

2. The implementation of the ADT queue that was introduced in Segment 11.10 uses a circular array with one unused location. Revise that implementation so that the unused location is always before the front of the queue, with `frontIndex` as the index of this unused location. Let `backIndex` be the index of the entry at the back of the queue. Initially, both `frontIndex` and `backIndex` are set to the maximum size of the queue (the array will be 1 larger than this number). You can distinguish an empty queue from a full queue by examining these indices. What tests should you perform to do so?

3. The array-based implementations of the ADT queue in this chapter used a circular array. One implementation counted the entries in the queue, while the other left one location in the array unused. We used these strategies to tell when the queue was empty and when it was full.

    A third strategy is possible. It does not count and does not have an unused location in the circular array. After initializing `frontIndex` to 0 and `backIndex` to `-1`, you do not use modulo arithmetic when you increment these fields. Instead, you use modulo arithmetic when you index the array, but without changing `frontIndex` and `backIndex`. Thus, if `queue` is the array, `queue[frontIndex % queue.length]` is the front entry, and the entry at the back of the queue is `queue[backIndex % queue.length]`.

    Now if `backIndex` is less than `frontIndex`, the queue is empty. The number of entries in the queue is `backIndex - frontIndex + 1`. You can compare this number with the size of the array to see whether the array is full.

Since frontIndex and backIndex can continue to grow, they might become too large to represent. To reduce the chance of this happening, set frontIndex to 0 and backIndex to -1 whenever the implementation detects an empty queue. Note that adding to a full queue invokes ensureCapacity, which sets frontIndex to 0 and backIndex to the index of the entry at the back of the queue.

Complete this array-based implementation of the ADT queue.

4. Implement the ADT queue by using a circular linked chain, as shown in Figure 11-12. Recall that this chain has only an external reference to its last node.

5. Consider a new kind of queue that allows only a single copy of an object in the queue. If an object is added to the queue, but it is already there, leave the queue unchanged. This queue has another operation moveToBack that takes an object in the queue and moves it to the back. If an object is not in the queue, the operation adds it at the back of the queue.

   Create an interface NoDuplicatesQueueInterface that extends QueueInterface. Then write an array-based implementation of NoDuplicatesQueueInterface. Finally, write a program that adequately demonstrates your new class.

6. Implement the ADT deque by using an array to contain its entries. Expand the array dynamically when necessary.

7. One difficulty with implementing the doubly linked chain described in Segment 11.35 is the number of special cases that occur at the beginning and end of the chain. You can eliminate these cases if the chain is never empty. Thus, you begin each chain with a **dummy node** that you do not use for data.

   Revise the implementation of the deque given in this chapter by using a dummy node.

8. Use a circular doubly linked chain (see the note at the end of Segment 11.40) to implement the ADT deque.

9. Repeat the previous project, but add a dummy node to the chain, as Project 7 describes.

10. In Project 5 you created a queue that does not allow duplicates. In this project you will create a deque that does not allow duplicates. The function of the deque's operations addToBack and addToFront should be analogous to the changed enqueue method in Project 5. Add two operations, moveToBack and moveToFront.

    Create an interface NoDuplicatesDequeInterface that extends DequeInterface. Then write a linked implementation of NoDuplicatesDequeInterface. Finally, write a program that adequately demonstrates your new class.

11. Implement the ADT priority queue by using an array, as pictured in Figure 11-20a.

12. Implement the ADT priority queue by using a chain of linked nodes, as pictured in Figure 11-20b.

13. Revise the interface for the ADT priority queue, as given in Segment 10.19 of the previous chapter, by replacing the method add with the following method:

```
public void add(T newEntry, Comparable<? super T> priorityValue)
```

    The client provides an entry and its priority value to this method. The priority queue does not use newEntry's compareTo method to assess its priority. Implement this version of the priority queue.

14. In Project 5 you created a queue that does not allow duplicates. In this project you will create a priority queue that does not allow duplicates. The function of the add operation should be analogous to the changed enqueue method in Project 5. In this case, the test for equals should not include the priority, so the header of the add method should be changed to the one given in the previous project. A new operation move will change the priority of a given item, if it is already in the priority queue. If the item is not in the priority queue, move will add it with the given priority.

    Create an interface for a priority queue that does not allow duplicates. Then write a class that implements this interface. Finally, write a program that adequately demonstrates your new class.

15. Implement a priority queue of queues, as described in Project 6 of the previous chapter.

16. The ADT *randomized queue* is like a queue, but the removal and retrieval operations involve an entry chosen at random instead of the entry at the beginning of the queue. These operations should return `null` if they encounter an empty randomized queue.

    a. Write a Java interface that specifies the methods for a randomized queue.

    b. Define a class of randomized queues, named `RandomizedQueue`, that implements the interface you created in Part *a*. Name the retrieval operation `get` instead of `getFront`.

Answers to Self-Test Questions

1. The back of the queue is at the end of the chain. Since you add to the back of a queue, you need to add a node to the end of the chain. A tail reference allows you to do this without first traversing the chain to locate its last node. Thus, a tail reference enables an efficient `enqueue` operation.

2. Entries in a bag are in no particular order within the bag and, thus, the array. Queue entries have an order relative to one another that must be maintained.

3.
```
public void clear()
{
   if (!isEmpty())
   {
      for (int index = frontIndex; index != backIndex;
           index = (index + 1) % queue.length)
      {
         queue[index] = null;
      } // end for

      queue[backIndex] = null;
   } // end if

   frontIndex = 0;
   backIndex = queue.length - 1;
} // end clear
```

4.
```
public void clear()
{
   while (!isEmpty())
      dequeue();
} // end clear
```

   This version of `clear` is easier to write than the version given in Question 3.

5. Each `enqueue` operation needs to move all of the entries in the queue to vacate `queue[0]` before it adds a new entry.

6. You place the new entry into the node that `freeNode` currently references. You then insert a new node after that node and make `freeNode` reference the new node. The new node is now the unused node.

7. You can repeatedly call `dequeue` until the queue is empty, as in the answer to Question 4. Or you can set the data fields of each node in the queue to `null` and then set `queueNode` equal to `freeNode`.

8.
```
public T getBack()
{
   T back = null;

   if (!isEmpty())
      back = lastNode.getData();

   return back;
} // end getBack
```

This page intentionally left blank

Chapter 12
Lists

Contents

Prerequisites

Introduction

Appendix B	Java Classes
Appendix D	Designing Classes
Appendix E	Exception Handling
Chapter 6	Stack Implementations

Objectives

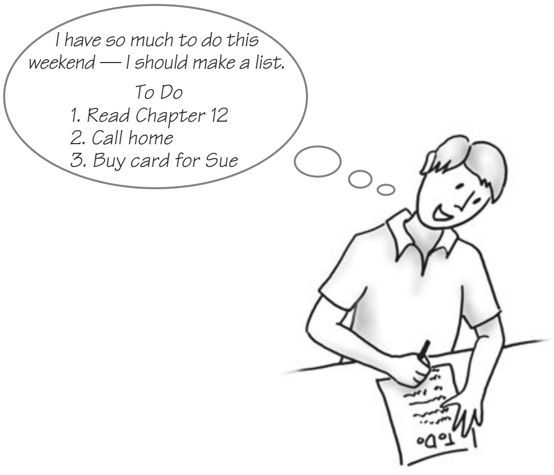
After studying this chapter, you should be able to

- Describe the ADT list
- Use the ADT list in a Java program

A list provides a way to organize data. We can have to-do lists, gift lists, address lists, grocery lists, even lists of lists. These lists provide a useful way for us to organize our lives, as illustrated in Figure 12-1. Each list has a first item, a last item, and usually items in between. That is, the items in a list have a position: first, second, and so on. An item's position might be important to you, or it might not. When adding an item to your list, you might always add it at the end, or you might insert it between two other items already in the list.

A **list** is a collection, and this chapter formalizes it as an ADT.

FIGURE 12-1 A to-do list



Specifications for the ADT List

12.1 Everyday lists such as to-do lists, gift lists, address lists, and grocery lists have entries that are strings. What can you do to such lists?

- Typically, you **add** a new entry **at the end** of the list.
- Actually, you can **add** a new entry **anywhere:** at the beginning, the end, or in between items.
- You can cross out an entry—that is, **remove** it.
- You can **remove all** entries.
- You can **replace** an entry.
- You can **look at** any entry.
- You can **look at all** of the entries.
- You can find out whether the list **contains** a particular entry.
- You can **count** the number of entries in the list.
- You can see whether the list is **empty**.

When you work with a list, you determine where an entry is or should be. You probably are not conscious of its exact position: Is it tenth? Fourteenth? However, when your program uses a list, a convenient way to identify a particular entry is by the entry's position within the list. It could be first, that is, at position 1, or second (position 2), and so on. This convention allows you to describe, or specify, the operations on a list more precisely.

VideoNote
The ADT list

12.2 To specify the ADT list, we describe its data and specify the operations on that data. Unlike common lists whose entries are strings, the ADT list is more general and has entries that are objects of the same type. The following is an initial specification of the ADT list:

Abstract Data Type: Queue

Data

- A collection of objects in a specific order and having the same data type
- The number of objects in the collection

Operations

Pseudocode	UML	Description
`add(newEntry)`	`+add(newEntry: T): void`	Task: Adds `newEntry` to the end of the list. Input: `newEntry` is an object. Output: None.
`add(newPosition, newEntry)`	`+add(newPosition: integer, newEntry: T): void`	Task: Adds `newEntry` at position `newPosition` within the list. Position 1 indicates the first entry in the list. Input: `newPosition` is an integer, `newEntry` is an object. Output: None.
`remove(givenPosition)`	`+remove(givenPosition: integer): T`	Task: Removes and returns the entry at position `givenPosition`. Input: `givenPosition` is an integer. Output: None.
`clear()`	`+clear(): void`	Task: Removes all entries from the list. Input: None. Output: None.
`replace(givenPosition, newEntry)`	`+replace(givenPosition: integer, newEntry: T): void`	Task: Replaces the entry at position `givenPosition` with `newEntry`. Input: `givenPosition` is an integer, `newEntry` is an object. Output: None.
`getEntry(givenPosition)`	`+getEntry(givenPosition: integer): T`	Task: Retrieves the entry at position `givenPosition`. Input: `givenPosition` is an integer. Output: Returns the entry at position `givenPosition`.

contains(anEntry)	+contains(anEntry: T): boolean	Task: Sees whether the list contains anEntry. Input: anEntry is an object. Output: Returns true if anEntry is in the list, or false if not.
getLength()	+getLength(): integer	Task: Gets the number of entries currently in the list. Input: None. Output: Returns the number of entries currently in the list.
isEmpty()	+isEmpty(): boolean	Task: Sees whether the list is empty. Input: None. Output: Returns true if the list is empty, or false if not.
toArray()	+toArray: T[]	Task: Retrieves all entries that are in the list in the order in which they occur. Input: None. Output: Returns a new array of the entries currently in the list.

We have only begun to specify the behaviors of these list operations, as the specifications just given leave some details to the imagination. Some examples will help us to better understand these operations so that we can improve the specifications. We'll need precise specifications before we implement the operations.

12.3 **Example.** When you first declare a new list, it is empty and its length is zero. If you add three objects—a, b, and c—one at a time and in the order given, to the end of the list, the list will appear as

```
a
b
c
```

The object a is first, at position 1, b is at position 2, and c is last at position 3.[1] To save space here, we will sometimes write a list's contents on one line. For example, we might write

```
a b c
```

to represent this list.

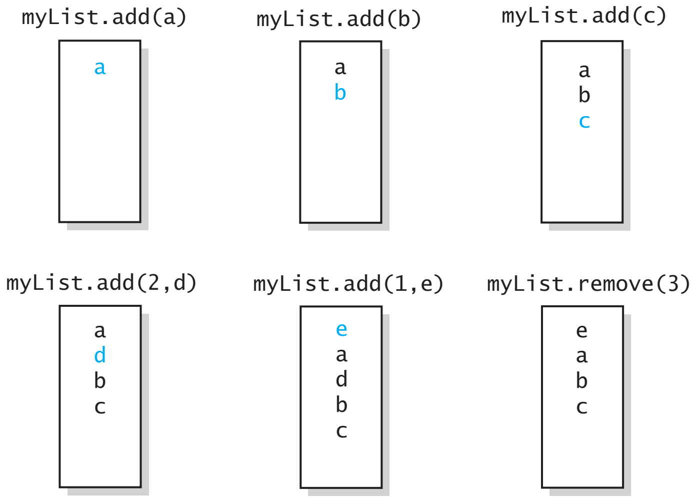
The following pseudocode represents the previous three additions to the specific list myList:

```
myList.add(a)
myList.add(b)
myList.add(c)
```

[1] Some people number the entries in a list beginning with 0 instead of 1.

At this point, myList is not empty, so myList.isEmpty() is false. Since the list contains three entries, myList.getLength() is 3. Notice that adding entries to the end of a list does not change the positions of entries already in the list. Figure 12-2 illustrates these add operations as well as the operations that we describe next.

FIGURE 12-2 The effect of ADT list operations on an initially empty list



12.4 Now suppose that we add entries at various positions within the list. For example,

```
myList.add(2, d)
```

places d second—that is, at position 2—within the list. Doing so, however, moves b to position 3 and c to position 4, so that the list now contains

```
a d b c
```

If we add e to the beginning of the list by writing

```
myList.add(1, e)
```

the current entries in the list move to the next higher position. The list then contains

```
e a d b c
```

Look at Figure 12-2 again to see the effect of these operations.

12.5 We can get the second entry in this list by writing

```
entry2 = myList.getEntry(2)
```

Remember that we are writing pseudocode here and ignoring details such as semicolons.

What happens when we remove an entry? For example,

```
myList.remove(3)
```

removes the third entry—d in the previous example—from the list. The list then contains

```
e a b c
```

Notice that entries after the one that was removed move to the next lower position within the list. Figure 12-2 illustrates this change to the list.

What if an application requires us to remove an entry from a list but retain the entry for another purpose? Our specification of remove would force us to first use getEntry to obtain the entry and then use remove to remove it from the list. We could refine the specification of remove to return the object removed from the list. To use this revised version of remove, we would write a pseudocode statement such as

```
oldEntry3 = myList.remove(3)
```

This change makes remove more versatile, as the client could either save or ignore the returned entry.

We can replace the third entry b of our list with f by writing

```
myList.replace(3, f)
```

No other entries move or change. We could refine the specification of replace to return the object that was replaced. So if we wrote

```
ref = myList.replace(3, f)
```

ref would reference the former entry b.

Note: The objects in an ADT list have an order determined by the client of the list. To add, remove, or retrieve an entry, you must specify the entry's position within the list. The first entry in the list is at position 1.

12.6 The previous specifications and examples ignore some difficulties that might arise during the use of the ADT list:

- The operations add, remove, replace, and getEntry are well behaved when the given position is valid for the current list. What happens when one of these operations receives an invalid position number?
- The methods remove, replace, and getEntry are not meaningful for empty lists. What happens when an empty list executes one of these operations?

As usual, we must decide how to handle these conditions and refine our specifications. The documentation for the ADT list should reflect both these decisions and the detail that the previous examples demonstrate.

As a reminder, we repeat the following note from Chapter 1.

Note: A first draft of an ADT's specifications often overlooks or ignores situations that you really need to consider. You might intentionally make these omissions to simplify this first draft. Once you have written the major portions of the specifications, you can concentrate on the details that make the specifications complete.

12.7 The Java interface in Listing 12-1 contains the methods for an ADT list and detailed comments that describe their behaviors. These comments address the situations we raised in the previous segment. The items in the list will be objects of the same class or classes related by inheritance.

LISTING 12-1 The interface ListInterface

```
/** An interface for the ADT list.
    Entries in the list have positions that begin with 1.
*/
```

```
public interface ListInterface<T>
{
   /** Adds a new entry to the end of this list.
       Entries currently in the list are unaffected.
       The list’s size is increased by 1.
       @param newEntry  the object to be added as a new entry */
   public void add(T newEntry);

   /** Adds a new entry at a specified position within this list.
       Entries originally at and above the specified position
       are at the next higher position within the list.
       The list’s size is increased by 1.
       @param newPosition  an integer that specifies the desired
                           position of the new entry
       @param newEntry     the object to be added as a new entry
       @return true if the addition is successful, or
               false if newPosition < 1, or newPosition > getLength()+1
   */
   public boolean add(int newPosition, T newEntry);

   /** Removes the entry at a given position from this list.
       Entries originally at positions higher than the given
       position are at the next lower position within the list,
       and the list’s size is decreased by 1.
       @param givenPosition  an integer that indicates the position of
                             the entry to be removed
       @return a reference to the removed entry or null, if either
               the list was empty, givenPosition < 1, or
               givenPosition > getLength() */
   public T remove(int givenPosition);

   /** Removes all entries from this list. */
   public void clear();

   /** Replaces the entry at a given position in this list.
       @param givenPosition  an integer that indicates the position of
                             the entry to be replaced
       @param newEntry  the object that will replace the entry at the
                        position givenPosition
       @return true if the replacement occurs, or false if either the
               list is empty, givenPosition < 1, or
               givenPosition > getLength() */
   public boolean replace(int givenPosition, T newEntry);

   /** Retrieves the entry at a given position in this list.
       @param givenPosition  an integer that indicates the position of
                             the desired entry
       @return a reference to the indicated entry or null, if either
```

```
                  the list is empty, givenPosition < 1, or
                  givenPosition > getLength() */
   public T getEntry(int givenPosition);

   /** Sees whether this list contains a given entry.
       @param anEntry  the object that is the desired entry
       @return true if the list contains anEntry, or false if not */
   public boolean contains(T anEntry);

   /** Gets the length of this list.
       @return the integer number of entries currently in the list */
   public int getLength();

   /** Sees whether this list is empty.
       @return true if the list is empty, or false if not */
   public boolean isEmpty();

   /** Retrieves all entries that are in this list in the order in which
       they occur in the list. */
   public T[] toArray();
} // end ListInterface
```

Question 1 Write pseudocode statements that add some objects to a list, as follows. First add c, then a, then b, and then d, such that the order of the objects in the list will be a, b, c, d.

Question 2 Write pseudocode statements that exchange the third and seventh entries in a list of 10 objects.

Note: The entries in a list of *n* entries are numbered from 1 to *n*. Although you cannot add a new entry at position 0, you can add one at position *n* + 1.

Using the ADT List

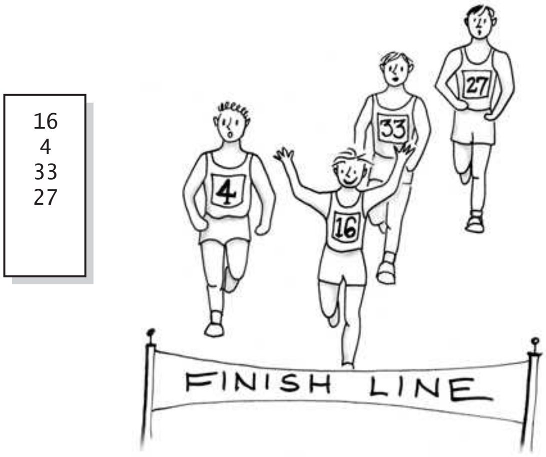
VideoNote
Using the ADT list

Imagine that we hire a programmer to implement the ADT list in Java, given the interface and specifications that we have developed so far. If we assume that these specifications are clear enough for the programmer to complete the implementation, we can use the ADT's operations in a program without knowing the details of the implementation. That is, we do not need to know *how* the programmer implemented the list to be able to use it. We only need to know *what* the ADT list does.

This section assumes that we have an implementation for the list and demonstrates how we can use a list in our program.The examples here can be part of a program that tests your implementation.

12.8 **Example.** Imagine that we are organizing a local road race. Our job is to note the order in which the runners finish the race. Since each runner wears a distinct identifying number, we can add each runner's number to the end of a list as the runners cross the finish line. Figure 12-3 illustrates such a list.

FIGURE 12-3 A list of numbers that identify runners in the order in which they finished a race



The Java program in Listing 12-2 shows how we can perform this task by using the ADT list. It assumes that the class `AList` implements the Java interface `ListInterface` that you saw in the previous section. Since `ListInterface` assumes that the items in the list are objects, we will treat each runner's identifying number as a string.

LISTING 12-2 A client of a class that implements `ListInterface`

```java
public class ListClient
{
   public static void main(String[] args)
   {
      testList();
   } // end main

   public static void testList()
   {
      ListInterface<String> runnerList = new AList<String>();
//  runnerList has only methods in ListInterface

      runnerList.add("16"); // winner
      runnerList.add(" 4"); // second place
      runnerList.add("33"); // third place
      runnerList.add("27"); // fourth place
      displayList(runnerList);
   } // end testList

   public static void displayList(ListInterface<String> list)
   {
      int numberOfEntries = list.getLength();
      System.out.println("The list contains " + numberOfEntries +
                         " entries, as follows:");

      for (int position = 1; position <= numberOfEntries; position++)
         System.out.println(list.getEntry(position) +
                            " is entry " + position);

      System.out.println();
   } // end displayList
} // end ListClient
```

Output

```
The list contains 4 entries, as follows:
16 is entry 1
 4 is entry 2
33 is entry 3
27 is entry 4
```

The data type of `displayList`'s input parameter `list` is `ListInterface<String>`. Thus, the argument of the method must be an object that satisfies both of the following conditions:

- The object's class must implement `ListInterface`.
- The object must be instantiated as a list of strings.

Although the method works for any implementation of the ADT list, it works only for lists of strings. You could remove the latter restriction by revising the header of the method as follows:

```
public static <T> void displayList(ListInterface<T> list)
```

Now the list passed to the method can contain objects of any one class.

Note: A reminder

Notice that the data type of `runnerList` is `ListInterface<String>`. This declaration obliges `runnerList` to call only methods in the interface and to add only strings to the list. If the data type was `AList<String>` instead, `runnerList` would be able to call any public methods in `AList` even if they were not declared in `ListInterface`.

SELF-TEST

Question 3 In the previous example, what changes to `testList` are necessary to represent the runner's numbers as `Integer` objects instead of strings? Use Java's auto-boxing feature, as described in Segment A.99 of Appendix A.

12.9 **Example.** A professor wants an alphabetical list of the names of the students who arrive for class today. As each student enters the room, the professor adds the student's name to a list. It is up to the professor to place each name into its correct position in the list so that the names will be in alphabetical order. The ADT list does *not* choose the order of its entries.

The following Java statements place the names Amy, Ellen, Bob, Drew, Aaron, and Carol in an alphabetical list. The comment at the end of each statement shows the list after the statement executes.

```
// make an alphabetical list of names as students enter a room
ListInterface<String> alphaList = new AList<String>();

alphaList.add(1, "Amy");     // Amy
alphaList.add(2, "Ellen");   // Amy Ellen
alphaList.add(2, "Bob");     // Amy Bob Ellen
alphaList.add(3, "Drew");    // Amy Bob Drew Ellen
alphaList.add(1, "Aaron");   // Aaron Amy Bob Drew Ellen
alphaList.add(4, "Carol");   // Aaron Amy Bob Carol Drew Ellen
```

After initially adding Amy to the beginning of the list and Ellen to the end of the list (at position 2), the professor inserts

- Bob between Amy and Ellen at position 2
- Drew between Bob and Ellen at position 3
- Aaron before Amy at position 1
- Carol between Bob and Drew at position 4

Recall that this technique of inserting each name into a collection of alphabetized names is called an insertion sort.

If we now remove the entry at position 4—Carol—by writing

```
alphaList.remove(4);
```

Drew and Ellen will then be at positions 4 and 5, respectively. Thus, `alphaList.getEntry(4)` would return a reference to Drew.

Finally, suppose that we want to replace a name in this list. We cannot replace a name with just any name and expect that the list will remain in alphabetical order. Replacing Bob with Ben by writing

```
alphaList.replace(3, "Ben");
```

would maintain alphabetical order, but replacing Bob with Nancy would not. The list's alphabetical order resulted from our original decisions about where to add names to the list. The order did not come about automatically as a result of list operations. That is, the client, not the list, maintained the order. We could, however, design an ADT that maintains its data in alphabetical order. You will see an example of such an ADT in Chapter 16.

Question 4 Suppose that `alphaList` contains a list of the four names Amy, Ellen, Bob, and Drew as strings. Write Java statements that swap Ellen and Bob and that then swap Ellen and Drew so that the list will be in alphabetical order.

12.10 **Example.** Let's look at a list of objects that are not strings. Suppose that we have the class `Name` from Appendix B that represents a person's first and last names. The following statements indicate how we could make a list of the names Amy Smith, Tina Drexel, and Robert Jones:

```
// make a list of names as you think of them
ListInterface<Name> nameList = new AList<Name>();
Name amy = new Name("Amy", "Smith");
nameList.add(amy);
nameList.add(new Name("Tina", "Drexel"));
nameList.add(new Name("Robert", "Jones"));
```

Now let's retrieve the name that is second in the list, Tina Drexel:

```
Name secondName = nameList.getEntry(2);
```

The definition of `getEntry` declares its return type as `T`, the generic type of the entries in the list. This type for `nameList` is `Name`, so `getEntry` returns a `Name` object.

Question 5 Suppose that the return type of getEntry was Object instead of a generic type. Would this change affect how you use the method? In particular, would the statement in the previous example that retrieved the second name in nameList be correct? Why?

12.11 **Example.** Let's talk a bit more about the previous example. The variable secondName is a reference to the second object in the list. Using this reference, we can modify the object. For example, we could change its last name by writing

```
secondName.setLast("Doe");
```

If the class Name did not have set methods like setLast, we would be unable to modify the objects in this list. For instance, if we had a list of strings, we would not be able to alter one of the strings in this way. The class String has no set methods, so once we create a String object, we cannot alter it. We could, however, replace an entire object in the list—regardless of its type—by using the ADT list operation replace.

Recall that Chapter 10 mentioned mutable and immutable objects. Since the class Name has set methods, its objects are mutable. The class String, on the other hand, does not define set methods, so its objects are immutable.

Java Class Library: The Interface List

12.12 The standard package java.util contains an interface List for an ADT list that is similar to the list that our interface describes. One difference between a list in the Java Class Library and our ADT list is the numbering of a list's entries. A list in the Java Class Library uses the same numbering scheme as a Java array: The first entry is at position, or index, 0. In contrast, we begin our list at position 1. The interface List also declares more methods than our interface does. You'll see a few of those additional methods in Chapter 15.

The following method headers from the interface List are for a selection of methods that are similar to the ones you have seen in this chapter. We have highlighted where they differ from our methods. Once again, T is the generic type of the entries in the list.

```
public boolean add(T newEntry)
public void add(int index, T newEntry)
public T remove(int index)
public void clear()
public T set(int index, T anEntry) // like replace
public T get(int index)            // like getEntry
public boolean contains(Object anEntry)
public int size()                  // like getLength
public boolean isEmpty()
```

The first add method, which adds an entry to the end of a list, returns a boolean value, whereas our analogous method is a void method. The second add method is a void method. It throws an exception if index is out of range, instead of returning the boolean value false, as our add method does. The methods remove and get also throw an exception if index is out of range. Our analogous methods return null instead. The method set is like our replace method, but it returns a reference to the entry that was replaced in the list instead of returning a boolean value. It also throws an exception if index is out of range. The data type of contains' parameter is Object instead of a generic type. In practice, this difference has little consequence. Lastly, the method get is like our getEntry, and size is like our getLength.

You can learn more about the interface List in the online documentation for the Java Class Library.

Java Class Library: The Class ArrayList

12.13 The Java Class Library contains an implementation of the ADT list that uses a resizable array. This class, called ArrayList, implements the interface java.util.List, that we just discussed. The class also is in the package java.util.

Two of the constructors available for `ArrayList` are as follows:

```
public ArrayList()
```

Creates an empty list with an initial capacity of 10. The list increases its capacity as needed by an unspecified amount.

```
public ArrayList(int initialCapacity)
```

Creates an empty list with the specified initial capacity. The list increases its capacity as needed by an unspecified amount.

12.14 The class `java.util.Vector`, which we described in Chapter 6, is similar to `ArrayList`. Both classes implement the same interfaces: `java.util.List`, as well as others. Even so, `Vector` contains a few more methods than `ArrayList`. We will ignore these extra methods, as they mostly are redundant.

You can use either `ArrayList` or `Vector` as an implementation of the interface `List`. For example, you could write the following statement to define a list of strings:

```
List<String> myList = new ArrayList<String>();
```

Now `myList` has only the methods declared in the interface `List`.

Our `ListInterface` is somewhat simpler than Java's `List`. It has fewer methods, and they do not throw an exception when given an illegal position. We can retain the simplicity of our interface and still make use of an existing class by using either `ArrayList` or `Vector` in an implementation of `ListInterface`. We will show you how in the next chapter. Although we will use `Vector`, you could use `ArrayList` just as easily.

Chapter Summary

- A list is an object whose data consists of ordered entries. Each entry is identified by its position within the list.
- The ADT list specifies operations that add an entry either to the end of a list or at a given position within the list. Among its other operations are those that retrieve, remove, or replace the entry at a given position.
- A client manipulates or accesses a list's entries by using only the operations defined for the ADT list.
- The entries in a bag are unordered, whereas the entries in a list, a stack, a queue, a deque, or a priority queue do have an order. A list, unlike these other collections, enables you to add, retrieve, remove, or replace an entry at any given position.

Exercises

1. If `myList` is an empty list of strings, what does it contain after the following statements execute?

```
myList.add("A");
myList.add("B");
myList.add("C");
myList.add("D");
myList.add(1, "one");
myList.add(1, "two");
myList.add(1, "three");
myList.add(1, "four");
```

2. If `myList` is an empty list of strings, what does it contain after the following statements execute?

```
myList.add("alpha");
myList.add(1, "beta");
myList.add("gamma");
myList.add(2, "delta");
myList.add(4, "alpha");
myList.remove(2);
myList.remove(2);
myList.replace(3, "delta");
```

3. Revise the method `displayList` in Listing 12-2 so that it uses the list method `toArray` instead of methods `getLength` and `getEntry`.

4. Suppose that you want an operation for the ADT list that returns the position of a given object in the list. The header of the method could be as follows:

```
public int getPosition(T anObject)
```

Write comments that specify this method.

5. Suppose that you want an operation for the ADT list that removes the first occurrence of a given object from the list. The header of the method could be as follows:

```
public boolean remove(T anObject)
```

Write comments that specify this method.

6. Suppose that you want an operation for the ADT list that moves the first item in the list to the end of the list. The header of the method could be as follows:

```
public void moveToEnd()
```

Write comments that specify this method.

7. Write Java statements at the client level that return the position of a given object in the list `myList`. Assume that the object is in the list.

8. Suppose that the ADT list did not have a method `replace`. Write Java statements at the client level that replace an object in the list `nameList`. The object's position in the list is `givenPosition` and the replacement object is `newObject`.

9. Suppose that the ADT list did not have a method `contains`. Suppose further that `nameList` is a list of `Name` objects, where `Name` is as defined in Appendix B. Write Java statements at the client level that see whether the `Name` object `myName` is in the list `nameList`.

10. Suppose that you have a list that is created by the following statement:

```
ListInterface<Student> studentList = new AList<Student>();
```

Imagine that someone has added to the list several instances of the class `Student` that Appendix C defined in Segment C.2.

a. Write Java statements that display the last names of the students in the list in the same order in which the students appear in the list. Do not alter the list.
b. Write Java statements that interchange the first and last students in the list.

11. Suppose that you have a list that is created by the following statement:

```
ListInterface<Double> quizScores = new AList<Double>();
```

Imagine that someone has added to this list the quiz scores received by a student throughout a course. The professor would like to know the average of these quiz scores, ignoring the lowest score.

a. Write Java statements at the client level that will find and remove the lowest score in the list.
b. Write Java statements at the client level that will compute the average of the scores remaining in the list.

12. Consider a class `Coin` that represents a coin. The class has methods such as `getValue`, `toss`, and `isHeads`. The method `getValue` returns the value, or denomination, of a coin. The method `toss` simulates a coin toss in which the coin lands either heads up or tails up. The method `isHeads` returns true if a coin is heads up.

Suppose that `coinList` is an ADT list of coins that have randomly selected denominations. Toss each of these coins. If the result of a coin toss is heads, move the coin to a second list called `headsList`; if it is tails, leave the coin in the original list. When you are finished tossing coins, compute the total value of the coins that came up heads. Assume that the list `headsList` has been created for you and is empty initially.

Projects

1. Define a class of bags that implements the interface `BagInterface`, as defined in Listing 1-1 in Chapter 1. Use an instance of the class `ArrayList` to contain a bag's entries. Then write a program that adequately demonstrates your new class. Note that you might have to handle exceptions thrown by methods of `ArrayList`.

2. Repeat Project 1, but instead define a class of stacks that implements the interface `StackInterface`, as defined in Listing 5-1 in Chapter 5.

3. Repeat Project 1, but instead define a class of queues that implements the interface `QueueInterface`, as defined in Listing 10-1 in Chapter 10.

4. Repeat Project 1, but instead define a class of deques that implements the interface `DequeInterface`, as defined in Listing 10-4 in Chapter 10.

5. Repeat Project 1, but instead define a class of sets that implements the interface `SetInterface`. Recall from Project 1 in Chapter 1 that a set is a bag whose entries are distinct. Define `SetInterface` by extending `BagInterface`, as defined in Listing 1-1 in Chapter 1.

6. Santa Claus allegedly keeps lists of those who are naughty and those who are nice. On the naughty list are the names of those who will get coal in their stockings. On the nice list are those who will receive gifts. Each object in this list contains a name (an instance of `Name`, as defined in Appendix B) and a list of that person's gifts (an instance of an ADT list).

Design an ADT for the objects in the nice list. Specify each ADT operation by stating its purpose, by describing its parameters, and by writing preconditions, postconditions, and a pseudocode version of its header. Then write a Java interface for the ADT that includes `javadoc`-style comments.

7. A recipe contains a title, a list of ingredients, and a list of directions. An entry in the list of ingredients contains an amount, a unit, and a description. For example, an object that represents *2 cups of flour* could be an entry in this list. An entry in the list of directions is a string.

Design an ADT that represents any entry in a list of ingredients, assuming that you have the class `MixedNumber`, which was described in the online Project 3 of Appendix D. Then design another ADT to represent any recipe. Specify each ADT operation by stating its purpose, by describing its parameters, and by writing preconditions, postconditions, and a pseudocode version of its header. Then write a Java interface for the ADT that includes `javadoc`-style comments.

8. Define and test a class that implements the interface for the ADT recipe that the previous project describes. Use an instance of `ArrayList` for each list that you need. Using a text editor, create a text file of recipes for your demonstration program to read.

9. Repeat Project 7 of Chapter 4, but use an instance of `ArrayList` instead of a bag.

10. As early as the tenth century, mathematicians studied the following triangular pattern of integers, now known as *Pascal's Triangle*:

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
         . . .
```

Despite the early advent of this pattern, it was named after the 17th-century mathematician Blaise Pascal.

Staggering the entries, as we have here, is traditional. Each row begins and ends with 1. Each interior entry is the sum of the two entries above it. For example, in the last row given here, 4 is the sum of 1 and 3, 6 is the sum of 3 and 3, and 4 is the sum of 3 and 1.

If we number both the rows and the entries in each row beginning with 0, the entry in position *k* of row *n* is often denoted as *C*(*n*, *k*). For example, the 6 in the last row is *C*(4, 2). Given *n* items, *C*(*n*, *k*) turns out to be the number of ways that you can select *k* of the *n* items. Thus, *C*(4, 2), which is 6, is the number of ways that you can select two of four given items. So if A, B, C, and D are the four items, here are the six possible choices:

A B, A C, A D, B C, B D, C D

Note that the order of the items in each pair is irrelevant. For instance, the choice A B is the same as the choice B A.

Design and implement the class `PascalTriangle`. Represent each row in a triangle as a list and the entire triangle as a list of these lists. Use the class `ArrayList` for these lists. Give your class constructors and at least the method `getChoices(n, k)`, which returns the integer value of *C*(*n*, *k*).

Answers to Self-Test Questions

1.

```
myList.add(c)
myList.add(1, a)
myList.add(2, b)
myList.add(4, d)
```

2.

```
seven = myList.remove(7)
three = myList.remove(3)
myList.add(3, seven)
myList.add(7, three)
```

Another solution:

```
seven = myList.getEntry(7)
three = myList.getEntry(3)
myList.replace(3, seven)
myList.replace(7, three)
```

3.

```
ListInterface<Integer> rList = new AList<Integer>();
rList.add(16);
rList.add(4);
rList.add(33);
rList.add(27);
rList.displayList();
```

4.

```
bob = alphaList.remove(3);
ellen = alphaList.remove(2);
alphaList.add(2, bob);
alphaList.add(3, ellen);
drew = alphaList.remove(4);
ellen = alphaList.remove(3);
alphaList.add(3, drew);
alphaList.add(4, ellen);
```

Another solution uses `getEntry` and `replace`, much like the second solution to Question 2.

5. The change to `getEntry` would affect its use by requiring a cast to the type of entry retrieved. Thus, you would write

```
Name secondName = (Name)nameList.getEntry(2);
```

A similar statement without the type cast would be incorrect. A reference to `Object` cannot be assigned to a `Name` variable without a cast.

Chapter 13

List Implementations That Use Arrays

Contents

Prerequisites

Appendix	D	Designing Classes
Chapter	2	Bag Implementations That Use Arrays
Chapter	4	The Efficiency of Algorithms
Chapter	12	Lists

Objectives

After studying this chapter, you should be able to

- Implement the ADT list by using either an array that you can resize or an instance of `Vector`
- Discuss the advantages and disadvantages of the two implementations presented

You have seen several examples of how to use the ADT list in a program. This chapter presents three different ways that you can implement a list in Java. We begin by using an array to represent the entries in a list. As in earlier implementations of other ADTs, when you use all of the space in an array, you can move the data to a larger array. Alternately, you can use an instance of the Java class `Vector` to represent the list entries. The result is like using an array that can expand, since the underlying implementation of `Vector` uses such an array. But this list implementation is simpler to write than our first one, because `Vector` does the work for you. Our final implementation uses a chain of linked nodes. Because you can insert and remove entries at any position within a list, the array-based and linked implementations are a bit more challenging than the previous ADT implementations we have encountered so far.

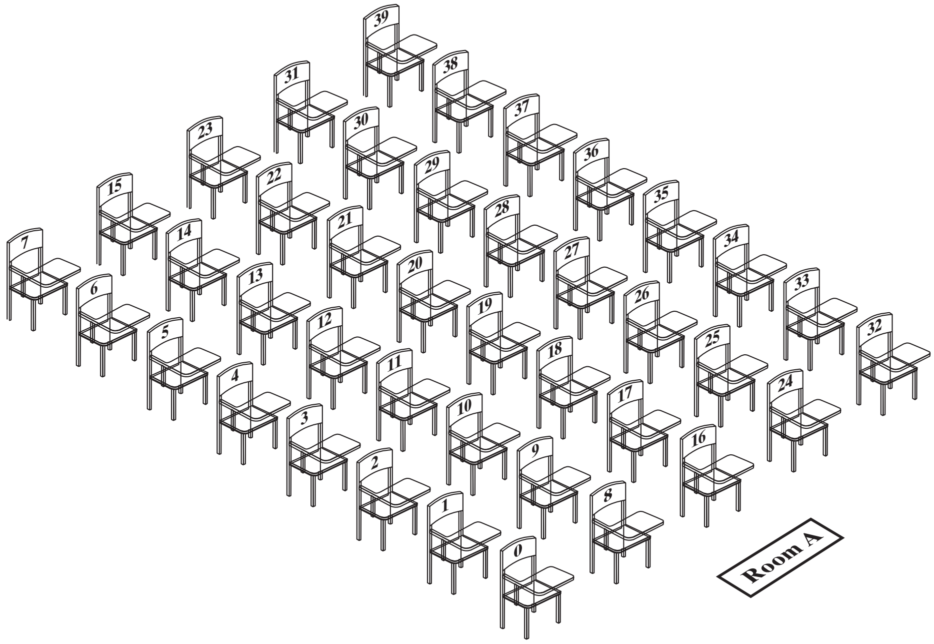
Using an Array to Implement the ADT List

We begin with the classroom analogy used in Chapter 2 to describe how an array can represent a bag, but this time we show how to represent a list. In doing so, we show how the `add` and `remove` methods would work. Subsequently, we present a corresponding Java implementation for the list.

An Analogy

13.1 Let's recall the classroom, room A, that we used in Chapter 2 and picture it again in Figure 13-1. The desks in the room are numbered sequentially, beginning with zero. An array is like this classroom, and each desk is like one location within an array. Again, we will treat the desks as a one-dimensional array and ignore that the desks are arranged in rows, as in a typical classroom.

FIGURE 13-1 A classroom that contains desks in fixed positions



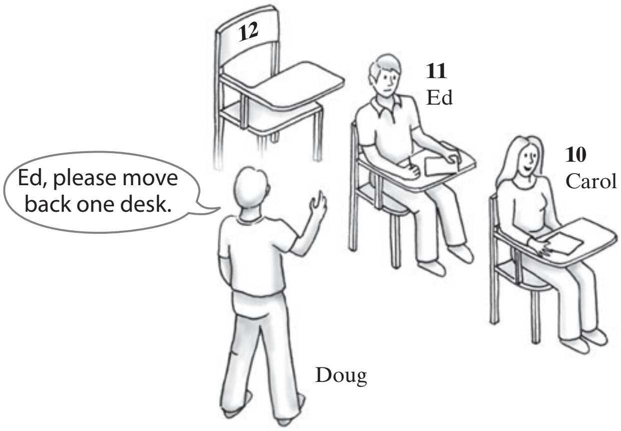
Suppose that the first student who arrives at the classroom sits at desk 0; the second student sits at desk 1, and so on. Eventually, 30 students occupy the desks numbered 0 through 29. They are organized by arrival time. The instructor knows immediately who arrived first (that person is at desk 0) and who arrived last (that person is at desk 29). Additionally, the instructor could ask for the name of the student seated at any particular desk, just as a programmer can access any array element directly. Thus, the instructor could ask for each student's name in order of arrival by polling desks 0 through 29, or in reverse order by polling desks 29 through 0.

Instead of arranging the students in room A by arrival time, suppose that we arrange them alphabetically by name. Doing so requires a sorting algorithm, such as the ones that Chapters 8 and 9 discuss. That is, the ADT list does not choose the order of its entries; the client must do so.

13.2 **Adding a new student.** Imagine that we have already arranged the students in room A alphabetically by name. Suppose that a new student wants to join the students already in the room. Recall that the 30 occupied desks are numbered sequentially from 0 to 29. Since 40 desks are in the room, the desk numbered 30 is available. When the students were arranged by arrival time, we would simply have assigned the new student to desk 30. Since the students are now arranged alphabetically by name, we must do more work.

Suppose that the new student belongs between the two students who occupy desks 10 and 11. That is, the new student's name is alphabetically between the names of the two students who occupy desks 10 and 11. Since the desks' positions are fixed, the new student must occupy desk 11. Before the new student can be seated, the student currently at desk 11 needs to move to desk 12, as Figure 13-2 illustrates. This requirement, however, causes a chain reaction: The student currently at desk 12 needs to move to desk 13, and so on. That is, each student seated in desks 11 through 29 must move to the next higher-numbered desk. If only one student moves at a time, the student in desk 29 must move to desk 30 before the student in desk 28 can move to desk 29, and so on. As you can see, adding a new student requires moving several other students. However, we do not disturb the students seated in the desks that are before the new student's desk—desks 0 through 10 in our example.

FIGURE 13-2 Seating a new student between two existing students: At least one other student must move



Question 1 In the previous example, under what circumstance could you add a new student alphabetically by name without moving any other student?

13.3 **Removing a student.** Now imagine that the student in desk 5 of room A drops the course. The desk stays in its fixed location within the room. If we still want students to sit in consecutively numbered desks, several students will need to move. In fact, each student in desks 6 through 30 must move to the next lower-numbered desk, beginning with the student in desk 6. That is, if only one student moves at a time, the student in desk 6 must move to desk 5 before the student in desk 7 moves to desk 6, and so on.

Question 2 What is an advantage of moving students as just described so that the vacated desk does not remain vacant?

Question 3 What is an advantage of leaving the vacated desk vacant?

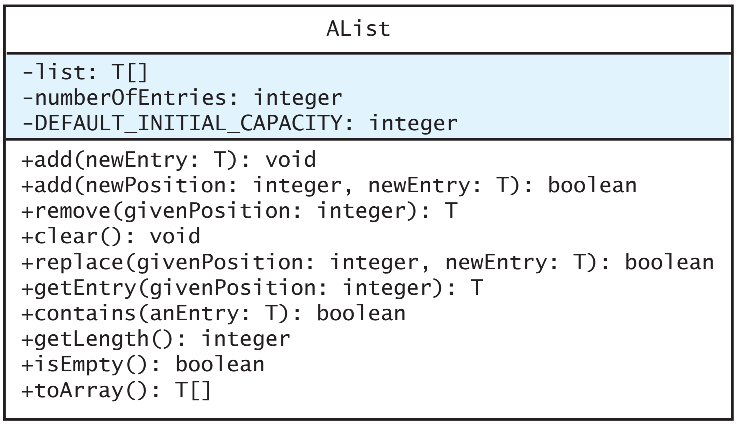
The Java Implementation

13.4 The Java array-based implementation for the ADT list incorporates some of the ideas that our classroom example illustrates. The implementation is a class `AList`[1] that implements the interface `ListInterface` that you saw in Chapter 12. Each public method within the class corresponds to an ADT list operation. The private data fields are

- An array of objects
- An integer that counts the number of entries in the list
- An integer constant that defines the size of the array, which is the maximum length of the list

We can describe the class using the UML notation shown in Figure 13-3.

FIGURE 13-3 UML notation for the class `AList`



```
AList
-list: T[]
-numberOfEntries: integer
-DEFAULT_INITIAL_CAPACITY: integer
+add(newEntry: T): void
+add(newPosition: integer, newEntry: T): boolean
+remove(givenPosition: integer): T
+clear(): void
+replace(givenPosition: integer, newEntry: T): boolean
+getEntry(givenPosition: integer): T
+contains(anEntry: T): boolean
+getLength(): integer
+isEmpty(): boolean
+toArray(): T[]
```

13.5 The class `AList` has the form shown in Listing 13-1. The generic type `T` represents the data type of the entries in the list. Notice the overall organization of the class, the private data, and the constructors. We use the class `Object` when allocating the array, but must cast it to an array whose entries have the generic type `T`. Also, notice the implementations of the first `add` method and the method `toArray`. These two methods are our core methods, as we will describe next. We will provide implementations for the other methods shortly.

VideoNote
The class `AList`

[1] Ordinarily we would name this class `ArrayList`. But as you saw in the previous chapter, Java already provides a class with that name. Although we certainly could have named our class `ArrayList` as well, we chose a different name to avoid confusion.

LISTING 13-1 The class `AList`

```
import java.util.Arrays;
/**
    A class that implements a list of objects by using an array.
    The list is never full.
    @author Frank M. Carrano
*/
public class AList<T> implements ListInterface<T>
{
   private T[] list;   // array of list entries
   private int numberOfEntries;
   private static final int DEFAULT_INITIAL_CAPACITY = 25;

   public AList()
   {
      this(DEFAULT_INITIAL_CAPACITY); // call next constructor
   } // end default constructor

   public AList(int initialCapacity)
   {
      numberOfEntries = 0;
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempList = (T[])new Object[initialCapacity];
      list = tempList;
   } // end constructor

   public void add(T newEntry)
   {
      ensureCapacity();
      list[numberOfEntries] = newEntry;
      numberOfEntries++;
   } // end add

   public boolean add(int newPosition, T newEntry)
   { < Implementation deferred >
   } // end add

   public T remove(int givenPosition)
   { < Implementation deferred >
   } // end remove

   public void clear()
   { < Implementation deferred >
   } // end clear

   public boolean replace(int givenPosition, T newEntry)
   { < Implementation deferred >
   } // end replace

   public T getEntry(int givenPosition)
   { < Implementation deferred >
   } // end getEntry

   public boolean contains(T anEntry)
   { < Implementation deferred >
   } // end contains

   public int getLength()
   {
      return numberOfEntries;
   } // end getLength

   public boolean isEmpty()
   {
      return numberOfEntries == 0; // or getLength() == 0
   } // end isEmpty

   public T[] toArray()
   {
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] result = (T[])new Object[numberOfEntries];

      for (int index = 0; index < numberOfEntries; index++)
      {
         result[index] = list[index];
      } // end for

      return result;
   } // end toArray

   // Doubles the size of the array list if it is full.
   private void ensureCapacity()
   {
      if (numberOfEntries == list.length)
         list = Arrays.copyOf(list, 2 * list.length);
   } // end ensureCapacity

   < This class will define two more private methods that will be discussed later. >
} // end AList
```

13.6 **The core methods.** As we just mentioned, we have chosen to implement the first `add` method and the method `toArray` before the others, as they are central, or core, to our class. Adding a new entry to the end of the list is easy; we simply add the entry to the array immediately after its last occupied location. Of course, adding a new entry is possible only if the array has available space. We call the private method `ensureCapacity` to resize the array if necessary. Thus, the first `add` method has an implementation much like the one you saw in Segment 2.37 of Chapter 2 for the class ResizableArrayBag. Moreover, the definition of ensureCapacity, which appears at the end of Listing 13-1, as well as the definition of the method toArray are analogous to the corresponding methods in ResizableArrayBag.

13.7 **Testing the partial implementation.** You should now write a main method to test what you have completed at this point. Testing a class should begin well before its implementation is complete. To avoid syntax errors in the incomplete class in Listing 13-1, make the incomplete methods stubs by adding to each one a return statement that returns a dummy value. For example, methods that return a boolean value could return true. Methods that return an object could return null. You can provide the actual definitions of methods, such as getLength and isEmpty, as they are just as simple as their stubs would be.

As you define more methods, test them by adding statements to main. As Appendix B notes, you can include your method main in the definition of AList for future use and reference.

13.8 **The second add method.** Adding a new entry at an arbitrary position within the list is like adding a student to room A in our example in Segment 13.2. Although that example positions students alphabetically by their names, remember that the list's client—not the list itself—determines the desired position of each entry. Thus, if that position is before the end of the list, we need to shift existing entries to vacate the desired location so that it can accommodate the new entry. If the addition is to the end of the list, no such shift is necessary. In either case, space must be available in the array to accommodate a new entry.

The following implementation of add uses a private method makeRoom to handle the details of moving data within the array. Remember that we can add to the list at positions that range from 1 to the length of the list plus 1. According to the method's specifications given in Segment 12.7 of the previous chapter, we must return false if the given position is invalid.

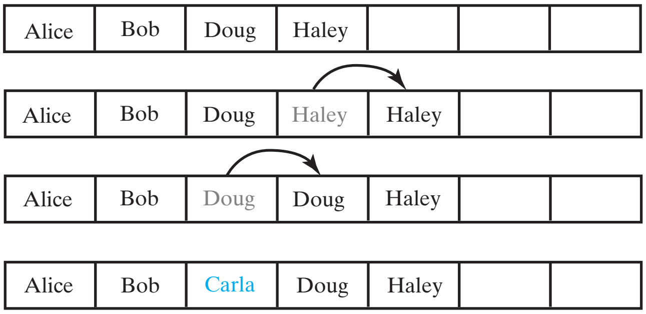
```
public boolean add(int newPosition, T newEntry)
{
   boolean isSuccessful = true;

   if ((newPosition >= 1) && (newPosition <= numberOfEntries + 1))
   {
      ensureCapacity();
      makeRoom(newPosition);
      list[newPosition - 1] = newEntry;
      numberOfEntries++;
   }
   else
      isSuccessful = false;

   return isSuccessful;
} // end add
```

13.9 **The private method makeRoom.** Now we must implement the private method makeRoom. Typically, the method shifts list entries toward the end of the array, beginning with the last entry, as Figure 13-4 illustrates. However, if newPosition is numberOfEntries + 1, the addition is at the end of the list, so no shift is necessary. In this case, makeRoom does nothing, since its for statement exits immediately.

```
// Makes room for a new entry at newPosition.
// Precondition: 1 <= newPosition <= numberOfEntries+1;
//               numberOfEntries is list's length before addition.
private void makeRoom(int newPosition)
{
   assert (newPosition >= 1) && (newPosition <= numberOfEntries + 1);
   int newIndex = newPosition - 1;
   int lastIndex = numberOfEntries - 1;

   // move each entry to next higher index, starting at end of
   // list and continuing until the entry at newIndex is moved
   for (int index = lastIndex; index >= newIndex; index--)
      list[index + 1] = list[index];
} // end makeRoom
```

Notice that the add method enforces the precondition of makeRoom. While testing makeRoom, however, we can enable the assertion to be sure.

FIGURE 13-4 Making room to insert Carla as the third entry in an array



Alice	Bob	Doug	Haley			
Alice	Bob	Doug	Haley	Haley		
Alice	Bob	Doug	Doug	Haley		
Alice	Bob	Carla	Doug	Haley		

Question 4 You could implement the first add method, which adds an entry to the end of the list, by invoking the second add method, as follows:

```
public void add(T newEntry)
{
   add(numberOfEntries + 1, newEntry);
} // end add
```

Discuss the pros and cons of this revised approach.

Question 5 Suppose that myList is a list that contains the five entries a b c d e.

a. What does myList contain after executing myList.add(5, w)?
b. Starting with the original five entries, what does myList contain after executing myList.add(6, w)?
c. Which of the operations in Parts *a* and *b* of this question require entries in the array to shift?

Question 6 If myList is a list of five entries, each of the following statements adds a new entry to the end of the list:

```
myList.add(newEntry);
myList.add(6, newEntry);
```

Which way requires fewer operations?

VideoNote
Completing the class `AList`

13.10 **The method `remove`.** Removing a list entry at an arbitrary position is like a student leaving room A in our example in Segment 13.3. We need to shift existing entries to avoid a gap in the array, except when removing the list's last entry. The following implementation uses a private method `removeGap` to handle the details of moving data within the array. Like the method `add`, `remove` is responsible for checking the validity of the given position. Note how this check also ensures that the list is not empty.

```
public T remove(int givenPosition)
{
   T result = null; // return value

   if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
   {
      assert !isEmpty();
      result = list[givenPosition - 1]; // get entry to be removed

      // move subsequent entries toward entry to be removed,
      // unless it is last in list
      if (givenPosition < numberOfEntries)
         removeGap(givenPosition);

      numberOfEntries--;
   } // end if

   return result; // return reference to removed entry, or
                  // null if either list is empty or givenPosition
                  // is invalid
} // end remove
```

SELF-TEST

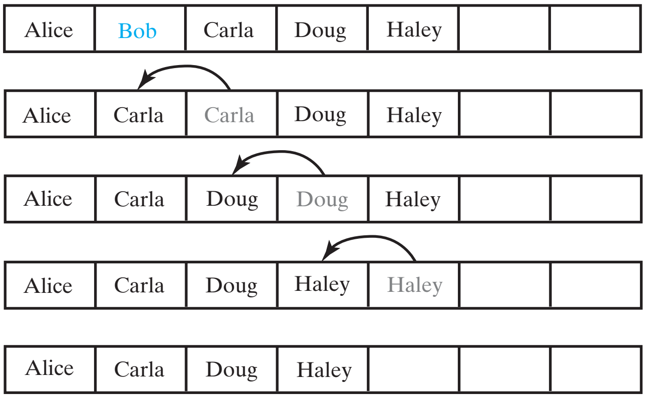
Question 7 Since the method `remove` does not explicitly check for an empty list, why is the assertion given in the method true?

Question 8 When a list is empty, how does `remove` return `null`?

13.11 **The private method `removeGap`.** The following private method `removeGap` shifts list entries within the array, as Figure 13-5 illustrates. Beginning with the entry after the one to be removed and continuing until the end of the list, `removeGap` moves each entry to its next lower position.

```
// Shifts entries that are beyond the entry to be removed to the
// next lower position.
// Precondition: 1 <= givenPosition < numberOfEntries;
//               numberOfEntries is list's length before removal.
private void removeGap(int givenPosition)
{
   assert (givenPosition >= 1) && (givenPosition < numberOfEntries);

   int removedIndex = givenPosition - 1;
   int lastIndex = numberOfEntries - 1;

   for (int index = removedIndex; index < lastIndex; index++)
      list[index] = list[index + 1];
} // end removeGap
```

FIGURE 13-5 Removing Bob by shifting array entries



Note that no shift is necessary if the deletion is at the end of the list. In that case, the last entry in the list is at position `numberOfEntries`, since the first entry is at position 1. If `givenPosition` equals `numberOfEntries`, the `for` statement in `removeGap` will exit immediately. Even so, `remove` will not call `removeGap` in this case. Notice that `removeGap`'s precondition requires `givenPosition` to be less than `numberOfEntries`, and the `remove` method enforces this precondition. You can enable the `assert` statement in `removeGap` to verify this enforcement.

This precondition of `removeGap` implies that the method should not be called if a list is empty. In fact, `remove` ensures that requirement is followed.

Question 9 Figure 13-5 shows Haley shifted toward the beginning of the array. Actually, the reference to Haley is copied, not moved, to its new location. Should we assign `null` to Haley's original location?

Question 10 The method `clear` could simply set the data field `numberOfEntries` to zero. Although the list methods would correctly behave as though the list was empty, the objects that were in the list would remain allocated. Suggest at least two ways that `clear` could deallocate these objects.

13.12 **The methods `replace` and `getEntry`.** Replacing a list entry and retrieving a list entry are two straightforward operations when an array is used to represent the entries. You simply replace or retrieve the object that is in the indicated array location. Like earlier methods, `replace` and `getEntry` are responsible for validating the given position. Like `remove`, these methods do not need an explicit test for an empty list to behave correctly. The `assert` statement is available to verify this claim during testing.

The following methods implement these two operations:

```
public boolean replace(int givenPosition, T newEntry)
{
   boolean isSuccessful = true;

   if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
   {
      assert !isEmpty();
      list[givenPosition - 1] = newEntry;
   }
```

```
      else
         isSuccessful = false;

      return isSuccessful;
   } // end replace

   public T getEntry(int givenPosition)
   {
      T result = null; // result to return

      if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
      {
         assert !isEmpty();
         result = list[givenPosition - 1];
      } // end if

      return result;
   } // end getEntry
```

13.13 **The method `contains`.** The method `getEntry` locates the entry at a given position by going directly to the appropriate array element. In contrast, the method `contains` is given an entry, not its position, and so must search the array for the entry. Beginning at index zero, the method examines each array entry until it either locates the desired one or reaches the end of the list without success. In the following implementation, we use a local boolean variable to terminate the loop when we find the desired entry:

```
   public boolean contains(T anEntry)
   {
      boolean found = false;

      for (int index = 0; !found && (index < numberOfEntries); index++)
      {
         if (anEntry.equals(list[index]))
           found = true;
      } // end for

      return found;
   } // end contains
```

This way of looking for a particular entry in an array is called a **sequential search**. Chapter 18 discusses this technique further and presents another algorithm that is generally faster.

The Efficiency of Using an Array to Implement the ADT List

Before we look at another implementation of the ADT list, let's examine the time complexity of some of the methods in our class `AList`.

13.14 **Adding to the end of a list.** Let's begin with the operation that adds a new entry to the end of a list. Listing 13-1 provided the following definition for this operation:

```
   public void add(T newEntry)
   {
      ensureCapacity();
      list[numberOfEntries] = newEntry;
      numberOfEntries++;
   } // end add
```

If the array of list entries is not full, each step in this method—ensuring the array's capacity, assigning a new entry to an array element, and incrementing the length—is an O(1) operation. By applying our knowledge of the material presented in Segments 4.16 and 4.17 of Chapter 4, we can show that this method is O(1), if the array is not resized. That is, we can add an entry to the end of a list independently of any other entries in the list.

As we have seen in previous chapters, resizing an array is an O(n) operation. Thus, the method `ensureCapacity` would require O(n) time if it encounters a full array. In this case, adding to the end of a list would be an O(n) operation. If we continued to add to the end of the list, the operations would be O(1) again.

13.15 **Adding to a list at a given position.** The ADT list has another method that adds a new entry to a list, but this one adds the entry at a position that the client specifies. The time-consuming portion of the method `add`, as given in Segment 13.8, is the following sequence of statements:

```
ensureCapacity();
makeRoom(newPosition);
list[newPosition - 1] = newEntry;
```

We already know that `ensureCapacity` is either O(n) or O(1), according to whether or not it resizes the array. Assigning the new entry to an array element is O(1). The difference between this method and the previous `add` method is that here we need to make room in the array for the new entry. This task is accomplished by the private method `makeRoom` shown in Segment 13.9.

After removing `makeRoom`'s `assert` statement and comments, we are left with the following code:

```
private void makeRoom(int newPosition)
{
   int newIndex = newPosition - 1;
   int lastIndex = numberOfEntries - 1;

   for (int index = lastIndex; index >= newIndex; index--)
      list[index + 1] = list[index];
} // end makeRoom
```

The method requires the most time when `newPosition` is 1, because it must shift all of the list entries. If the list contains n entries, the body of the loop is repeated n times in this case. Therefore, the method `makeRoom` is O(n) in the worst case. This observation implies that the method `add` is also O(n) in the worst case.

The best case occurs when `newPosition` is `numberOfEntries` + 1, that is, when we add to the end of the list. In this event, `makeRoom`'s loop exits immediately, so `makeRoom` is O(1) in the best case. This result is consistent with our findings in the previous segment for the first `add` method.

Note: Adding to the beginning of an array-based list is an O(n) operation. Adding to the end is O(1) if the underlying array is not resized; otherwise it is O(n). The time required to add at other positions depends on the position. As the position number increases, the time needed for an addition decreases.

SELF-TEST

Question 11 What is the Big Oh of the list method `remove` in the best case and the worst case?

Question 12 Repeat Question 11 for the list method `replace`.

Question 13 Repeat Question 11 for the list method `getEntry`.

Question 14 Repeat Question 11 for the list method `contains`.

Using a Vector to Implement the ADT List

In Chapter 6, we used a vector—that is, an instance of the class `java.util.Vector`—instead of an array to contain the entries in a stack. We can use a vector here to contain the entries in a list.

13.16 **Beginning the class.** We begin our new class as follows:

```
import java.util.Vector;
public class VectorList<T> implements ListInterface<T>
```

```
{
   private Vector<T> list; // entries in list
   . . .
```

The data field `list` is an instance of `Vector` instead of an array, as it was earlier in this chapter. Since `Vector` is defined in terms of a generic type, you provide a data type when declaring a vector. However, within our class `VectorList`, the data type of the list's objects is still unknown, so we use the generic type `T` in the declaration of the field `list`.

Since a vector keeps track of the number of entries it contains, a data field to count them is not required. Any time we want to know the number of entries in the vector, and hence the list, we can write `list.size()`.

13.17 **The constructors.** The constructors for our class create an instance of `Vector` by invoking `Vector`'s constructors. Our default constructor simply invokes `Vector`'s default constructor with the generic type `T`:

```
public VectorList()
{
   list = new Vector<T>();
} // end default constructor
```

`Vector`'s default constructor creates a vector that can hold 10 entries. This vector will double in size after it becomes full.

Our second constructor enables the client to specify the initial capacity of the list. It invokes a corresponding constructor of `Vector`:

```
public VectorList(int initialSize)
{
   list = new Vector<T>(initialSize);
} // end constructor
```

Here, `Vector`'s constructor creates a vector that can hold `initialSize` entries. This vector also will double in size after it becomes full.

If we wanted a different initial capacity for our default list, we could define the default constructor, as follows, instead of as given previously:

```
public VectorList()
{
   this(DEFAULT_INITIAL_CAPACITY);
} // end default constructor
```

This version of the default constructor calls `VectorList`'s second constructor, passing it a constant `DEFAULT_INITIAL_CAPACITY`, which the class must define.

13.18 **The `add` methods.** To add to the end of a list, you use `Vector`'s `add` method. This method adds a given object to the end of a vector. If necessary, the vector increases its capacity to accommodate the new entry. Thus, our `add` method does not test whether the vector is full:

```
public void add(T newEntry)
{
   list.add(newEntry);
} // end add
```

To add an entry at a given position in the list, we use another of `Vector`'s `add` methods. This method throws an exception if `newPosition` is illegal, but we simply want to return false in this case. By checking the validity of `newPosition`, we can avoid the exception. Remember that `Vector` numbers its entries beginning with 0 instead of 1.

```
public boolean add(int newPosition, T newEntry)
{
   boolean isSuccessful = true;
```

```
      if ((newPosition >= 1) && (newPosition <= list.size() + 1))
         list.add(newPosition - 1, newEntry);
      else
         isSuccessful = false;

      return isSuccessful;
   } // end add
```

13.19 **The method `replace`.** The method `replace` has an implementation similar to the one you just saw. It uses the statement

```
   list.set(givenPosition - 1, newEntry);
```

to replace a designated entry in the list.

```
   public boolean replace(int givenPosition, T newEntry)
   {
      boolean isSuccessful = true;

      if ((givenPosition >= 1) && (givenPosition <= list.size()))
      {
         assert !isEmpty();
         list.set(givenPosition - 1, newEntry);
      }
      else
         isSuccessful = false;

      return isSuccessful;
   } // end replace
```

13.20 **The method `toArray`.** We call `Vector`'s method `toArray` within the definition of `toArray` for a list. The only worry is to take care of the unchecked cast of an array of type `Object[]` to an array of type `T[]`. Thus, the method has the following definition:

```
   public T[] toArray()
   {
      @SuppressWarnings(“unchecked”);
      T[] result = (T[])list.toArray();

      return result;
   } // end toArray
```

13.21 **The method `remove`.** Our method `remove` uses `Vector`'s method `remove`, which returns the object it removes from the vector. Like `add`, `Vector`'s `remove` throws an exception when given an illegal position. Our method, however, will return `null` in this case. Thus, our `remove` method has the following implementation:

```
   public T remove(int givenPosition)
   {
      T result = null; // return value

      if ((givenPosition >= 1) && (givenPosition <= list.size()))
      {
         assert !isEmpty();
         result = list.remove(givenPosition - 1);
      } // end if

      return result;
   } // end remove
```

13.22 **The method `getEntry`.** The method `getEntry` has an implementation similar to that of `remove`. It uses the expression

```
   list.get(givenPosition - 1);
```

to retrieve a particular entry from the list.

```
public T getEntry(int givenPosition)
{
   T result = null; // return value

   if ((givenPosition >= 1) && (givenPosition <= list.size()))
   {
      assert !isEmpty();
      result = list.get(givenPosition - 1);
   } // end if

   return result;
} // end getEntry
```

13.23 **Remaining methods.** The method `clear` uses the statement

```
list.clear();
```

to remove all entries from the vector and hence from the list. Likewise, the implementations of the methods `contains`, `getLength`, and `isEmpty` are simple and left as exercises.

Note: The methods in our class `VectorList` function similarly to methods in Java's class `Vector`, but their specifications differ. In implementing `ListInterface`, `VectorList` simply invokes methods of the class `Vector`. `VectorList` is an example of an adapter class, which we described in Segment C.3 of Appendix C.

Note: Since Java's class `Vector` uses an array in its implementation, the two implementations of the ADT list that we have covered so far are array based. Both implementations use a resizable array to contain the list's entries, and so a list can grow in size as needed.

Note: Writing `VectorList` is certainly easier than writing the array-based implementation that this chapter describes. Since the methods of `VectorList` invoke the methods of `Vector`, they can require more execution time than those of `AList`. However, this time increase typically is insignificant.

Note: When you use an array or a vector to implement the ADT list,

- Retrieving the entry at a given position is fast
- Adding an entry at the end of a list is fast
- Adding or removing an entry that is between other entries requires shifting them within the array
- Increasing the size of the array or vector requires copying entries

Chapter Summary

- The two implementations of the ADT list in this chapter use an array to store the items in a list.
- Using an array results in a straightforward implementation of the list, but it is somewhat more involved than the implementations of either the ADT bag or the ADT stack.
- An array provides direct access to any of its elements, so a method such as `getEntry` has a simple, efficient implementation.

- Adding an entry to or removing an entry from an array-based list typically requires that other entries shift by one position within the array. This data movement degrades the time efficiency of these operations, particularly when the list is long and the position of the addition or removal is near the beginning of the list.
- Expanding the size of an array adds to the time required by the affected `add` method, since doing so requires copying the contents of the array to a larger array.
- Using an instance of `java.util.Vector` has the same advantages and disadvantages as using a resizable array but results in an implementation that is much easier to write.

Exercises

1. Add a constructor to each of the classes `AList` and `VectorList` that creates a list from a given array of objects.

2. Suppose that you want an operation for the ADT list that returns the position of a given object in the list. The header of the method could be as follows:

```
public int getPosition(T anObject)
```

where `T` is the generic type of the objects in the list. Write an implemention of this method for each of the two classes described in this chapter.

3. Suppose that you want an operation for the ADT list that removes the first occurrence of a given object from the list. The header of the method could be as follows:

```
public boolean remove(T anObject)
```

where `T` is the generic type of the objects in the list. The method returns true if the list contained `anObject` and that object was removed. Write an implementation of this method for each of the two classes described in this chapter.

4. Suppose that you want an operation for the ADT list that moves the first item in the list to the end of the list. The header of the method could be as follows:

```
public void moveToEnd()
```

Write an implemention of this method for each of the two classes described in this chapter.

5. Exercise 8 in the previous chapter asked you to write statements at the client level that replace an object in a given list. Write a method at the client level that performs such a replacement. How does your method compare with the method `replace` of the ADT list?

6. The method `replace` for the ADT list returns a boolean value. Implement a method `replace` that returns the replaced object. Do this for each of the two classes described in this chapter.

7. Suppose that a list contains `Comparable` objects. Implement the following methods for each of the two classes described in this chapter:

```
/** Returns the smallest object in the list. */
public T getMin()

/** Removes and returns the smallest object in the list. */
public T removeMin()
```

8. Implement an `equals` method for the ADT list that returns true when the entries in one list equal the entries in a second list. In particular, add this method to the classes `AList` and `VectorList`.

9. Repeat the previous exercise, but have the `equals` method call a private recursive method that detects equality.

10. Consider the method `contains` in the class `AList`. Revise the method's definition so that it calls a private recursive method that detects whether the list contains the given object.

11. Implement the methods `contains`, `getLength`, and `isEmpty` in the class `VectorList`.

12. The class `AList` has an array that can grow in size as objects are added to the list. Consider a similar class whose array also can shrink in size as objects are removed from the list. Accomplishing this task will require two new private methods.

The first new method checks whether we should reduce the size of the array:

```
private boolean isTooBig()
```

This method returns true if the number of entries in the list is less than half the size of the array and the size of the array is greater than 20.

The second new method creates a new array that is three quarters the size of the current array and then copies the objects in the list to the new array:

```
private void reduceArray()
```

Implement each of these two methods for our new class. Then use these methods in the definition of the method `remove`.

13. Consider the two private methods described in the previous question.

a. The method `isTooBig` requires the size of array to be greater than 20. What problem could occur if this requirement is dropped?

b. The method `reduceArray` is not analogous to the method `ensureCapacity` in that it does not reduce the size of the array by one half. What problem could occur if the size of the array is reduced by half instead of three quarters?

Projects

1. Write a program that thoroughly tests the class `AList`. Next, revise your program to perform the same tests on the class `VectorList`.

2. Define a class of bags by using an instance of the class `AList` to contain its entries. Then write a program that adequately demonstrates your new class.

3. Repeat Project 2, but define a class of stacks instead.

4. Repeat Project 2, but define a class of queues instead.

5. Repeat Project 2, but define a class of deques instead.

6. Repeat Project 2, but define a class of sets instead. Recall from Project 1 in Chapter 1 that a set is a bag whose entries are distinct.

7. Implement the interface `ListInterface` by using an array in which you ignore the first array location. Thus, you store the list's i^{th} entry in the array location at index `i`.

8. Using a fixed-size array to implement the ADT list limits the size of the list. Some applications can use a list that has a limited length. For example, the length of a list of airline passengers or a list of ticket holders to a movie should not exceed a known maximum.

Define an interface `FixedSizeListInterface` that is similar to `ListInterface`, but adds the method `isFull` and revises the specifications of other methods as necessary to accommodate a fixed-size list. Consider whether your new interface should extend `ListInterface`. Then define and demonstrate a class that implements `FixedSizeListInterface`.

9. Implement as the class `Shoe` the ADT shoe that Project 4 of Chapter 1 describes. *Hint*: To shuffle the shoe, use two private lists of cards, a source list and a shuffled list. Put all the available cards into the source list. Initially, this will be every card. Later, only those cards not held by a player will be available. Use the class `java.util.Random` to repeatedly generate a random position in the source list, remove the card at that position, and put it at the end of the shuffled list.

Write a program that adequately demonstrates the operation of the class `Shoe`.

10. Implement the ADT bid that Project 5 of Chapter 1 describes.

11. Implement the ADT for the objects in Santa's nice list that Project 6 of Chapter 12 describes. Then create some instances of your class and place them on the nice list.

12. Implement the ADT recipe that Project 7 of Chapter 12 describes.

13. Repeat Project 5 of Chapter 4, but use an instance of `AList` instead of a bag.

14. Revise `ListInterface`, as given in Listing 12-1 of the previous chapter, so that each of the methods `add`, `remove`, `replace`, and `getEntry` throws an exception if the position passed to it is out of range. Then revise the class `AList` so that it implements your revised interface.

15. The popular social network Facebook™ was founded by Mark Zuckerberg and his classmates at Harvard University in 2004. At the time, he was a sophomore studying computer science.

Design and implement an application that maintains the data for a simple social network. Each person in the network should have a profile that contains the person's name, optional image, current status, and a list of friends. Your application should allow a user to join the network, leave the network, create a profile, modify the profile, search for other profiles, and add friends.

Answers to Self-Test Questions

1. When the name comes after the name of the student in the last occupied desk; the new student then sits at the desk after the last one that is currently occupied.

2. The students remain in consecutively numbered positions. You do not have to keep track of the locations of the empty desks.

3. Time is saved by not moving the students.

4. Advantage: It is easier to implement this `add` method. Your code will more likely be correct if the other `add` method is correct.
Disadvantage: Invoking another method uses more execution time. Additionally, the second `add` method invokes `makeRoom` needlessly.

5. **a.** `a b c d w e`
b. `a b c d e w`
c. The operation in Part *a*

6. `myList.add(newEntry)`. The other `add` method validates the position 6 and then needlessly invokes `makeRoom`.

7. If the list is empty, `numberOfEntries` is zero, so `(givenPosition >= 1) && (givenPosition <= 0)` is always false. Thus, the only way that the previous expression can be true is if the list is not empty.

8. By the reasoning given in the previous answer, if the list is empty, the initial value of `result` is returned.

9. We could, but it is not necessary. After `remove` decrements `numberOfEntries`, `list[numberOfEntries]` is the array location that Haley originally occupied. So `list[numberOfEntries]` and `list[numberOfEntries - 1]` both reference Haley. Since we are not deallocating Haley, we do not have to set `list[numberOfEntries]` to `null`. We can simply ignore its contents.

10. The method `clear` could take one of the following actions to deallocate the objects currently in a list: (1) Set elements of the array `list` to `null`; (2) repeatedly remove the last entry by repeatedly calling `remove(numberOfEntries)`; (3) reallocate the array `list`.

11. O(1); O(n)

12. O(1); O(1)

13. O(1); O(1)

14. O(1) when the entry is found immediately in position 1. O(n) when the entry is found in the last position or is not found at all.

A List Implementation That Links Data

Chapter 14

Contents

Prerequisites

Chapter	3	A Bag Implementation That Links Data
Chapter	11	Queue, Deque, and Priority Queue Implementations (partial)
Chapter	12	Lists
Chapter	13	List Implementations That Use Arrays

Objectives

After studying this chapter, you should be able to

- Describe a linked organization of data
- Implement the `add` methods of the ADT list by using a linked chain of nodes
- Test a partially complete implementation of a class

Using an array to implement the ADT list has both advantages and disadvantages, as you saw in the previous chapter. An array can either have a fixed size or be moved to a larger array when it becomes full. Since a fixed-size array can lead to a full list, our class `AList` uses a resizable array to provide as much space as the list needs. This strategy, however, requires data to move each time the array expands. In addition, any array requires you to move data either to make room for a new entry or to close up a gap after a deletion.

This chapter describes a linked implementation of the list. Like our previous linked implementations, this one uses memory only as needed for a new entry and returns any unneeded memory to the system after an entry is removed. Moreover, it avoids moving data when adding or removing list entries. These features make this way of implementing a list an important alternative to array-based approaches.

Operations on a Chain of Linked Nodes

We used a chain of linked nodes in Chapter 2 to implement the ADT bag and in Chapter 6 to implement the ADT stack. In both of those cases, we added a node to and removed a node from the beginning of the chain. We added a node to the end of a chain in Chapter 11 for one implementation of the ADT queue. While those operations are still needed for a list, we also must be able to add a node between existing nodes and to delete a node from positions other than the beginning or end. Let's talk about these operations. We use the same class, `Node`, that appears in Listing 3-4 of Chapter 3.

Adding a Node at Various Positions

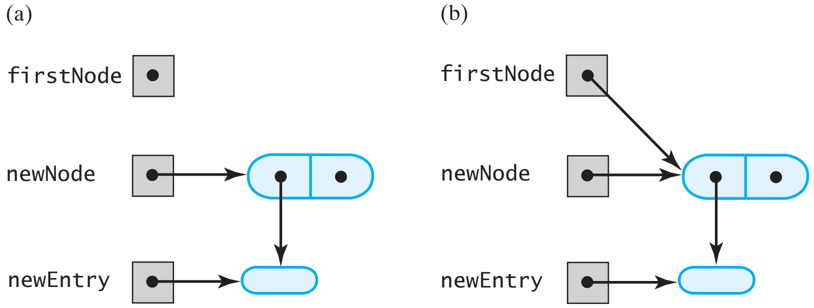
To add a node to a chain at a specified position, we must consider the following cases:

- Case 1: The chain is empty
- Case 2: Adding a node at the chain's beginning
- Case 3: Adding a node between adjacent nodes
- Case 4: Adding a node to the chain's end

As you will see, we will be able to combine these four cases into two. To that end, we examine each case, even though some of the detail will be familiar to you.

14.1 **Case 1: Adding a node to an empty chain.** Although we have added nodes to an empty chain before, let's recall the necessary steps. If `firstNode` is the head reference to the chain, it will contain `null` when the chain is empty. Figure 14-1a illustrates this state, along with a node that we want to add to the chain.

FIGURE 14-1 (a) An empty chain and a new node; (b) after adding the new node to a chain that was empty



The following pseudocode establishes a new node for the given data—referenced by `newEntry`—and inserts it into the empty chain:

```
newNode references a new instance of Node
Place newEntry in newNode
firstNode = address of newNode
```

Figure 14-1b shows the result of this operation. In Java, these steps appear as follows, where `newEntry` references the entry to be added to the list:

```
Node newNode = new Node(newEntry);
firstNode = newNode;
```

Notice that in Figure 14-1b, `firstNode` and `newNode` reference the same node. After the insertion of the new node is complete, only `firstNode` should reference it. We could set `newNode` to `null`, but as you will see, `newNode` will be a local variable of the method `add`. As such, `newNode` will not exist after `add` ends its execution.

14.2 **Case 2: Adding a node to the beginning of a chain.** This case should be familiar to you, as well. The following pseudocode describes the steps needed to add a node to the beginning of a chain:

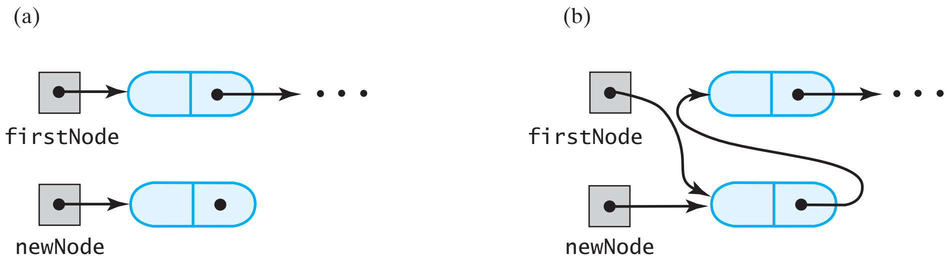
```
newNode references a new instance of Node
Place newEntry in newNode
Set newNode's link to firstNode
Set firstNode to newNode
```

The new node is now the first node. Figure 14-2 illustrates these steps, and the following Java statements implement them:

```
Node newNode = new Node(newEntry);
newNode.setNextNode(firstNode);
firstNode = newNode;
```

To simplify the figure, we have omitted the actual entries in the list. These entries are objects that the nodes reference.

FIGURE 14-2 A chain of nodes (a) just prior to adding a node at the beginning; (b) just after adding a node at the beginning



Note: Recall that adding a node to an empty chain, as Figure 14-1 depicts, is actually the same as adding a node to the beginning of a chain.

14.3 **Case 3: Adding a node between adjacent nodes of a chain.** Because the ADT list allows additions between two existing entries, we must be able to add a node to a chain between two existing, consecutive nodes. The necessary steps for this task are described by the following pseudocode:

```
newNode references the new node
Place newEntry in newNode
```

```
Let nodeBefore reference the node that will be before the new node
Set nodeAfter to nodeBefore's link
Set newNode's link to nodeAfter
Set nodeBefore's link to newNode
```

To indicate where in the chain the new node should be inserted, let's number the nodes, beginning with 1. We need to locate the node at a given position within the chain and get a reference to it. Suppose that the method `getNodeAt` performs this task for us. Since the method returns a reference to a node, and the class `Node` will be an inner class of a class of lists, `getNodeAt` is an implementation detail that we would not want a client to use. Thus, `getNodeAt` should be a private method. Let's specify `getNodeAt` as follows:

```java
// Returns a reference to the node at a given position.
// Precondition: The chain is not empty;
//               1 <= givenPosition <= numberOfNodes
private Node getNodeAt(int givenPosition)
```

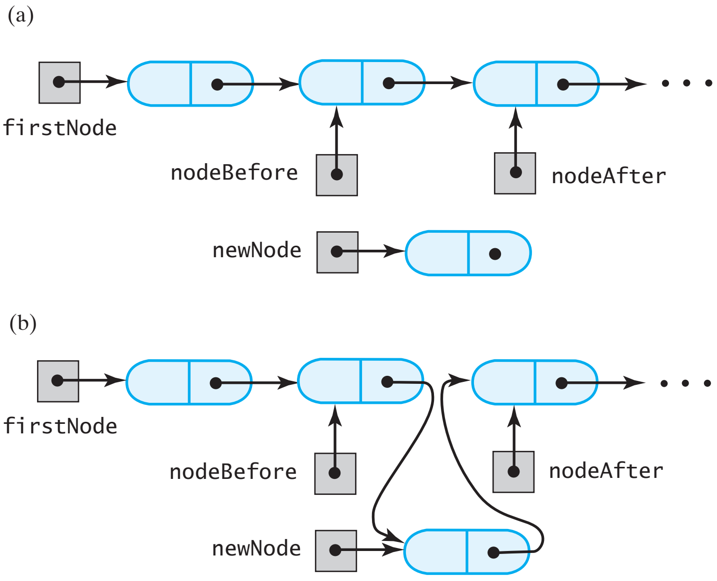
We can define the method later.

In the meantime, knowing only what `getNodeAt` does, and not how it does it, we can use it in the implementation of the previous pseudocode. If `newPosition` is the number of the new node after its insertion, the following Java statements add the new node to the chain:

```java
Node newNode = new Node(newEntry);
Node nodeBefore = getNodeAt(newPosition - 1);
Node nodeAfter = nodeBefore.getNextNode();
newNode.setNextNode(nodeAfter);
nodeBefore.setNextNode(newNode);
```

Figure 14-3a shows the chain after the first three statements execute, and Figure 14-3b shows it after the node has been added. In this figure, `newPosition` is 3.

FIGURE 14-3 A chain of nodes (a) just prior to adding a node between two adjacent nodes; (b) just after adding a node between two adjacent nodes



Question 1 Describe the steps that the method `getNodeAt` must take to locate the node at a given position.

14.4 **Case 4: Adding a node to the end of a chain.** To add a node to the end of an existing chain, we can take the following steps:

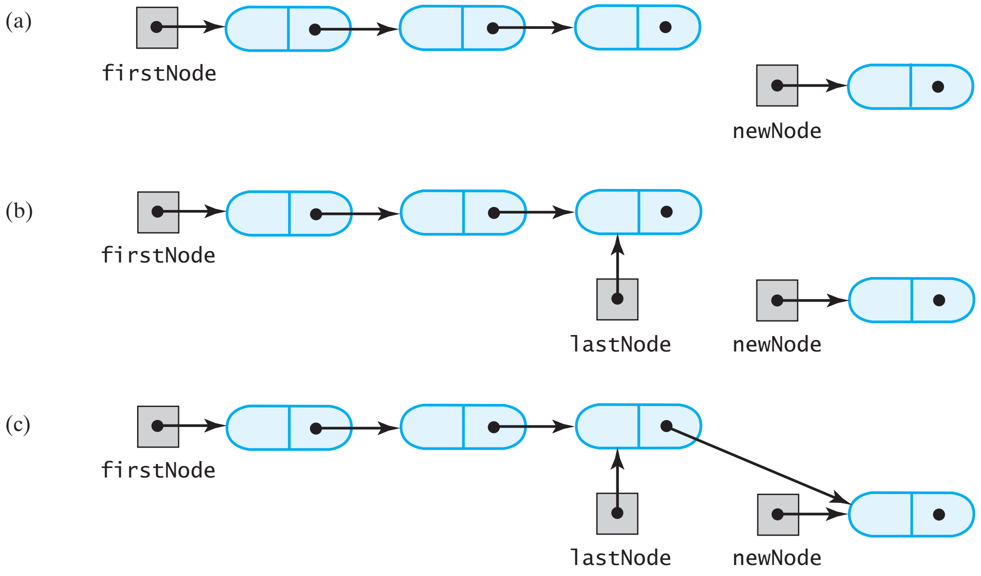
```
newNode references a new instance of Node
Place newEntry in newNode
Locate the last node in the chain
Place the address of newNode in this last node
```

That is, we make the last node in the chain reference the new node. Using the same method, `getNodeAt`, that the previous segment described, we can implement these steps in Java as follows:

```
Node newNode = new Node(newEntry);
Node lastNode = getNodeAt(numberOfNodes);
lastNode.setNextNode(newNode);
```

Figure 14-4 illustrates this addition to the end of a chain of nodes.

FIGURE 14-4 A chain of nodes (a) prior to adding a node at the end; (b) after locating its last node; (c) after adding a node at the end



Note: Adding a new node to the end of a chain of *n* nodes can be thought of as adding the node at position *n* + 1.

SELF-TEST

Question 2 The code that we developed in Segment 14.3 to add a node between two adjacent nodes of a chain is

```
Node newNode = new Node(newEntry);
Node nodeBefore = getNodeAt(newPosition - 1);
Node nodeAfter = nodeBefore.getNextNode();
newNode.setNextNode(nodeAfter);
nodeBefore.setNextNode(newNode);
```

Is it possible to use this code instead of the following code, which we just developed, to add a node to the end of a chain? Explain your answer.

```
Node newNode = new Node(newEntry);
Node lastNode = getNodeAt(numberOfNodes);
lastNode.setNextNode(newNode);
```

Question 3 Adding a node to an empty chain could be thought of as adding a node to the end of a chain that is empty. Can you use the statements in Segment 14.4 instead of

```
Node newNode = new Node(newEntry);
firstNode = newNode;
```

which we developed in Segment 14.1 to add a node to an empty chain? Why or why not?

Removing a Node from Various Positions

To remove a node at a specified position within a nonempty chain, we must consider two cases:

- Case 1: Removing the first node
- Case 2: Removing a node other than the first one

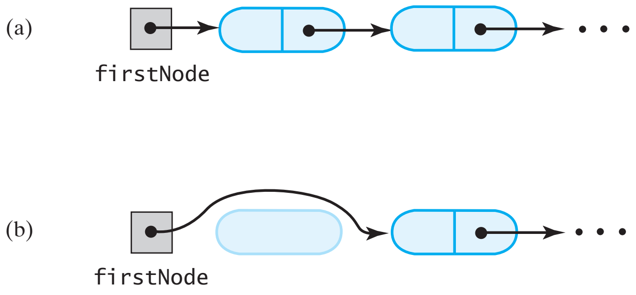
14.5 **Case 1: Removing the first node.** This case should be familiar to you, as we removed the first node in the linked implementations of the ADTs bag, stack, queue, deque, and priority queue. The steps to take are

Set `firstNode` to the link in the first node.
Since references to the first node no longer exist, the system automatically recycles its memory.

Figure 14-5 illustrates these steps, and the following Java statement implements them:

```
firstNode = firstNode.getNextNode();
```

FIGURE 14-5 A chain of nodes (a) just prior to removing the first node; (b) just after removing the first node



14.6 **Case 2: Removing a node other than the first one.** In the second case, we remove a node at a position other than the beginning of the chain. Here are the steps to take:

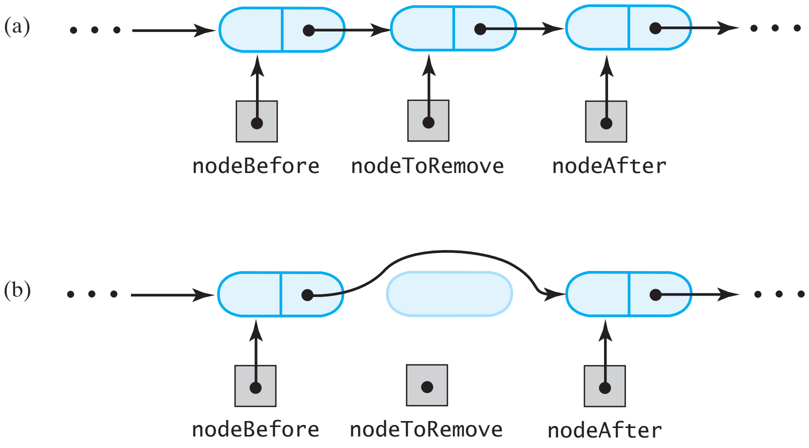
> *Let* `nodeBefore` *reference the node before the one to be removed.*
> *Set* `nodeToRemove` *to* `nodeBefore`*'s link;* `nodeToRemove` *now references the node to be removed.*
> *Set* `nodeAfter` *to* `nodeToRemove`*'s link;* `nodeAfter` *now references the node after the one to be removed.*
> *Set* `nodeBefore`*'s link to* `nodeAfter`*. (*`nodeToRemove` *is now disconnected from the chain.)*
> *Set* `nodeToRemove` *to* `null`*.*
> *Since references to the disconnected node no longer exist, the system automatically recycles its memory.*

The following Java statements implement these steps, assuming that the node to remove is at position `givenPosition`:

```
Node nodeBefore = getNodeAt(givenPosition - 1);
Node nodeToRemove = nodeBefore.getNextNode();
Node nodeAfter = nodeToRemove.getNextNode();
nodeBefore.setNextNode(nodeAfter);
nodeToRemove = null;
```

Figure 14-6a illustrates the chain after the first three statements execute, and Figure 14-6b shows it after the node is removed.

FIGURE 14-6 A chain of nodes (a) just prior to removing an interior node; (b) just after removing an interior node



The Private Method `getNodeAt`

14.7 The previous operations on a chain depended on the method `getNodeAt`, which returns a reference to the node at a given position within the chain. Recall the specifications for this method:

```
// Returns a reference to the node at a given position.
// Precondition: The chain is not empty;
//                1 <= givenPosition <= numberOfNodes
private Node getNodeAt(int givenPosition)
```

To locate a particular node within a chain, we begin at the chain's first node and traverse the chain from one node to another. We know that `firstNode` contains a reference to the first node in the chain. That first node contains a reference to the second node in the chain, the second node contains a reference to the third node, and so on.

We can use a temporary variable, `currentNode`, to reference the nodes one at a time as we traverse the chain from the first node to the desired node. Initially, we set `currentNode` to `firstNode` so that it references the first node in the chain. If we are seeking the first node, we are done. Otherwise, we move to the next node by executing

```
currentNode = currentNode.getNextNode();
```

If we are seeking the second node, we are done. Otherwise, we move to the next node by executing

```
currentNode = currentNode.getNextNode();
```

once again. We continue in this manner until we locate the node at the desired position within the list.

The implementation for `getNodeAt` follows:

```
private Node getNodeAt(int givenPosition)
{
   assert (firstNode != null) &&
          (1 <= givenPosition) && (givenPosition <= numberOfNodes);
   Node currentNode = firstNode;

   // traverse the chain to locate the desired node
   for (int counter = 1; counter < givenPosition; counter++)
      currentNode = currentNode.getNextNode();

   assert currentNode != null;

   return currentNode;
} // end getNodeAt
```

Within the `for` loop, `currentNode` should never become `null` if the method's precondition is met. Thus, `currentNode.getNextNode()` never executes if `currentNode` is `null`. You can enable the `assert` statements during the testing of `getNodeAt` to verify these claims.

? SELF-TEST

Question 4 The statements in Segment 14.4 that add an entry to the end of a chain invoke the method `getNodeAt`. Suppose that you use these statements repeatedly to create a chain by adding entries to its end.

a. How efficient of time is this approach?
b. Is there a faster way to repeatedly add entries to the end of a chain? Explain.

Question 5 How does `getNodeAt`'s precondition prevent `currentNode` from becoming `null`?

Beginning the Implementation

Design Decision: The structure of the chain of linked nodes

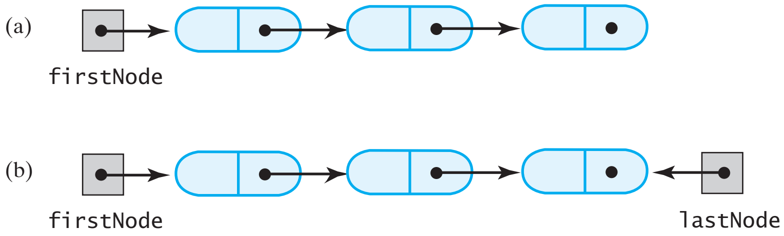
Imagine that we have a collection of data from which we will create a list. That is, our data will be the list's entries. If the data is in the order in which the entries will appear in the list, we create the list by repeatedly adding the next entry to the end of the list.

We could do this by using the list's first add method. However, if add contains the statements in Segment 14.4 that add an entry to the end of a list, the method getNodeAt will be invoked to locate the last node in the chain. To accomplish this task, getNodeAt must begin at the first node and traverse the chain until it locates the last node. Given a reference to the last node, add can insert the new entry at the end of the chain. If this reference is not retained when the method completes its task, adding another entry to the end of the list forces add to invoke getNodeAt again. The result is another traversal of the chain from its beginning. Since we plan to add entries repeatedly to the end of the list, many repetitious traversals will occur.

In such cases, maintaining a reference to the end of the chain—as well as a reference to the beginning of the chain—is advantageous. Such a reference to the end of a chain is called a tail reference, and was introduced in Chapter 11 for the linked implementation of a queue. Figure 14-7 illustrates two linked chains: one with a head reference and the other with both head and tail references.

Maintaining both head and tail references is somewhat more involved for a list than it was for a queue. Thus, our first class of lists using a linked implementation will not define a tail reference. After we successfully complete this simpler definition, we will modify it by adding a tail reference and thereby improve its time efficiency. Recall that solving a simpler problem first is often a reasonable strategy.

FIGURE 14-7 A linked chain with (a) a head reference; (b) both a head reference and a tail reference



The Data Fields and Constructor

14.8 Listing 14-1 contains an outline of the class LList[1] that implements the ADT list. Recall that Chapter 12 defined the interface ListInterface. It and the classes that implement it define a generic type T for the objects in the list. We plan to define a chain of linked nodes to contain the list's entries. Thus, we need the class Node, which we have used in our previous discussion, and so we define it as an inner class of LList. The generic type T that appears in LList's header is the same one that we use in the class Node.

LISTING 14-1 An outline of the class LList

```
/**
   A linked implementation of the ADT list.
   @author Frank M. Carrano
*/
public class LList<T> implements ListInterface<T>
```

[1] We named this class LList instead of LinkedList to avoid confusion with Java's class LinkedList in the package java.util. You will see Java's LinkedList at the end of this chapter.

```
{
   private Node firstNode; // reference to first node
   private int  numberOfEntries;

   public LList()
   {
      clear();
   } // end default constructor

   public final void clear() // note the final method
   {
      firstNode = null;
      numberOfEntries = 0;
   } // end clear
   < Implementations of the public methods add, remove, replace, getEntry, contains,
     getLength, isEmpty, and toArray go here. >
   . . .

   // Returns a reference to the node at a given position.
   // Precondition: List is not empty;
   //               1 <= givenPosition <= numberOfEntries.
   private Node getNodeAt(int givenPosition)
   {
      < See Segment 14.7. >
   } // end getNodeAt

   private class Node // private inner class
   {
      < See Listing 3-4 in Chapter 3. >
   } // end Node
} // end LList
```

VideoNote
The class LList

As we discussed earlier, this version of the class will maintain only a head reference to the chain of nodes. The data field `firstNode` is this head reference. Another data field, `numberOfEntries`, records the number of entries in the current list. This number is also the number of nodes in the chain. The default constructor simply 5initializes these data fields by calling `clear`. So initially, the list is empty, `firstNode` is `null`, and `numberOfEntries` is 0.

As was noted in Appendix C, when a constructor cnalls another public method such as `clear`, that method should be final so that no subclass can override it, thereby changing the effect of the constructor. Adding `final` to `clear`'s header is an implementation detail that is not reflected in `ListInterface`. Recall from Appendix D that an interface cannot declare a method to be final.

Adding to the End of the List

14.9 Let's choose the methods `add` and `toArray` as the core methods that we will implement first. We have already defined the method `clear`, because the constructor calls it.

We begin with the first add method. This method adds a new entry to the end of the list. Like the statements in Segment 14.4, the following statements make this addition:

```
Node newNode = new Node(newEntry);
Node lastNode = getNodeAt(numberOfEntries);
lastNode.setNextNode(newNode);
```

Assuming that we have the private method getNodeAt, we can complete the method add as follows:

```
public void add(T newEntry)
{
   Node newNode = new Node(newEntry);

   if (isEmpty())
      firstNode = newNode;
   else                              // add to end of nonempty list
   {
      Node lastNode = getNodeAt(numberOfEntries);
      lastNode.setNextNode(newNode); // make last node reference new node
   } // end if

   numberOfEntries++;
} // end add
```

This method first creates a new node for the new entry. If the list is empty, it adds the new node by making firstNode reference it. If the list is not empty, however, we must locate the end of the list. Because we have a reference only to the first node, we must traverse the list until we locate the last node and obtain a reference to it. We will call the private method getNodeAt, which we defined in Segment 14.7, to accomplish this task. Since the data field numberOfEntries contains the size of the list, and since we identify list entries by their positions within the list beginning with 1, the last node is at position numberOfEntries. We need to pass this value to getNodeAt. Once getNodeAt gives us a reference to the last node, we can set the last node's link to reference the new node.

Note that we must define isEmpty, since add calls it, and so we add it to our core group of methods that we define first.

Adding at a Given Position Within the List

14.10 The second add method adds a new entry at a specified position within the list. After creating a new node that newNode references, we see whether the existing list is empty. If it is, we add the new node to the list by writing firstNode = newNode, as we did in the first add method. If the list is not empty, we must consider two cases:

- Case 1: Adding the entry to the beginning of the list
- Case 2: Adding the entry at a position other than the beginning of the list

Segment 14.2 lists the following statements for an addition to the beginning of a list:

```
Node newNode = new Node(newEntry);
newNode.setNextNode(firstNode);
firstNode = newNode;
```

Recall that these statements apply whether the list is empty or not. Additions anywhere else are performed by the statements shown in Segment 14.3:

```
Node newNode = new Node(newEntry);
Node nodeBefore = getNodeAt(newPosition - 1);
Node nodeAfter = nodeBefore.getNextNode();
newNode.setNextNode(nodeAfter);
nodeBefore.setNextNode(newNode);
```

14.11 **The Java method.** The following implementation of the `add` method is based on the previous code fragments. We begin by checking the validity of the insertion position, `newPosition`. If it is within range, we create the new node. We then insert the new node into the chain according to its intended position, `newPosition`: The insertion is either at the beginning of the list or somewhere else in the list.

```java
public boolean add(int newPosition, T newEntry)
{
   boolean isSuccessful = true;

   if ((newPosition >= 1) && (newPosition <= numberOfEntries + 1))
   {
      Node newNode = new Node(newEntry);

      if (newPosition == 1)                  // case 1
      {
         newNode.setNextNode(firstNode);
         firstNode = newNode;
      }
      else                                   // case 2: list is not empty
      {                                      // and newPosition > 1
         Node nodeBefore = getNodeAt(newPosition - 1);
         Node nodeAfter = nodeBefore.getNextNode();
         newNode.setNextNode(nodeAfter);
         nodeBefore.setNextNode(newNode);
      } // end if

      numberOfEntries++;
   }
   else
      isSuccessful = false;

   return isSuccessful;
} // end add
```

Question 6 Consider the first `else` clause of the previous method `add`.

a. What `assert` statement could you add to this clause?
b. What call to `getNodeAt` could replace the value assigned to `nodeAfter`?
c. Should we make the change suggested in Part *b*?

Question 7 In the previous method `add`, the second `if` statement tests the value of `newPosition`. Should the boolean expression it tests be `isEmpty() || (newPosition == 1)`? Explain.

Question 8 How do the `add` methods given in Segments 14.9 and 14.11 enforce the precondition of `getNodeAt`?

The Methods `isEmpty` and `toArray`

14.12 **The method `isEmpty` and assertions.** The implementation of the method `isEmpty` could simply test that the length of the list is zero, as it did in the array-based implementations that you saw in the previous chapter. However, we have another criterion that we could use here. When a list is empty, the reference `firstNode` is `null`. If our implementation is correct, either criterion is fine, but what happens during development when some part of our class might contain an error in logic? We can use assert statements involving the second criterion to help us catch an error, as in the following version of isEmpty:

```
public boolean isEmpty()
{
   boolean result;
   if (numberOfEntries == 0) // or getLength() == 0
   {
      assert firstNode == null;
      result = true;
   }
   else
   {
      assert firstNode != null;
      result = false;
   } // end if

   return result;
} // end isEmpty
```

14.13 **Example.** Let's look at an example of how the previous implementation of isEmpty can help us find an error in logic. Consider the definition of the first add method given in Segment 14.9. If we had been concerned that we might forget to increment the data field numberOfEntries, we might have written numberOfEntries++ as the method's first action instead of as one of its last. This change would have caused an error. If the list was empty when the method was called, numberOfEntries would have been given the value 1, and isEmpty would have been invoked. Since firstNode would have been null, the second assertion within isEmpty would have produced an error message like the following one, assuming that we had enabled assertions:

```
Exception in thread "main" java.lang.AssertionError
        at LList.isEmpty(LList.java:167)
        at LList.add(LList.java:36)
        at Driver.testList(driver.java:16)
        at Driver.main(driver.java:5);
```

This message indicates that the method add called isEmpty, which produced the assertion error. We could clarify this message by adding to the assert statements in isEmpty. For example, if the second assert statement is

```
assert firstNode != null : "numberOfEntries is not 0 but firstNode is null";
```

the previous error message would begin as follows:

```
Exception in thread "main" java.lang.AssertionError:
numberOfEntries is not 0 but firstNode is null
```

If we ran our program without enabling assertions, isEmpty would simply test numberOfEntries. Since numberOfEntries would not be zero, isEmpty would return false, so add's else clause would execute. When add invoked getNodeAt(1), null would be returned—since firstNode would be null—and assigned to lastNode. As a result, lastNode.setNextNode(newNode) would cause an exception and would produce an error message such as

```
Exception in thread "main" java.lang.NullPointerException
        at LList$Node.access$102(LList.java:212)
        at LList.add(LList.java:41)
        at Driver.testList(driver.java:16)
        at Driver.main(driver.java:5);
```

This message is not as clear as the previous one, so more effort would be needed to discover the problem.

Question 9 Suppose that the implementation of the method `isEmpty` contains the following single statement:

```
return (numberOfEntries == 0) && (firstNode == null);
```

If we make the error in the method `add` that is described in the previous segment, what happens when `add` is called and the list is empty? Assume that assertions are enabled.

14.14 **The method `toArray`.** By implementing the method `toArray`, we will be able to test the previous methods that we have written before we complete the rest of `LList`. The method must traverse the chain and copy the data in each node to an element within an array. Thus, it needs a local variable to reference each node in the chain. For example, `currentNode` could reference the node whose data we want to copy. That data is `currentNode.getData()`.

Initially, we want `currentNode` to reference the first node in the chain, so we set it to `firstNode`. To make `currentNode` reference the next node, we would execute the statement

```
currentNode = currentNode.getNextNode();
```

Thus, we can write a loop that iterates until `currentNode` becomes `null`.

The following method `toArray` uses these ideas:

```
public T[] toArray()
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] result = (T[])new Object[numberOfEntries];

   int index = 0;
   Node currentNode = firstNode;
   while ((index < numberOfEntries) && (currentNode != null))
   {
      result[index] = currentNode.getData();
      currentNode = currentNode.getNextNode();
      index++;
   } // end while

   return result;
} // end toArray
```

Question 10 In the previous implementation of `toArray`, the `while` statement tests the values of both `index` and `currentNode`. Can you replace the `while` statement with

a. `while (index < numberOfEntries)`

b. `while (currentNode != null)`

Explain your responses.

Question 11 Compare the work required by the loop in the previous method `toArray` with that required by the following version of the loop:

```
int index = 0;
Node currentNode = firstNode;
while ((index < numberOfEntries) && (currentNode != null))
{
   currentNode = getNodeAt(index + 1);
   result[index] = currentNode.getData();
   index++;
} // end while
```

Testing the Core Methods

14.15 Earlier, we realized that the add methods are fundamental to our class, so they are part of the core group of methods that we implement and test first. The method toArray lets us see whether add works correctly, so it too is in our core group. The constructor is also fundamental, and so is the method clear, since the constructor calls it. Similarly, since add calls isEmpty and getNodeAt, they are among the core methods that we implement and test first. Lastly, we define the method getLength as a check that the add methods correctly maintain the field numberOfEntries. Although it is not really an essential method right now, its definition is simple and is the same as for the array-based implementation that you saw in the previous chapter.

Now that we have implemented these core methods, we can test them. Since LList implements ListInterface, however, we first must write stubs for the remaining methods in the interface. We assume that we have completed that simple task.

Let’s choose the add method that adds to the end of the list for our first tests. Listing 14-2 contains an example of a main method that we could use for this purpose. Notice how the descriptive output makes it easier to see whether our implementation is correct. The method displayList is the same as the one in Listing 12-2 of Chapter 12. Recall that this method calls the methods getLength and getEntry.

LISTING 14-2 A main method that tests part of the implementation of the ADT list

```
public static void main(String[] args)
{
   System.out.println("Create an empty list.");
   ListInterface<String> myList = new LList<String>();
   System.out.println("List should be empty; isEmpty returns" +
                      myList.isEmpty() + ".");
   System.out.println("\nTesting add to end:");
   myList.add("15");
   myList.add("25");
   myList.add("35");
   myList.add("45");
   System.out.println("List should contain 15 25 35 45.");
   displayList(myList);
   System.out.println("List should not be empty; isEmpty() returns" +
                      myList.isEmpty() + ".");
   System.out.println("\nTesting clear():");
   myList.clear();
   System.out.println("List should be empty; isEmpty returns" +
                      myList.isEmpty() + ".");
} // end main
```

Output

```
Create an empty list.
List should be empty; isEmpty returns true.

Testing add to end:
List should contain 15 25 35 45.
List contains 4 entries, as follows:
15 is entry 1
25 is entry 2
35 is entry 3
45 is entry 4

List should not be empty; isEmpty() returns false.

Testing clear():
List should be empty; isEmpty returns true.
```

SELF-TEST

Question 12 Consider the method `displayList`, as given in Listing 12-2 of Chapter 12.

a. Why is the method less efficient of time in the context of Listing 14-2 than it was in Listing 12-2?

b. Revise `displayList` so that it is time efficient when it is used with either `LList` or `AList`.

Continuing the Implementation

To complete the class `LList`, we now define the methods `remove`, `replace`, `getEntry`, and `contains`.

14.16 **The method `remove`.** To remove the first entry from our list, we execute the statement

```
firstNode = firstNode.getNextNode();
```

To remove an entry after the first one, we execute the following statements:

```
Node nodeBefore = getNodeAt(givenPosition - 1);
Node nodeToRemove = nodeBefore.getNextNode();
Node nodeAfter = nodeToRemove.getNextNode();
nodeBefore.setNextNode(nodeAfter);
nodeToRemove = null;
```

Recall that the `remove` method returns the entry that it deletes from the list. Although the node that contains this entry is recycled, the entry itself is not as long as the client saves the reference to it.

VideoNote
Completing the class `LList`

```
public T remove(int givenPosition)
{
   T result = null;                          // return value

   if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
   {
      assert !isEmpty();
      if (givenPosition == 1)                // case 1: remove first entry
      {
         result = firstNode.getData();       // save entry to be removed
         firstNode = firstNode.getNextNode();
      }
      else                                   // case 2: not first entry
      {
         Node nodeBefore = getNodeAt(givenPosition - 1);
         Node nodeToRemove = nodeBefore.getNextNode();
         Node nodeAfter = nodeToRemove.getNextNode();
         nodeBefore.setNextNode(nodeAfter);
```

```
            result = nodeToRemove.getData();      // save entry to be removed
         } // end if

         numberOfEntries--;
      } // end if

      return result;                              // return removed entry, or
                                                  // null if operation fails
   } // end remove
```

Notice that we use the private method `getNodeAt`, which we wrote originally for the `add` methods, to locate the node before the one to be removed. This method is called only when we remove an entry other than the first one. Thus, its argument `givenPosition - 1` will always be greater than zero, as its precondition requires.

Also notice that we do not explicitly set `nodeToRemove` to `null` after disconnecting the node. This variable is local to the `remove` method and so does not exist after the method completes execution. Although we could set `nodeToRemove` to `null`, doing so is not necessary.

Question 13 Why is the assertion in the previous method true?

14.17 **The method `replace`.** Replacing a list entry requires us to replace the data portion of a node with other data. We can use the private method `getNodeAt` to locate the node and then simply replace its data portion. Before calling `getNodeAt`, we check that the list is not empty and the given position is valid. The implementation appears as follows:

```
public boolean replace(int givenPosition, T newEntry)
{
   boolean isSuccessful = true;
   if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
   {
      assert !isEmpty();
      Node desiredNode = getNodeAt(givenPosition);
      desiredNode.setData(newEntry);
   }
   else
      isSuccessful = false;

   return isSuccessful;
} // end replace
```

Note: The method `replace` replaces the data in a node, but not the node itself.

Question 14 Compare the time required to replace an entry in a list using the previous method `replace` with the time required for the array-based version given in Segment 13.12.

14.18 **The method `getEntry`.** Retrieving a list entry is straightforward:

```
public T getEntry(int givenPosition)
{
   T result = null; // result to return
```

```
      if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
      {
         assert !isEmpty();
         result = getNodeAt(givenPosition).getData();
      } // end if

      return result;
   } // end getEntry
```

The method getNodeAt returns a reference to the desired node, so

```
   getNodeAt(givenPosition).getData()
```

is the data portion of that node.

Although our implementations of getEntry and replace are easy to write, each does more work than if we had used an array to represent the list. Here, getNodeAt starts at the first node in the chain and moves from node to node until it reaches the desired one. In Segment 13.12, you saw that replace and getEntry can reference the desired array entry directly, without involving any other entry in the array.

14.19 **The method contains.** The method contains for a list could have the same definition as the one given in Segment 3.17 of Chapter 3 for a bag. However, the inner class Node now has set and get methods, so contains can appear as follows:

```
   public boolean contains(T anEntry)
   {
      boolean found = false;
      Node currentNode = firstNode;

      while (!found && (currentNode != null))
      {
         if (anEntry.equals(currentNode.getData()))
            found = true;
         else
            currentNode = currentNode.getNextNode();
      } // end while

      return found;
   } // end contains
```

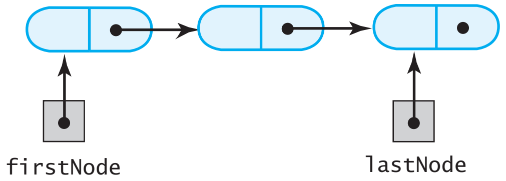
Because the ADT bag has a remove method that removes a given entry, it must do the same search as contains. For that reason, we revised the definition of contains in Chapter 3 so that the search is performed by a private method that both contains and remove can call. Our version of the ADT list, however, removes entries by position, not by the value of the entry. Thus, the search that contains does is performed only by contains.

Note: Test the class LList
The class LList is now complete and should be thoroughly tested before we continue. We leave this test to you as an exercise. You will be able to use your program later when you test the improved version of LList that we are about to write.

A Refined Implementation

Currently, the chain of linked nodes that contains a list's entries has only a head reference. When we began writing the class LList, we noted that the first add method, which adds a new entry at the end of the chain, must invoke the private method getNodeAt to locate the chain's last node. To do so, getNodeAt must traverse the chain from its beginning. We can improve the time efficiency of this add method by maintaining a reference to the end of the chain, as well as a reference to the beginning of the chain, as was pictured earlier in Figure 14-7b and repeated here in Figure 14-8. In this way, we avoid a traversal of the entire chain each time that add is called.

FIGURE 14-8 A linked chain with both a head reference and a tail reference



Question 15 Examine the implementation of the class LList given in this chapter. Which methods would require a new definition if you use both a head reference and a tail reference?

The Tail Reference

14.20 The tail reference, like the head reference, is a private data field of the class. Thus, the private data fields of the revised class are now

```
private Node firstNode;       // head reference to first node
private Node lastNode;        // tail reference to last node
private int  numberOfEntries; // number of entries in list
```

By examining the class LList, as described earlier in this chapter, you should find that the two add methods and the methods remove and clear are the ones that will involve the head and tail references and, thus, need to be revised. We should also revise the assertions in the method isEmpty. The rest of the original implementation, including the constructor, remains the same. Let's examine these revisions. We will name the revised class LList2 to distinguish it from the original one.

14.21 **The method clear.** We begin with the method clear, because the constructor calls it. It must initialize both the head and tail references as well as the field numberOfEntries:

```
public final void clear()
{
   firstNode = null;
   lastNode = null;
   numberOfEntries = 0;
} // end clear
```

Here, and in the rest of the revision, changes to the original implementation are highlighted.

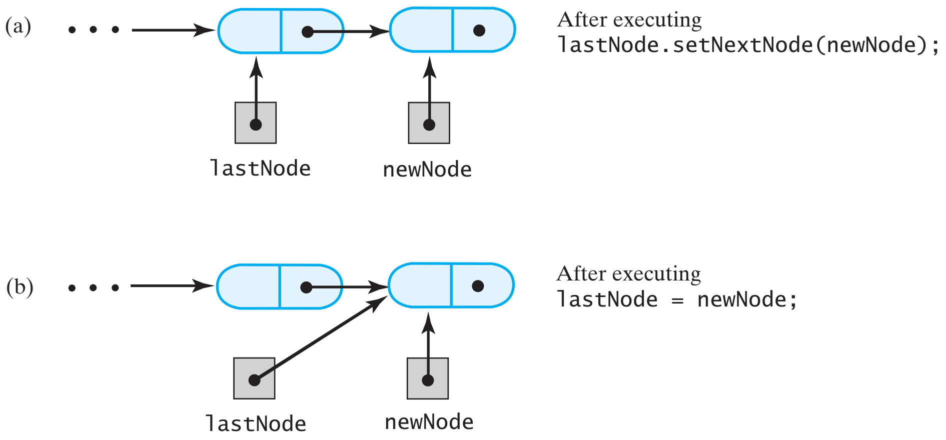
14.22 **Adding to the end of the list.** The steps required to add an entry to the end of a list depend upon whether the list is empty or not. After an item is added to the end of an empty list, both the head and tail references must reference the new solitary node. Thus, after creating a new node that newNode references, the add method would execute

```
firstNode = newNode;
lastNode = newNode;
```

Adding to the end of a nonempty list no longer requires a traversal to locate the last entry: The tail reference lastNode provides this information. After the addition has been made, the tail reference must change to refer to the new last entry. The following statements perform these steps, as Figure 14-9 illustrates:

```
lastNode.setNextNode(newNode);
lastNode = newNode;
```

FIGURE 14-9 Adding a node to the end of a nonempty chain that has a tail reference



The following revision of the first add method reflects the previous comments:

```
public void add(T newEntry)
{
   Node newNode = new Node(newEntry);

   if (isEmpty())
      firstNode = newNode;
   else
      lastNode.setNextNode(newNode);

   lastNode = newNode;
   numberOfEntries++;
} // end add
```

Note that the method no longer calls getNodeAt to establish lastNode, as it did in Segment 14.9.

14.23 **Adding to the list at a given position.** Adding to a list by position affects the tail reference only when we are adding to an empty list or adding to the end of a nonempty list. Other cases do not affect the tail reference, so we treat them as we did in Segment 14.11 when we did not have a tail reference.

Thus, the implementation of the method that adds by position is

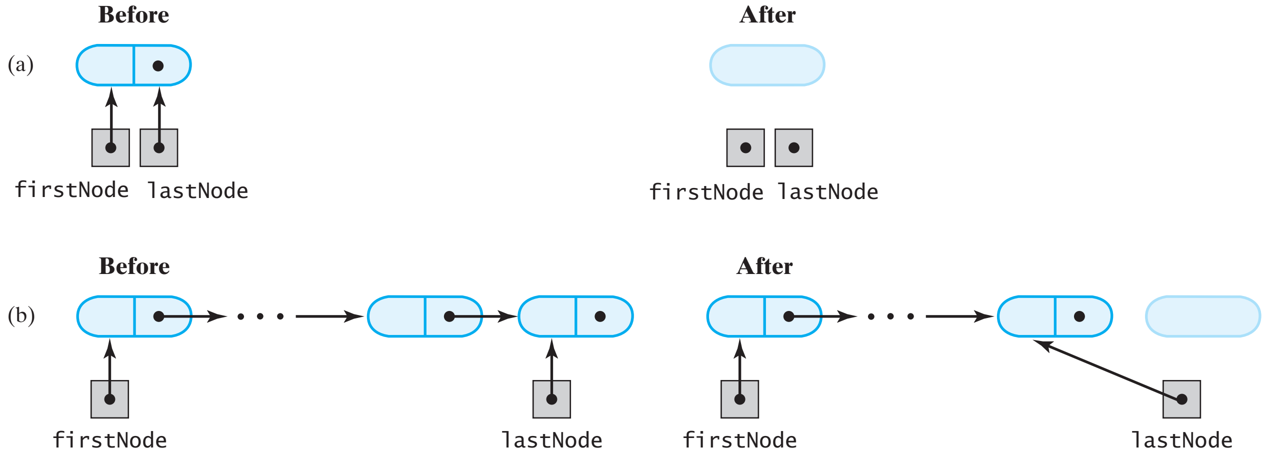
```
public boolean add(int newPosition, T newEntry)
{
   boolean isSuccessful = true;

   if ((newPosition >= 1) && (newPosition <= numberOfEntries + 1))
   {
      Node newNode = new Node(newEntry);

      if (isEmpty())
      {
         firstNode = newNode;
         lastNode = newNode;
      }
      else if (newPosition == 1)
      {
         newNode.setNextNode(firstNode);
         firstNode = newNode;
      }
      else if (newPosition == numberOfEntries + 1)
      {
         lastNode.setNextNode(newNode);
         lastNode = newNode;
      }
      else
      {
         Node nodeBefore = getNodeAt(newPosition - 1);
         Node nodeAfter = nodeBefore.getNextNode();
         newNode.setNextNode(nodeAfter);
         nodeBefore.setNextNode(newNode);
      } // end if

      numberOfEntries++;
   }
   else
      isSuccessful = false;

   return isSuccessful;
} // end add
```

14.24 **Removing an entry from a list.** Removing an entry can affect the tail reference in two cases:

- Case 1: If the list contains one entry and we remove it, an empty list results, and we must set both the head and tail references to `null`.
- Case 2: If the list contains several entries and we remove the last one, we must change the tail reference so that it references the new last entry.

Figure 14-10 illustrates these two cases.

FIGURE 14-10 Removing the last node from a chain that has both head and tail references when the chain contains (a) one node; (b) more than one node



The following method treats the previous two cases in its implementation of the remove operation:

```
public T remove(int givenPosition)
{
   T result = null;                          // return value

   if ((givenPosition >= 1) && (givenPosition <= numberOfEntries))
   {
      assert !isEmpty();
      if (givenPosition == 1)                // case 1: remove first entry
      {
         result = firstNode.getData();       // save entry to be removed
         firstNode = firstNode.getNextNode();
         if (numberOfEntries == 1)
            lastNode = null;                 // solitary entry was removed
      }
      else                                   // case 2: not first entry
      {
         Node nodeBefore = getNodeAt(givenPosition - 1);
         Node nodeToRemove = nodeBefore.getNextNode();
         Node nodeAfter = nodeToRemove.getNextNode();
         nodeBefore.setNextNode(nodeAfter);
         result = nodeToRemove.getData();    // save entry to be removed

         if (givenPosition == numberOfEntries)
            lastNode = nodeBefore;           // last node was removed
      } // end if

      numberOfEntries--;
   } // end if

   return result;                            // return removed entry, or
                                             // null if operation fails
} // end remove
```

Note: Adding to the end of a chain of linked nodes requires less work when you maintain a tail reference because you avoid a traversal of the chain. Removing the last node from a chain requires a traversal to locate the next-to-last node whether or not you have a tail reference.

SELF-TEST **Question 16** In light of the tail reference, what changes should you make to the assertions in the method `isEmpty`, as given in Segment 14.12?

The Efficiency of Using a Chain to Implement the ADT List

Let's consider the time complexity of some of the methods in our classes `LList` and `LList2`. Several of these methods call the private method `getNodeAt`, as given in Segment 14.7. The loop in `getNodeAt` cycles *i* times in its search for the i^{th} node in the chain. Thus, `getNodeAt` is O(*n*). We will use this fact in our analysis of the public methods.

14.25 **Adding to the end of the list.** Because the chain in the class `LList` does not have a tail reference, `LList`'s `add` method, as described in Segment 14.9, must traverse the entire chain of linked nodes to locate the last one before it can add an entry at the end of the list. The method calls `getNodeAt` to locate this node. Since `getNodeAt` is O(*n*), this `add` method is also O(*n*).

The class `LList2`, on the other hand, does maintain a tail reference for its chain of linked nodes. Thus, its `add` method, as given in Segment 14.22, does not call `getNodeAt`, and so is O(1).

14.26 **Adding to a list at a given position.** The `add` method, as given in Segment 14.11, for the class `LList` can add an entry to the beginning of a list in O(1) time. Adding to a list at other positions depends on the position. As the position number increases, the time needed for an addition increases. In other words, since `add` calls `getNodeAt`, and `getNodeAt` is O(*n*), `add` is O(*n*) when the addition is beyond the beginning of the list.

The `add` method, as given in Segment 14.23, for the class `LList2` can add an entry to either the beginning or the end of a list in O(1) time. It requires O(*n*) time for other additions.

Question 17 What is the Big Oh of the method `toArray`, as given in Segment 14.14?

Question 18 What is the Big Oh of the method `remove`, as given in Segment 14.16?

Question 19 What is the Big Oh of the method `replace`, as given in Segment 14.17?

Question 20 What is the Big Oh of the method `getEntry`, as given in Segment 14.18?

Question 21 What is the Big Oh of the method `contains`, as given in Segment 14.19?

Question 22 In light of the tail reference, what changes can you make to the method `getNodeAt`, as given in Segment 14.7, to improve the time complexity of the methods `replace`, `getEntry`, and `contains`?

14.27 Using Big Oh notation, Figure 14-11 summarizes the time complexities of the operations of the ADT list for the implementations that use an array and a chain of linked nodes. For some operations, two or three complexities are given: The first one indicates the time requirement for an operation at the beginning of the list, the second is for operations at other positions within the list, and the third one, if it exists, is for operations at the end of the list.

FIGURE 14-11 The time efficiencies of the ADT list operations for three implementations, expressed in Big Oh notation

Operation	`AList`	`LList`	`LList2`
`add(newEntry)`	O(1)	O(*n*)	O(1)
`add(newPosition, newEntry)`	O(*n*); O(1)	O(1); O(*n*)	O(1); O(*n*); O(1)
`toArray()`	O(*n*)	O(*n*)	O(*n*)
`remove(givenPosition)`	O(*n*); O(1)	O(1); O(*n*)	O(1); O(*n*)
`replace(givenPosition, newEntry)`	O(1)	O(1); O(*n*)	O(1); O(*n*); O(1)
`getEntry(givenPosition)`	O(1)	O(1); O(*n*)	O(1); O(*n*); O(1)
`contains(anEntry)`	O(*n*)	O(*n*)	O(*n*)
`clear(), getLength(), isEmpty()`	O(1)	O(1)	O(1)

For example, the first `add` method in an array-based implementation is O(1), and the second `add` method is O(*n*), unless it adds at the end of the list, in which case it is O(1). The method `toArray` is always O(*n*), and `getEntry` is always O(1).

For the linked implementation `LList`, which maintains only a head reference to the chain of nodes, the first `add` method and `toArray` are each O(*n*). The second `add` method is O(*n*) unless it adds to the beginning of the list, in which case it is O(1).

For the linked implementation `LList2`, which maintains both a head reference and a tail reference to the chain of nodes, the first `add` method is O(1), and `toArray` is O(*n*). The second `add` method is O(*n*), unless it adds to either the beginning or the end of the list, in which case it is O(1).

As you can see, some of the operations have the same time complexity for each implementation. However, operations that add to the end of a list, replace an entry, or retrieve an entry take less time when you use an array to represent a list than when you use a chain of linked nodes. If your application uses these particular operations frequently, an array-based implementation could be attractive.

The operations that add or remove an entry at a given position have time requirements that depend on this position regardless of their implementation. If your application primarily adds and removes entries at or near the beginning of a list, use a linked implementation. If these operations are mostly at or near the end of the list, use an array-based implementation. The operations that you use the most in an application should influence your choice of implementation for an ADT.

Design Decision: **Which implementation of an ADT should you use?**

Even small changes to an ADT's underlying data structure can increase or decrease the time efficiency of the ADT's operations. When choosing an implementation for an ADT, you should consider the operations that your application requires. If you use a particular operation frequently, you want its implementation to be efficient. Conversely, if you rarely use an operation, you can afford to use a class that has an inefficient implementation of that operation.

Like the linked implementations you have seen in previous chapters, the classes `LList` and `LList2` enable their instances to grow as large as necessary. You can add as many nodes to a chain—and, therefore, entries to a list—as computer memory allows. Although resizing the array in an array-based implementation gives you the same advantage, it comes with the cost of copying data from one array to another. No such copying occurs in a linked implementation.

In addition, a chain enables you to add and remove nodes without moving any existing entries that are already in the list. With an array, adding and removing entries usually requires that other entries be moved within the array. However, you must traverse a chain from its beginning to determine where to make the addition or deletion.

Retrieving an existing entry from a chain requires a similar traversal to locate the desired entry. When you use an array instead of a chain, you can access any element directly by position, without searching the array. However, a method such as `contains` that does not have the position of an entry must perform a search regardless of whether an array or a chain represents the list.

Finally, as you have seen before, a chain stores additional references compared to an array. For each entry in a list, a chain stores two references, compared to an array's one. This additional memory requirement is somewhat offset by the fact that a chain uses memory only as needed for each list entry, whereas an array often is larger than necessary, thereby wasting memory.

Any implementation of an ADT has its advantages and disadvantages. You should choose the implementation that best suits your particular application.

Java Class Library: The Class `LinkedList`

14.28 Recall from Chapter 12 that the Java Class Library contains the interface `java.util.List`. This interface is like our `ListInterface`, but it declares more methods. Also, some methods have different names or specifications, and the list entries begin at position 0 instead of 1. Segment 12.12 of Chapter 12 summarized these differences.

The same package `java.util` contains the class `LinkedList`. This class implements the interface `List`, as well as the interfaces `Queue` and `Deque` that are described in Chapter 10. Thus, `LinkedList` defines more methods than are in the interface `List`. Furthermore, you can use the class `LinkedList` as an implementation of the ADT queue, deque, or list.

Chapter Summary

- When a chain has only a head reference or has both head and tail references, the following are true:
    - Adding a node to the chain's beginning is a special case.
    - Removing the first node from the chain is a special case.
    - Adding or removing a node that is last in the chain requires a traversal of the entire chain.
    - Adding a node anywhere within a chain requires a change of at most two references.
    - Removing any node from a chain requires a change of at most two references.
- When a chain has both head and tail references, the following are true:
    - Adding a node to an empty chain is a special case.
    - Adding a node to the chain's end is a special case.
    - Removing the last node from a chain is a special case.
- Maintaining a reference to a chain's last node as well as to its first node eliminates the need for a traversal when adding a node at the end of the chain. Thus, adding to the end of a list is faster when the chain has both head and tail references than when it has only a head reference. For this reason, we have used a chain that has both head and tail references to implement the ADT list.

Exercises

1. Add a constructor to the class `LList` that creates a list from a given array of objects. Consider at least two different ways to implement such a constructor. Which way requires the least amount of work?

2. Write a program that tests the first version of `LList`.

3. Consider the definition of the `add` method that adds an entry to a list at a given position and appears in Segment 14.11. Replace the statements that execute in case 1 with the following ones:

```
if (isEmpty() || (newPosition == 1)) // case 1
{
   firstNode = newNode;
   newNode.next = firstNode;
}
```

a. What is displayed by the following statements in a client of the modified `LList`?

```
ListInterface<String> myList = new LList<String>();
myList.add(1, "30");

myList.add(2, "40");
myList.add(3, "50");
myList.add(1, "10");
myList.add(5, "60");
myList.add(2, "20");
int numberOfEntries = myList.getLength();
for (int position = 1; position <= numberOfEntries; position++)
   System.out.print(myList.getEntry(position) + " ");
```

b. What methods, if any, in `LList` could be affected by the change to the method `add` when they execute? Why?

4. Suppose that you want an operation for the ADT list that adds an array of items to the end of the list. The header of the method could be as follows:

```
public void addAll(T[] items)
```

Write an implementation of this method for the class `LList`.

5. Define the method `getPosition`, as described in Exercise 2 of the previous chapter, for the class `LList`. Compare the execution time required by this method with the version of `getPosition` defined in the class `AList`.

6. Implement an `equals` method for the class `LList` that returns true when the entries in one list equal the entries in a second list.

7. Repeat Exercise 10 in the previous chapter, but use the class `LList` instead of `AList`.

8. Suppose that a list contains `Comparable` objects. Implement a method that returns a new list of items that are less than some given item. The header of the method could be as follows:

```
public LList<T> getAllLessThan(Comparable<T> anObject)
```

Write an implementation of this method for the class `LList`. Make sure that your method does not affect the state of the original list.

9. Define the method `remove`, as described in Exercise 3 of the previous chapter, for the class `LList`.

10. Repeat the previous exercise, but remove all occurrences of `anObject` from the list.

11. Define the method `moveToEnd`, as described in Exercise 4 of the previous chapter, for the class `LList`.

12. Implement a `replace` method for the class `LList` that returns the replaced object.

13. Suppose that a list contains `Comparable` objects. Define the methods `getMin` and `removeMin`, as described in Exercise 7 of the previous chapter, for the class `LList`.

14. Consider an instance `arrayList` of `AList`, as given in the previous chapter. Let the list have an initial size of 10. Also, consider an instance of `LList` called `chainList`.

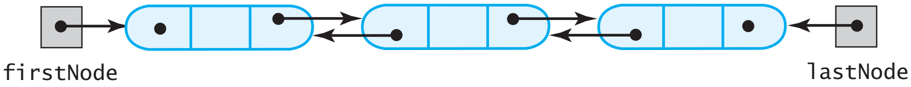
    a. How large is the underlying array after adding 145 items to `arrayList`?
    b. How large is the underlying array after adding 20 more items to `arrayList`?
    c. How many nodes are in the chain after adding 145 items to `chainList`?
    d. How many nodes are in the chain after adding 20 more items to `chainList`?
    e. Each node in a chain has two references, so a chain of *n* nodes has 2*n* references. An array of size *n*, on the other hand, has *n* references. Count the number of references in each of the situations described in Parts *a* through *d*.
    f. When will `arrayList` use fewer references than `chainList`?
    g. When will `chainList` use fewer references than `arrayList`?

15. A doubly linked chain, like the one described in Exercise 12 and Figure 3-11 of Chapter 3, has nodes that each can reference a previous node and a next node. In Chapter 3, the doubly linked chain has only a head reference, but it can have both a head reference and a tail reference, as Figure 14-12 illustrates.

    List the steps necessary to add a node to a doubly linked chain when the new node is

    a. First in the chain
    b. Last in the chain
    c. Between existing nodes in the chain

FIGURE 14-12 A doubly linked chain for Exercises 15 and 16 and Project 8



16. List the steps necessary to remove a node from the doubly linked chain shown in Figure 14-12 when the node is

    a. First in the chain
    b. Last in the chain
    c. Between existing nodes in the chain.

Projects

1. Write a program that thoroughly tests the class `LList2`.

2. Listing 3-5 of Chapter 3 shows the class `Node` as part of a package. Create another package that contains `Node`, `LList`, and `ListInterface`. Revise `LList` to use this version of `Node`.

3. Create a Java interface that declares the following methods:

```
/** Adds a new entry to the beginning of this list. */
public void addFirst(T newEntry)

/** Adds a new entry to the end of this list. */
public void addLast(T newEntry)

/** Removes and returns the first entry in this list. */
public T removeFirst()

/** Removes and returns the last entry in this list. */
public T removeLast()

/** Returns the first entry in this list. */
public T getFirst()

/** Returns the last entry in this list. */
public T getLast()

/** Moves the first entry in this list to the end of the list. */
public void moveToEnd()
```

Then define `DoubleEndedListInterface` by extending this interface and `ListInterface`. Write a class that implements `DoubleEndedListInterface`. Represent the list's entries by using a chain of nodes that has both a head reference and a tail reference. Write a program that thoroughly tests your class.

4. Repeat the previous project, but do not use a tail reference.

5. Adding nodes to or removing nodes from a linked chain requires a special case when the operation is at the beginning of the chain. To eliminate the special case, you can add a **dummy node** at the beginning of the chain. The dummy node is always present but does not contain a list entry. The chain, then, is never empty, and so the head reference is never `null`, even when the list is empty. Modify the class `LList`, as presented in this chapter, by adding a dummy node to the chain.

6. In a circularly linked chain, the last node references the first node. Commonly, only one external reference—to the last node—is maintained, since the first node is found easily from the last one.

   Modify the class `LList`, as presented in this chapter, by using a circular linked chain and a tail reference.

7. Implement as the class `Ring` the ADT ring that Project 3 of Chapter 1 described. Represent the ring as a chain of linked nodes. Consider using a circular linked chain, as described in the previous project.

8. Implement and test a class of lists using a doubly linked chain, as shown in Figure 14-12, to represent the entries in the list. Use an inner class of nodes like the one that Exercise 12 of Chapter 3 asked you to define, but include set and get methods.

9. You can add a dummy head node, as Project 5 describes, to the beginning of a doubly linked chain. Modify the implementation of the ADT list described in the previous project by adding a dummy head node to the chain.

10. Define a class that is a linked implementation of the interface `FixedSizeListInterface`, as described in Project 8 of the previous chapter.

11. Repeat any of the projects 1 through 5 and 8 through 15 in the previous chapter by either using a linked chain or involving the class `LList` instead of the class `AList`.

Answers to Self-Test Questions

1. To locate the n^{th} node in a chain, `getNodeAt` starts at the first node and counts nodes as it traverses the chain from node to node, until it reaches the n^{th} one. The following pseudocode describes the steps in more detail:

```
currentNode = firstNode
for (counter = 1 to n)
   currentNode = currentNode.getNextNode()

The desired node is at currentNode
```

2. Yes. With `newPosition` equal to `numberOfNodes + 1`, `nodeBefore` will reference the last node in the chain. Moreover, `nodeAfter` will be `null`, `newNode`'s link field will be set to `null`, and the last node's link will reference the new node.

3. No. The statements given in Segment 14.4 do not assign a new value to `firstNode`. Also, when the chain is empty, `numberOfNodes` is zero. The precondition of `getNodeAt` given in Segment 14.3 requires a positive argument. Even if you redesign `getNodeAt`, the empty chain will remain a special case.

4. **a.** This approach is inefficient of time, since each addition causes `getNodeAt` to traverse the chain from its beginning until it locates the chain's last node. Thus, each addition depends on the length of the chain.
   b. Maintaining a tail reference would allow additions to the end of the chain to occur in O(1) time, that is, independently of the length of the chain.

5. Since the chain is not empty, `firstNode` is not `null`. Thus, `currentNode`'s initial value is not `null`. The loop in `getNodeAt` can iterate no more than `numberOfNodes - 1` times. After the first iteration, `currentNode` references the second node. After the second iteration, it references the third node. If the loop were to iterate `numberOfNodes - 1` times, `currentNode` would reference the last node. It would not be `null`.

6. **a.** `assert !isEmpty() && (newPosition > 1);`
   b. `getNodeAt(newPosition)`
   c. No. Calling `getNodeAt` to get a reference to the node after the one that `nodeBefore` references results in another traversal of the chain from its beginning. The expression `nodeBefore.getNextNode()` provides a much faster way to get the required reference.

7. It could be, but testing for an empty list is unnecessary. The first `if` statement ensures that the value of `newPosition` ranges from 1 to `numberOfEntries` + 1. If the list is empty, `numberOfEntries` is 0, therefore `newPosition` must be 1. Since adding to an empty list is the same as adding to the beginning of any list, we need not be concerned with empty lists.

8. The first method `add` invokes `getNodeAt` only if the list is not empty. Thus, the argument `numberOfEntries` passed to `getNodeAt` is ≥ 1 and ≤ itself. The second method `add` checks the validity of `newPosition` and then calls `getNodeAt` only if the list is not empty and `newPosition` > 1. It is possible for `newPosition` to equal `numberOfEntries` + 1, but since `getNodeAt`'s argument is `newPosition - 1`, the argument's value is ≤ `numberOfEntries` as required.

9. When the method add calls the method isEmpty, isEmpty will return false because numberOfEntries incorrectly is not zero. Thus, the method getNodeAt will be invoked. The second assert statement in getNodeAt will cause an AssertionError, because currentNode is null.

10. Either version of the while statement would control the loop in toArray correctly, assuming the rest of the class is correct. Testing both index and currentNode in the while statement guards against a mistake somewhere else in the class.

11. The version of toArray that calls getNodeAt performs much more work than the original version. Each call to getNodeAt results in a traversal of the chain of nodes from its first node to the desired node. The original version of toArray traverses the chain only once.

12. **a.** The method getEntry in the class AList retrieves any entry from a list in O(1) time, because the list's entries are stored in an array. The corresponding method in LList must traverse a chain of linked nodes to retrieve a list entry. It is an O(*n*) operation.

b. Instead of calling getEntry, we invoke the method toArray to get all of the list's entries.

```
public static void displayList(ListInterface<String> list)
{
   int numberOfEntries = list.getLength();
   System.out.println("The list contains " + numberOfEntries +
                      " entries, as follows:");
   Object[] listArray = list.toArray();
   for (int index = 0; index < listArray.length; index++)
   {
      System.out.print(listArray[index] + " is entry " + (index + 1));
   } // end for
   System.out.println();
} // end displayList
```

13. If the list is empty, numberOfEntries is zero. Thus, (givenPosition >= 1) && (givenPosition <= 0) is always false, and the method would then return null. The only way the body of the first if statement can execute is if the list is not empty.

14. The method replace given in this chapter performs more work than an array-based replace because it must traverse the chain to locate the entry to replace. An array-based replace can locate the desired entry directly, given its array index.

15. The methods clear, both add methods, and the method remove would need to be revised to accommodate the addition of a tail reference.

16. The first assertion should be

```
assert (firstNode == null) && (lastNode == null);
```

The second assertion should be

```
assert (firstNode != null) && (lastNode != null);
```

17. O(*n*).

18. O(1) when removing the first entry, or O(*n*) otherwise.

19. O(1) when replacing the first entry, or O(*n*) otherwise.

20. O(1) when retrieving the first entry, or O(*n*) otherwise.

21. O(*n*).

22.
```
private Node getNodeAt(int givenPosition)
{
   assert (firstNode != null) && (1 <= givenPosition) && (givenPosition <= numberOfEntries);
   Node currentNode = firstNode;

   if (givenPosition == numberOfEntries)
      currentNode = lastNode;
   else if (givenPosition > 1) // traverse the chain to locate the desired node
   {
      for (int counter = 1; counter < givenPosition; counter++)
         currentNode = currentNode.getNextNode();

   } // end if

   assert currentNode != null;
   return currentNode;
} // end getNodeAt
```

Chapter 15

Iterators

Contents

Prerequisites

Appendix	E	Exception Handling
Chapter	13	List Implementations That Use Arrays
Chapter	14	A List Implementation That Links Data

Objectives

After studying this chapter, you should be able to

- Describe the concept of an iterator
- Use an iterator to traverse or manipulate a list
- Implement in Java a separate class iterator and an inner class iterator for a list
- Describe the pros and cons of separate class iterators and inner class iterators

An iterator is an object that traverses a collection of data. During the traversal, you can look at the data entries, modify them, add entries, and remove entries. The Java Class Library contains two interfaces, `Iterator` and `ListIterator`, that specify methods for an iterator. While you could add these iterator methods to the operations of the ADT list, you should instead implement them as a distinct class that interacts with the ADT list. This iterator class can be outside of the ADT list or hidden within its implementation. We will explore both of these approaches in this chapter.

What Is an Iterator?

15.1 How would you count the number of lines on this page? You could use your finger to point to each line as you counted it. Your finger would keep your place on the page. If you paused at a particular line, your finger would be on the current line, and there would be a previous line and a next line. If you think of this page as a list of lines, you would be traversing the list as you counted the lines.

VideoNote
Iterators and their use

An **iterator** is a program component that enables you to step through, or **traverse**, a collection of data such as a list, beginning with the first entry. During one complete traversal, or **iteration**, each data item is considered once. You control the progress of the iteration by repeatedly asking the iterator to give you a reference to the next entry in the collection. You also can modify the collection as you traverse it by adding, removing, or simply changing entries.

You are familiar with iteration because you have written loops. For example, if `nameList` is a list of strings, you can write the following `for` loop to display the entire list:

```
int listSize = nameList.getLength();
for (int position = 1; position <= listSize; position++)
   System.out.println(nameList.getEntry(position));
```

Here the loop traverses, or **iterates**, through the entries in the list. Instead of simply displaying each entry, we could do other things to or with it.

15.2 Notice that the previous loop is at the client level, since it uses the ADT operation `getEntry` to access the list. For an array-based implementation of the list, `getEntry` can retrieve the desired array entry directly and quickly. But if a chain of linked nodes represents the list's entries, `getEntry` must move from node to node until it locates the desired one. For example, to retrieve the n^{th} entry in the list, `getEntry` would begin at the first node in the chain and then move to the second node, the third node, and so on until it reached the n^{th} node. At the next repetition of the loop, `getEntry` would retrieve the $n + 1^{st}$ entry in the list by beginning again at the first node in the chain and stepping from node to node until it reached the $n + 1^{st}$ node. This wastes time.

Iteration is such a common operation that we could include it as part of the ADT list. Doing so would enable a more efficient implementation than we were just able to achieve at the client level. Notice that the operation `toArray` of the ADT list performs a traversal. It is an example of a traversal controlled by the ADT. A client can invoke `toArray` but cannot control its traversal once it begins.

But `toArray` only returns the list's entries. What if we want to do something else with them as we traverse them? We do not want to add another operation to the ADT each time we think of another way to use an iteration. We need a way for a client to step through a collection of data and retrieve or modify the entries. The traversal should keep track of its progress; that is, it should know where it is in the collection and whether it has accessed each entry. An iterator provides such a traversal.

Note: Iterators

An iterator is a program component that steps through, or traverses, a collection of data. The iterator keeps track of its progress during the traversal, or iteration. It can tell you whether a next entry exists and, if so, return a reference to it. During one cycle of the iteration, each data item is considered once.

The Interface Iterator

15.3 The package java.util in the Java Class Library contains two interfaces—Iterator and ListIterator—that specify methods appropriate for an iterator. Let's begin by examining the interface Iterator, given in Listing 15-1. Like many of the interfaces we have considered, Iterator specifies a generic type to represent the data type of the entries involved in the iteration. It specifies only three methods—hasNext, next, and remove—that an iterator can have. These methods enable you to traverse a collection of data from its beginning.

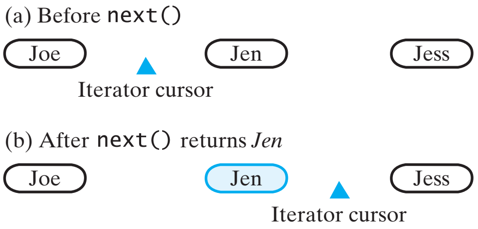
LISTING 15-1 Java's interface java.util.Iterator

```
package java.util;
public interface Iterator<T>
{
   /** Detects whether this iterator has completed its traversal
       and gone beyond the last entry in the collection of data.
       @return true if the iterator has another entry to return */
   public boolean hasNext();

   /** Retrieves the next entry in the collection and
       advances this iterator by one position.
       @return a reference to the next entry in the iteration,
              if one exists
       @throws NoSuchElementException if the iterator had reached the
               end already, that is, if hasNext() is false */
   public T next();

   /** Removes from the collection of data the last entry that
       next() returned. A subsequent call to next() will behave
       as it would have before the removal.
       Precondition: next() has been called, and remove() has not been
           called since then. The collection has not been altered
           during the iteration except by calls to this method.
       @throws IllegalStateException if next() has not been called, or
             if remove() was called already after the last call to
             next().
       @throws UnsupportedOperationException if the iterator does
             not permit a remove operation. */
   public void remove(); // Optional method
} // end Iterator
```

15.4 An iterator marks its current position within a collection much as your finger can point to an entry in a list or to a line on this page. However, in Java, the position of an iterator is not *at* an entry. Instead, it is positioned either before the first entry in the collection, between two entries, or after the last entry. The **next entry** in an iteration is the one right after the position of the iterator's **cursor**. The method hasNext sees whether a next entry exists and returns true or false accordingly.

As long as hasNext returns true, the method next moves the iterator's cursor over the next entry and returns a reference to it, as Figure 15-1 illustrates. Repeated calls to next traverse through the list. As the iteration progresses, the iterator returns entry after entry. Once `next` has returned the last entry in the collection, a subsequent call to it causes a `NoSuchElementException`.

FIGURE 15-1 The effect of a call to `next` on a list iterator



The method `remove` removes the entry that `next` just returned. Contrast this with the ADT list operation `remove`, which removes the entry at a given position within the list. When you implement the `Iterator` interface, you do not have to provide a `remove` operation—it is optional—but you do need to define a method `remove`, because it appears in the interface. Such a method should throw the exception `UnsupportedOperationException` if the client invoked it.

Programming Tip: All of the exceptions mentioned in the interface `Iterator` are runtime exceptions, so no `throws` clause is necessary in any of the methods' headers. In addition, you do not have to write `try` and `catch` blocks when you invoke these methods. However, you will need to import `NoSuchElementException` from the package `java.util`. The other exceptions are in `java.lang`, so no `import` statement is necessary for them.

Using the Interface `Iterator`

15.5 Some details of using an iterator depend on the approach used to implement the iterator methods. A possible, but not optimal, way to provide an ADT with traversal operations is to define them as ADT operations. For example, if `ListInterface` extends `Iterator`, a list object would have iterator methods as well as list methods. Although such an approach provides efficient traversals, it has disadvantages, as you will see.

A better way is to implement the iterator methods within their own class. In one approach, this class is public and separate from the class that implements the ADT in question. The two classes must, of course, interact in some way. We will call an instance of such an iterator class a **separate class iterator**. Alternatively, the iterator class can be a private inner class of the class that implements the ADT. We'll call an instance of this inner class an **inner class iterator**. As you will see, an inner class iterator is usually preferable. This chapter will discuss both approaches.

But first, let's focus on how the methods in the interface `Iterator` behave. A separate class iterator of a list and an inner class iterator of a list are objects distinct from the list. Both of these iterators invoke their methods in the same way. The following examples arbitrarily use a separate class iterator.

15.6 **Example.** Let's look at an example of how the methods `hasNext` and `next` of the interface `Iterator` work with the ADT list. Suppose we create a list of names. We will use strings for the names, but we could instead use instances of the class `Name` that Appendix B presented. The following Java statements create such a list:

```
ListInterface<String> nameList = new LList<String>();
nameList.add("Jamie");
nameList.add("Joey");
nameList.add("Rachel");
```

At this point, nameList contains the strings

Jamie
Joey
Rachel

Suppose that the public class SeparateIterator implements the interface Iterator. To create a separate class iterator for nameList, we create an instance of SeparateIterator, as follows:

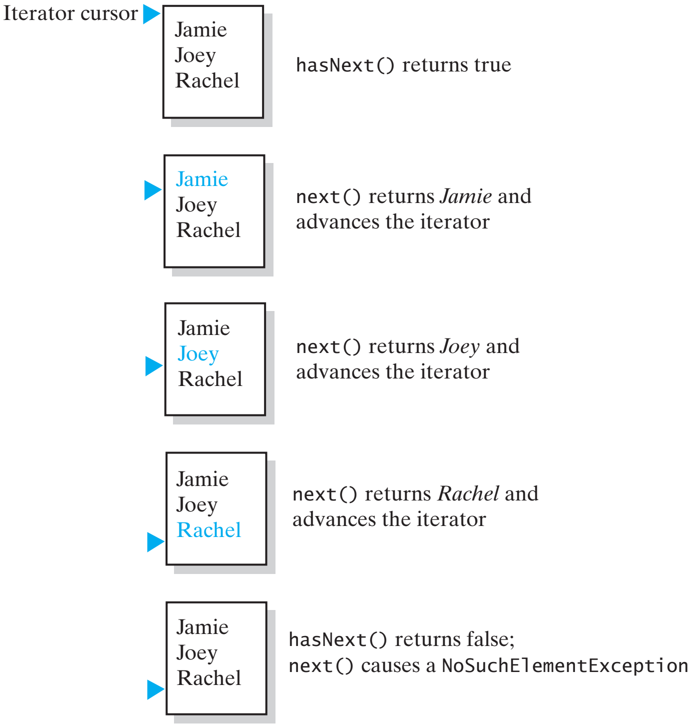
```
Iterator<String> nameIterator = new SeparateIterator<String>(nameList);
```

This invocation of SeparateIterator's constructor connects the iterator nameIterator to the list nameList and positions the iterator just before the first entry in the list. The following sequence of events demonstrates the iterator methods:

- nameIterator.hasNext() returns true because a next entry exists.
- nameIterator.next() returns the string *Jamie* and advances the iterator.
- nameIterator.next() returns the string *Joey* and advances the iterator.
- nameIterator.next() returns the string *Rachel* and advances the iterator.
- nameIterator.hasNext() returns false because the iterator is beyond the end of the list.
- nameIterator.next() causes a NoSuchElementException.

Figure 15-2 illustrates these events.

FIGURE 15-2 The effect of the iterator methods hasNext and next on a list


Iterator cursor
Jamie
Joey
Rachel
hasNext() returns true
Jamie
Joey
Rachel
next() returns *Jamie* and advances the iterator
Jamie
Joey
Rachel
next() returns *Joey* and advances the iterator
Jamie
Joey
Rachel
next() returns *Rachel* and advances the iterator
Jamie
Joey
Rachel
hasNext() returns false;
next() causes a NoSuchElementException

15.7 **Example.** We can use an iterator to display all the entries in a list. The following statements display the strings in the list `nameList`, one per line:

```
Iterator<String> nameIterator = new SeparateIterator<String>(nameList);
while (nameIterator.hasNext())
   System.out.println(nameIterator.next());
```

The iterator `nameIterator` begins just before the first entry in the list. As long as `hasNext` returns true, `next` returns the next entry in the list and advances the iterator. Thus, every entry in the list is retrieved and displayed.

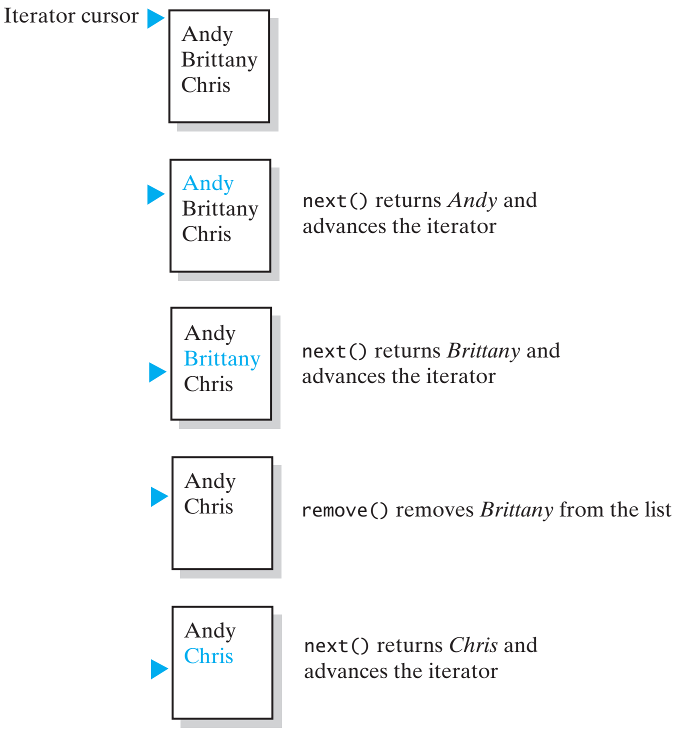
15.8 **Example.** The interface `Iterator` provides an operation to remove an entry from a data collection. This entry is the one returned by the last call to the method `next`. Thus, you must invoke `next` before you can call `remove`.

If `nameList` contains the strings *Andy*, *Brittany*, and *Chris*, and `nameIterator` is defined as in the previous example,

- `nameIterator.next()` returns the string *Andy* and advances the iterator.
- `nameIterator.next()` returns the string *Brittany* and advances the iterator.
- `nameIterator.remove()` removes *Brittany* from the list.
- `nameIterator.next()` returns the string *Chris* and advances the iterator.

Figure 15-3 shows the list during the previous iteration.

FIGURE 15-3 The effect of the iterator methods `next` and `remove` on a list


Iterator cursor
Andy
Brittany
Chris
Andy
Brittany
Chris
next() returns *Andy* and advances the iterator
Andy
Brittany
Chris
next() returns *Brittany* and advances the iterator
Andy
Chris
remove() removes *Brittany* from the list
Andy
Chris
next() returns *Chris* and advances the iterator

15.9 **Example.** The requirement that you invoke next before you call remove results in two situations that cause the exception IllegalStateException. If nameList is defined as in the previous example, and we write

```
nameIterator = new SeparateIterator<String>(nameList);
nameIterator.hasNext();
nameIterator.remove();
```

an IllegalStateException occurs because we did not call next before we called remove. Similarly, if we write

```
nameIterator.next();
nameIterator.remove();
nameIterator.remove();
```

the second remove causes an IllegalStateException because remove had been called already since the most recent call to next.

SELF-TEST

Question 1 Assume that nameList contains the names Jamie, Joey, and Rachel, as it does in Segment 15.6. What output is produced by the following Java statements?

```
Iterator<String> nameIterator = new SeparateIterator<String>(nameList);
nameIterator.next();
nameIterator.next();
nameIterator.remove();
System.out.println(nameIterator.hasNext());
System.out.println(nameIterator.next());
```

Question 2 Assume that nameList is an instance of a class that implements ListInterface, and nameIterator is defined as in the previous question. If nameList contains at least three strings, write Java statements that display the list's third entry.

Question 3 Given nameList and nameIterator as described in the previous question, write statements that display the even-numbered entries in the list. That is, display the second entry, the fourth entry, and so on.

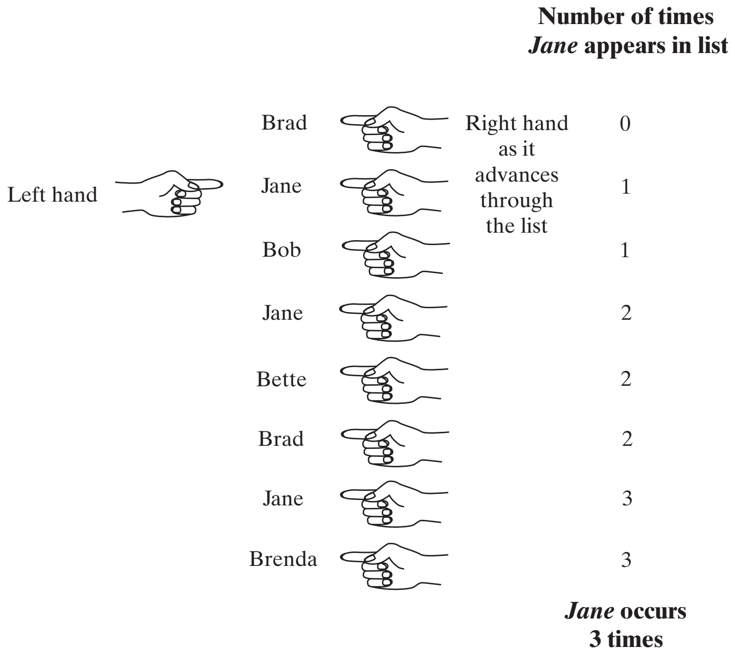
Question 4 Given nameList and nameIterator as described in Question 2, write statements that remove all entries from the list.

Note: Java's interface java.util.Iterator specifies three methods: hasNext, next, and remove. The method hasNext sees whether the iterator has a next entry to return. If so, next returns a reference to it. The method remove can remove the entry last returned by a call to next, or it can simply throw an UnsupportedOperationException if you choose to disallow removals by the iterator.

15.10 **Multiple iterators.** Although the previous examples show one iterator traversing a list, we can have several iterations of the same list in progress simultaneously. For example, imagine a printed list of names that are not distinct and are in no particular order. Running one finger down that list to count the names is like having one iteration of a list. Now suppose that you want to count the number of times each name occurs in the printed list. You can use two fingers, as follows. With your left hand, use one finger to point to the first name in the list. With your right hand, use one finger to point to each of the names in the list, starting with the first one. As you traverse the list with your right hand, compare each name to the name that your left hand marks. In this way, you can count the number of times the first name occurs in the list. Now move your left-hand finger to the next name in the list and use your right hand to point to the beginning of the list. Repeat the previous process to count the number of times that the second name appears in the list. Try it with the names in Figure 15-4. (Since your left hand will encounter Jane three times, you will repeat the computation needlessly unless you are careful. We consid*er thi*s detail a bit later.)

Each of your two fingers can traverse the list independently of the other. They are like two independent iterators that traverse the same list, as you will see in the next example.

FIGURE 15-4 Counting the number of times that *Jane* appears in a list of names



15.11 **Example.** Let's write some code that counts the occurrences of each name in the list in Figure 15-4. Let `nameIterator` correspond to your left hand in the figure. Now we'll define a second iterator, `countingIterator`, that corresponds to your right hand. For each name that your left hand marks, your right hand traverses the entire list to count the occurrences of that name. Thus, we have the following nested loops, assuming that `nameList` is the list:

```
Iterator<String> nameIterator = new SeparateIterator<String>(nameList);
while (nameIterator.hasNext())
{
   String currentName = nameIterator.next();

   int nameCount = 0;

   Iterator<String> countingIterator = new SeparateIterator<String>(nameList);
   while (countingIterator.hasNext())
   {
      String nextName = countingIterator.next();
      if (currentName.equals(nextName))
         nameCount++;
   } // end while

   System.out.println(currentName + " occurs " + nameCount + " times.");
} // end while
```

To reset countingIterator to the list's beginning, we call the constructor again, since Iterator does not have a method for this purpose.

With the names given in Figure 15-4, these statements produce the following output:

```
Brad occurs 2 times.
Jane occurs 3 times.
Bob occurs 1 times.
Jane occurs 3 times.
Bette occurs 1 times.
Brad occurs 2 times.
Jane occurs 3 times.
Brenda occurs 1 times.
```

As you can see, since nameIterator (your left hand) encounters *Brad* twice and *Jane* three times, the computation in the inner loop is repeated needlessly. For example, we compute that *Brad* occurs twice each time nameIterator encounters *Brad*.

If SeparateIterator supports a remove operation, and if we are allowed to destroy the list, we can remove the duplicate entries—and thereby prevent the repeated computations—by modifying the if statement as follows:

```
if (currentName.equals(nextName))
{
   nameCount++;
   if (nameCount > 1)
      countingIterator.remove();
} // end if
```

When nameCount exceeds 1, nextName must be a name that the iterator countingIterator has retrieved from the list more than once. Thus, we remove that entry so that nameIterator will not encounter it. We do so by invoking countingIterator.remove(). The iterator countingIterator then continues with the next entry. Exercise 8 at the end of this chapter considers the case when we cannot destroy the list.

A Separate Class Iterator

We will now examine an implementation of the public class SeparateIterator used in the previous examples. This class implements the interface java.util.Iterator.

15.12 **An outline of the class SeparateIterator.** In the previous examples, we connect an iterator—which is an instance of the class SeparateIterator—with a list by invoking the class's constructor. To accomplish this connection, the class needs a data field that references the list. As you can see in Listing 15-2, the constructor assigns this reference to the field. Also, notice that we make the definition of SeparateIterator independent of a particular implementation of the list, such as AList or LList, by defining the field list as an instance of ListInterface.

In addition to connecting the iterator to the list in question, the constructor initializes it so the iteration will begin at the first entry in the list. To enable this, the class has another data field nextPosition that tracks where we are in the iteration. This field is simply the integer position of the entry in the list that the method `next` last returned. It is convenient to initialize this field to zero.

LISTING 15-2 An outline of the class `SeparateIterator`

```
import java.util.Iterator;
import java.util.NoSuchElementException;
public class SeparateIterator<T> implements Iterator<T>
{
   private ListInterface<T> list;
   private int nextPosition; // position of entry last returned by next()
   private boolean wasNextCalled; // needed by remove

   public SeparateIterator(ListInterface<T> aList)
   {
      list = aList;
      nextPosition = 0;
      wasNextCalled = false;
   } // end constructor

   < Implementations of the methods hasNext, next, and remove go here >
   . . .
} // end SeparateIterator
```

Providing an iterator with a remove operation is optional; however, we shall do so here because the previous examples used one. This desire complicates our class somewhat, because the client must call the method `next` before each call to `remove`. This requirement isn't simply a precondition. The `remove` method must throw an exception if it isn't met. Therefore, we need an additional data field—a boolean flag—that enables `remove` to check whether `next` was called. We name this data field `wasNextCalled`. The constructor initializes this field to false.

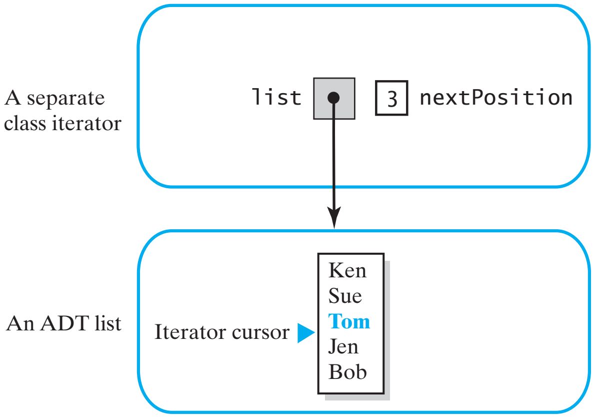
15.13 **The method `hasNext`.** The class `SeparateIterator` has no special access to the private data fields of the class that implements the list. It is a client of the list and so can process the list only by using the list's ADT operations. Figure 15-5 shows a separate class iterator with a reference to a list but with no knowledge of the list's implementation. The implementations of the iterator methods will use methods specified in `ListInterface`. The resulting implementations are rather straightforward but take longer to execute, in general, than the implementation of an inner class iterator. For example, the method `hasNext` calls the list's `getLength` method:

```
public boolean hasNext()
{
   return nextPosition < list.getLength();
} // end hasNext
```

SELF-TEST

Question 5 What does the method `hasNext` return when the list is empty? Why?

FIGURE 15-5 A separate class iterator with a reference to an ADT, an indicator of its position within the iteration, and no knowledge of the ADT's implementation



15.14 **The method `next`.** As long as the iteration has not ended—that is, as long as `hasNext` returns true—the method `next` retrieves the iteration's next entry by calling the list's `getEntry` method. If, however, the iteration has ended, `next` throws an exception.

```
public T next()
{
   if (hasNext())
   {
      wasNextCalled = true;
      nextPosition++;
      return list.getEntry(nextPosition);
   }
   else
      throw new NoSuchElementException("Illegal call to next(); " +
                                       "iterator is after end of list.");
} // end next
```

Since `nextPosition` begins at zero, we must increment it before passing it to `getEntry`. Doing so advances the iterator as required. Notice that we also set the field `wasNextCalled` to true so that the method `remove` can tell that `next` was called.

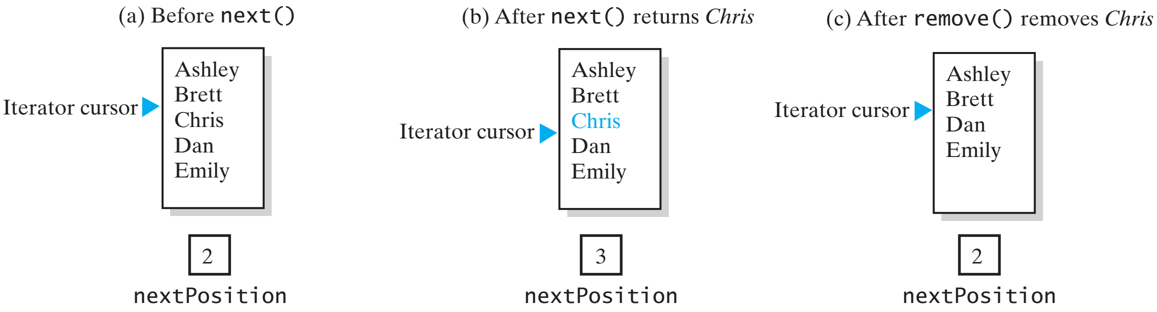
Question 6 The work performed by the method `next` depends upon the implementation of the ADT list that is ultimately used. For which implementation of the list, array-based or linked, will `next` use the most execution time? Why?

15.15 **The method `remove`.** The iterator's method `remove` removes from the list the entry that the most recent call to `next` returned. If `next` was not called, or if `remove` has been called since the last call to `next`, `remove` throws an `IllegalStateException`. The class's data field `wasNextCalled` helps us to implement this aspect of the method. If the field is true, we know that `next` has been called. Then, by setting the field to false, we enable a subsequent invocation of `remove` to require another call to `next`.

The field `nextPosition` is the position of the entry just returned by `next`, so it is the position of the entry to be removed. Thus, we pass it to the list's `remove` method. Then, since a subsequent call to `next` must behave as it would have before the removal, we must decrement `nextPosition`.

Figure 15-6 shows a list and the field `nextPosition` just before the call to `next`, just after the call to `next` but before the call to `remove`, and just after the call to `remove`. Notice in Part *b* that `next` increments `nextPosition` and then returns a reference to *Chris*, the entry at that position and the next entry in the iteration. A call to `remove` in Part *c* removes the entry—*Chris*—at `nextPosition`. Afterwards, the next entry—*Dan* in the figure—moves to the next lower-numbered position in the list. Thus, `remove` must decrement `nextPosition` so that a subsequent call to `next` will return *Dan*.

FIGURE 15-6 A list and `nextPosition` (a) just before the call to `next`; (b) just after the call to `next` but before the call to `remove`; (c) after the call to `remove`



The following implementation of `remove` reflects this discussion.

```
public void remove()
{
   if (wasNextCalled)
   {
      // nextPosition was incremented by the call to next(), so
      // it is the position number of the entry to be removed
      list.remove(nextPosition);
      nextPosition--;        // a subsequent call to next() must be
                             // unaffected by this removal
      wasNextCalled = false; // reset flag
   }
   else
      throw new IllegalStateException("Illegal call to remove(); " +
                                      "next() was not called.");
} // end remove
```

Note: Separate class iterators

A separate class iterator must access an ADT's data by using the public methods of the ADT. However, certain ADTs, such as a stack, do not provide sufficient public access to their data to make such an iterator possible. In addition, the typical separate class iterator takes longer to perform its operations than do other kinds of iterators because of the indirect access to the ADT's data. On the other hand, the implementation of a separate class iterator is usually straightforward. You can also have several independent separate class iterators in existence at the same time for a given ADT.

To provide an iterator for an ADT's implementation that exists and cannot be altered, you might need to define a separate class iterator.

An Inner Class Iterator

15.16 By using separate class iterators, you can have multiple and distinct iterations of a list exist simultaneously. However, separate class iterators belong to a public class, so they can access the list's data fields only indirectly via ADT operations. As a result, the iterations take more time than those performed by other kinds of iterators. For ADTs other than a list, a separate class iterator might have insufficient access to the data fields to perform an iteration.

VideoNote
Alternative iterator implementations

A desirable alternative is to define the iterator class as an inner class of the ADT. Because the resulting iterator objects are distinct from the ADT, you can have multiple iterations in progress at the same time. Moreover, since the iterator belongs to an inner class, it has direct access to the ADT's data fields. For these reasons, an inner class iterator is usually preferable to a separate class iterator.

In this section, we will implement the interface `Iterator` by adding an inner class to each of two implementations of the ADT list. First, we will use a linked implementation of the list but will provide only the iterator operations `hasNext` and `next`. Then we will use an array-based list and support all three operations of `Iterator`.

A Linked Implementation

15.17 To achieve our goal, we must define the methods specified in `Iterator` within a new inner class of the class that implements the ADT list. We'll name this inner class `IteratorForLinkedList` and name the outer class `LinkedListWithIterator`. The outer class will be much like the class `LList` of Chapter 14. However, it needs another method that the client can use to create an iterator. This method, `getIterator`, has the following simple implementation:

```
public Iterator<T> getIterator()
{
   return new IteratorForLinkedList();
} // end getIterator
```

We will show you how to use this method shortly.

To accommodate this new method, we create a new interface—shown in Listing 15-3—that extends `ListInterface` instead of changing it. This interface has all the list methods of `ListInterface` and the new method `getIterator`.

LISTING 15-3 The interface `ListWithIteratorInterface`

```
import java.util.Iterator;
public interface ListWithIteratorInterface<T> extends ListInterface<T>
{
   public Iterator<T> getIterator();
} // end ListWithIteratorInterface
```

Because a class can implement more than one interface, we could define the class `LinkedListWithIterator` without using our new interface. But having this interface enables us to declare an object of type `ListWithIteratorInterface` and know that the object will have the list methods as well as the method `getIterator`.

15.18 **Example: Using the iterator to display a list.** Once again, let's create a list of strings. Since we've defined the interface `ListWithIteratorInterface` that includes the method `getIterator` and the methods of `ListInterface`, we can use it to create the new list:

```
ListWithIteratorInterface<String> myList =
        new LinkedListWithIterator<String>();
```

We add entries to this list using the list's `add` methods, as we have done before.

We now can display the list by using an iterator. We first create an iterator object by invoking the new list method `getIterator`:

```
Iterator<String> myIterator = myList.getIterator();
```

The resulting iterator is ready to access the first entry in the list.

We then write a loop like the one you saw in Segment 15.7:

```
while (myIterator.hasNext())
   System.out.println(myIterator.next());
```

15.19 **An outline of the class.** Listing 15-4 outlines the class `LinkedListWithIterator` with its inner classes `IteratorForLinkedList` and `Node`. We will define the methods declared in the interface `Iterator` within the inner class `IteratorForLinkedList`. However, we will not give iterators the ability to remove entries from the data collection.

LISTING 15-4 An outline of the class `LinkedListWithIterator`

```
import java.util.Iterator;
import java.util.NoSuchElementException;
public class LinkedListWithIterator<T> implements
                                       ListWithIteratorInterface<T>
{
   private Node firstNode;
   private int  numberOfEntries;

   public LinkedListWithIterator()
   {
      clear();
   } // end default constructor

   < Implementations of the methods of the ADT list go here;
     you can see them in Chapter 14, beginning at Segment 14.7 >
   . . .
   public Iterator<T> getIterator()
   {
      return new IteratorForLinkedList();
   } // end getIterator

   < Segment 15.20 begins a description of the following inner class.>
   private class IteratorForLinkedList implements Iterator<T>
   {
      private Node nextNode;
      private IteratorForLinkedList()
      {
         nextNode = firstNode;
      } // end default constructor

      < Implementations of the methods in the interface Iterator go here;
         you can see them in Segments 15.21 through 15.23.>
     . . .
   } // end IteratorForLinkedList

   < Implementation of the private class Node (Listing 3-4 of Chapter 3) goes here. >
   . . .
} // end LinkedListWithIterator
```

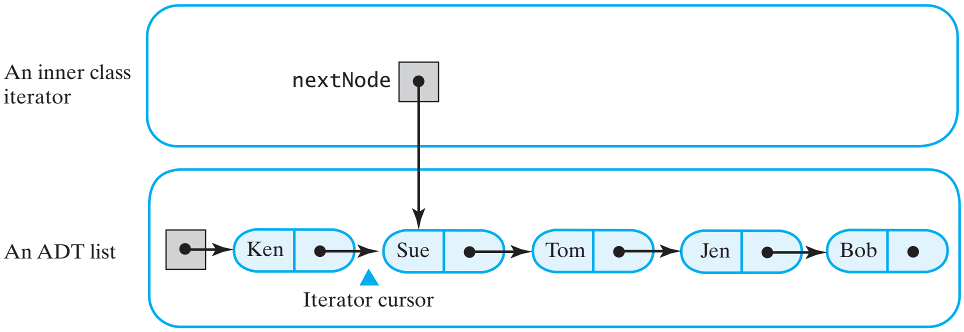
15.20 **The inner class `IteratorForLinkedList`.** As you can see in Listing 15-4, the private inner class `IteratorForLinkedList` has a data field `nextNode` to track an iteration. The constructor initializes this field to `firstNode`, which is a data field of the outer class and references the first node in the chain that contains the list's entries. We cannot position the iterator between nodes, even though we imagine its position to be between entries. Nor can `nextNode` reference the node before the one that `next` will access, because the first node has no node before it. Thus, `nextNode` references the next node in the iteration, that is, the node that the method `next` must access to get the next entry.

Note: An inner class can refer to its outer class's data fields by name alone, if it does not also use the same names for its own definitions. For example, the constructor of the inner class `IteratorForLinkedList` references the field `firstNode` directly by name since no other `firstNode` exists. But we could have written `LinkedListWithIterator.this.firstNode` instead.

Figure 15-7 illustrates an inner class iterator. The iterator has direct access to the ADT's underlying data structure—a linked chain, in this example. Since the data field `nextNode` maintains the current position of the iteration, the iterator can quickly retrieve the next entry in the iteration without first returning to the beginning of the chain.

We now implement the methods of the interface `Iterator` within the inner class. These methods will be public, even though they appear within a private class, because they are public in `Iterator` and will be used by clients of `LinkedListWithIterator`.

FIGURE 15-7 An inner class iterator with direct access to the linked chain that implements the ADT



15.21 **The method `next`.** If the iteration has not ended, `nextNode` references the node containing the next entry in the iteration. Thus, `next` can easily get a reference to this entry. The method then must advance `nextNode` to the next node and return the retrieved list entry. However, `next` must throw an exception if the iteration has already ended.

```
public T next()
{
   if (hasNext())
   {
      Node returnNode = nextNode;          // get next node
      nextNode = nextNode.getNextNode();   // advance iterator

      return returnNode.getData();         // return next entry in iteration
   }
   else
      throw new NoSuchElementException("Illegal call to next(); " +
                                        "iterator is after end of list.");
} // end next
```

15.22 **The method `hasNext`.** After the method `next` returns the last entry in the iteration, `nextNode` will be `null`, since `null` is in the link portion of the last node in the chain. The method `hasNext` can simply compare `nextNode` with `null` to see whether the iteration has ended:

```
public boolean hasNext()
{
   return nextNode != null;
} // end hasNext
```

SELF-TEST

Question 7 What does the method `hasNext` return when the list is empty? Why?

15.23 **The method `remove`.** Even though we decided not to support a `remove` operation for this iterator, we must implement the method because it is declared in the interface `Iterator`. If the client invokes `remove`, the method simply throws the run-time exception `UnsupportedOperationException`. Here is an example of how you can define `remove`:

```
public void remove()
{
   throw new UnsupportedOperationException("remove() is not " +
                                           "supported by this iterator");
} // end remove
```

This exception is in the package `java.lang` and so is included automatically in every Java program. Thus, an `import` statement is unnecessary.

Note: The `remove` method

An iterator that does not allow the removal of items during a traversal is not unusual. In such cases, the `remove` method is defined, but it throws an exception if invoked.

Note: Inner class iterators

An inner class iterator has direct access to an ADT's data, so it typically can execute faster than a separate class iterator. Its implementation is usually more involved, however. Both of these iterators have another advantage: Several iterator objects can be in existence at the same time and traverse a list independently of one another.

Question 8 Given the class `LinkedListWithIterator`, what Java statements create the iterators `nameIterator` and `countingIterator` mentioned in Segment 15.11?

Question 9 Revise the method `displayList`, as shown in Listing 12-2 of Chapter 12, for use in a client of the class `LinkedListWithIterator` by using iterator methods to display the list.

An Array-Based Implementation

15.24 For the array-based implementation, our iterator will support the `remove` method. Let's begin with the array-based implementation of the ADT list, `AList`, as given in Chapter 13. Our new class, whose form is shown in Listing 15-5, has the same data fields and methods as the class `AList`. But since our new class implements the interface `ListWithIteratorInterface`, it also includes the method `getIterator`. Our class also contains the inner class `IteratorForArrayList`, which implements the interface `Iterator`.

LISTING 15-5 An outline of the class `ArrayListWithIterator`

```
import java.util.Iterator;
import java.util.NoSuchElementException;
public class ArrayListWithIterator<T> implements
                                      ListWithIteratorInterface<T>
{
   private T[] list; // array of list entries
   private int numberOfEntries;
   private static final int DEFAULT_INITIAL_CAPACITY = 25;

   public ArrayListWithIterator()
   {
      this(DEFAULT_INITIAL_CAPACITY);
   } // end default constructor

   public ArrayListWithIterator(int initialCapacity)
   {
      numberOfEntries = 0;
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempList = (T[])new Object[initialCapacity];
      list = tempList;
   } // end constructor

   < Implementations of the methods of the ADT list go here;
     you can see them in Chapter 13, beginning at Segment 13.5.>
   . . .
   public Iterator<T> getIterator()
   {
      return new IteratorForArrayList();
   } // end getIterator
   < Segment 15.25 begins a description of the following inner class. >
   private class IteratorForArrayList implements Iterator<T>
   {
      private int nextIndex;
      private boolean wasNextCalled; // needed by remove

      private IteratorForArrayList()
      {
         nextIndex = 0;
         wasNextCalled = false;
      } // end default constructor

      < Implementations of the methods in the interface Iterator go here;
        you can see them in Segments 15.26 through 15.28. >
      . . .
   } // end IteratorForArrayList
} // end ArrayListWithIterator
```

15.25 **The inner class `IteratorForArrayList`.** Just as you can use your finger to keep track of your place on this page, our iterator implementation uses an index to keep track of the iterator's position within the array of list entries. This index, which we call `nextIndex`, is a data field of the private inner class `IteratorForArrayList`. It will be the index of the next entry in the iteration. The constructor initializes `nextIndex` to zero, as Listing 15.5 shows.

Just as you saw earlier in Segments 15.12 and 15.15, providing an iterator with a remove operation requires an additional data field that the `remove` method can use to see whether `next` was called. Again, we name this data field `wasNextCalled`, but here it is defined within the inner class. The constructor initializes this field to false.

15.26 **The method `hasNext`.** The iterator has a next entry to retrieve if `nextIndex` is less than the length of the list. Thus, `hasNext` has the following straightforward implementation:

```
public boolean hasNext()
{
   return nextIndex < numberOfEntries;
} // end hasNext
```

Notice that `hasNext` returns false when the list is empty, that is, when `numberOfEntries` is zero.

15.27 **The method `next`.** The implementation of the method `next` has the same general form as the version given in Segment 15.14 for the separate class iterator. If `hasNext` returns true, `next` returns the next entry in the iteration. Here, the next entry is `list[nextIndex]`. The method also advances the iteration by incrementing `nextIndex` and sets the flag `wasNextCalled` to true. On the other hand, if `hasNext` returns false, `next` throws an exception.

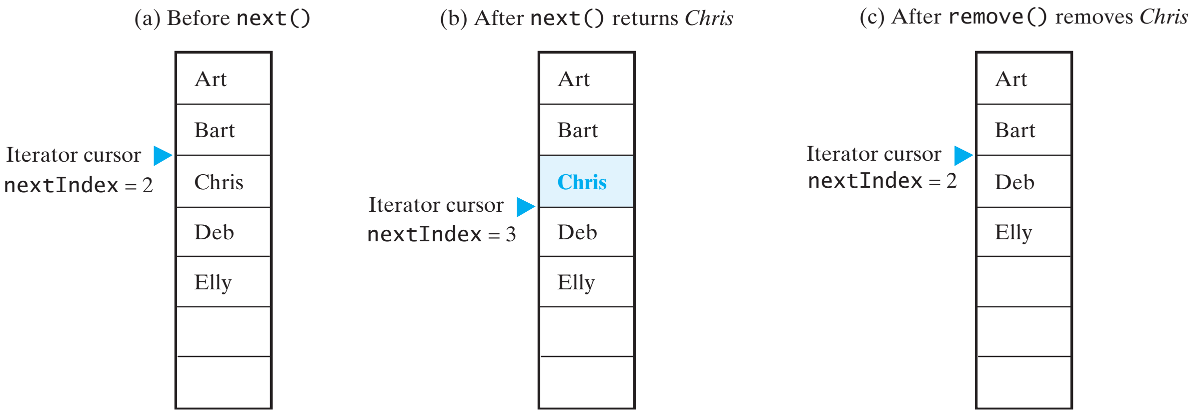
```
public T next()
{
   if (hasNext())
   {
      wasNextCalled = true;
      T nextEntry = list[nextIndex];
```

```
            nextIndex++; // advance iterator
            return nextEntry;
         }
         else
            throw new NoSuchElementException("Illegal call to next(); " +
                                             "iterator is after end of list.");
      } // end next
```

15.28 **The `remove` method.** Removing an entry from the list involves shifting entries within the array `list`. Since we have already developed that code for the list's `remove` method, we will call it instead of accessing the array `list` directly. To do that, we need the position number of the list entry to be removed, rather than its array index. Recall from Segment 12.1 that the position number of an entry in a list begins at 1, so it is 1 larger than the corresponding array index.

Figure 15-8 illustrates how to use `nextIndex` in this implementation. The figure shows the array of list entries and the index `nextIndex` just before the call to `next`, just after the call to `next` but before the call to `remove`, and just after the call to `remove`. Part *b* shows that `next` returns a reference to the next entry, *Chris*, in the iteration and then increments `nextIndex`. The method `remove` must remove this entry from the list. Since `nextIndex` is now 1 larger than the index of *Chris*, it is the position number of the list entry that must be removed. After *Chris* is removed in Part *c*, the next entry—*Deb*—moves to the next lower-numbered position in the array. Thus, `remove` decrements `nextIndex` so that it remains the index of the next entry in the iteration.

FIGURE 15-8 The array of list entries and `nextIndex` (a) just before the call to `next`; (b) just after the call to `next` but before the call to `remove`; (c) after the call to `remove`



The method `remove` has the following implementation within the inner class `IteratorForArrayList`:

```
public void remove()
{
   if (wasNextCalled)
   {
      // nextIndex was incremented by the call to next, so it
      // is the position number of the entry to be removed
      ArrayListWithIterator.this.remove(nextIndex);
      nextIndex--;            // index of next entry in iteration
      wasNextCalled = false; // reset flag
   }
   else
      throw new IllegalStateException("Illegal call to remove(); " +
                                      "next() was not called.");
} // end remove
```

To call the list's method `remove`, which is defined in the outer class, from within the iterator's `remove`, we must precede the name of the list's method with `ArrayListWithIterator.this`.

Question 10 Consider the list and the calls to `next` and `remove` in Figure 15-8.

a. What would a call to `next` return if it occurred after the call to `remove` in Figure 15-8c?
b. What would a call to `next` return if it occurred after the call to `next` in Figure 15-8b?

Question 11 What changes would be necessary to the methods in the inner class `IteratorForArrayList` if its constructor set `nextIndex` to -1 instead of 0?

Why Are Iterator Methods in Their Own Class?

15.29 Both separate class iterators and inner class iterators enable us to have several distinct iterations of a data collection in progress at the same time. Because inner class iterators have direct access to the structure containing the ADT's data, they can execute faster than separate class iterators, and so are usually preferable.

Why didn't we simply consider the iterator operations as additional ADT operations? To answer this question, let's modify the linked implementation of the list given in the previous chapter by including the methods specified in Java's interface `Iterator`. To keep this implementation simple, we will not provide the remove operation specified in `Iterator`. The resulting class, outlined in Listing 15-6, is actually quite similar to the class `LinkedListWithIterator`, described in Segment 15.19, which implements an inner class iterator. The differences between these classes are highlighted.

The inner class `IteratorForLinkedList` shown in Segment 15.19 does not appear in our new class, but its data field `nextNode` and its iterator methods `hasNext`, `next`, and `remove` do appear unchanged. Instead of the inner class's constructor, we have the public method `resetTraversal`, which sets `nextNode` to `firstNode`. We'll call this method before we begin a traversal.

LISTING 15-6 An outline of the class `ListWithTraversal`

```
import java.util.Iterator;
import java.util.NoSuchElementException;
public class ListWithTraversal<T> implements ListInterface<T>,
                                             Iterator<T>
{
   private Node firstNode;
   private int  numberOfEntries;
   private Node nextNode; // node containing next entry in iteration

   public ListWithTraversal()
   {
      clear();
   } // end default constructor
   public final void clear()
   {
      firstNode = null;
      numberOfEntries = 0;
      nextNode = null;
   } // end clear

   < Implementations of the remaining methods of the ADT list go here;
     you can see them in Chapter 14, beginning at Segment 14.7.>
   . . .

   < Implementations of the methods in the interface Iterator go here;
     you can see them in Segments 15.21 through 15.23.>
   . . .
   public void resetTraversal()
   {
      nextNode = firstNode;
   } // end resetTraversal

   < Implementation of the private class Node (Listing 3-4 of Chapter 3) goes here.>
} // end ListWithTraversal
```

15.30 **Example: Traversing a list.** If `myList` is an instance of the previous class `ListWithTraversal`, it has methods of the ADT list as well as the methods in `Iterator`. Thus, if we add strings to `myList` using invocations such as `myList.add("Chris")`, we can display the list as follows:

```
myList.resetTraversal();
while (myList.hasNext())
   System.out.println(myList.next());
```

Invoking `resetTraversal` is essential to set the traversal to the beginning of the list. Notice that you use the list `myList`, not a separate iterator object, to invoke the `Iterator` methods. The reverse was true in Segments 15.7 and 15.18.

Question 12 Revise the method `displayList`, as shown in Listing 12-2 of Chapter 12, for use in a client of the class `ListWithTraversal` by using the approach of the previous example to display the list. Is there any disadvantage to this implementation? Explain.

Question 13 Suppose that you want to omit the method `resetTraversal`.

a. Could the default constructor initialize `nextNode` to `firstNode`? Explain.
b. Could the `add` methods initialize `nextNode` to `firstNode`? Explain.

15.31 **What's wrong with this approach?** Although these traversal methods can execute quickly because they have direct access to the underlying data structure of the list, including them as list operations has disadvantages. Only one traversal can be in progress at a time. Moreover, an operation like `resetTraversal`, which is not in the interface `Iterator`, is necessary to initialize the traversal. The resulting ADT has too many operations; it suffers from **interface bloat**.

With a little additional programming effort, you can organize the iterator methods as an inner class. In doing so, you retain the speed of execution and suffer none of the disadvantages.

The Interface ListIterator

15.32 The Java Class Library provides a second interface for iterators—ListIterator—in the package java.util. This type of iterator enables you to traverse a list in either direction and to modify the list during the iteration. In addition to the three methods hasNext, next, and remove that the interface Iterator specifies, ListIterator contains methods such as hasPrevious, previous, add, and set.

We begin by looking at the interface ListIterator, which is shown in Listing 15-7.

LISTING 15-7 Java's interface java.util.ListIterator

```
package java.util;
public interface ListIterator<T> extends Iterator<T>
{
   /** Detects whether this iterator has gone beyond the last
       entry in the list.
       @return true if the iterator has another entry to return when
               traversing the list forward; otherwise returns false */
   public boolean hasNext();

   /** Retrieves the next entry in the list and
       advances this iterator by one position.
       @return a reference to the next entry in the iteration,
               if one exists
      @throws NoSuchElementException if the iterator had reached the
              end already, that is, if hasNext() is false */
   public T next();

   /** Removes from the list the last entry that either next()
       or previous() has returned.
       Precondition: next() or previous() has been called, but the
             iterator's remove() or add() method has not been called
             since then. That is, you can call remove only once per
             call to next() or previous(). The list has not been altered
             during the iteration except by calls to the iterator's
             remove(), add(), or set() methods.
       @throws IllegalStateException if next() or previous() has not
             been called, or if remove() or add() has been called
             already after the last call to next() or previous()
       @throws UnsupportedOperationException if the iterator does not
             permit a remove operation */
   public void remove(); // Optional method

// The previous three methods are in the interface Iterator; they are
// duplicated here for reference and to show new behavior for remove.

   /** Detects whether this iterator has gone before the first
       entry in the list.
```

```
    @return true if the iterator has another entry to visit when
          traversing the list backward; otherwise returns false */
public boolean hasPrevious();

/** Retrieves the previous entry in the list and moves this
    iterator back by one position.
    @return a reference to the previous entry in the iteration, if
          one exists
    @throws NoSuchElementException if the iterator has no previous
          entry, that is, if hasPrevious() is false */
public T previous();

/** Gets the index of the next entry.
    @return the index of the list entry that a subsequent call to
            next() would return. If next() would not return an entry
            because the iterator is at the end of the list, returns
            the size of the list. Note that the iterator numbers
            the list entries from 0 instead of 1. */
public int nextIndex();

/** Gets the index of the previous entry.
    @return the index of the list entry that a subsequent call to
          previous() would return. If previous() would not return
          an entry because the iterator is at the beginning of the
          list, returns -1. Note that the iterator numbers the
          list entries from 0 instead of 1. */
public int previousIndex();

/** Adds an entry to the list just before the entry, if any,
    that next() would have returned before the addition. This
    addition is just after the entry, if any, that previous()
    would have returned. After the addition, a call to
    previous() will return the new entry, but a call to next()
    will behave as it would have before the addition.
    Further, the addition increases by 1 the values that
    nextIndex() and previousIndex() will return.
    @param newEntry  an object to be added to the list
    @throws ClassCastException if the class of newEntry prevents the
            addition to the list
    @throws IllegalArgumentException if some other aspect of newEntry
            prevents the addition to the list
    @throws UnsupportedOperationException if the iterator does not
            permit an add operation */
public void add(T newEntry); // Optional method
```

```
   /** Replaces the last entry in the list that either next()
       or previous() has returned.
       Precondition: next() or previous() has been called, but the
             iterator's remove() or add() method has not been called
             since then.
       @param newEntry  an object that is the replacement entry
       @throws ClassCastException if the class of newEntry prevents the
               addition to the list
       @throws IllegalArgumentException if some other aspect of newEntry
               prevents the addition to the list
       @throws IllegalStateException if next() or previous() has not
               been called, or if remove() or add() has been called
               already after the last call to next() or previous()
       @throws UnsupportedOperationException if the iterator does not
               permit a set operation */
   public void set(T newEntry); // Optional method
} // end ListIterator
```

15.33 **Observations.** Notice that `ListIterator` extends `Iterator`. Thus, `ListIterator` would include the methods `hasNext`, `next`, and `remove` from the interface `Iterator`, even if we did not write them explicitly. We have done so for your reference and to indicate `remove`'s additional behavior.

The methods `remove`, `add`, and `set` are optional in the sense that you can choose not to provide one or more of these operations. In that case, however, each such operation must have an implementation that throws the exception `UnsupportedOperationException` if the client invokes the operation. An iterator of type `ListIterator` that does not support `remove`, `add`, and `set` is still useful, since it enables you to traverse a list in both directions. It is also easier to implement without these operations.

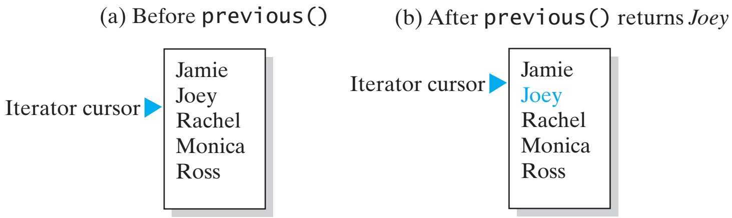
The programming tip given in Segment 15.4 for the interface `Iterator` applies here as well. We repeat it here in terms of `ListInterface`.

Programming Tip: All of the exceptions mentioned in the interface `ListIterator` are run-time exceptions, so no `throws` clause is necessary in any of the methods' headers. In addition, you do not have to write `try` and `catch` blocks when you invoke these methods. However, you will need to import `NoSuchElementException` from the package `java.util`. The other exceptions are in `java.lang`, so no `import` statement is necessary for them.

15.34 **The next entry.** Like `Iterator`, `ListIterator` positions an iterator either before the first entry in a list, between two entries, or after the last entry. Recall that the method `hasNext` sees whether a next entry exists after the iterator's position. If one exists, `next` returns a reference to it and advances the iterator's cursor by one position, as Figure 15-1 illustrated. Repeated calls to `next` step through the list. So far, nothing is different from what you learned about the interface `Iterator` earlier in this chapter.

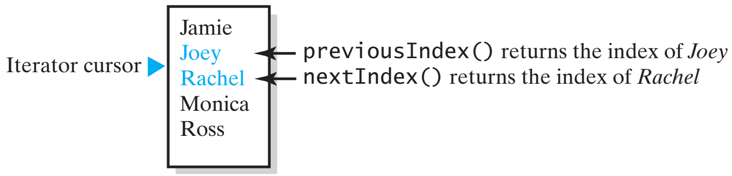
15.35 **The previous entry.** `ListIterator` also provides access to the entry just before the iterator's position—that is, to the previous entry. The method `hasPrevious` sees whether a previous entry exists. If so, the method `previous` returns a reference to it and moves the iterator's cursor back by one position. Figure 15-9 shows the effect of `previous` on a list. Intermixing calls to `previous` and `next` enables you to move back and forth within the list. If you call `next` and then call `previous`, each method returns the same entry. Like `next`, `previous` throws an exception when called after it has completed its traversal of the list.

FIGURE 15-9 The effect of a call to previous on a list



15.36 **The indices of the current and previous entries.** As Figure 15-10 shows, the methods `nextIndex` and `previousIndex` each return the index of the entry that a subsequent call to `next` or `previous`, respectively, would return. Note that the iterator numbers the list's entries beginning with 0, instead of 1 as the ADT list operations do. If a call to `next` would throw an exception because the iterator is at the end of the list, `nextIndex` returns the size of the list. Similarly, if a call to `previous` would throw an exception because the iterator is at the beginning of the list, `previousIndex` returns `-1`.

FIGURE 15-10 The indices returned by the methods nextIndex and previousIndex


Iterator cursor
Jamie
Joey
Rachel
Monica
Ross
previousIndex() returns the index of *Joey*
nextIndex() returns the index of *Rachel*

Note: The interface `ListIterator` specifies nine methods, including the three methods that `Iterator` specifies. They are `hasNext`, `hasPrevious`, `next`, `previous`, `nextIndex`, `previousIndex`, `add`, `remove`, and `set`.

Using the Interface ListIterator

15.37 **Example: Traversals.** Let's look at an example of the methods that work with the current and previous entries and then use it to describe the remaining methods in the interface. We make the following assumptions:

- The interface `ListIterator` is implemented as an inner class of the class that implements the ADT list.
- The iterator includes the operations `add`, `remove`, and `set`.
- The method `getIterator` is added to the ADT list.
- The list `nameList` contains the following names:

  Jess
  Jim
  Josh

- The iterator `traverse` is defined as follows:

```
ListIterator<String> traverse = nameList.getIterator();
```

Since `traverse` is at the beginning of the list, the Java statements

```
System.out.println("nextIndex     " + traverse.nextIndex());
System.out.println("hasNext       " + traverse.hasNext());
System.out.println("previousIndex " + traverse.previousIndex());
System.out.println("hasPrevious   " + traverse.hasPrevious());
```

produce the output

```
nextIndex     0
hasNext       true
previousIndex -1
hasPrevious   false
```

If we then execute the statements

```
System.out.println("next      " + traverse.next());
System.out.println("nextIndex " + traverse.nextIndex());
System.out.println("hasNext   " + traverse.hasNext());
```

the output is

```
next      Jess
nextIndex 1
hasNext   true
```

Finally, the statements

```
System.out.println("previousIndex " + traverse.previousIndex());
System.out.println("hasPrevious   " + traverse.hasPrevious());
System.out.println("previous      " + traverse.previous());
System.out.println("nextIndex     " + traverse.nextIndex());
System.out.println("hasNext       " + traverse.hasNext());
System.out.println("next          " + traverse.next());
```

produce the output

```
previousIndex 0
hasPrevious   true
previous      Jess
nextIndex     0
hasNext       true
next          Jess
```

SELF-TEST

Question 14 Suppose that `traverse` is an iterator as defined in the previous segment, but the contents of `nameList` are unknown. Write Java statements that display the names in `nameList` in reverse order, beginning at the end of the list.

15.38 **Example: The method `set`.** The method `set` replaces the entry that either `next` or `previous` just returned. At the end of the preceding segment, `next` had just returned *Jess*, so

```
traverse.set("Jen");
```

replaces *Jess* with *Jen.* Since *Jess* was the first entry in the list, the list now appears as

Jen
Jim
Josh

Note that this replacement operation does not affect the position of the iterator within the list. Thus, calls to `nextIndex` and `previousIndex`, for example, are not affected. In this case, since the iterator is between *Jen* and *Jim,* `nextIndex` returns 1 and `previousIndex` returns 0. Also note that we can call `set` again; doing so here will replace *Jen.*

Question 15 If the iterator's position is between the first two entries of the previous list, write Java statements that replace *Josh* with *Jon.*

15.39 **Example: The method `add`.** The method `add` inserts an entry into the list just before the iterator's current position. Thus, the insertion is made immediately before the entry, if any, that `next` would have returned before `add` was called and just after the entry, if any, that `previous` would have returned. Note that if the list is empty, `add` inserts a new entry as the only entry in the list.

If the iterator's position is currently between the first two entries of the previous list, the statement

```
traverse.add("Ashley");
```

adds *Ashley* to the list just before *Jim*—that is, at index 1 or, equivalently, at list position 2. After this addition, the list is as follows:

Jen
Ashley
Jim
Josh

A call to `next` at this point returns *Jim,* since `next` would have returned *Jim* had we not called `add`. If, however, we call `previous` instead of `next`, the new entry *Ashley* will be returned. Furthermore, the addition increases by 1 the values that `nextIndex` and `previousIndex` will return. Thus, immediately after the addition, `nextIndex` will return 2 and `previousIndex` will return 1.

Question 16 If the iterator's position is between *Ashley* and *Jim,* write Java statements that add *Miguel* right after *Jim.*

15.40 **Example: The method `remove`.** The behavior of the method `remove` is similar to that of `remove` in the interface `Iterator`, which you saw earlier in this chapter. But in the interface `ListIterator`, `remove` is affected by the method `previous` as well as by `next`. Thus, `remove` removes the list entry that the last call to either `next` or `previous` returned.

If the list contains

Jen
Ashley
Jim
Josh

and the iterator `traverse` is positioned between *Ashley* and *Jim,* the statements

```
traverse.previous();
traverse.remove();
```

remove *Ashley* from the list, since `previous` returns *Ashley.* The iterator's position remains just before *Jim.*

Notice that both `set` and `remove` will throw the exception `IllegalStateException` if neither `next` nor `previous` has been called, or if either `remove` or `add` has been called already since the last call to `next` or `previous`. As you will see in the next section, this behavior complicates the implementation somewhat.

An Array-Based Implementation of the Interface `ListIterator`

15.41 As we did for the interface `Iterator` earlier in this chapter, we will implement the interface `ListIterator` as an inner class of a class that uses an array to represent the ADT list. First, we define an interface in Listing 15-8 that declares the operations of the ADT list and the method `getIterator`. In this case, the method's return type is `ListIterator<T>` instead of `Iterator<T>`.

LISTING 15-8 The interface `ListWithListIteratorInterface`

```
import java.util.ListIterator;
public interface ListWithListIteratorInterface<T> extends
                                                  ListInterface<T>
{
   public ListIterator<T> getIterator();
} // end ListWithListIteratorInterface
```

15.42 **The class that implements the ADT list.** Our class has the same data fields and methods as the class `AList` given in Chapter 13, and includes the method `getIterator`. The class also contains the inner class `IteratorForArrayList`, which implements the interface `ListIterator`. Listing 15-9 shows the form of our new class of lists.

LISTING 15-9 An outline of the class `ArrayListWithListIterator`

```
import java.util.ListIterator;
import java.util.NoSuchElementException;
public class ArrayListWithListIterator<T>
             implements ListWithListIteratorInterface<T>
{
   private T[] list;  // array of list entries
   private int numberOfEntries;
   private static final int DEFAULT_INITIAL_CAPACITY = 25;

   public ArrayListWithListIterator()
   {
      this(DEFAULT_INITIAL_CAPACITY);
   } // end default constructor

   public ArrayListWithListIterator(int initialCapacity)
   {
      numberOfEntries = 0;
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      T[] tempList = (T[])new Object[initialCapacity];
      list = tempList;
   } // end constructor

   < Implementations of the methods of the ADT list go here;
     you can see them in Chapter 13, beginning at Segment 13.5.>
   . . .
```

```
      public ListIterator<T> getIterator()
      {
         return new IteratorForArrayList();
      } // end getIterator

      private class IteratorForArrayList implements ListIterator<T>
      {
         < The description of this implementation begins with Segment 15.43. >
         . . .
      } // end IteratorForArrayList
   } // end ArrayListWithListIterator
```

The Inner Class

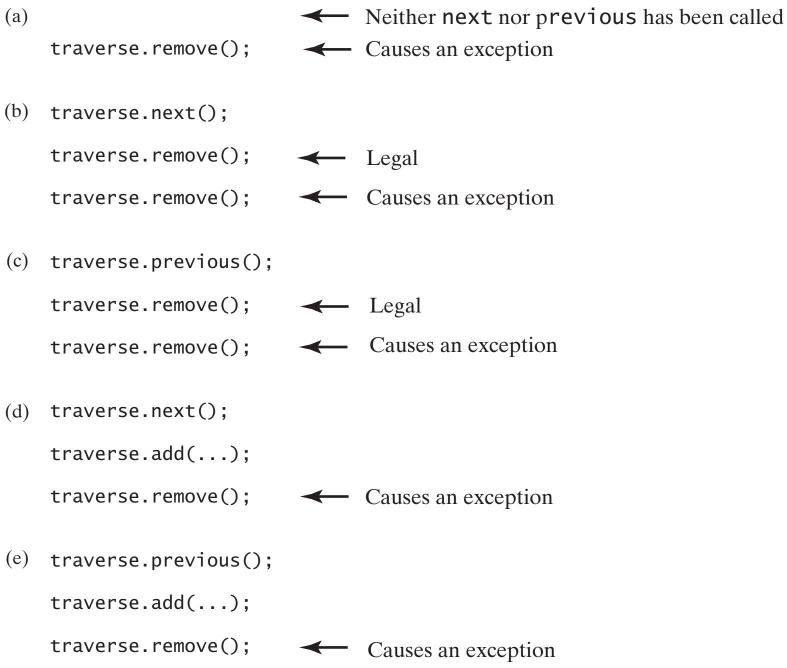
15.43 **The data fields and constructor.** We begin implementing the inner class IteratorForArrayList by thinking about how the methods remove and set will throw the exception IllegalStateException. Both of these methods throw this exception for the same reasons, that is, if either

- next or previous was not called or
- remove or add has been called since the last call to next or previous

Figure 15-11 shows calls to remove in various contexts that cause an IllegalStateException.

This aspect of the implementation might be intimidating at first, but it need not be difficult. When we implemented Iterator's remove method in Segment 15.28, we tested the boolean data field wasNextCalled to see whether next had been called. We could do that here and define analogous

FIGURE 15-11 Possible contexts in which the method remove of the iterator traverse throws an exception when called



```
(a)                          ← Neither next nor previous has been called
      traverse.remove();     ← Causes an exception

(b)   traverse.next();
      traverse.remove();     ← Legal
      traverse.remove();     ← Causes an exception

(c)   traverse.previous();
      traverse.remove();     ← Legal
      traverse.remove();     ← Causes an exception

(d)   traverse.next();
      traverse.add(...);
      traverse.remove();     ← Causes an exception

(e)   traverse.previous();
      traverse.add(...);
      traverse.remove();     ← Causes an exception
```

fields for the methods previous and add, but the logic would be more involved than necessary. Instead, let's define a boolean field to indicate whether a call to either remove or set is legal:

```
private boolean isRemoveOrSetLegal;
```

If either remove or set finds this field to be false, it should throw an IllegalStateException. This field should be initialized to false by the constructor. The methods next and previous should set it to true, and the methods add and remove should set it to false.

Both remove and set must know which of next or previous was called so that they can access the correct list entry. Thus, we define a data field to track the last call to these methods and an enumeration to provide values for the field. The following enumeration will suffice:

```
private enum Move {NEXT, PREVIOUS}
```

Since an enumeration is really a class, we define it outside of the inner class IteratorForArrayList, but within ArrayListWithListIterator. The data field then is simply

```
private Move lastMove;
```

In addition to these two data fields, we need a field nextIndex to track the index of the next entry in the iteration. This field is just like the one we described earlier in Segment 15.25. Thus, the inner class begins as follows:

```
private class IteratorForArrayList implements ListIterator<T>
{
   private int nextIndex;
   private boolean isRemoveOrSetLegal;
   private Move lastMove;

   private IteratorForArrayList()
   {
      nextIndex = 0;
      isRemoveOrSetLegal = false;
      lastMove = null;
   } // end default constructor
   . . .
```

15.44 **The method hasNext.** The method hasNext has the same implementation that it had earlier in Segment 15.26. Recall that it returns true if the iterator has not reached the end of the list.

```
public boolean hasNext()
{
   return nextIndex < numberOfEntries;
} // end hasNext
```

15.45 **The method next.** The implementation of next is similar to the one given in Segment 15.27. Here, however, it has different fields to set. We set lastMove to Move.NEXT and isRemoveOrSetLegal to true.

```
public T next()
{
   if (hasNext())
   {
      lastMove = Move.NEXT;
      isRemoveOrSetLegal = true;

      T nextEntry = list[nextIndex];
      nextIndex++;

      return nextEntry;
   }
   else
      throw new NoSuchElementException("Illegal call to next(); " +
                                       "iterator is after end of list.");
} // end next
```

15.46 **The methods hasPrevious and previous**. The methods hasPrevious and previous have implementations that are analogous to those of hasNext and next, respectively.

```
public boolean hasPrevious()
{
   return (nextIndex > 0) && (nextIndex <= numberOfEntries);
} // end hasPrevious

public T previous()
{
   if (hasPrevious())
   {
      lastMove = Move.PREVIOUS;
      isRemoveOrSetLegal = true;

      nextIndex--;
      return list[nextIndex];
   }
   else
      throw new NoSuchElementException("Illegal call to previous(); " +
                                       "iterator is before beginning of list.");
} // end previous
```

15.47 **The methods nextIndex and previousIndex**. The method nextIndex returns either the index of the entry that the method next would return if called or the size of the list if the iterator is after the end of the list.

```
public int nextIndex()
{
   int result;

   if (hasNext())
      result = nextIndex;
   else
      result = numberOfEntries;

   return result;
} // end nextIndex
```

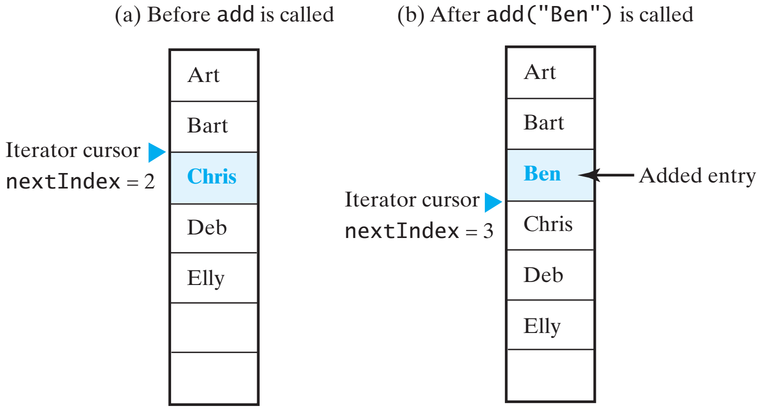
The method previousIndex returns either the index of the entry that the method previous would return if called or -1 if the iterator is before the beginning of the list.

```
public int previousIndex()
{
   int result;

   if (hasPrevious())
      result = nextIndex - 1;
   else
      result = -1;

   return result;
} // end previousIndex
```

15.48 **The method add**. The method add inserts an entry into the list just before the iterator's current position, that is, immediately before the entry in list[nextIndex], as Figure 15-12 illustrates. To avoid duplicate code and effort, we call the list's add method to add an entry at position nextIndex + 1 within the list. Recall that entries after the new entry will be shifted and renumbered. Therefore, we need to increment nextIndex so that it will continue to mark the entry that a subsequent call to next would return. If we increment nextIndex before calling add, we can pass nextIndex to add as the position of the insertion. Thus, add has the following implementation:

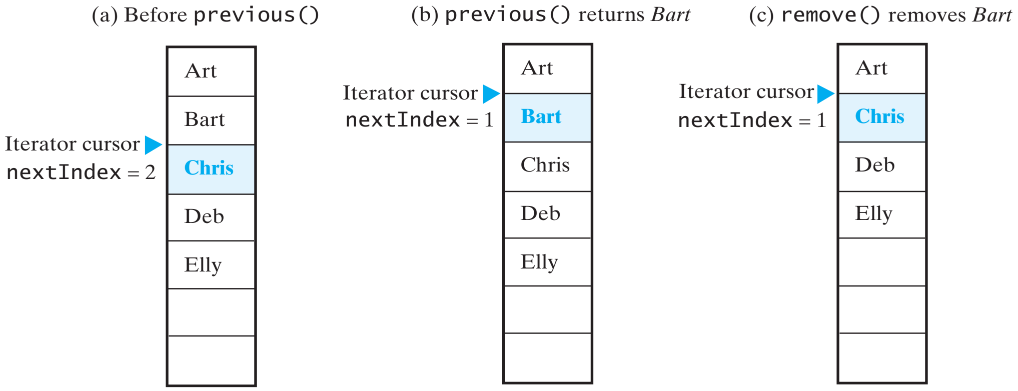
```
public void add(T newEntry)
{
   isRemoveOrSetLegal = false;
   nextIndex++;
   ArrayListWithListIterator.this.add(nextIndex, newEntry);
} // end add
```

FIGURE 15-12 The array of list entries and `nextIndex` (a) just before the call to `add`; (b) just after the call to `add`



15.49 **The method `remove`.** The logic for the `remove` method when a call to `next` precedes its call is somewhat like the logic for the `remove` method in the interface `Iterator`, which you saw in Segment 15.28. Recall that Figure 15-8 illustrated the array of list entries and the index `nextIndex` before and after the calls to `next` and `remove`. Figure 15-13 provides a similar illustration, showing what happens when a call to `previous` precedes the call to `remove`. In Part *b*, `previous` returns a reference to the previous entry—*Bart*—in the iteration and decrements `nextIndex`. The method `remove` must remove this entry from the list. Notice that `nextIndex` is now 1 smaller than the position number of the list entry that must be removed. After the entry *Bart* has been removed in Figure 15-13c, the next entry—*Chris*—moves to the next lower-numbered position in the array. Thus, `nextIndex` remains the index of the next entry in the iteration and so is unchanged.

FIGURE 15-13 The array of list entries and `nextIndex` (a) just before the call to `previous`; (b) just after the call to `previous` but before the call to `remove`; (c) after the call to `remove`



Remember, remove must throw an exception if the field isRemoveOrSetLegal is false. If the field is true, the method must set it to false. An implementation of remove follows:

```
public void remove()
{
   if (isRemoveOrSetLegal)
   {
      isRemoveOrSetLegal = false;

      if (lastMove.equals(Move.NEXT))
      {
         // next() called, but neither add() nor remove() has been
         // called since

         // nextIndex is 1 more than the index of the entry returned
         // by next(), so it is the position number of the entry
         // to be removed
         ArrayListWithListIterator.this.remove(nextIndex);
         nextIndex--;
      }
      else
      {
         // previous() called, but neither add() nor remove() has been
         // called since
         assert lastMove.equals(Move.PREVIOUS);

         // nextIndex is the index of the entry returned by previous(),
         // so it is 1 less than the position number of the entry
         // to be removed
         ArrayListWithListIterator.this.remove(nextIndex + 1);
      } // end if
   }
   else
      throw new IllegalStateException("Illegal call to remove(); " +
                                      "next() or previous() was not called, OR "
                                      "add() or remove() called since then.");
} // end remove
```

15.50 **The method set**. The method set replaces the last entry in the list that either next or previous has returned. It uses nextIndex, as updated by either of the methods next or previous. Since the method next returns list[nextIndex] and then increments nextIndex, the method set would replace the object in list[nextIndex - 1] after a call to next. Likewise, since previous decrements nextIndex and then returns list[nextIndex], the method set would replace list[nextIndex] after a call to previous.

The following implementation of set reflects these observations and uses the same logic that we used in remove to see whether to throw IllegalStateException:

```
public void set(T newEntry)
{
   if (isRemoveOrSetLegal)
   {
      if (lastMove.equals(Move.NEXT))
         list[nextIndex - 1] = newEntry;
      else
      {
         assert lastMove.equals(Move.PREVIOUS);
         list[nextIndex] = newEntry;
      } // end if
   }
   else
```

```
         throw new IllegalStateException("Illegal call to set(); " +
                                         "next() or previous() was not called, OR "
                                         "add() or remove() called since then.");
   } // end set
```

Note: Implementing the entire interface `ListIterator` as an inner class is easier when the associated ADT has an array-based implementation rather than a linked implementation. (See Exercise 17.)

Note: An iterator of type `ListIterator` is simpler to implement when it does not support the operations `add`, `remove`, and `set`. Such an iterator is useful, as it enables you to traverse a list in both directions. We leave this implementation as an exercise.

Java Class Library: The Interface `Iterable`

In a sense, this entire chapter has been about the Java Class Library, since the interfaces `Iterator` and `ListIterator` are components of it. This last section introduces the interface `Iterable` and shows its relation to for-each loops and the interface `java.util.List`.

15.51 The package `java.lang` of the Java Class Library contains the interface `Iterable`. This interface declares only one method, as Listing 15-10 shows.

LISTING 15-10 The interface `java.lang.Iterable`

```
package java.lang;
public interface Iterable<T>
{
   /** @return an iterator for a collection of objects of type T */
   Iterator<T> iterator()
} // end Iterable
```

The method `iterator` returns an iterator that adheres to the interface `Iterator`. This method has the same purpose as our method `getIterator`, as declared in the interface `ListWithIteratorInterface` in Segment 15.17. Recall that our classes `LinkedListWithIterator` (Segment 15.19) and `ArrayListWithIterator` (Segment 15.24) implement `ListWithIteratorInterface`. These classes could certainly implement the method `iterator` in addition to `getIterator`. To do so, we could modify the definition of `ListWithIteratorInterface`, as shown in Listing 15.11.

LISTING 15-11 The interface `ListWithIteratorInterface` modified to extend `Iterable`

```
import java.util.Iterator;
public interface ListWithIteratorInterface<T> extends ListInterface<T>,
                                                      Iterable<T>
{
   public Iterator<T> getIterator();
} // end ListWithIteratorInterface
```

Declaring the method iterator explicitly within the interface is permissible but not necessary, because the interface extends Iterable. Since Iterable is in java.lang, no import statement is needed for it.

As both iterator and getIterator have the same purpose, and since we have already implemented getIterator, the implementation of iterator should call getIterator.

Iterable and for-each loops

15.52 A class that implements the interface Iterable has a distinct advantage over classes that do not: You can use a for-each loop to traverse the objects in an instance of such a class. For example, suppose that we have ListWithIteratorInterface and LinkedListWithIterator, as described in the previous segment. That is, the class LinkedListWithIterator implements Iterable. You can use a for-each loop to display the items in an instance of this class.

Let's form a list, as follows:

```
ListWithIteratorInterface<String> nameList =
                                   new LinkedListWithIterator<String>();
nameList.add("Joe");
nameList.add("Jess");
nameList.add("Josh");
nameList.add("Jen");
```

The statements

```
for (String name : nameList)
   System.out.print(name + " ");
System.out.println();
```

then produce the following output:

```
Joe Jess Josh Jen
```

Programming Tip: A class that defines an inner class iterator should implement the interface Iterable. A client of the class then can use a for-each loop to traverse the objects in an instance of the class.

The Interface List Revisited

15.53 The interface java.util.List that we described in Segment 12.12 of Chapter 12 extends the interface Iterable, so it has the method iterator. Additionally, List declares the following methods related to iterators:

```
public ListIterator<T> listIterator(int index);
public ListIterator<T> listIterator();
```

Each of the listIterator methods returns an iterator whose methods are specified in the interface ListIterator. The iterator returned by the first version of listIterator begins at the list entry indicated by index, where zero indicates the first entry in the list. The second version of this method has the same effect as listIterator(0).

Since the classes ArrayList, LinkedList, and Vector of the package java.util implement the interface List, they have these two listIterator methods as well as the method iterator.

Chapter Summary

- The interface `Iterator` specifies three methods: `hasNext`, `next`, and `remove`. An iterator that implements this interface need not provide a `remove` operation. Instead, the method `remove` would throw the exception `UnsupportedOperationException`.
- The interface `ListIterator` specifies nine methods, including the three methods that `Iterator` specifies. They are `hasNext`, `next`, `hasPrevious`, `previous`, `nextIndex`, `previousIndex`, `add`, `remove`, and `set`. The methods `add`, `remove`, and `set` are optional in the sense that they can throw the exception `UnsupportedOperationException` instead of affecting the list.
- You can implement each of the interfaces `Iterator` and `ListIterator` as its own class. This class could be an inner class of the class that implements the ADT in question, or it could be public and separate from the ADT's class.
- An inner class iterator enables you to have several independent iterators that traverse a collection. It also allows the iterator direct access to the underlying data structure, so its implementation can be efficient.
- A separate class iterator also allows multiple and distinct iterations to exist simultaneously. However, since the iterator can access the list's data fields only indirectly via ADT operations, the iteration takes more time than one performed by an inner class iterator. On the other hand, the implementation is usually straightforward.
- Certain ADTs do not provide sufficient public access to their data to make a separate class iterator possible. However, to provide an iterator for an ADT's implementation that exists and cannot be altered, you might need to define a separate class iterator.

Programming Tips

- All of the exceptions mentioned in the interfaces `Iterator` and `ListIterator` are run-time exceptions, so no `throws` clause is necessary in any of the methods' headers. In addition, you do not have to write `try` and `catch` blocks when you invoke these methods. However, you will need to import `NoSuchElementException` from the package `java.util`. The other exceptions are in `java.lang`, so no `import` statement is necessary for them.
- A class that defines an inner class iterator should implement the interface `Iterable`. A client of the class can then use a for-each loop to traverse the objects in an instance of the class.

Exercises

1. Suppose that `nameList` is a list that contains the following strings: *Kyle, Cathy, Sam, Austin, Sara.* What output is produced by the following sequence of statements?

```
Iterator<String> nameIterator = nameList.getIterator();
System.out.println(nameIterator.next());
nameIterator.next();
System.out.println(nameIterator.next());
nameIterator.remove();
System.out.println(nameIterator.next());
displayList(nameList);
```

2. Repeat Exercise 1, but instead use the following statements:

```
Iterator<String> nameIterator = nameList.getIterator();
nameIterator.next();
nameIterator.remove();
nameIterator.next();
nameIterator.next();
nameIterator.remove();
System.out.println(nameIterator.next());
displayList(nameList);
System.out.println(nameIterator.next());
System.out.println(nameIterator.next());
```

3. Suppose that `nameList` is a list of at least one string and that `nameIterator` is defined as follows:

```
Iterator<String> nameIterator = nameList.getIterator();
```

Write Java statements that use `nameIterator` to display only the last string in the list.

4. Given `nameList` and `nameIterator` as described in Exercise 3, write statements that display all strings in the list from last to first.

5. Given `nameList` and `nameIterator` as described in Exercise 3, write statements that use `nameIterator` to remove all the entries from the list.

6. Given `nameList` and `nameIterator` as described in Exercise 3, write statements that remove all occurrences of the string *CANCEL* from the list.

7. Given `nameList` and `nameIterator` as described in Exercise 3, write statements that remove any duplicates in the list.

8. Given `nameList` and `nameIterator` as described in Exercise 3, write statements that count the number of times each string occurs in the list, without altering the list and without repeating the computations.

9. Suppose that `aList` and `bList` are instances of `java.util.ArrayList`. Use two iterators to find and display all the objects that are common to both lists. Do not alter the contents of either list.

10. Assume that `aList` and `bList` are instances of `java.util.ArrayList` that contain `Comparable` objects in order from smallest to largest. Use two iterators to move the objects from `bList` to the appropriate locations in `aList`. When you are done, the objects in `aList` should be in order, and `bList` should be empty.

11. Revise the class `SeparateIterator` outlined in Segment 15.12 so that it does not support a remove operation. Simplify the class as much as possible.

12. Imagine a class that implements the interface `ListWithIteratorInterface`, as given in Listing 15-11 of Segment 15.51. Suppose that `aList` is an instance of this class and contains `Comparable` objects in no particular order. Using an iterator, implement the following two methods within the class:

a. `getMin` returns the smallest object in the list
b. `removeMin` removes and returns the smallest object in the list

13. Repeat the previous exercise, but use a for-each loop instead of an iterator.

14. Suppose that `nameList` is a list that contains the following strings: *Kyle, Cathy, Sam, Austin, Sara.* What output is produced by the following sequence of statements?

```
ListIterator<String> nameIterator = nameList.getIterator();
System.out.println(nameIterator.next());
nameIterator.next();
nameIterator.next();
System.out.println(nameIterator.next());
nameIterator.set("Brittany");
nameIterator.previous();
nameIterator.remove();
System.out.println(nameIterator.next());
displayList(nameList);
```

15. Repeat the previous exercise, but instead use the following statements:

```
ListIterator<String> nameIterator = nameList.getIterator();
nameIterator.next();
nameIterator.remove();
nameIterator.next();
nameIterator.next();
nameIterator.previous();
nameIterator.remove();
System.out.println(nameIterator.next());
nameIterator.next();
nameIterator.set("Brittany");
System.out.println("Revised list:");
displayList(nameList);
System.out.println(nameIterator.previous());
System.out.println(nameIterator.next());
```

16. Given a list of strings and an iterator `nameIterator` whose data type is `ListIterator`, write statements that add the string *Bob* after the first occurrence of the string *Sam*.

17. If you wanted to implement the interface `ListIterator` as an inner class iterator by using a linked implementation, what difficulties would you face?

18. Implement an iterator for the array-based implementation of a bag, using an inner class. Include a `remove` operation.

19. Repeat the previous exercise for the linked implementation of a bag.

20. If you were to add an iterator to the ADT stack, should the iterator support the `remove` operation?

21. Implement an iterator for the linked implementation of a stack, using an inner class.

22. Implement an iterator for the array-based implementation of a stack, using an inner class.

PROJECTS

1. Revise the class `LinkedListWithIterator` described in Segment 15.19 so that the inner class `IteratorForLinkedList` provides a `remove` operation. You will need another data field `priorNode` to reference the node before the next one.

2. Implement all of the methods in the interface `ListIterator` as a separate class iterator.

3. Implement the interface `ListIterator` as an inner class, but do not support the operations `add`, `remove`, and `set`.

4. Consider a solitaire matching game in which you have a list of random integer values between 10 and 99. You remove from the list any pair of consecutive integers whose first or second digits match. If all values are removed, then you win.

For example, consider the following sequence of 10 integers:

10 82 43 23 89 12 43 84 23 32

The integers in the pair 10 and 82 do not match in either digit and so cannot be removed. However, the integers in the pair 43 and 23 match in the second digit and are removed, leaving the following sequence:

10 82 89 12 43 84 23 32

Continue checking for pairs from 89, the value after the removed pair. No other pairs have matching integers. Now return to the beginning of the list and check the pairs. The integers in the pair 82 and 89 match in the first digit and can be removed:

10 12 43 84 23 32

No other pairs can be removed, so we lose.

Write a program that simulates this game. It should generate 40 random two-digit integers and place them in an instance of `java.util.ArrayList`, using an instance of `ListIterator`. Then, using this iterator, scan the list and remove matching pairs of values. After each pair is removed, use an iterator to display the values remaining in the list.

5. One statistical operation that is sometimes performed on a set of data values is to remove values that are far from the average. Write a program that reads real values from a text file, one per line. Store the data values as `Double` objects in an instance of the class `java.util.ArrayList`. Then

- Use an iterator to compute the average and standard deviation of the values. Display these results.
- Use a second iterator to remove any value that is more than two standard deviations away from the average.
- Use a for-each loop to display the remaining values and compute the new average. Display the new average.

If the data values are x_1, x_2, ..., x_n, their average μ is their sum divided by n, and their standard deviation is

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$

6. Consider the following situation. You create a list, and then you add 10 items to it. You get an iterator to the list and call `next` twice to advance it. You remove the first five items from the list, using the list's `remove` method. You then call the iterator's `remove` method, expecting to remove the item last returned by the method `next`. However, this entry has already been removed from the list. Changing the state of the list while using the iterator, as you have done here, may result in unpredictable behavior of the iterator.

Modify the interface `Iterator` so that the methods will throw the exception `StateChangedException` if the state of the list is changed after the iterator was created but before the method is called. Modify the implementation of `LinkedListWithIterator` that Project 1 describes to accommodate the changes to `Iterator`.

7. Revise the class `ArrayListWithIterator` outlined in Segment 15.24 so that it extends the class `AList`, as discussed in Chapter 13.

8. Repeat Projects 7 and 8 of Chapter 12, adding an iterator to the ADT recipe.

Answers to Self-Test Questions

1. The output is

```
true
Rachel
```

2.

```
nameIterator.next();
nameIterator.next();
System.out.println(nameIterator.next());
```

3.

```
nameIterator.next(); // skip first entry; list has > 1 entry
while (nameIterator.hasNext())
{
   System.out.println(nameIterator.next()); // display even-numbered entry
   if (nameIterator.hasNext())
      nameIterator.next();                  // skip odd-numbered entry
} // end while
```

4.

```
while (nameIterator.hasNext())
{
  nameIterator.next();
  nameIterator.remove();
} // end while
```

5. False. When the list is empty, both nextPosition and list.getLength() are zero.

6. Linked. The particular implementation of the list affects the amount of work that the method getEntry must perform. For an array-based implementation, getEntry accesses the required entry directly and immediately. For a linked implementation, getEntry must traverse a chain of nodes to find the desired entry. This takes more time to accomplish than accessing an array entry.

7. False. When the list is empty, firstNode, and therefore nextNode, is null.

8. Create the iterators by writing

```
Iterator<String> nameIterator = nameList.getIterator();
Iterator<String> countingIterator = nameList.getIterator();
```

9.

```
public static void displayListQ9(ListWithIteratorInterface<String> list)
{
   int numberOfEntries = list.getLength();
   System.out.println("The list contains " + numberOfEntries +
                      " entries, as follows:");
   Iterator<String> traverser = list.getIterator();
   int position = 0;
   while (traverser.hasNext())
   {
      position++;
      System.out.println(traverser.next() + " is entry " + position);
   } // end while

   System.out.println();
} // end displayListQ9
```

10. **a.** Deb.
b. Deb.

11. Originally, nextIndex is the index of the next entry that next will return. The change makes nextIndex the index of the last entry that next returned. Thus, the following changes are needed:

- hasNext should compare nextIndex to numberOfEntries - 1 instead of numberOfEntries
- next should increment nextIndex before accessing list[nextIndex]
- remove should remove the entry at nextIndex + 1

12.

```
public static void displayListQ12(ListWithTraversal<String> list)
{
   int numberOfEntries = list.getLength();
   System.out.println("The list contains " + numberOfEntries +
                      " entries, as follows:");
   list.resetTraversal();
   int position = 0;
   while (list.hasNext())
   {
      position++;
      System.out.println(list.next() + " is entry " + position);
   } // end while

   System.out.println();
} // end displayListQ12
```

A disadvantage is that if you have an iteration in progress, and you pause it to call displayListQ12, you will not be able to resume your iteration.

13. **a.** No. The default constructor creates an empty list. If it set `nextNode` to `firstNode`, `nextNode` would be set to `null`.

b. Yes, but with a disadvantage. Each addition to the list would set `nextNode` to `firstNode`. After creating a list, you could traverse it. However, the only way you could reset the traversal to the list's beginning would be to add another entry to the list.

14.

```
while (traverse.hasNext())
  traverse.next();

while (traverse.hasPrevious())
  System.out.println(traverse.previous());
```

15.

```
traverse.next();     // return Jim
traverse.next();     // return Josh
traverse.set("Jon"); // replace Josh
```

16.

```
traverse.next();        // return Jim
traverse.add("Miguel"); // add Miguel after Jim
```

This page intentionally left blank

Chapter 16

Sorted Lists

Contents

Prerequisites

Chapter	4	The Efficiency of Algorithms
Chapter	7	Recursion
Chapter	12	Lists
Chapter	14	A List Implementation That Links Data

Objectives

After studying this chapter, you should be able to

- Use a sorted list in a program
- Describe the differences between the ADT list and the ADT sorted list
- Implement the ADT sorted list by using a chain of linked nodes
- Implement the ADT sorted list by using the operations of the ADT list

Chapter 12 introduced you to the ADT list. The entries in a list are ordered simply by their positions within the list. Thus, a list has a first entry, a second entry, and so on. This ADT enables you to order entries according to any criterion you want—alphabetical or chronological, for instance. In fact, Chapter 12 showed you an example that used a list to organize names in alphabetical order. To do so, the client had to determine where in the list a particular entry belonged.

If your application creates a list and then at some point needs to sort the list's entries into numerical or alphabetical order, for example, you can add a sort operation to the ADT list. You can use one of the algorithms given in Chapters 8 and 9 to implement this operation. But when your application requires only sorted data, having an ADT that orders the data for you would be more convenient than the ADT list. The sorted list is such an ADT.

When you either add an entry to or remove an entry from a sorted list, you provide only the entry. You do not specify where in the list the entry belongs or exists. The ADT determines this for you.

This chapter describes the operations of the ADT sorted list, provides examples of using a sorted list, and presents two possible Java implementations. One of these implementations uses the ADT list, but it is not especially efficient. The next chapter addresses the reuse of a class and provides a more efficient implementation of the sorted list as it discusses the use of inheritance.

Specifications for the ADT Sorted List

16.1 The ADT list leaves it up to the client to arrange the objects in a given collection. The client can maintain the objects in any order that meets its needs. Suppose that you want a list of names or other strings that are in alphabetical order. You could certainly use the ADT list for this task, but you would have to determine the position that each string should have within the list. Wouldn't it be more convenient if the list itself alphabetized the entries as you added them? What you need is a different ADT, namely the **sorted list**.

Recall that to use the `add` operation of the ADT list, you must specify both the new entry and its desired position within the list. Such an operation is not desirable for the ADT sorted list, since the sorted list is responsible for organizing its entries. If you were allowed to specify a new entry's position, you might destroy the order of the sorted list's entries. Instead, the `add` operation of the ADT sorted list requires only the new entry. The operation compares the new entry to other entries in the sorted list to determine the new entry's position. Thus, the entries in a sorted list must be objects that can be compared with one another.

What, then, can you place in a sorted list? One possibility is strings, since the class `String` provides a `compareTo` method for comparing two strings. In general, you can have a sorted list of any objects of a class that has a `compareTo` method. As you saw in Segment D.20 of Appendix D and again at the beginning of Chapter 8, such classes implement the interface `Comparable`. Since Java's wrapper classes, such as `Integer` and `Double`, implement the `Comparable` interface, you can place instances of them into a sorted list.

16.2 Let's examine the possible operations for this ADT. For simplicity, we will allow the sorted list to contain duplicate items. Insisting that the sorted list contain only unique items is somewhat more complicated, and we will leave this variation as an exercise.

We've already mentioned that you can add an entry to the sorted list. Since the sorted list determines the position of a new entry, you could ask the ADT for this position. That is, you could ask for the position of an existing entry or for the position in which a proposed entry would occur if you added it to the list. You could also ask the ADT whether it contained a particular entry. And clearly you should be able to remove an entry.

Let's specify these operations more carefully.

Abstract Data Type: Sorted List

Data

- A collection of objects in sorted order and having the same data type
- The number of objects in the collection

Operations

Pseudocode	UML	Description
`add(newEntry)`	`+add(newEntry: T): void`	Task: Adds `newEntry` to the sorted list so that the list remains sorted. Input: `newEntry` is the object to be added. Output: None.
`remove(anEntry)`	`+remove(anEntry: T): boolean`	Task: Removes the first or only occurrence of `anEntry` from the sorted list. Input: `anEntry` is the object to be removed. Output: Returns true if `anEntry` was located and removed, or false if not. In the latter case, the list remains unchanged.
`getPosition(anEntry)`	`+getPosition(anEntry: T): integer`	Task: Gets the position of the first or only occurrence of `anEntry`. Input: `anEntry` is the object to be found. Output: Returns the position of `anEntry` if it occurs in the list. Otherwise, returns the position where `anEntry` would occur in the list, but as a negative integer.

The following operations behave as they do for the ADT list and are described in Chapter 12:

```
getEntry(givenPosition)
contains(anEntry)
remove(givenPosition)
clear()
getLength()
isEmpty()
toArray()
```

16.3 The first two methods are straightforward, but `getPosition` deserves some comment. Given an entry in the sorted list, the method `getPosition` returns the entry's position number within the list, as you would expect. We number the entries beginning with 1, just as we do for the ADT list. But what if the given entry is not in the sorted list? In this case, `getPosition` returns the position number where the entry belongs in the list. The returned number is negative, however, to signal that the entry is not in the list. For example, if `missingObject` is not in the sorted list `sList` but belongs at position 3, `sList.getPosition(missingObject)` would return -3.

The sorted list also has some, but not all, of the operations of an ADT list. We have already mentioned that adding an entry at a given position is not possible, because otherwise the client could destroy the order of the sorted list. For the same reason, the list's `replace` method is not available to a sorted list. The other operations of the ADT list, however, are useful for a sorted list as well, including the ones that retrieve or remove the entry at a given position. The methods `getEntry` and `remove` each have a position number as a parameter, but they will not alter the relative order of the entries in the sorted list.

Although the list's `remove` method returns the object removed from the list, it is not necessary for the sorted list's `remove` method to do so. The client already has at least a copy of this entry to enable it to invoke sorted list's `remove`.

16.4 The Java interface in Listing 16-1 specifies these operations in more detail. The notation `? super T`, which Segment 8.2 introduced, means any superclass of the generic type `T`.

LISTING 16-1 The interface `SortedListInterface`

```
/** An interface for the ADT sorted list.
    Entries in the list have positions that begin with 1.
*/
public interface SortedListInterface<T extends Comparable<? super T>>
{
   /** Adds a new entry to this sorted list in its proper order.
       @param newEntry  the object to be added as a new entry */
   public void add(T newEntry);

   /** Removes a specified entry from this sorted list.
       @param anEntry  the object to be removed
       @return true if anEntry was located and removed */
   public boolean remove(T anEntry);

   /** Gets the position of an entry in this sorted list.
       @param anEntry  the object to be found
       @return the position of the first or only occurrence of anEntry
               if it occurs in the list; otherwise returns the position
               where anEntry would occur in the list, but as a negative
               integer */
   public int getPosition(T anEntry);

   // The following methods are described in Segment 12.7 of Chapter 12
   // as part of the ADT list:

   public T getEntry(int givenPosition);
   public boolean contains(T anEntry);
   public T remove(int givenPosition);
   public void clear();
   public int getLength();
   public boolean isEmpty();
   public T[] toArray();
} // end SortedListInterface
```

Note: The ADT sorted list can add, remove, or locate an entry, given the entry as an argument. The sorted list has several operations that are the same as ADT list operations, namely `getEntry`, `contains`, `remove` (by position), `clear`, `getLength`, `isEmpty`, and `toArray`. However, a sorted list will not let you add or replace an entry by position.

Using the ADT Sorted List

16.5 **Example.** To demonstrate the operations of the ADT sorted list that the previous section specifies, we first create a sorted list of strings. We begin by declaring and allocating the list `nameList`, where we assume that `SortedList` is an implementation of the ADT operations specified by the interface `SortedListInterface`:

```
SortedListInterface<String> nameList = new SortedList<String>();
```

Next, we add names in an arbitrary order, realizing that the ADT will organize them alphabetically:

```
nameList.add("Jamie");
nameList.add("Brenda");
nameList.add("Sarah");
nameList.add("Tom");
nameList.add("Carlos");
```

The sorted list now contains the following entries:

Brenda
Carlos
Jamie
Sarah
Tom

16.6 Assuming the list just given, here are some examples of the ADT operations on the sorted list:

`nameList.getPosition("Jamie")` returns 3, the position of *Jamie* in the list
`nameList.contains("Jill")` returns false, because *Jill* is not in the list
`nameList.getPosition("Jill")` returns –4, because *Jill* belongs at position 4 in the list
`nameList.getEntry(2)` returns *Carlos*, because he is at position 2 in the list

Now remove *Tom* and the first name in the list by writing

```
nameList.remove("Tom");
nameList.remove(1);
```

The list now contains

Carlos
Jamie
Sarah

Removing the last entry, *Tom*, did not change the positions of the other entries in the list, but removing the first entry did. *Carlos* is now at position 1, instead of 2.

Question 1 Suppose that `wordList` is an unsorted list of words. Using the operations of the ADT list and the ADT sorted list, create a sorted list of these words.

Question 2 Assuming that the sorted list you created in the previous question is not empty, write Java statements that

a. Display the last entry in the sorted list.
b. Add the sorted list's first entry to the sorted list again.

A Linked Implementation

VideoNote
The class `SortedLinkedList`

As with all ADTs, you have a choice of several ways in which to implement the sorted list. You could store a sorted list's entries in, for example, an array, a chain of linked nodes, an instance of a vector, or an instance of an ADT list. In this chapter, we will consider a chain of linked nodes and an instance of an ADT list. In the next chapter, we will use inheritance to develop a completely different implementation.

16.7 **An outline of the class.** An implementation that uses a chain of linked nodes to store the entries in a sorted list has several details in common with the linked implementation of the ADT list that you studied in Chapter 14. In particular, it has the same data fields, similar constructors, the same implementations for several of its methods, and the same definition of the inner class `Node`. Thus, we outline in Listing 16-2 a class definition that implements the ADT sorted list.

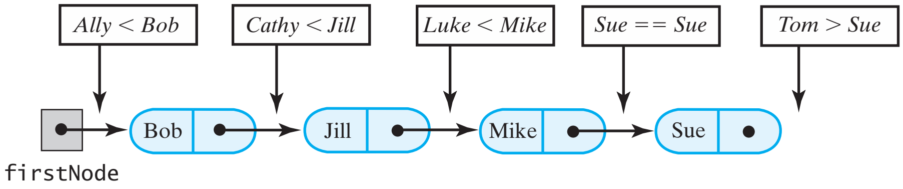
LISTING 16-2 An outline of a linked implementation of the ADT sorted list

```
public class SortedLinkedList<T extends Comparable<? super T>>
             implements SortedListInterface<T>
{
   private Node firstNode; // reference to first node of chain
   private int  numberOfEntries;

   public SortedLinkedList()
   {
      firstNode = null;
      numberOfEntries = 0;
   } // end default constructor

   < Implementations of the sorted list operations go here.>
   . . .

   private class Node
   {
      private T    data;
      private Node next;

      < Constructors >
      . . .
      < Accessor and mutator methods: getData, setData, getNextNode, setNextNode >
      . . .

   } // end Node
} // end SortedLinkedList
```

The Method add

16.8 **Locating the insertion point.** Adding an entry to a sorted list requires that you find where in the list the new entry belongs. Since the entries are sorted, you compare the new entry with the entries in the sorted list until you reach an entry that is not smaller than the new entry. Figure 16-1 depicts a chain of linked nodes, each containing a name, sorted alphabetically. The figure shows where the additional names *Ally*, *Cathy*, *Luke*, *Sue*, and *Tom* would be inserted into the chain and the comparisons that would have to occur to arrive at those locations.

FIGURE 16-1 Places to insert names into a sorted chain of linked nodes



You can see from the figure that, in a string comparison, *Ally* is less than *Bob*, and so it would be inserted at the beginning of the chain. To see where to insert *Luke*, you would find that *Luke* is greater than both *Bob* and *Jill* but less than *Mike*. Thus, *Luke* belongs before *Mike* in the chain. *Sue*, on the other hand, is already in one of the nodes. You would discover that *Sue* is greater than *Bob*, *Jill*, and *Mike* but not greater than *Sue*. So you would insert the new entry *Sue* just before the existing entry *Sue*. Finally, *Tom* is greater than all the current names in the list, so you would add it to the end of the chain.

Note: Given a sorted list with entries in ascending order, you insert a new entry just before the first entry that is not smaller than the new entry.

16.9 **The algorithm.** Recall from Segment 14.10 of Chapter 14 that you handle the addition of a new node to the beginning of a chain differently from an addition at other points in the chain. Adding to the beginning is easy, since `firstNode` references the first node in the chain. To add anywhere else, you need a reference to the node that will ultimately occur before the new node. Thus, while you traverse the chain of linked nodes to discover where the new entry belongs, you must retain a reference to the node prior to the one under consideration.

A high-level algorithm that describes our strategy follows:

```
Algorithm add(newEntry)
// Adds a new entry to the sorted list.

Allocate a new node containing newEntry
Search the chain until either you find a node containing newEntry or you pass the point
   where it should be
Let nodeBefore reference the node before the insertion point
if (the chain is empty or the new node belongs at the beginning of the chain)
   Add the new node to the beginning of the chain
else
   Insert the new node after the node referenced by nodeBefore

Increment the length of the sorted list
```

16.10 **An iterative implementation of `add`.** A Java implementation of the previous algorithm follows. We use a private method, `getNodeBefore`, to search the chain for the node before the insertion point.

```
public void add(T newEntry)
{
   Node newNode = new Node(newEntry);
   Node nodeBefore = getNodeBefore(newEntry);

   if (isEmpty() || (nodeBefore == null)) // add at beginning
   {
      newNode.setNextNode(firstNode);
      firstNode = newNode;
   }
   else                                   // add after nodeBefore
   {
      Node nodeAfter = nodeBefore.getNextNode();
      newNode.setNextNode(nodeAfter);
      nodeBefore.setNextNode(newNode);
   } // end if

   numberOfEntries++;
} // end add
```

16.11 **The private method `getNodeBefore`.** We still need to implement the private method `getNodeBefore`. We will need two references as we traverse the list. Clearly we need a reference to the current node so we can compare its entry to the desired entry. But we also must retain a reference to the previous node, because it is this reference that the method returns. In the following implementation, these references are `currentNode` and `nodeBefore`:

```
/** Finds the node that is before the node that should or does
    contain a given entry.
    @param anEntry  the object to be located
    @return either a reference to the node that is before the node
            that contains or should contain anEntry, or null if
            no prior node exists (that is, if anEntry belongs at
            the beginning of the list) */
private Node getNodeBefore(T anEntry)
{
   Node currentNode = firstNode;
   Node nodeBefore = null;

   while ( (currentNode != null) &&
           (anEntry.compareTo(currentNode.getData()) > 0) )
```

```
      {
         nodeBefore = currentNode;
         currentNode = currentNode.getNextNode();
      } // end while

      return nodeBefore;
   } // end getNodeBefore
```

Recall that the method `compareTo` returns a negative, zero, or positive integer according to whether the comparison is, respectively, less than, equal to, or greater than.

Question 3 In the `while` statement of the method `getNodeBefore`, how important is the order of the two boolean expressions that the operator `&&` joins? Explain.

Question 4 What does `getNodeBefore` return if the sorted list is empty? How can you use this fact to simplify the implementation of the method `add` given in Segment 16.10?

Question 5 Suppose that you use the previous method `add` to add an entry to a sorted list. If the entry is already in the list, where in the list will `add` insert it? Before the first occurrence of the entry, after the first occurrence of the entry, after the last occurrence of the entry, or somewhere else?

Question 6 What would be the answer to the previous question if you changed `>` to `>=` in the `while` statement of the method `getNodeBefore`?

16.12 **Thinking recursively.** Using recursion to process a chain of linked nodes can be an attractive alternative to an iterative approach. The basic concept is easy, but, as you will see, the implementation is more involved, since Java passes objects to methods as references.

Recall from Segment 7.20 of Chapter 7 that you can process the chain's first node and then process the rest of the chain recursively. Thus, to add a new node to a sorted chain of linked nodes, you use the following logic:

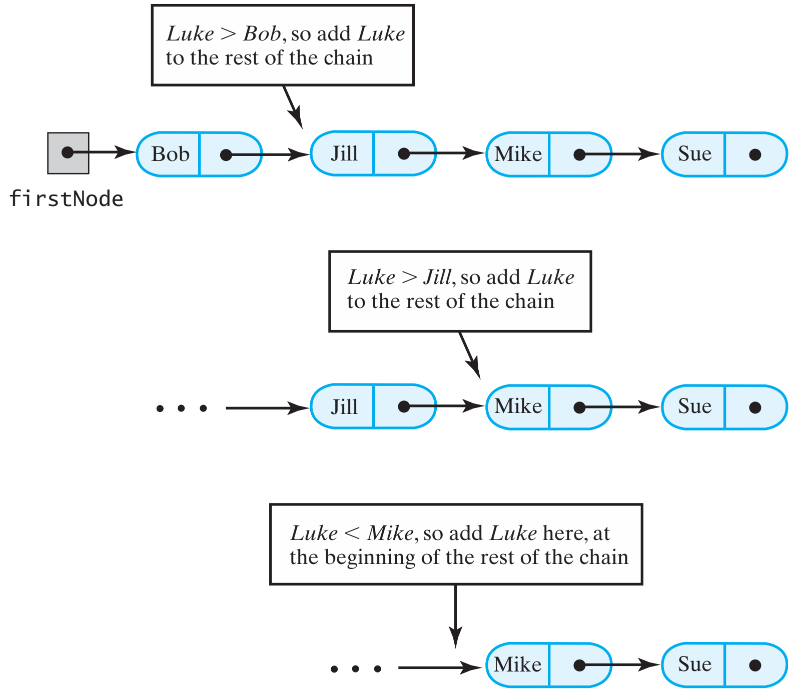
```
if (the chain is empty or the new node belongs at the beginning of the chain)
   Add the new node to the beginning of the chain
else
   Ignore the first node and add the new node to the rest of the chain
```

Figure 16-2 illustrates the logic needed to recursively add the name *Luke* to a sorted chain of names. Since *Luke* is greater than *Bob,* you recursively consider the subchain that begins at *Jill*. *Luke* is also greater than *Jill*, so you now consider the subchain beginning at *Mike*. Finally, *Luke* is less than *Mike*, so you make the actual addition at the beginning of this subchain—that is, before *Mike*. Adding to the beginning of a chain—or subchain—is the base case of this recursion. Happily, the beginning of a chain is the easiest place to make an addition.

If `currentNode` initially references the chain and later references the rest of the chain, we can add some detail to the previous logic, as follows:

```
if ( (currentNode == null) or (newEntry <= currentNode.getData()) )
   currentNode = new Node(newEntry, currentNode)
else
   Recursively add newEntry to the chain beginning at currentNode.getNextNode()
```

FIGURE 16-2 Recursively adding *Luke* to a sorted chain of names


Luke > *Bob*, so add *Luke* to the rest of the chain
firstNode
Bob
Jill
Mike
Sue
Luke > *Jill*, so add *Luke* to the rest of the chain
. . .
Jill
Mike
Sue
Luke < *Mike*, so add *Luke* here, at the beginning of the rest of the chain
. . .
Mike
Sue

16.13 **A recursive implementation of add.** The example in Segment 7.20 displayed the contents of a chain. Since that operation does not alter the chain, its recursive formulation was straightforward. Obviously, in our present situation, the method add does alter the chain. Getting the recursive method to make these changes is the challenge in Java.

Let's look at the recursive implementation of the method add before we describe why it works. You learned in Segment 7.19 that you write a private method to perform the recursion and you write a public method—typically the one that implements the ADT operation—to invoke this private method. Thus, we have the following method definitions:

```
public void add(T newEntry)
{
   firstNode = add(newEntry, firstNode);
   numberOfEntries++;
} // end add

private Node add(T newEntry, Node currentNode)
{
   if ( (currentNode == null) ||
        (newEntry.compareTo(currentNode.getData()) <= 0) )
   {
      currentNode = new Node(newEntry, currentNode);
   }
   else
   {
      Node nodeAfter = add(newEntry, currentNode.getNextNode());
      currentNode.setNextNode(nodeAfter);
   } // end if
```

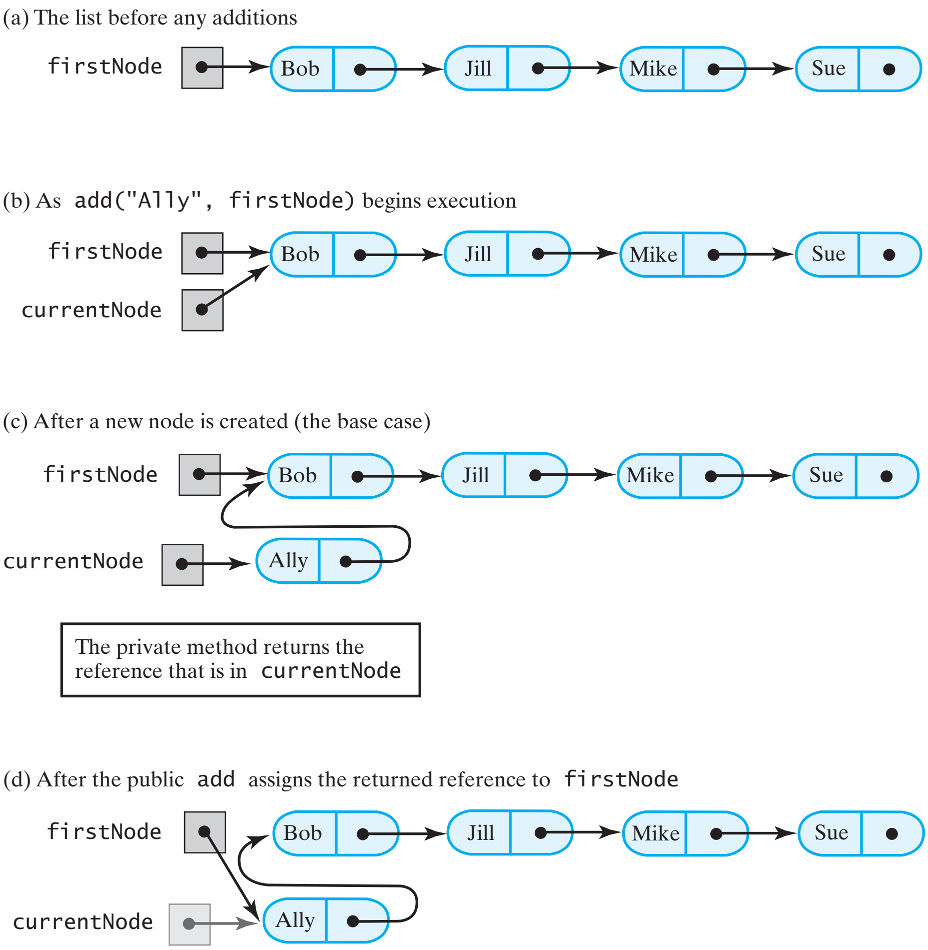
```
      return currentNode;
   } // end add
```

The private method add adds newEntry to the subchain that begins at currentNode. We will trace and explain its logic in a moment.

SELF-TEST **Question 7** Repeat Question 5, using the method add that was just given.

16.14 **Tracing an addition to the list's beginning.** Suppose that nameList is the sorted list that the chain in Figure 16-3a represents. Let's invoke nameList.add("Ally") to add *Ally* to this list. This addition will occur at the beginning of the chain. The public method add will call the private method add with the invocation add("Ally", firstNode). The reference in the argument firstNode is copied to the parameter currentNode, and so it also references the first node in the chain, as Figure 16-3b illustrates.

FIGURE 16-3 Recursively adding a node at the beginning of a chain



(a) The list before any additions
firstNode
Bob
Jill
Mike
Sue
(b) As add("Ally", firstNode) begins execution
firstNode
currentNode
Bob
Jill
Mike
Sue
(c) After a new node is created (the base case)
firstNode
currentNode
Bob
Jill
Mike
Sue
Ally
The private method returns the reference that is in currentNode
(d) After the public add assigns the returned reference to firstNode
firstNode
currentNode
Bob
Jill
Mike
Sue
Ally

Since *Ally* will be added to the beginning of the chain, the statement

```
currentNode = new Node("Ally", currentNode);
```

executes and creates a new node for *Ally*. This node is linked to the original chain, as Figure 16-3c shows. Notice that `firstNode` is unchanged, even though it is the argument that corresponds to the parameter `currentNode`.

The private method now returns the value of `currentNode`, and the public method `add` assigns that value to `firstNode`. Thus, the chain with the completed addition appears as in Figure 16-3d.

16.15 **Tracing an addition to the list's interior: the recursive calls.** What happens when the addition is not at the beginning of the original chain? Let's trace what happens when we add *Luke* to the chain in Figure 16-4a. The public method `add` calls the private method `add` with the invocation `add("Luke", firstNode)`. As in the previous segment, the reference in `firstNode` is copied to the parameter `currentNode`, and so it also references the first node in the chain, as Figure 16-4a illustrates.

Since *Luke* comes after *Bob*, another recursive call occurs:

```
add("Luke", currentNode.getNextNode())
```

The second argument is a reference to the chain's second node, the one containing *Jill*. This reference is copied to the parameter `currentNode`, as Figure 16-4b depicts.

Luke comes after *Jill*, so the recursive process is repeated again, and `currentNode` references the chain's third node—*Mike*'s node—as shown in Figure 16-4c. *Luke* is less than *Mike,* so no recursive call occurs. We are at the base case. A new node is created that contains *Luke* and references *Mike*'s node, as Figure 16-4d illustrates.

16.16 **Tracing the returns from the recursive method.** Having just created a new node, the private method `add` returns a reference to it, as Figure 16-4d indicates. The statement that invoked `add` now resumes execution:

```
nodeAfter = add("Luke", currentNode.getNextNode());
```

Thus, `nodeAfter` is assigned a reference to the new node containing *Luke*, as Figure 16-4e illustrates.

At this point, `currentNode` references *Jill*'s node, as it did in Part *b* of the figure. The next statement to execute is

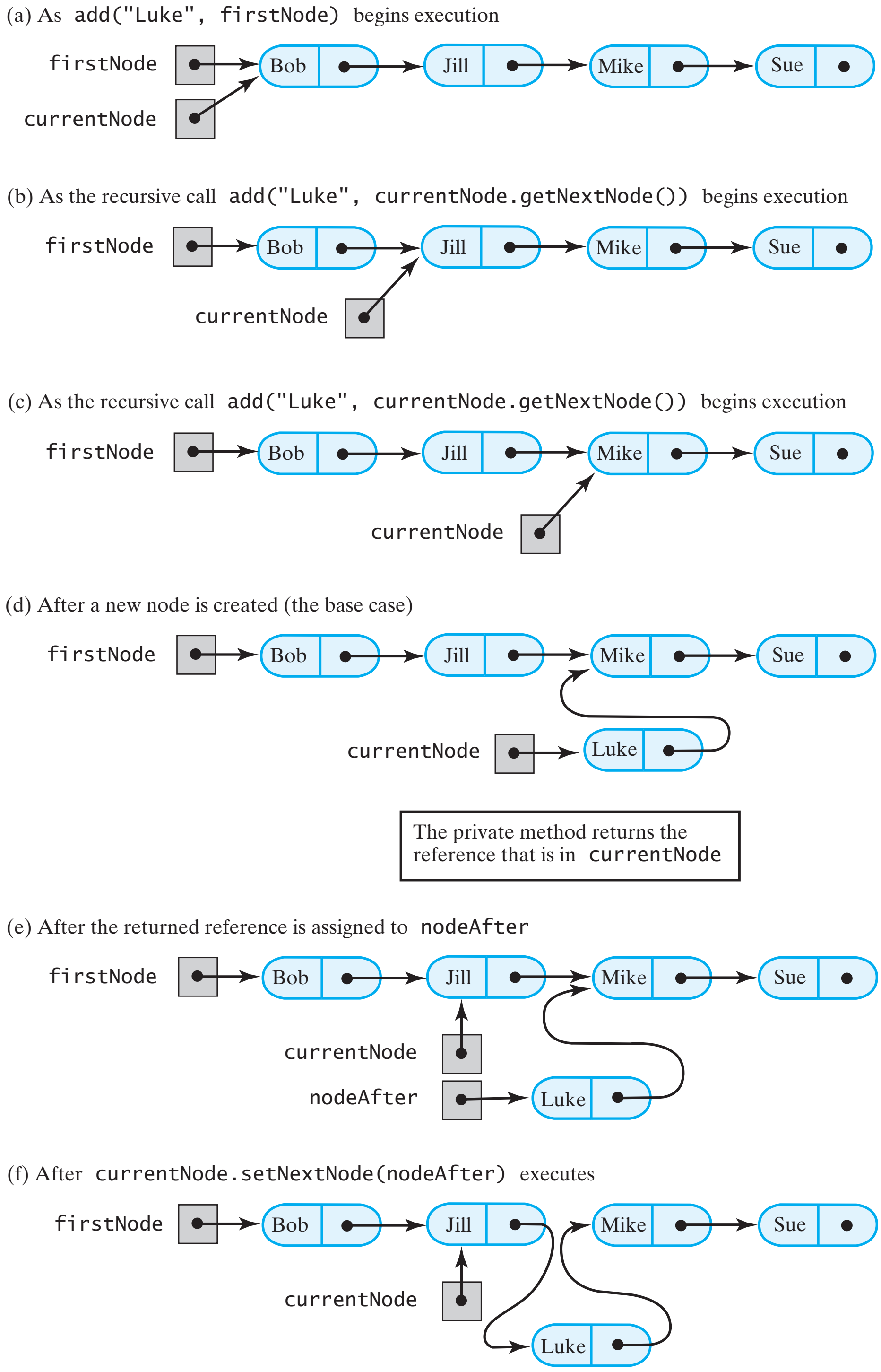
```
currentNode.setNextNode(nodeAfter);
```

Thus, the data field `next` in *Jill*'s node is changed to reference *Luke*'s node, as shown in Figure 16-4f.

The private method `add` now returns a reference to *Jill*'s node. If we continue the trace, we will make *Bob*'s node reference *Jill*'s node and `firstNode` reference *Bob*'s node, even though these references are already in place.

Note: A recursive addition to a chain of nodes locates and remembers the nodes prior to the insertion point. After the portion of the chain that follows the insertion point is linked to the new node, the recursion links the remembered nodes back into the chain.

FIGURE 16-4 Recursively adding a node between existing nodes in a chain



(a) As `add("Luke", firstNode)` begins execution

(b) As the recursive call `add("Luke", currentNode.getNextNode())` begins execution

(c) As the recursive call `add("Luke", currentNode.getNextNode())` begins execution

(d) After a new node is created (the base case)

The private method returns the reference that is in `currentNode`

(e) After the returned reference is assigned to `nodeAfter`

(f) After `currentNode.setNextNode(nodeAfter)` executes

16.17 Projects 1 and 2 at the end of this chapter ask you to complete the iterative and recursive implementations of the sorted list. Notice that many of the sorted list operations are the same as operations of the ADT list and so would have implementations like those you saw in Chapter 14.

Note: Since the ADTs sorted list and list share many of the same operations, portions of their implementations are identical.

SELF-TEST

Question 8 The linked implementation of the ADT sorted list, as given in this chapter, does not maintain a tail reference. Why is a tail reference more significant for a linked implementation of the ADT list than it is for a sorted list?

The Efficiency of the Linked Implementation

16.18 If you consider the analysis of the linked implementation of the ADT list given in Chapter 14, you will see that the performance of the `add` method depends on the efficiency of the method `getNodeAt`. The latter method locates the insertion point by traversing the chain of nodes. It is an O(n) operation. The `add` method for the sorted list does its own traversal of the list to locate where to make the addition. This traversal is also O(n), making the addition to a sorted list an O(n) operation.

Figure 16-5 summarizes the performance of the sorted list operations. Deriving these results is left as an exercise. When comparing implementations, you should realize that the worst cases can occur under different circumstances. For example, a worst-case addition to an array-based sorted list occurs at the list's beginning, whereas for a linked implementation, it occurs at the list's end.

VideoNote
An array-based sorted list

FIGURE 16-5 The worst-case efficiencies of the operations on the ADT sorted list for two implementations

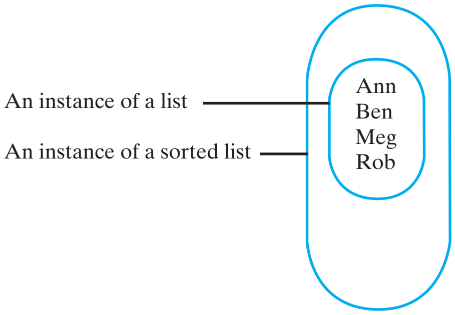
ADT Sorted List Operation	Array	Linked
`add(newEntry)`	O(n)	O(n)
`remove(anEntry)`	O(n)	O(n)
`getPosition(anEntry)`	O(n)	O(n)
`getEntry(givenPosition)`	O(1)	O(n)
`contains(anEntry)`	O(n)	O(n)
`remove(givenPosition)`	O(n)	O(n)
`toArray()`	O(n)	O(n)
`clear(), getLength(), isEmpty()`	O(1)	O(1)

An Implementation That Uses the ADT List

16.19 As we noted in Segment 16.17, the linked implementation of the ADT sorted list repeats much of the corresponding implementation of the ADT list. Can we avoid this duplication of effort and reuse portions of the list's implementation? The answer to this question is yes, as you will soon see.

You can certainly use the ADT list to create and maintain an alphabetical list of strings. It is natural, then, to consider using the ADT list when implementing the ADT sorted list. Basically, you can do this in one of two ways. Here we will use a list as a data field within the class that implements the sorted list. Figure 16-6 shows an instance of such a sorted list. Recall from Segment C.1 of Appendix C that this approach is called composition and illustrates the *has-a* relationship between two classes. The next chapter considers the second approach, using inheritance to derive the sorted list from the list.

FIGURE 16-6 An instance of a sorted list that contains a list of its entries



16.20 Our class `SortedList` will implement the interface `SortedListInterface`. We begin this class by declaring a list as a data field and defining a default constructor. We assume that the class `LList`, as discussed in Chapter 14, is an implementation of the interface `ListInterface` for the ADT list. Thus, our class begins as follows:

```
public class SortedList<T extends Comparable<? super T>>
             implements SortedListInterface<T>
{
   private ListInterface<T> list;

   public SortedList()
   {
      list = new LList<T>();
   } // end default constructor
      . . .
} // end SortedList
```

Note our use of the generic type `T` when we declare the data field `list` and create an instance of `LList`.

16.21 **The method `add`.** The implementations of the operations of the ADT sorted list are brief, as the list does most of the work. To add a new entry to the sorted list, we first use the method `getPosition`, which is an operation of the sorted list. We assume that it is already implemented, even though we have not written it yet. Recall that `getPosition` finds the position of an existing entry within a sorted list, or the position at which we should insert a new entry that does not occur in the sorted list. The method sets the sign of the integer it returns to indicate whether the entry exists in the list already. When adding an entry to a sorted list that can contain duplicate entries, it does not matter whether the entry exists in the sorted list already. Thus, we can ignore the sign of the integer that `getPosition` returns. Notice that the following implementation uses the method `abs` of the class `Math` to discard this sign. It also uses the `add` operation of the ADT list. (In this section, calls to ADT list operations are highlighted.)

```
public void add(T newEntry)
{
   int newPosition = Math.abs(getPosition(newEntry));
   list.add(newPosition, newEntry);
} // end add
```

? SELF-TEST

Question 9 Repeat Question 5, using the method `add` that was just given.

Question 10 Can a client of `SortedList` invoke the operation `add(position, entry)` of the ADT list? Explain.

16.22 **The method `remove`.** We also use `getPosition` when removing an object from a sorted list. This time, however, we do need to know whether the given entry exists in the sorted list. If it does not exist, we cannot remove it. In such cases, `remove` returns false. Also notice that the method uses the operation `remove` of the ADT list to make the deletion. Thus, the method has the following implementation:

```
public boolean remove(T anEntry)
{
   boolean result = false;
   int position = getPosition(anEntry);

   if (position > 0)
   {
      list.remove(position);
      result = true;
   } // end if

   return result;
} // end remove
```

SELF-TEST

Question 11 If a sorted list contains five duplicate objects and you use the previous method `remove` to remove one of them, what will be removed from the list: the first occurrence of the object, the last occurrence of the object, or all occurrences of the object?

16.23 **The logic for `getPosition`.** Implementing `getPosition` is somewhat harder than implementing the previous two methods. To decide where in the list `anEntry` is or belongs, we need to compare `anEntry` to the entries already in the list, beginning with the first one. If `anEntry` is in the list, we obviously compare entries until we find a match. However, if `anEntry` is not in the list, we want to stop the search at the point where it belongs in the sorted list. We take advantage of the sorted order of the objects by using logic similar to that described in Segment 16.8.

For example, suppose that the sorted list contains the four names *Brenda*, *Carlos*, *Sarah*, and *Tom*. If we want to see where *Jamie* belongs in the sorted list, we discover that, as strings,

Jamie > Brenda
Jamie > Carlos
Jamie < Sarah

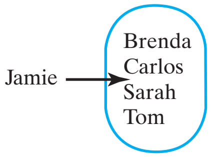
Thus, *Jamie* belongs after *Carlos* but before *Sarah*—that is, at position 3 in the sorted list, as Figure 16-7 illustrates.

To compare `anEntry` to an entry in the sorted list, we first use the list operation `getEntry` to return the entry at a given position within the sorted list. Then the expression

```
anEntry.compareTo(list.getEntry(position))
```

makes the comparison.

FIGURE 16-7 A sorted list in which *Jamie* belongs after *Carlos* but before *Sarah*



16.24 **The implementation of `getPosition`.** In the following implementation of `getPosition`, the `while` loop finds `anEntry`'s position in the sorted list, and the `if` statement sees whether `anEntry` is in the list.

```
public int getPosition(T anEntry)
{
   int position = 1;
   int length = list.getLength();

   // find position of anEntry
   while ( (position <= length) &&
           (anEntry.compareTo(list.getEntry(position)) > 0) )
   {
      position++;
   } // end while

   // see whether anEntry is in list
   if ( (position > length) ||
        (anEntry.compareTo(list.getEntry(position)) != 0) )
   {
      position = -position; // anEntry is not in list
   } // end if

   return position;
} // end getPosition
```

SELF-TEST

Question 12 Assume that the sorted list `nameList` contains the four names *Brenda*, *Carlos*, *Sarah*, and *Tom* as strings. By tracing the code for `getPosition`, see what `getPosition` returns when `anEntry` represents

a. *Carlos* **b.** *Alan* **c.** *Wendy* **d.** *Tom* **e.** *Jamie*

Question 13 Since you can decide whether a given entry is in a particular sorted list by testing the sign of the integer that `getPosition` returns, you can use `getPosition` to implement the method `contains`. Write such an implementation.

16.25 Each of the remaining methods—`contains`, `remove`, `getEntry`, `clear`, `getLength`, `isEmpty`, and `toArray`—has the same specifications as in the ADT list. Each can simply invoke the corresponding list method. For example, the method `getEntry` has the following implementation in `SortedList`:

```
public T getEntry(int givenPosition)
{
   return list.getEntry(givenPosition);
} // end getEntry
```

SELF-TEST

Question 14 You can implement the method `contains` by invoking either `getPosition`, as Question 13 suggests, or the ADT list's `contains` method. Which of these implementations will execute faster when the entry sought is not present in the sorted list? Why?

Efficiency Issues

Except perhaps for some subtle logic in `getPosition`, you can write the previous implementation quickly and with few, if any, errors. Saving human time is an attractive feature of using an existing class to build another. But does the implementation use computer time efficiently? In this particular implementation, several methods invoke `getPosition`, so their efficiency depends on `getPosition`'s efficiency.

16.26 **The efficiency of `getPosition`.** As we examine `getPosition`, as given in Segment 16.24, we note that the list method `getLength` is an O(1) operation. Therefore, we need not be concerned with it. On the other hand, a loop examines the entries in the list one at a time by invoking `getEntry` until the desired entry is located. Thus, the efficiency of `getPosition` depends in part on the efficiency of `getEntry`. However, the efficiency of `getEntry` depends upon which implementation of the ADT list you use. We will examine two list implementations that lead to rather different efficiencies for `getPosition`.

Chapter 14 discussed the efficiencies of the ADT list operations. Figure 16-8 recalls the worst-case performance of the list operations that we need to complete our analysis of the sorted list. If you use an array to represent the entries in a list, `getEntry` is always an O(1) operation. The loop in `getPosition` is therefore O(n) in the worst case, and so `getPosition` is O(n) when the list has an array-based implementation.

If you use a chain of linked nodes to contain the entries in a list, the method `getEntry` is O(n). Since `getPosition`'s loop invokes `getEntry`, we see that `getPosition` is O(n^2) in the worst case. Each time `getEntry` retrieves the next entry in the list, it starts its search at the beginning of the chain. This fact is the cause of `getPosition`'s inefficiency.

FIGURE 16-8 The worst-case efficiencies of selected ADT list operations for array-based and linked implementations

ADT List Operation	Array	Linked
`getEntry(givenPosition)`	O(1)	O(n)
`add(newPosition, newEntry)`	O(n)	O(n)
`remove(givenPosition)`	O(n)	O(n)
`contains(anEntry)`	O(n)	O(n)
`toArray()`	O(n)	O(n)
`clear(), getLength(), isEmpty()`	O(1)	O(1)

16.27 **The efficiency of `add`.** The implementation of the sorted list method `add` given in Segment 16.21 contains the following statements:

```
int newPosition = Math.abs(getPosition(newEntry));
list.add(newPosition, newEntry);
```

For an array-based implementation of the ADT list, both `getPosition` and the list operation `add` are O(n) operations. Thus, the sorted list operation `add` is O(n) in the worst case. For a linked implementation of the list, `getPosition`'s worst-case behavior is O(n^2) and dominates the list operation `add`, which is only O(n). Thus, the sorted list operation `add` is O(n^2) in the worst case.

16.28 Figure 16-9 summarizes the efficiencies of the sorted list operations for array-based and linked implementations of the ADT list. Confirmation of these results is left as an exercise. As you can see, the implementation of the sorted list given in this section is easy to write but is not very efficient if the underlying list uses a chain of linked nodes. The next chapter will show you how you can reuse the ADT list in the implementation of the sorted list without sacrificing efficiency.

SELF-TEST

Question 15 Give an advantage and a disadvantage of using composition in the implementation of the class `SortedList`.

FIGURE 16-9 The worst-case efficiencies of the ADT sorted list operations when implemented using an instance of the ADT list

ADT Sorted List Operation	List Implementation	
	Array	Linked
`add(new Entry)`	O(n)	O(n^2)
`remove(anEntry)`	O(n)	O(n^2)
`getPosition(anEntry)`	O(n)	O(n^2)
`getEntry(givenPosition)`	O(1)	O(n)
`contains(anEntry)`	O(n)	O(n)
`remove(givenPosition)`	O(n)	O(n)
`toArray()`	O(n)	O(n)
`clear(), getLength(), isEmpty()`	O(1)	O(1)

Note: Using composition to implement the ADT sorted list

When you use an instance of an ADT list to represent the entries in the ADT sorted list, you must use the list's operations to access the sorted list's entries, instead of accessing them directly. Such an implementation of the sorted list is easy to write but is inefficient when the underlying list uses a chain of linked nodes to store its entries.

Chapter Summary

- The ADT sorted list maintains its entries in sorted order. It, not the client, determines where to place an entry.
- The ADT sorted list can add, remove, or locate an entry, given the entry as an argument.
- The sorted list has several operations that are the same as the corresponding operations of the ADT list. However, a sorted list will not let you add or replace an entry by position.
- A chain of linked nodes provides a reasonably efficient implementation of the sorted list.
- An implementation of the sorted list that uses an ADT list as a data field is easy to write. However, depending upon how the ADT list is implemented, its efficiency can suffer.

Exercises

1. Suppose that `nameList` is a sorted list of names. Using the operations of the ADT list and the ADT sorted list, create a list of these names without changing their order.

2. As specified in this chapter, the sorted list can contain duplicate entries. Specify a sorted list of unique items. For example, `add` could return true if it added an entry to the list but return false if the entry is in the list already.

3. The *mode* of a list of values is the value having the greatest frequency.

   a. Write an algorithm to find the mode of a sorted list using only methods of the ADT sorted list.
   b. What is the Big Oh of the algorithm if the sorted list has an array-based implementation?
   c. What is the Big Oh of the algorithm if the sorted list has a linked implementation?

4. The schedule of activities for a room consists of an *activity list*. Each activity has a description, a start time, and an end time. You can add activities to the list, but they must be compatible with the other activities. Two activities are incompatible if their time intervals overlap. Specify the ADT activity list.

5. Imagine you are working for a geologist who has records for earthquakes that occurred during the past 50 years. Each record includes a date, location, strength, and duration. Design and specify an ADT for this collection of data.

6. Explain how you can use the ADT sorted list in the implementations of the ADTs described in Exercises 4 and 5.

7. Consider an array-based implementation of the sorted list. To implement the method `add`, you must add an entry to a sorted array so that the array remains sorted.

   a. Describe the steps in this implementation.
   b. On which sort have you based your logic?
   c. Analyze the worst-case efficiency of this implementation of `add`.

8. Figure 16-5 tabulates the worst-case efficiencies of the sorted list operations for both array-based and linked implementations. Derive these Big Oh expressions.

9. Figure 16-9 tabulates the worst-case efficiencies of the sorted list operations when implemented using an instance of the ADT list. Derive these Big Oh expressions.

10. Consider an array-based implementation of the sorted list. Let the array `list` be the data field that represents the list's entries. If a constructor is given an array of unsorted list entries, the constructor must place them into `list` in sorted order. To do so, it could repeatedly use the sorted list's `add` method to add the entries to the sorted list (and hence to the array `list`) in their proper order. Or it could copy the entries to `list` and sort them by using a sort algorithm from Chapters 8 and 9.

    a. If you use the first approach, what sort are you actually using?
    b. Would you ever want to use the second approach? Explain.

11. Consider the implementation of the sorted list that uses an instance of the ADT list. In particular, consider the method `contains`. One implementation of `contains` could invoke `getPosition` (see Question 13 at the end of Segment 16.24). Another implementation could simply invoke `list.contains`. Compare the efficiencies of these two implementations.

12. Write a linked implementation of the sorted list method `contains`. Your search of the chain should end when it either locates the desired entry or passes the point at which the entry should have occurred.

13. Compare the efficiency of the method `contains` that Exercise 12 describes with that of the list's version of `contains`.

14. Segment 9.2 of Chapter 9 described how to merge two sorted arrays into one sorted array. Add an operation to the ADT sorted list that merges two sorted lists. Implement the merge in three ways, as follows:

    a. Use only sorted list operations.
    b. Assume an array-based implementation.
    c. Assume a linked implementation.

PROJECTS

1. Complete the linked implementation of the ADT sorted list that was begun in this chapter. Use iteration instead of recursion.

2. Repeat Project 1, but use recursion wherever possible.

3. Implement the ADT sorted list by using an array to represent the ADT's entries. Use array resizing so that the sorted list can grow as large as necessary.

4. Implement the ADT sorted list by using an instance of `Vector` to represent the ADT's entries. Recall that Chapter 13 presented a similar implementation for the ADT list.

5. Exercise 2 asked you to specify an ADT sorted list of unique items. Implement such an ADT using one of the approaches described in this chapter or in the previous projects.

6. Add an iterator to the ADT sorted list by defining an inner class within the class that implements the ADT.

7. A *polynomial* in *x* is an algebraic expression that involves integer powers of *x*, as follows:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

The *a*'s are called *coefficients*. The *degree* of the polynomial is *n*, the highest exponent of *x* that appears in *P*(*x*). Although a_n cannot be zero in a degree *n* polynomial, any other coefficient can be zero.

Specify an ADT polynomial that includes operations such as `getDegree`, `getCoefficient`, `setCoefficient`, `add`, and `subtract`. Implement this ADT by using a sorted list. The sorted list should not contain any coefficients that are zero.

8. Exercise 3 asked you to create an algorithm to find the mode of a sorted list. Let's add a method to the implementation of a sorted list that finds the list's mode. The header of such a method could be

```
public T mode()
```

Implement this method in three ways, as follows:

a. Use only sorted list operations.
b. Assume an array-based implementation.
c. Assume a linked implementation.

9. You can use a *substitution code* to encode a message. In this scheme, a *key* maps each letter to another letter. Each letter in the *plain-text* message is replaced according to the key to produce the encoded message, or *cipher text*.

Suppose you are given some cipher text, but not the key. One method of breaking such a code is to count the frequency of letters in the cipher text and then make guesses about the mapping based on the frequencies of letters in typical English text. Write a program that reads characters from a file and uses a sorted list to find the frequency of each letter.

10. Implement the ADT priority queue by using a sorted list to contain the priority queue's entries.

11. Project 1 of Chapter 1 defines a set as a bag that does not allow duplicate entries. Implement the ADT set by using a sorted list to contain its entries. Include the operations union, intersection, and difference, as described respectively in Exercises 5, 6, and 7 of Chapter 1.

12. In certain computer networks, a message is not sent as a continuous stream of data. Instead, it is divided into pieces, called *packets*, and sent a packet at a time. The packets might not arrive at their destination in the same order as the one in which they were sent. To enable the receiver to assemble the packets in their correct order, each packet contains a sequence number.

For example, to send the message "Meet me at 6 o'clock" three characters at a time, the packets would appear as follows:

```
1 Mee
2 t m
3 e a
4 t 6
5  o'
6 clo
7 ck
```

Regardless of when the packets arrive, the receiver can order the packets by their sequence numbers to determine the message.

Given a text file containing the packets of data in the order they were received, write an application that reads the file and extracts the message by using a sorted list. Design and create auxiliary classes such as `Packet` and `Message`.

Answers to Self-Test Questions

1.
```
SortedListInterface<String> sortedWordList = new SortedList<String>();
int numberOfWords = wordList.getLength();
for (int position = 1; position <= numberOfWords; position++)
   sortedWordList.add(wordList.getEntry(position));
```

2. **a.**
```
int length = sortedWordList.getLength();
String lastEntry = sortedWordList.getEntry(length);
System.out.println(lastEntry);
```
b.
```
sortedList.add(sortedList.getEntry(1));
```

3. The order is critical. When `currentNode` is `null`, `currentNode != null` is false. Thus, the entire expression in the `while` statement is false without executing the call `currentNode.getData()`. If the latter call were to execute first when `currentNode` was `null`, an exception would occur. Thus, the `while` statement should remain as written.

4. When the sorted list is empty, `getNodeBefore` returns `null`. Thus, in the definition of `add`, you can omit the call to `isEmpty` in the `if` statement.

5. Before the first occurrence of the entry.

6. After the last occurrence of the entry.

7. Before the first occurrence of the entry.

8. The method `add(newEntry)` for a list adds a new entry at the end of the list. A tail reference makes this method O(1) instead of O(*n*). For a sorted list, `add(newEntry)` must traverse the chain to locate the point of insertion. If the insertion is at the end of the chain, the traversal will give you a reference to the last node. A separate tail reference is not needed.

9. Before the first occurrence of the entry. Note that `getPosition` returns the position of the first occurrence of the entry within the list.

10. No. The field `list` is private so its methods are unavailable to a client of `SortedList`.

11. The first occurrence of the object. Note that `getPosition` returns the position of the first occurrence of the entry within the list.

12. **a.** 2; **b.** -1; **c.** -5; **d.** 4; **e.** -3.

13.
```
public boolean contains(T anEntry)
{
   return getPosition(anEntry) > 0;
} // end contains
```

14. The implementation that invokes `getPosition` will execute faster. Because the list is sorted, the method `getPosition` does not always search the entire list when the entry is not present. However, the list's method `contains` always searches the entire list in this case.

15. Advantage: The implementation is easy to write.
Disadvantage: The implementation is not efficient when the implementation of the underlying list is linked.

Chapter 17

Inheritance and Lists

Contents

Prerequisites

Appendix C		Creating Classes from Other Classes
Chapter	12	Lists
Chapter	13	List Implementations That Use Arrays
Chapter	14	A List Implementation That Links Data
Chapter	16	Sorted Lists

Objectives

After studying this chapter, you should be able to

- Describe how a class implementation that uses inheritance differs from one that uses composition
- Design a class that contains protected methods to make it suitable for use as a base class
- Write an efficient implementation of a sorted list by using inheritance

Chapter 16 introduced you to the ADT sorted list, which maintains its entries in a sorted order. As with many other ADTs, you can implement the sorted list by using either an array or a chain of linked nodes. The advantage of such implementations is their time efficiency. However, they require you to repeat a portion of the implementation of the ADT list, since the ADTs sorted list and list have several operations in common.

In an attempt to avoid this duplication of effort, Chapter 16 used an instance of the ADT list to contain the entries of the sorted list. This list was a data field of the class implementing the sorted list. The result was an implementation that you could write quickly, because the implementation of the list had done most of the work. But since the sorted list operations used the list in the same way that a client would, these operations were inefficient of time when the ADT list had a linked implementation.

But what if, instead of using composition, as we did in Chapter 16, we use inheritance? This chapter looks at the implications of deriving a sorted list from a list. In doing so we'll find that a subclass (derived class) can be more efficient if it can access the underlying data structures of its superclass (base class). This is possible if the superclass includes methods that enable future subclasses to examine or modify its data fields. A class designer should plan for the future use of a class as well as the present need.

Using Inheritance to Implement a Sorted List

VideoNote
Inheritance and ADT implementations

17.1 Recall the implementation of the class `SortedList` that we developed in Chapter 16 beginning at Segment 16.20. `SortedList` has an instance of another class, `LList` in this case, as a data field. `SortedList` and `LList` have a *has-a* relationship. Several of `SortedList`'s methods—namely `remove` (by position), `getEntry`, `contains`, `clear`, `getLength`, `isEmpty`, and `toArray`—behave like `LList`'s methods. If `SortedList` inherited these methods from `LList`, we would not have to implement them again, as we did in the previous chapter. Thus, we could revise `SortedList` as follows:

```
public class SortedList<T extends Comparable<? super T>>
             extends LList<T> implements SortedListInterface<T>
{
   public void add(T newEntry)
   {
      int newPosition = Math.abs(getPosition(newEntry));
      super.add(newPosition, newEntry);
   } // end add

   < Implementations of remove(anEntry) and getPosition(anEntry) go here. >
   . . .
} // end SortedList
```

The notation `T extends Comparable<? super T>`, introduced in Segments 8.1 and 8.2, defines the generic type `T`. The class that `T` represents must implement the interface `Comparable`. Writing `? super T`, which means any superclass of `T`, allows some flexibility when using the method `compareTo`.

You can see that `SortedList` is derived from `LList`. Also notice that we have omitted the data field `list` and the default constructor that appeared in Segment 16.20. To revise the `add` method given in Segment 16.21, we simply replaced `list` with `super`. That is, we wrote

```
super.add(newPosition, newEntry);
```

to invoke the `add` operation of the ADT list, instead of

```
list.add(newPosition, newEntry);
```

Coincidentally, `SortedList`'s `add` method overrides the other `add` method in `LList` that adds to the end of a list.

We would make similar changes to the methods `remove` and `getPosition`. The remaining methods of the sorted list are inherited from `LList`, and so they do not appear explicitly in `SortedList`.

Question 1 Although `SortedList` inherits the method `contains` from `LList`, the method is not as efficient as it could be. Why? Show how you could override `contains` with a more efficient version.

17.2 **A pitfall.** This implementation contains a pitfall that is the direct result of using inheritance. Although `SortedList` conveniently inherits methods such as `isEmpty` from `LList`, it also inherits two methods that a client can use to destroy the order of a sorted list. These two methods appear in `ListInterface` as follows:

```
/** Adds newEntry to the list at position newPosition. */
public boolean add(int newPosition, T newEntry);

/** Replaces the entry at givenPosition with newEntry. */
public boolean replace(int givenPosition, T newEntry);
```

If a client writes

```
SortedList<String> sList = new SortedList<String>();
```

for example, `sList` can invoke any method declared in either `SortedListInterface` or `ListInterface`, including the previous methods `add` and `replace`. Thus, a client could destroy the order of the entries in a sorted list either by adding an entry out of order or by replacing an entry.

17.3 **Possible ways to avoid the pitfall.** What can we do to avoid this pitfall? Here are three possibilities:

- Use `SortedListInterface` in the declaration of the sorted list. For example, if the client contains

  ```
  SortedListInterface<String> sList = new SortedList<String>();
  ```

  `sList` can invoke only methods declared within `SortedListInterface`. Notice that the list operations `add` and `replace` do not appear in `SortedListInterface`. Although this can be a good programming practice, that is all it is. A client need only ignore this practice and define the data type of `sList` as `SortedList` to have all operations of the ADT list available to it. You have already seen how a client can sabotage the sorted list in this case.
- Implement the list's `add` and `replace` methods within the class `SortedList`, but have them return false. For example, `add` could appear as follows:

  ```
  public boolean add(int newPosition, T newEntry)
  {
     return false;
  } // end add
  ```

  This version of `add` overrides the version that `LList` implements. If the client invokes this method, the sorted list will remain unchanged. The client can detect that the method was unsuccessful, but not why.

If the list's method were a void method, we could give the overriding version an empty body, but then the client would be unaware that the method did not do anything.

- Implement the list's add and replace methods within the class SortedList and have them throw an exception when invoked. For example, add could appear as follows:

```
public boolean add(int newPosition, T newEntry)
{
   throw new UnsupportedOperationException("Illegal attempt to add " +
                       "at a specified position within a sorted list.");
} // end add
```

This version of add also overrides the version that LList implements. If the client invokes this method, an exception occurs. This approach is a common practice, and it is the one we prefer.

Note: If SortedList overrides the list's method add, the class's implementation still can invoke the method, as happens in Segment 17.1. The use of super in the call indicates that we are invoking the list's version of the method, not the overriding version in SortedList.

SELF-TEST **Question 2** As a variation of the second possibility just given, you could implement the ADT list's two add methods within SortedList so that each one calls the add method specified in SortedListInterface. In this way, the new entry is added in its correct position within the sorted list. Why is this not a good idea?

Programming Tip: If your class inherits methods that are inappropriate, you can override them with methods that throw an exception when invoked. In such a case, examine your design and consider whether inheritance was the right choice. Do the benefits of inheritance outweigh the inconvenience of overriding the inappropriate methods, or would composition provide a cleaner design?

17.4 **Efficiency.** The implementation of SortedList given here has the same efficiency—or inefficiency in this case—as the version that uses composition given in the previous chapter. If LList had been designed with inheritance in mind, SortedList could access LList's underlying data structure and provide faster operations. To this end, we revise the class LList in the next section.

Note: The implementation of the sorted list that extends the class LList is as inefficient as the implementation that uses composition given in the previous chapter.

SELF-TEST **Question 3** Give at least one advantage and one disadvantage of using inheritance in the way shown in this section to implement the class SortedList.

Designing a Base Class

17.5 Let's examine the class LList that we developed in Chapter 14 as a linked implementation of the ADT list. Recall that the class places each of the list's entries into its own node. These nodes are linked so that the first entry's node references the node of the second entry, and so on. A data field firstNode of the class references the first node, and another data field numberOfEntries counts the number of entries in the list.

Like most classes, `LList` has data fields that are private. The client cannot access these fields directly by name. The class designer must decide whether to provide public methods that give the client indirect access to the data fields. In the case of `LList`, the public method `getLength` enables the client to get the length of the list. The client, however, cannot directly change the list's length. Only other member methods, such as `add` and `remove`, can alter the length. In addition, `LList` denies the client access to the field `firstNode` by not providing public accessor or mutator methods for this field. This design is appropriate, as `firstNode` is an implementation detail that should be hidden from the client.

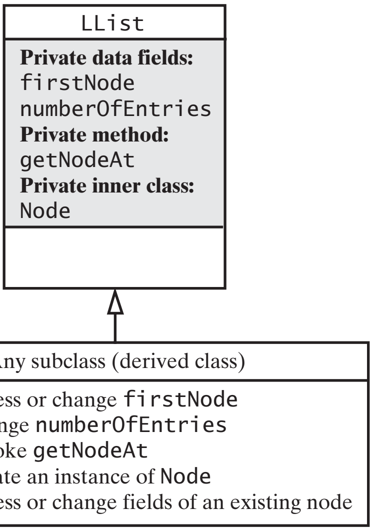
17.6 The excerpt of the class `LList` given in Listing 17-1 shows aspects of the class that are relevant to this discussion. Each node is represented by the private class `Node`, which is defined within `LList` and hidden from the client. The method `getNodeAt` facilitates the implementation of other member methods by returning a reference to the node at a given position. We do not want the client to have access to this node, since it is part of the underlying representation of the list, so we make the method private.

LISTING 17-1 Relevant aspects of the class `LList`

```
public class LList<T> implements ListInterface<T>
{
   private Node firstNode; // reference to first node
   private int  numberOfEntries;

   public LList()
   {
      clear();
   } // end default constructor

   public final void clear()
   {
      firstNode = null;
      numberOfEntries = 0;
   } // end clear

   public int getLength()
   {
      return numberOfEntries;
   } // end getLength

   < Implementations of the public methods add, remove, replace, getEntry, contains,
     isEmpty, and toArray go here. >
   . . .

   // Returns a reference to the node at a given position.
   private Node getNodeAt(int givenPosition)
   {
      . . .
   } // end getNodeAt
   private class Node
   {
      private T data;
      private Node next;
      . . .
   } // end Node
} // end LList
```

17.7 So far, nothing should be new to you. Now imagine that we want `LList` to serve as a base class for another class that you are developing. You saw in the previous section of this chapter that a subclass of `LList`—just like a client of `LList`—cannot access by name anything declared as private within `LList`. That is, a subclass cannot access the data field `firstNode`, the method `getNodeAt`, or the class `Node`, as Figure 17-1 illustrates. If we want to extend the capability of `LList` and do so efficiently, the subclass will need access to these aspects of the class—in other words, to the underlying data structure.

FIGURE 17-1 A derived class of the class `LList` cannot access or change anything that is private within `LList`


LList
Private data fields:
firstNode
numberOfEntries
Private method:
getNodeAt
Private inner class:
Node
Any subclass (derived class)
• Cannot access or change firstNode
• Cannot change numberOfEntries
• Cannot invoke getNodeAt
• Cannot create an instance of Node
• Cannot access or change fields of an existing node

We can revise `LList` to make it more suitable as a base class by providing its subclasses controlled access to items that are hidden from a client. First, let's recall protected access, which we discussed in Segment C.13 of Appendix C:

Note: Protected access

You can access a protected method or data field by name only within its own class definition `C`, within any subclass of `C`, or within any class in the same package as `C`.

Our goal, then, is to provide a subclass protected—but limited—access to the underlying chain of nodes. The subclass should be able to traverse or modify the chain efficiently. However, modifications to the chain must be done in a way that helps to maintain its integrity.

17.8 To enable a subclass to access the data fields by name without giving this access to the client, we could declare `firstNode` and `numberOfEntries` to be protected. It is more typical, however, to keep them private and to provide protected methods for only the access we desire. The subclass will need to access the head reference `firstNode`, so we provide a protected get method to do this. Since `getLength` is public, the subclass can get the value of `numberOfEntries`.

A subclass likely will need to change `firstNode` and `numberOfEntries`, so we could provide protected methods that do so. But while we want an efficient subclass, we also want to keep our data structure intact. So we will not allow a subclass to change these fields directly. Instead, we can provide protected methods that modify our chain of nodes in a way that can satisfy both of our desires. For example, protected methods can add or delete nodes, updating the chain's length in the process. No mutators will be available to directly change either the field `numberOfEntries` or a node's link. Thus, the subclass can alter the chain efficiently, but we are assured that the nodes will be linked correctly and the chain's length will be accurate.

17.9 As a result of this discussion, we make the following changes to `LList`:

1. We define a protected method `getFirstNode`, enabling the subclass to access the head reference `firstNode`:

```
protected Node getFirstNode()
{
   return firstNode;
} // end getFirstNode
```

2. We define protected methods to add and remove nodes, changing `firstNode` and `numberOfEntries` as necessary:

```
/** Adds a node to the beginning of a chain. */
protected void addFirstNode(Node newNode)

/** Adds a node to a chain after a given node. */3
protected void addAfterNode(Node nodeBefore, Node newNode)

/** Removes a chain's first node. */
protected T removeFirstNode()

/** Removes the node after a given one. */
protected T removeAfterNode(Node nodeBefore)
```

The implementations of these methods use the techniques presented in Chapter 14. For example, `addFirstNode` has the following definition, assuming that `newNode` is not `null` and the inner class `Node` has set and get methods:

```
protected void addFirstNode(Node newNode)
{
   assert newNode != null : "null argument in addFirstNode";
   newNode.setNextNode(firstNode);
   firstNode = newNode;
   numberOfEntries++;
} // end addFirstNode
```

To prevent a subclass from overriding these protected methods, you can declare them as final.

3. The public methods of `LList` and its subclasses can call the previous methods, and thereby reduce the chance of error. For example, we can revise `LList`'s method `remove` as follows:

```
public T remove(int givenPosition)
{
   T result = null;

   if ((givenPosition >= 1) && (givenPosition <= getLength()))
   {
      assert !isEmpty();

      if (givenPosition == 1)        // case 1: remove first entry
         result = removeFirstNode();
      else                           // case 2: givenPosition > 1
      {
         Node nodeBefore = getNodeAt(givenPosition - 1);
         result = removeAfterNode(nodeBefore);
      } // end if
   } // end if

   return result;
} // end remove
```

4. Next, we make `getNodeAt` protected instead of private. The client still cannot use this method, but the implementations of the class and any subclass can.
5. We also make the class `Node` protected instead of private. `Node` will remain hidden from the client but will be available to any subclass of `LList`. We could make `Node`'s data fields `data` and `next` protected instead of private, but just as we did for `LList`, we instead make them private and provide protected accessor methods. We also provide a protected set method for a node's data. To ensure the integrity of our chain, we do not allow a subclass to alter the link portion of a node, and so we make that set method private. Thus, `Node` has the following four methods:

```
protected T getData()
protected void setData(T newData)
protected Node getNextNode()
private void setNextNode(Node nextNode)
```

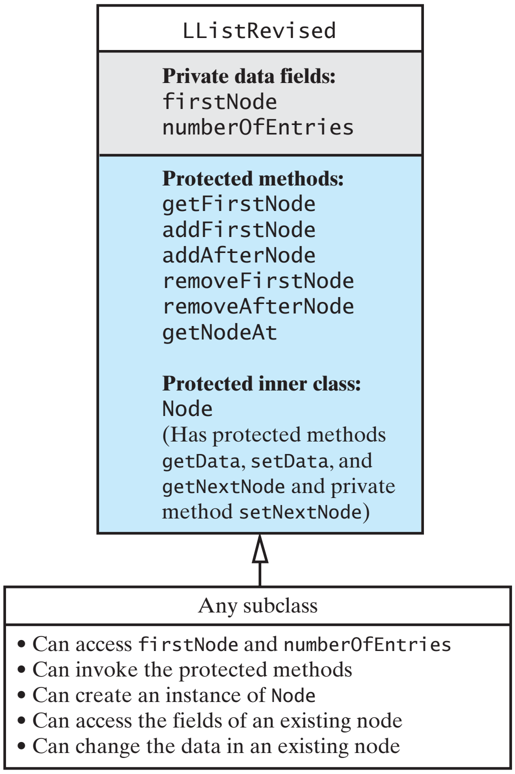
Finally, we make `Node`'s first constructor protected but leave its second constructor private, since it sets a node's link portion.

Making these changes to the class `LList` results in a new class, which we will name `LListRevised`. Figure 17-2 illustrates this class and the access that a derived class has to it. Later in this chapter, we will use `LListRevised` as the base class for a sorted list.

Programming Tip: Planning for the future

When designing a class, you should plan for its future use as well as the present need. If public accessor methods are not already in your design, provide protected accessor methods. Decide whether you want any future subclass to manipulate your class's data fields. If you do, provide protected methods that enable a subclass to make changes to the data fields both efficiently and safely.

FIGURE 17-2 Access available to a class derived from the class `LListRevised`


LListRevised
Private data fields:
firstNode
numberOfEntries
Protected methods:
getFirstNode
addFirstNode
addAfterNode
removeFirstNode
removeAfterNode
getNodeAt
Protected inner class:
Node
(Has protected methods getData, setData, and getNextNode and private method setNextNode)
Any subclass
• Can access firstNode and numberOfEntries
• Can invoke the protected methods
• Can create an instance of Node
• Can access the fields of an existing node
• Can change the data in an existing node

Question 4 Imagine a subclass of the class `LListRevised`.

a. Implement a method within the subclass that adds an entry to the beginning of the list.

b. Implement a method within the subclass that adds an entry right after the entry at the midpoint of the list. If the list contains *n* entries, the entry at the midpoint is at position *n* / 2, where the division is truncated to an integer.

Creating an Abstract Base Class

17.10 We can simplify the previous class `LListRevised` by organizing the portion of it that deals with the chain of linked nodes into an abstract base class. Listing 17-2 outlines such a class, `LinkedChainBase`. Note that this class is abstract by virtue of the keyword `abstract`. All of its methods are implemented, but we do not allow instances of this class.

VideoNote
Creating a base class

LISTING 17-2 The abstract base class `LinkedChainBase`

```
public abstract class LinkedChainBase<T>
{
   private Node firstNode; // reference to first node
   private int  numberOfEntries
   public LinkedChainBase()
   {
      clear();
   } // end default constructor

   < Implementations of the public methods clear, getLength, isEmpty, and toArray
     go here. >
   . . .

   < Implementations of the protected methods getNodeAt, getFirstNode, addFirstNode,
     addAfterNode, removeFirstNode, and removeAfterNode go here. >
   . . .

   protected class Node
   {
      private T data;    // data portion
      private Node next; // next to next node

      protected Node(T dataPortion)
      {
         data = dataPortion;
         next = null;
      } // end constructor

      private Node(T dataPortion, Node nextNode)
      {
         data = dataPortion;
         next = nextNode;
      } // end constructor

   < Implementations of the protected methods getData, setData, and getNextNode go here. >
   . . .

   < Implementation of the private method setNextNode goes here. >
   . . .

   } // end Node
} // end LinkedChainBase
```

17.11 We can now revise the class `LListRevised`, as shown in Listing 17-3.

LISTING 17-3 A revision of `LListRevised` that extends `LinkedChainBase`

```
public class LListRevised<T> extends LinkedChainBase<T>
                             implements ListInterface<T>
{
   public LListRevised()
   {
      super();
   } // end default constructor

   < Implementations of the public methods add, remove, replace, getEntry, and contains
     go here. >
   . . .

} // end LListRevised
```

The base class `LinkedChainBase` can be useful in other contexts as well. You can use it or `LListRevised` to define an efficient implementation of the ADT sorted list, as you will see next.

An Efficient Implementation of a Sorted List

17.12 Instead of calling ADT list operations to perform operations on an ADT sorted list, our implementation will execute faster if it can be similar to the linked implementation that we wrote in the previous chapter, beginning at Segment 16.7. The protected methods defined in the class `LinkedChainBase` will enable us to manipulate the list's underlying data structure faster than if we had to rely solely on the operations of the ADT list to do so. Thus, we want our class to extend `LinkedChainBase`. We begin it by writing

```
public class SortedLinkedList<T extends Comparable<? super T>>
             extends LinkedChainBase<T>
             implements SortedListInterface<T>
```

As before, we will implement a sorted list of `Comparable` objects.

The Method add

17.13 The `add` operation in our new class is quite similar to the one given in Segment 16.10 for the class `SortedLinkedList`. However, the details of that previous addition now are hidden within the protected methods `addFirstNode` and `addAfterNode` of `LinkedChainBase`. Thus, our revised method appears as follows (changes to the `add` method of Segment 16.10 are highlighted):

```
public void add(T newEntry)
{
   Node newNode = new Node(newEntry);
   Node nodeBefore = getNodeBefore(newEntry);

   if (nodeBefore == null) // no need to call isEmpty
      addFirstNode(newNode);
   else
      addAfterNode(nodeBefore, newNode);
} // end add
```

Preceding each of the protected methods with `super` is optional, since no other methods have their names.

Question 4 in Segment 16.11 of Chapter 16 suggested the simplification for empty lists that we made here. When a list is empty, `getNodeBefore` returns `null`. Thus, we can omit a call to `isEmpty` in the `if` statement.

17.14 **The private method `getNodeBefore`.** We still need to implement the private method `getNodeBefore`. The implementation is like the one given in Segment 16.11, but it uses `getFirstNode()` instead of `firstNode`:

```
private Node getNodeBefore(T anEntry)
{
   Node currentNode = getFirstNode();
   Node nodeBefore = null;

   while ((currentNode != null) &&
          (anEntry.compareTo(currentNode.getData()) > 0))
   {
      nodeBefore = currentNode;
      currentNode = currentNode.getNextNode();
   } // end while

   return nodeBefore;
} // end getNodeBefore
```

17.15 **Efficiency.** This version of the method `add` executes faster than the versions given in Segments 17.1 and 16.21. Those earlier versions can use only the operations of the ADT list—that is, the public methods of the class `LList`. Recall that those `add` methods first invoke `getPosition` to find where in the list the new entry belongs, and then they invoke the list's `add` method. The implementation of `getPosition` given in Segment 16.24 traverses the sorted list to determine the position for the new entry. Within the O(n) loop that performs this traversal is an invocation of the method `getEntry`. When `getEntry` has a linked implementation, it also traverses the sorted list, and so it is O(n). Thus, `getPosition` is O(n^2). It follows that the `add` methods in Segment 17.1 and Segment 16.21 are each O(n^2).

Our improved `add` method in Segment 17.13 adds a new node in its proper location by traversing the chain of nodes at most once. Even though the method must use the protected methods to alter the chain of linked nodes, it can add the new node as soon as it finds its proper location, without traversing the chain repeatedly. Thus, it is an O(n) operation.

17.16 **The rest of the class.** To implement `remove` and `getPosition`, we would make similar changes to their linked implementations. Recall that Chapter 16 left these implementations as an exercise. Finally, we need to implement the remaining operations of the ADT list that are common to the ADT sorted list but not inherited from `LinkedChainBase`. They are `getEntry`, `contains`, and `remove` (by position).

Note: You can use inheritance *and* maintain efficiency if your base class provides protected access to its underlying data structure.

Note: More than one implementation of an ADT is often possible. When choosing a particular implementation for a given application, you should consider all of the factors relevant to your situation. Execution time, memory use, and extensibility are just some of the issues that you should weigh. These same considerations should also be examined when you implement an ADT.

Chapter Summary

- This chapter demonstrated the difference between implementations that use composition and those that use inheritance. The basic ideas are the same as those described in Appendix C. With composition, a class has an object as a data field. The class's methods must act as clients of the object, so they use only the object's public methods. With inheritance, a class inherits all the public methods of its base class. Its implementation, as well as its client, can use these public methods.
- A base class can provide protected methods that enable its subclasses to manipulate its data fields in ways that its client cannot. In this way, a subclass's methods can be more efficient than if they had to use only public methods, as the client must.
- You can derive the sorted list from a base class that has appropriate protected methods and still have an efficient implementation.

Programming Tips

- If your class inherits methods that are inappropriate, you can override them with methods that throw an exception when invoked. In such a case, examine your design and consider whether inheritance was the right choice. Do the benefits of inheritance outweigh the inconvenience of overriding the inappropriate methods, or would composition provide a cleaner design?
- When designing a class, you should plan for its future use as well as the present need. If public accessor methods are not already in your design, provide protected accessor methods. Decide whether you want any future subclass to manipulate your class's data fields. If you do, provide protected methods that enable a subclass to make changes to the data fields both efficiently and safely.

Exercises

1. Implement the method `contains` for the class `SortedLinkedList`, as described in Segment 17.12. Take advantage of the list's sorted nature.

2. Write a constructor for the class `SortedLinkedList`, as described in Segment 17.12, that has as a parameter an instance of a class that implements `ListInterface`. The new sorted list should contain all the elements of the list, but in sorted order.

3. Write an `equals` method for the class `SortedLinkedList`, as described in Segment 17.12, that overrides the method `equals` inherited from the class `Object`. Assuming that objects in the list have an appropriate implementation of `equals`, your new method should return true if each entry in one list equals the corresponding entry in a second list.

4. Repeat Exercise 3 for the class `LListRevised`, as described in Segment 17.11, instead of the class `SortedLinkedList`.

5. If the class `LinkedChainBase` had the method `getIterator`, as described in Segment 15.17 of Chapter 15, what would you need to do to define an iterator for the class `SortedLinkedList`?

6. Compare the time efficiency of the sorted list method

```
    public void add(T newEntry)
```

   as given in Segment 17.13, with that of the list method

```
    public boolean add(int newPosition, T newEntry)
```

7. Exercise 5 in Chapter 16 asked you to design an ADT for a list of earthquake records. Can you implement this ADT by using inheritance, with `LinkedChainBase` as the base class? Which methods would you need to override? Is containment more appropriate?

8. Repeat Exercise 7, but consider the class SortedLinkedList, as described in Segment 17.12, instead of LinkedChainBase.

9. Suppose that the class LinkedChainBase, as given in Segment 17.10, implements the interface java.util.ListIterator, as described in Segment 15.32. If LinkedChainBase is a base class of SortedLinkedList, which of the iterator methods are appropriate for a sorted list?

Projects

1. Complete the implementation of the class SortedLinkedList that Segment 17.12 began.

2. Derive the class SortedLinkedList from the class LListRevised, as described in Segment 17.11. What is the disadvantage of this approach?

3. Exercise 4 in Chapter 16 asked you to design the ADT activity list. Show how you would implement such a class by using inheritance, with LListRevised as the base class.

4. Project 8 in Chapter 16 asked you to implement the method mode. Show how you would implement this method for the class SortedLinkedList, as described in Segment 17.12.

5. Using inheritance, derive the class LinkedListWithIterator, as described in Chapter 15 in Segment 15.19, from the class LListRevised.

6. Define a class of bags that implements the interface BagInterface, as given in Listing 1-1 of Chapter 1, and is a subclass of LinkedChainBase, as given in Listing 17-2.

Answers to Self-Test Questions

1. The list's method contains searches the entire list when the desired entry is not present. By calling getPosition, contains can take advantage of the sorted order of the entries. The answer to Question 13 of Chapter 16 gives such a method. By adding this method to SortedList, you can override LList's contains.

2. The client will be unaware of what has happened and will not know where in the sorted list the addition was made.

3. Advantages: The implementation is easy to write. You inherit methods such as isEmpty that you do not have to implement.
Disadvantages: The implementation is not efficient, especially when the implementation of the underlying list is linked. Using inheritance in this way is as inefficient as using composition. You inherit methods (add and replace by position) that you do not want. Moreover, since inheritance implies an *is a* relationship between the sorted list and the list, type compatibility dictates that the sorted list be able to behave like a list. This clearly is not the case, since you cannot insert or replace entries at any given position. Thus, a sorted list is not really a list.

4. **a.**
```
public void addToBeginning(T newEntry)
{
   addFirstNode(new Node(newEntry));
} // end addToBeginning
```

b.
```
public void addAfterMidpoint(T newEntry)
{
   Node nodeBefore = getNodeAt(getLength() / 2);
   addNodeAfter(nodeBefore, new Node(newEntry));
} // end addAfterMidpoint
```

Chapter 18

Searching

Contents

Prerequisites

Chapter 4	The Efficiency of Algorithms	Chapter 13	List Implementations That Use Arrays
Chapter 7	Recursion	Chapter 14	A List Implementation That Links Data
Chapter 12	Lists	Chapter 16	Sorted Lists

Objectives

After studying this chapter, you should be able to

- Search an array by using a sequential search
- Search an array by using a binary search
- Search a chain of linked nodes sequentially
- Describe the time efficiency of a search

People are always looking for something, be it a date, a mate, or a lost sock. In fact, searching is one of the most common tasks done for us by computers. Just think of how many times you search the Internet. This chapter looks at two simple search strategies, the sequential search and the binary search. You can use these strategies when implementing the method `contains` for either the ADT list or the ADT sorted list. A binary search is usually much faster than a sequential search when the data is in an array rather than a chain of linked nodes and when the data is sorted. Sorting data, however, usually takes much more time than searching it. This fact should influence your choice of search method in a given situation.

The Problem

18.1 Like the people in Figure 18-1, you can search your desk for a pen, your closet for your favorite sweater, or a list of names to see whether you are on it. Searching for a particular item—called the **target**—among a collection of many items is a common task.

FIGURE 18-1 Searching is an everyday occurrence

Let's find your name on that list. If `nameList` is an instance of an ADT list whose entries are names, we can search it by using the list operation `contains`. Recall that this method is boolean-valued and returns true if a given item is in the list.

The implementation of `contains` depends upon how we store the list entries. Among the implementations of the ADT list given in Chapters 13 and 14 is one that stores the list's entries in an array and another that uses a chain of linked nodes. Let's look at the array first.

Searching an Unsorted Array

18.2 As Segment 13.13 of Chapter 13 mentioned, a sequential search of a list compares the desired item—the target—with the first entry in the list, the second entry in the list, and so on until it either locates the desired entry or looks at all the entries without success. In an array-based implementation of the list, we search the array that contains the list's entries. We can implement this search either iteratively or recursively. This section looks at both approaches and examines their efficiencies.

VideoNote
Searching an array

Recall that two data fields of our list implementation in Chapter 13 are the array `list`, which contains the list's entries, and the integer `numberOfEntries`, which is the number of entries.

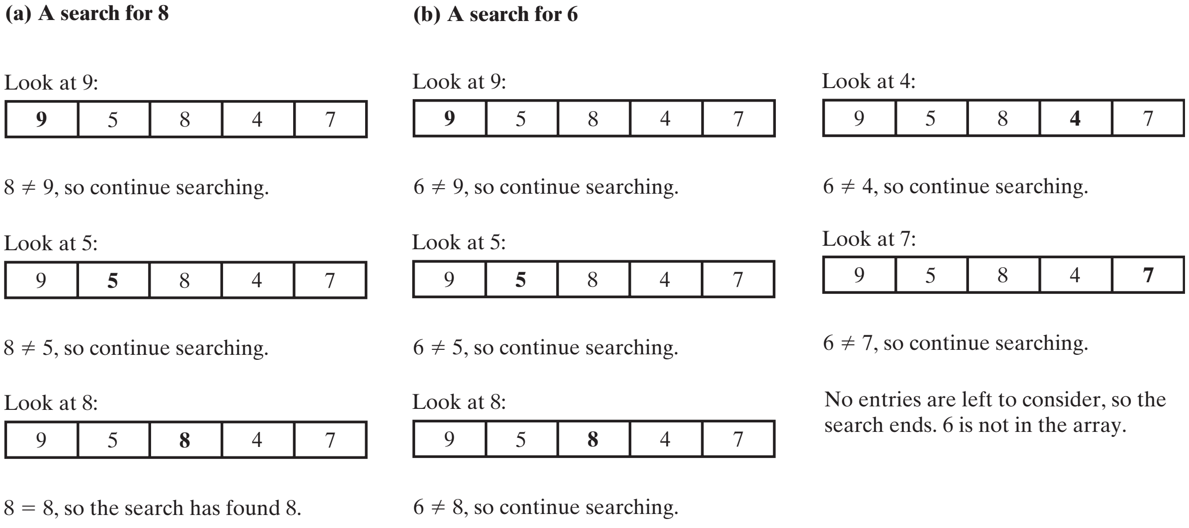
An Iterative Sequential Search of an Unsorted Array

18.3 The following implementation of `contains` was given in Segment 13.13. It uses a loop to search the array `list` containing `numberOfEntries` objects having the generic type `T` for a particular object `anEntry`:

```
public boolean contains(T anEntry)
{
   boolean found = false;

   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(list[index]))
         found = true;
   } // end for

   return found;
} // end contains
```

The loop exits as soon as it locates the first entry in the array that matches the desired item. In this case, `found` is true. On the other hand, if the loop examines all the entries in the list without finding one that matches `anEntry`, `found` remains false. Figure 18-2 provides an example of these two outcomes. For simplicity, our illustrations use integers.

FIGURE 18-2 An iterative sequential search of an array that (a) finds its target; (b) does not find its target


(a) A search for 8
Look at 9:
9 5 8 4 7
8 ≠ 9, so continue searching.
Look at 5:
9 5 8 4 7
8 ≠ 5, so continue searching.
Look at 8:
9 5 8 4 7
8 = 8, so the search has found 8.
(b) A search for 6
Look at 9:
9 5 8 4 7
6 ≠ 9, so continue searching.
Look at 5:
9 5 8 4 7
6 ≠ 5, so continue searching.
Look at 8:
9 5 8 4 7
6 ≠ 8, so continue searching.
Look at 4:
9 5 8 4 7
6 ≠ 4, so continue searching.
Look at 7:
9 5 8 4 7
6 ≠ 7, so continue searching.
No entries are left to consider, so the search ends. 6 is not in the array.

SELF-TEST

Question 1 Write a method `contains` that returns the index of the first array entry that equals `anEntry`. If the array does not contain such an entry, return -1.

Question 2 Write a method `contains` that performs an iterative sequential search of a list by using only operations of the ADT list. The method should return true if a given item is in a given list.

A Recursive Sequential Search of an Unsorted Array

18.4 We begin a sequential search of an array by looking at the first entry in the array. If that entry is the desired one, we end the search. Otherwise we search the rest of the array. Since this new search is also sequential and since the rest of the array is smaller than the original array, we have a recursive description of a solution to our problem. Well, almost. We need a base case. An empty array could be the base case because it never contains the desired item.

For the array `a`, we search the `n` elements `a[0]` through `a[n - 1]` by beginning with the first element, `a[0]`. If it is not the one we seek, we need to search the rest of the array—that is, we search array elements `a[1]` through `a[n - 1]`. In general, we search the array elements `a[first]` through `a[n - 1]`. To be even more general, we can search array elements `a[first]` through `a[last]`, where `first` ≤ `last`.

18.5 The following pseudocode describes the logic of our recursive algorithm:

```
Algorithm to search a[first] through a[last] for desiredItem
if (there are no elements to search)
   return false
else if (desiredItem equals a[first])
   return true
else
   return the result of searching a[first + 1] through a[last]
```

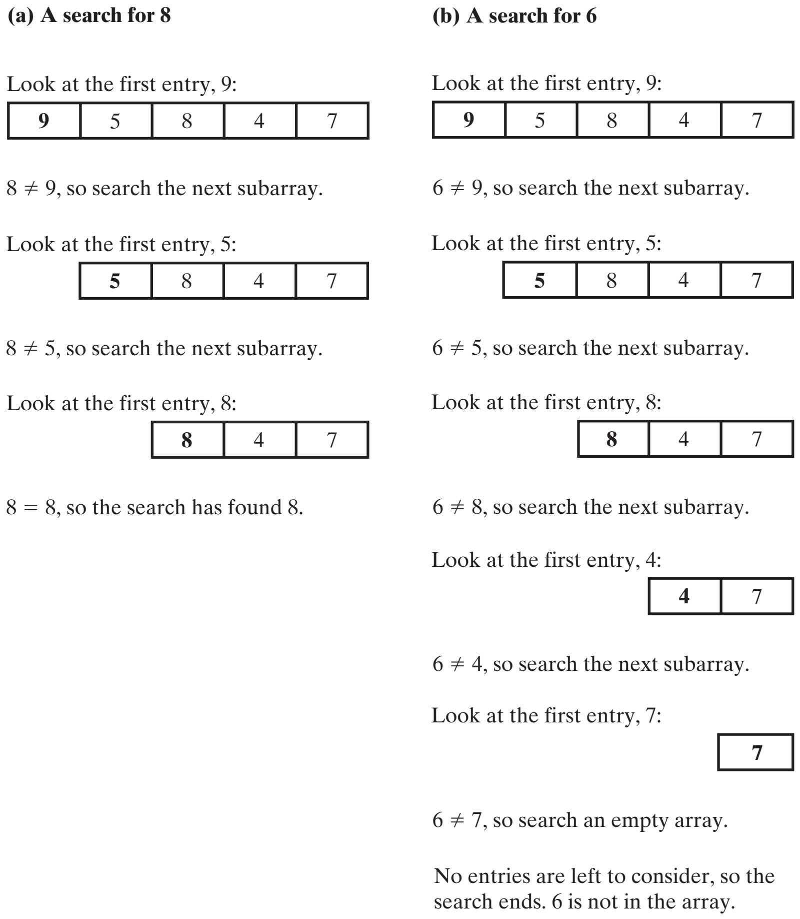
Figure 18-3 illustrates a recursive search of an array.

18.6 The method that implements this algorithm will need parameters `first` and `last`. To spare the client the detail of providing values for these parameters, and to allow the method `contains` to have the same header as it did in Segment 18.3, we implement the algorithm as a private method `search` that `contains` invokes. Since we again assume the array-based list implementation from Chapter 13, the array `list` takes the place of the array `a` in the previous algorithm, and `numberOfEntries` is the number of elements to search. Because `list` and `numberOfEntries` are data fields of the class that implements the list, they are not parameters of the methods that follow.

```
/** Searches the list for anEntry. */
public boolean contains(T anEntry)
{
   return search(0, numberOfEntries - 1, anEntry);
} // end contains

/** Searches list[first] through list[last] for desiredItem.
    @param first        an integer index >= 0 and < numberOfEntries
    @param last         an integer index >= 0 and < numberOfEntries
    @param desiredItem  the object to be found
    @return true if desiredItem is found */
```

```
private boolean search(int first, int last, T desiredItem)
{
   boolean found;

   if (first > last)
      found = false; // no elements to search
   else if (desiredItem.equals(list[first]))
      found = true;
   else
      found = search(first + 1, last, desiredItem);

   return found;
} // end search
```

FIGURE 18-3 A recursive sequential search of an array that (a) finds its target; (b) does not find its target



(a) A search for 8

Look at the first entry, 9:

9	5	8	4	7

8 ≠ 9, so search the next subarray.

Look at the first entry, 5:

5	8	4	7

8 ≠ 5, so search the next subarray.

Look at the first entry, 8:

8	4	7

8 = 8, so the search has found 8.

(b) A search for 6

Look at the first entry, 9:

9	5	8	4	7

6 ≠ 9, so search the next subarray.

Look at the first entry, 5:

5	8	4	7

6 ≠ 5, so search the next subarray.

Look at the first entry, 8:

8	4	7

6 ≠ 8, so search the next subarray.

Look at the first entry, 4:

4	7

6 ≠ 4, so search the next subarray.

Look at the first entry, 7:

7

6 ≠ 7, so search an empty array.

No entries are left to consider, so the search ends. 6 is not in the array.

Question 3 List the comparisons that the previous method `search` makes while searching for the object o in the array of objects

o1 o2 o3 o4 o5

Question 4 Implement at the client level a recursive method `search` by using only operations of the ADT list. The method should return true if a given item is in a given list.

The Efficiency of a Sequential Search of an Array

18.7 Whether you implement a sequential search iteratively or recursively, the number of comparisons will be the same. In the best case, you will locate the desired item first in the array. You will have made only one comparison, and so the search will be O(1). In the worst case, you will search the entire array. Either you will find the desired item at the end of the array or you will not find it at all. In either event, you will have made n comparisons for an array of n entries. The sequential search in the worst case is therefore O(n). Typically, you will look at about one-half of the entries in the array. Thus, the average case is O(n/2), which is just O(n).

Note: The time efficiency of a sequential search of an array

Best case	O(1)
Worst case:	O(n)
Average case:	O(n)

Searching a Sorted Array

A sequential search of an unsorted array is rather easy to understand and to implement. When the array contains relatively few entries, the search is efficient enough to be practical. However, when the array contains many entries, a sequential search can be time-consuming. For example, imagine that you are looking through a jar of coins for one minted during the year of your birth. A sequential search of 10 coins is not a problem. With 1000 coins, the search could be lengthy; with 1 million coins, it is overwhelming. A faster search method would be welcome. Fortunately, faster searches are possible.

A Sequential Search of a Sorted Array

18.8 Suppose that before you begin searching your coins, someone arranges them in sorted order by their dates. If you search the sorted coins in Figure 18-4 sequentially for the date 1998, you would look at the coins dated 1992, 1995, and 1997 before arriving at 1998. If, instead, you look for the date 2000, you would look at the first five coins without finding it. Should you keep looking? If the coins are sorted into ascending order and you have reached the one dated 2005, you will not find 2000 beyond it. If the coins were not sorted, you would have to examine all of them to see that 2000 was not present.

Note: A sequential search can be more efficient if the data is sorted.

FIGURE 18-4 Coins sorted by their mint dates

If our array is sorted into either ascending or descending order, we can use the previous ideas to revise the sequential search. This modified search can tell whether an item does not occur in an array faster than a sequential search of an unsorted array. The latter search always examines the entire array in this case. With a sorted array, however, the modified sequential search often makes far fewer comparisons to make the same determination. Exercise 2 at the end of this chapter asks you to implement a sequential search of a sorted array.

After expending the effort to sort an array, you often can search it even faster by using the method that we discuss next.

A Binary Search of a Sorted Array

18.9 Think of a number between 1 and 1 million. When I guess at your number, tell me whether my guess is correct, too high, or too low. At most, how many attempts will I need before I guess correctly? You should be able to answer this question by the time you reach the end of this section!

If you had to find a new friend's telephone number in a printed directory, what would you do? Typically you would open the book to a page near its middle, glance at the entries, and quickly see whether you were on the correct page. If you were not, you would decide whether you had to look at earlier pages—those in the left "half" of the book—or later pages—those in the right "half." What aspect of a telephone directory enables you to make this decision? The alphabetical order of the names does.

If you decided to look in the left half, you could ignore the entire right half. In fact, you could tear off the right half and discard it, as Figure 18-5 illustrates. You have reduced the size of the search problem dramatically, as you have only half of the book left to search. You then would repeat the process on this half. Eventually you would either find the telephone number or discover that it is not there. This approach—called a **binary search**—sounds suspiciously recursive.

18.10 Let's adapt these ideas to searching an array `a` of `n` integers that are sorted into ascending order. (Descending order would also work with a simple change in our algorithm.) We know that

`a[0] ≤ a[1] ≤ a[2] ≤ . . . ≤ a[n-1]`

Because the array is sorted, we can rule out whole sections of the array that could not possibly contain the number we are looking for—just as you ruled out an entire half of the telephone directory.

For example, if we are looking for the number 7 and we know that `a[5]` is equal to 9, then, of course, we know that 7 is less than `a[5]`. But we also know that 7 cannot appear after `a[5]` in the array, because the array is sorted. That is,

`7 < a[5] ≤ a[6] ≤ . . . ≤ a[n-1]`

FIGURE 18-5 Ignoring one half of the data when the data is sorted

I don't need this half of the book. I'll just throw it away.

We know this without looking at the elements beyond `a[5]`. We therefore can ignore these elements as well as `a[5]`. Similarly, if the sought-after number were greater than `a[5]` (for example, if we were looking for 10), we could ignore `a[5]` and all the elements before it.

Replacing the index `5` in the preceding example with whatever index is in the middle of the array leads to a first draft of an algorithm for a binary search of an array:

```
Algorithm to search a[0] through a[n - 1] for desiredItem
mid  = approximate midpoint between 0 and n - 1
if (desiredItem equals a[mid])
   return true
else if (desiredItem < a[mid])
   return the result of searching a[0] through a[mid - 1]
else if (desiredItem > a[mid])
   return the result of searching a[mid + 1] through a[n - 1]
```

Notice that to

Search `a[0]` *through* `a[n - 1]`

you have to either

Search `a[0]` *through* `a[mid - 1]`

or

Search `a[mid + 1]` *through* `a[n - 1]`

These two searches of a portion of the array are smaller versions of the very task we are solving, and so can be accomplished by calling the algorithm itself recursively.

18.11 One complication arises, however, when we write the recursive calls in the previous pseudocode. Each call searches a subrange of the array. In the first case, it is the elements indexed by 0 through mid - 1. In the second case, it is the elements indexed by mid + 1 through n - 1. Thus, we need two extra parameters—first and last—to specify the first and last indices of the subrange of the array that is to be searched. That is, we search a[first] through a[last] for desiredItem.

Using these parameters and making the recursive calls look more like Java, we can express the pseudocode as follows:

```
Algorithm binarySearch(a, first, last, desiredItem)
mid = approximate midpoint between first and last
if (desiredItem equals a[mid])
   return true
else if (desiredItem < a[mid])
   return binarySearch(a, first, mid - 1, desiredItem)
else if (desiredItem > a[mid])
   return binarySearch(a, mid + 1, last, desiredItem)
```

To search the entire array, we initially set first to 0 and last to n - 1. Each recursive call will then use some other values for first and last. For example, the recursive call that appears first would set first to 0 and last to mid - 1.

When you write any recursive algorithm, you should always check that the recursion is not infinite. Let's check whether every possible invocation of the algorithm will lead to a base case. Consider the three cases in the nested if statement in the previous pseudocode. In the first case, the sought-after item is found in the array, so there is no recursive call, and the process terminates. In each of the other two cases, a smaller portion of the array is searched by a recursive call. If the sought-after item is in the array, the algorithm uses smaller and smaller portions of the array until it finds the item. But what if the item is not anywhere in the array? Will the resulting series of recursive calls eventually lead to a base case? Unfortunately not, but that is not hard to fix.

18.12 Note that in each recursive call, either the value of first is increased or the value of last is decreased. If they ever pass each other and first actually becomes larger than last, we will have run out of array elements to check. In that case, desiredItem is not in the array. If we add this test to our pseudocode and refine the logic a bit, we get the following more complete algorithm:

```
Algorithm binarySearch(a, first, last, desiredItem)
mid = (first + last) / 2 // approximate midpoint
if (first > last)
   return false
else if (desiredItem equals a[mid])
   return true
else if (desiredItem < a[mid])
   return binarySearch(a, first, mid - 1, desiredItem)
else // desiredItem > a[mid]
   return binarySearch(a, mid + 1, last, desiredItem)
```

Figure 18-6 provides an example of a binary search.

Question 5 When the previous binary search algorithm searches the array in Figure 18-6 for 8 and for 16, how many comparisons to an array entry are necessary in each case?

FIGURE 18-6 A recursive binary search of a sorted array that (a) finds its target; (b) does not find its target

(a) A search for 8

Look at the middle entry, 10:

2	4	5	7	8	**10**	12	15	18	21	24	26
0	1	2	3	4	5	6	7	8	9	10	11

8 < 10, so search the left half of the array.

Look at the middle entry, 5:

2	4	**5**	7	8
0	1	2	3	4

8 > 5, so search the right half of the array.

Look at the middle entry, 7:

7	8
3	4

8 > 7, so search the right half of the array.

Look at the middle entry, 8:

8
4

8 = 8, so the search ends. 8 is in the array.

(b) A search for 16

Look at the middle entry, 10:

2	4	5	7	8	**10**	12	15	18	21	24	26
0	1	2	3	4	5	6	7	8	9	10	11

16 > 10, so search the right half of the array.

Look at the middle entry, 18:

12	15	**18**	21	24	26
6	7	8	9	10	11

16 < 18, so search the left half of the array.

Look at the middle entry, 12:

12	15
6	7

16 > 12, so search the right half of the array.

Look at the middle entry, 15:

15
7

16 > 15, so search the right half of the array.

The next subarray is empty, so the search ends. 16 is not in the array.

18.13 Imagine an array-based implementation of the ADT sorted list. An array `list`—which is a private data field—holds the list's entries in sorted order. Another field `numberOfEntries` records the number of entries. When implementing the ADT's method `contains`, the algorithm `binarySearch` becomes a private method that `contains` invokes. The array `list` takes the place of the array `a` in the algorithm, and `numberOfEntries` takes the place of `n`. As before in Segment 18.6, since `list` and `numberOfEntries` are data fields, they are not parameters of `contains` and `binarySearch`.

Although the implementations of the sequential search that were given in Segments 18.3 and 18.6 use the method `equals` to make the necessary comparisons, the binary search requires more than a test for equality. To make the necessary comparisons, we need the method `compareTo`. Since all classes inherit `equals` from the class `Object` and can override it, all objects can invoke `equals`. But for an object to invoke `compareTo`, it must belong to a class that implements the interface `Comparable`. Such is the case for objects in a sorted list, as Segment 16.1 indicated.

Like the class `SortedLinkedList` in Segment 16.7, our implementation of a sorted list could begin as follows:

```
public class SortedArrayList<T extends Comparable<? super T>>
             implements SortedListInterface<T>
```

Thus, the method `binarySearch` can have the following implementation:

```
private boolean binarySearch(int first, int last, T desiredItem)
{
   boolean found;
   int mid = first + (last - first) / 2;

   if (first > last)
      found = false;
   else if (desiredItem.equals(list[mid]))
      found = true;
   else if (desiredItem.compareTo(list[mid]) < 0)
      found = binarySearch(first, mid - 1, desiredItem);
   else
      found = binarySearch(mid + 1, last, desiredItem);

   return found;
} // end binarySearch
```

Now `contains` appears as follows:

```
public boolean contains(T anEntry)
{
   return binarySearch(0, numberOfEntries - 1, anEntry);
} // end contains
```

Note: Notice that the Java computation of the midpoint `mid` is

```
int mid = first + (last - first) / 2;
```

instead of

```
int mid = (first + last) / 2;
```

as the pseudocode would suggest. If you were to search an array of at least 2^{30}, or about one billion, elements, the sum of `first` and `last` could exceed the largest possible `int` value of 2^{30} - 1. Thus, the computation `first + last` would overflow to a negative integer and result in a negative value for `mid`. If this negative value of `mid` was used as an array index, an `ArrayIndexOutOfBoundsException` would occur. The computation `first + (last - first) / 2`, which is algebraically equivalent to `(first + last) / 2`, avoids this error.

Programming Tip: Classes that implement the `Comparable` interface must define a `compareTo` method. Such classes should also define an `equals` method that overrides the `equals` method inherited from `Object`. Both `compareTo` and `equals` should use the same test for equality. The previous method `binarySearch` calls both the method `equals` and the method `compareTo`. If the objects in the array did not have an appropriate `equals` method, `binarySearch` would not execute correctly. Note, however, that you could use `compareTo` instead of `equals` to test for equality.

Question 6 During a binary search, which elements in the array

4 8 12 14 20 24

are compared to the target when the target is **a.** 2; **b.** 8; **c.** 15.

Question 7 Modify the previous method `contains` so that it returns the index of the first array entry that equals `anEntry`. If the array does not contain such an entry, return −1. You will have to modify `binarySearch` also.

Question 8 What changes to the binary search algorithm are necessary when the array is sorted in descending order (from largest down to smallest) instead of ascending order, as we have assumed during our discussion?

Java Class Library: The Method `binarySearch`

18.14 The class `Arrays` in the package `java.util` defines several versions of a static method `binarySearch` with the following specification:

```
/** Searches an entire array for a given item.
    @param array        an array sorted in ascending order
    @param desiredItem  the item to be found in the array
    @return index of the array entry that equals desiredItem;
            otherwise returns -belongsAt - 1, where belongsAt is
            the index of the array element that should contain
            desiredItem */
public static int binarySearch(type[] array, type desiredItem);
```

Here, both occurrences of *type* must be the same; *type* can be `Object` or any of the primitive types `byte`, `char`, `double`, `float`, `int`, `long`, or `short`.

The Efficiency of a Binary Search of an Array

18.15 The binary search algorithm eliminates about half of the array from consideration after examining only one element. It then eliminates another quarter of the array, and then another eighth, and so on. Thus, most of the array is not searched at all, saving much time. Intuitively, the binary search algorithm is very fast.

But just how fast is it? Counting the comparisons that occur will provide a measure of the algorithm's efficiency. To see the algorithm's worst-case behavior, you count the maximum number of comparisons that can occur when searching an array of *n* items. Comparisons are made each time the algorithm divides the array in half. After each division, half of the items are left to search.

That is, beginning with n items, we would be left with $n/2$ items, then $n/4$ items, and so on. In the worst case, the search would continue until only one item was left. That is, $n/2^k$ would equal 1 for some integer value of k. This value of k gives us the number of times the array is divided in half, or the number of recursive calls to `binarySearch`.

If n is a power of 2, n is 2^k for some positive k. By the definition of a logarithm, k is $\log_2 n$. If n is not a power of 2, you can find a positive integer k so that n lies between 2^{k-1} and 2^k. For example, if n is 14, $2^3 < 14 < 2^4$. Thus, we have for some $k \geq 1$,

$$
\begin{aligned}
&2^{k-1} < n < 2^k \\
&k - 1 < \log_2 n < k \\
&k = 1 + \log_2 n \text{ rounded down} \\
&\quad = \log_2 n \text{ rounded up}
\end{aligned}
$$

To summarize,

$$
\begin{aligned}
&k = \log_2 n \text{ when } n \text{ is a power of 2} \\
&k = \lceil \log_2 n \rceil \text{ when } n \text{ is not a power of 2}
\end{aligned}
$$

In general, k—the number of recursive calls to `binarySearch`—is $\lceil \log_2 n \rceil$.

Note: Ceiling and floors

The *ceiling* of a number x, denoted as $\lceil x \rceil$, is the smallest integer greater than or equal to x. For example, $\lceil 4.1 \rceil$ is 5. The *floor* of a number x, denoted as $\lfloor x \rfloor$, is the largest integer less than or equal to x. For example, $\lfloor 4.9 \rfloor$ is 4. When you *truncate* a positive real number to an integer, you actually are computing the number's floor by discarding any fractional portion.

Each call to `binarySearch`, with the possible exception of the last one, makes two comparisons between the target and the middle element in the array: One tests for equality and one for less than or greater than. Thus, the binary search performs at most $2\lceil \log_2 n \rceil$ comparisons, and so in the worst case is O(log n).

To search an array of 1000 elements, the binary search will compare the target to about 10 array entries in the worst case. In contrast, a simple sequential search could compare the target to as many as all 1000 array entries, and on average will compare it to about 500 array elements.

Note: The time efficiency of a binary search of an array

Best case:	O(1)
Worst case:	O(log n)
Average case:	O(log n)

SELF-TEST

Question 9 Think of a number between 1 and 1 million. When I guess at your number, tell me whether my guess is correct, too high, or too low. At most, how many attempts will I need before I guess correctly? *Hint*: You are counting guesses, not comparisons.

18.16 **Another approach.** The binary search makes comparisons each time it locates the midpoint of the array. Thus, to search n items, the binary search looks at the middle item and then searches $n/2$ items. If we let $t(n)$ represent the time requirement for searching n items, we find that at worst

$$t(n) = 1 + t(n/2) \text{ for } n > 1$$
$$t(1) = 1$$

We encountered this recurrence relation in Segment 7.25 of Chapter 7. There, we showed that

$$t(n) = 1 + \log_2 n$$

Thus, the binary search is O(log n) in the worst case.

Searching an Unsorted Chain

18.17 Within a linked implementation of either the ADT list or the ADT sorted list, the method `contains` would search a chain of linked nodes for the target. As you will see, a sequential search is really the only practical choice. We begin with a chain whose data is unsorted, as typically would be the case for the ADT list.

VideoNote
Searching a linked chain

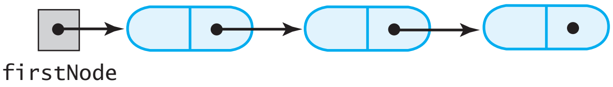
Regardless of a list's implementation, a sequential search of the list looks at consecutive entries in the list, beginning with the first one, until either it finds the desired entry or it looks at all entries without success. When the implementation is linked, however, moving from node to node is not as simple as moving from one array location to another. Despite this fact, you can implement a sequential search of a chain of linked nodes either iteratively or recursively and with the same efficiency as that of a sequential search of an array.

An Iterative Sequential Search of an Unsorted Chain

18.18 Figure 18-7 illustrates a chain of linked nodes that contain the list's entries. Recall from Segment 14.8 of Chapter 14 that `firstNode` is a data field of the class that implements the list. While it is clear that a method can access the first node in this chain by using the reference `firstNode`, how can it access the subsequent nodes? Since `firstNode` is a data field that always references the first node in the chain, we would not want our search to alter it or any other aspect of the list. Thus, an iterative method `contains` should use a local reference variable `currentNode` that initially contains the same reference as `firstNode`. To make `currentNode` reference the next node, we would execute the statement

```
currentNode = currentNode.getNextNode();
```

FIGURE 18-7 A chain of linked nodes that contain the entries in a list



The iterative sequential search has the following straightforward implementation:

```
public boolean contains(T anEntry)
{
   boolean found = false;
   Node currentNode = firstNode;

   while (!found && (currentNode != null))
   {
      if (anEntry.equals(currentNode.getData()))
         found = true;
      else
         currentNode = currentNode.getNextNode();
   } // end while

   return found;
} // end contains
```

This implementation is like the one given in Segment 14.19 of Chapter 14.

A Recursive Sequential Search of an Unsorted Chain

18.19 When done recursively, a sequential search looks at the first entry in the list and, if it is not the desired entry, searches the rest of the list. This recursive approach is the same regardless of whether you implement the search at the client level by using only the list's ADT operations—as you did in Question 4—or as a public method of an array-based implementation of the list—as we did in Segment 18.6. We use the same approach for a linked implementation of the list, as follows.

How would you implement the step *search the rest of the list* when the list's entries are in a chain of linked nodes? The iterative method `contains` that you saw in the previous segment uses a local variable `currentNode` to move from node to node. A recursive method could not have `currentNode` as a local variable, since `currentNode` would get reset to an initial value at each recursive call. Instead, such a method needs `currentNode` as a formal parameter. But then we would have a method whose parameter depends on the list's implementation, making it unsuitable as a public method. Just as we did earlier in Segments 18.6 and 18.13, we would make this search method private and call it from the public method `contains`.

18.20 The private recursive method `search` examines the list entry in the node that its parameter `currentNode` references. If the entry is not the desired one, the method recursively calls itself with an argument that references the next node in the chain. Thus, the method `search` has the following implementation:

```
// Recursively searches a chain of nodes for desiredItem,
// beginning with the node that currentNode references.
private boolean search(Node currentNode, T desiredItem)
{
   boolean found;

   if (currentNode == null)
      found = false;
   else if (desiredItem.equals(currentNode.getData()))
      found = true;
   else
      found = search(currentNode.getNextNode(), desiredItem);

   return found;
} // end search
```

Now we write the public method `contains` as follows:

```
public boolean contains(T anEntry)
{
   return search(firstNode, anEntry);
} // end contains
```

Notice that the call to the method `search` initializes the parameter `currentNode` to `firstNode`, much as an iterative method initializes its local variable `currentNode` to `firstNode`.

The Efficiency of a Sequential Search of a Chain

18.21 The efficiency of a sequential search of a chain is really the same as that of a sequential search of an array. In the best case, the desired item will be first in the chain. Thus, at best the search will be O(1), since you will have made only one comparison. In the worst case, you will search the entire chain, making *n* comparisons for a chain of *n* nodes. Therefore, the sequential search in the worst case is O(*n*). Typically, you will look at about half of the nodes in the chain. Thus, the average-case search is O(*n*/2), which is just O(*n*).

Note: The time efficiency of a sequential search of a chain of linked nodes

Best case:	O(1)
Worst case:	O(*n*)
Average case:	O(*n*)

Searching a Sorted Chain

We now search a chain whose data is sorted. Such a chain would occur in a linked implementation of the ADT sorted list.

A Sequential Search of a Sorted Chain

18.22 Searching a chain of linked nodes whose data is sorted is similar to sequentially searching a sorted array, as described in Segment 18.8. Here, we incorporate that logic into the following implementation of `contains`:

```
public boolean contains(T anEntry)
{
   Node currentNode = firstNode;

   while ( (currentNode != null) &&
           (anEntry.compareTo(currentNode.getData()) > 0) )
   {
      currentNode = currentNode.getNextNode();
   } // end while

   return (currentNode != null) &&
           anEntry.equals(currentNode.getData());
} // end contains
```

The method traverses the chain until it either reaches a node that could contain the desired object or examines all nodes without success. Following the traversal, a final test is necessary to draw a conclusion.

A Binary Search of a Sorted Chain

18.23 A binary search of an array looks first at the element that is at or near the middle of the array. It is easy to determine the index `mid` of this element by computing `first + (last - first) / 2`, where `first` and `last` are the indices of the first and last elements, respectively, in the array. Accessing this middle element is also easy: For an array `a`, it is simply `a[mid]`.

Now consider searching a chain of linked nodes, such as the one you saw earlier in Figure 18-7, whose nodes are sorted. How would you access the entry in the middle node? Since this chain has only three nodes, you can get to the middle node quickly, but what if the chain contained 1000 nodes? In general, you need to traverse the chain, beginning at the first node, until you reach the middle node. How will you know when you get there? If you know the length of the chain, you can divide the length in half and count nodes as you traverse. The details are not as important as a realization that it takes a bit of work to access the middle node.

After looking at the entry in the middle node, you probably need to ignore half of the chain and search the other half. Do not change the chain when ignoring part of it. Remember that you want to search the chain, not destroy it. Once you know which half to search, you must find its middle node, again by traversing the chain. It should be clear to you that a binary search of a linked chain of nodes would be challenging to implement and less efficient than a sequential search.

Note: A binary search of a chain of linked nodes is impractical.

Choosing a Search Method

18.24 **Choosing between a sequential search and a binary search.** You just saw that you should use a sequential search to search a chain of linked nodes. But if you want to search an array of objects, you need to know which algorithms are applicable. To use a sequential search, the objects must have a method `equals` that ascertains whether two distinct objects are equal in some sense. Since all objects inherit `equals` from the class `Object`, you must ensure that the objects you search have overridden `equals` with an appropriate version. To perform a binary search on an array of objects, on the other hand, the objects must have a `compareTo` method and the array must be sorted. If these conditions are not met, you must use a sequential search.

If both search algorithms are applicable to your array, what search should you use? If the array is small, you can simply use a sequential search. If the array is large and already sorted, a binary search is typically much faster than a sequential search. But if the array is not sorted, should you sort it and then use a binary search? The answer depends on how often you plan to search the array. Sorting takes time, typically more time than a sequential search would. If you plan to search an unsorted array only a few times, sorting the array so that you can use a binary search likely will not save you time; use a sequential search instead.

Figure 18-8 summarizes the time efficiencies of the sequential search and the binary search. Only the sequential search is applicable to unsorted data. The efficiencies given for the binary search are for an array-based sorted list. For a large, sorted list, the binary search is typically much faster than a sequential search.

FIGURE 18-8 The time efficiency of searching, expressed in Big Oh notation

	Best Case	**Average Case**	**Worst Case**
Sequential search (unsorted data)	O(1)	O(n)	O(n)
Sequential search (sorted data)	O(1)	O(n)	O(n)
Binary search (sorted array)	O(1)	O(log n)	O(log n)

18.25 **Choosing between an iterative search and a recursive search.** Since the recursive sequential search is tail recursive, you can save some time and space by using the iterative version of the search. The binary search is fast, so using recursion will not require much additional space for the recursive calls. Also, coding the binary search recursively is somewhat easier than coding it iteratively. To convince yourself of this, try to code an iterative version of the binary search. (See Exercise 6 at the end of this chapter.)

Chapter Summary

- A sequential search of either a list, an array, or a chain looks at the first item, the second item, and so on until it either finds a particular item or discovers that the item does not occur in the group.
- The average-case performance of a sequential search is O(*n*).
- Typically, you perform a sequential search iteratively, although a simple recursive approach is also possible.
- A binary search of an array requires that the array be sorted. It looks first to see whether the desired item is at the middle of the array. If it is not, the search decides in which half of the array the item can occur and repeats this strategy on only this half.
- A binary search is O(log *n*) in the worst case.
- Typically, you perform a binary search recursively, although an iterative approach is also possible.
- A binary search of a linked chain of nodes is impractical.

Programming Tip

- Classes that implement the `Comparable` interface must define a `compareTo` method. Such classes should also define an `equals` method that overrides the `equals` method inherited from `Object`. Both `compareTo` and `equals` should use the same test for equality. The method `binarySearch` in Segment 18.13 calls both the method `equals` and the method `compareTo`. If the objects in the array did not have an appropriate `equals` method, `binarySearch` would not execute correctly. Note, however, that you could use `compareTo` instead of `equals` to test for equality.

Exercises

1. Revise the recursive method `search`, as given in Segment 18.6, so that it looks at the last entry in the array instead of the first one.

2. When searching a sorted array sequentially, you can ascertain that a given item does not appear in the array without searching the entire array. For example, if you search the array

   2 5 7 9

   for 6, you can use the approach described in Segment 18.8. That is, you compare 6 to 2, then to 5, and finally to 7. Since you did not find 6 after comparing it to 7, you do not have to look further, because the other entries in the array are greater than 7 and therefore cannot equal 6. Thus, you do not simply ask whether 6 equals an array entry, you also ask whether it is greater than the entry. Since 6 is greater than 2, you continue the search. Likewise for 5. Since 6 is less than 7, you have passed the point in the array where 6 would have had to occur, so 6 is not in the array.

   a. Write an iterative method `contains` to take advantage of these observations when searching a sorted array sequentially.
   b. Write a recursive method `search` that a method `contains` can call to take advantage of these observations when searching a sorted array sequentially.

3. How many comparisons are made by the recursive method `search` described in Part *b* of the previous exercise when searching the array in Figure 18-6 for 8 and for 16?

4. Trace the method `binarySearch`, as given in Segment 18.13, when searching for 4 in the following array of values:

   5 8 10 13 15 20 22 26 30 31 34 40

   Repeat the trace when searching for 34.

5. Modify the method `binarySearch` in Segment 18.13 so that it returns the index of the first array entry that equals `desiredItem`. If the array does not contain such an entry, return `-(belongsAt + 1)`, where `belongsAt` is the index of the array location that should contain `desiredItem`. At the end of Segment 18.13, Question 7 asked you to return `-1` in this case. Notice that both versions of the method return a negative integer if and only if `desiredItem` is not found.

6. Implement a binary search of an array iteratively. Model your methods after the ones given in Segment 18.13.

7. Write a recursive method to find the largest object in an array-based list of `Comparable` objects. Like the binary search, your method should divide the array into halves. Unlike the binary search, your method should search both halves for the largest object. The largest object in the array will then be the larger of these two largest objects.

8. Suppose that you are searching an unsorted array of objects that might contain duplicates. Devise an algorithm that returns a list of the indices of all objects in the array that match a given object. If the desired object is not in the list, return an empty list.

9. Repeat the previous exercise for a sorted array. Your algorithm should be recursive and efficient.

10. In Segment 18.13, the method `contains` calls a private method that performs a binary search of an array. Assuming a linked implementation of the ADT sorted list, revise this private method to perform a binary search of a chain of nodes. Do not alter the chain.

11. Consider the number $f(n)$ of comparisons that a sequential search makes in the worst case.

    a. Write a recurrence relation for $f(n)$.
    b. Prove by induction on n that $f(n) = n$.

12. At the end of Segment 18.3, Question 2 asked you to write a method that performs an iterative sequential search of a list by using only operations of the ADT list. Compare the time efficiency of this method with the ADT operation `contains`.

13. In Segment 18.7, we said that a sequential search of an array will examine on average about half of the n entries. Let's look a little more carefully at this computation. A sequential search is either successful or not. Let α be the probability that we will find the desired value in the array and $1 - \alpha$ be the probability that we will not. We further assume that the value, if found, is equally likely to be in each of the locations of the array. We need to consider each possibility.

    For each case, we count the comparisons and determine its probability of occurrence. To find the average number of comparisons made by the search, we first multiply each probability by the number of comparisons in each case. The following table summarizes these results:

	Probability	**Number of Comparisons**	**Product**
Found at index 0	α / n	1	α / n
Found at index 1	α / n	2	$2 \alpha / n$
Found at index 2	α / n	3	$3 \alpha / n$
...	...	...	...
Found at index n - 2	α / n	$n - 1$	$(n - 1) \alpha / n$
Found at index n - 1	α / n	n	α
Not found	$1 - \alpha$	n	$(1 - \alpha) n$

a. Compute the average number of comparisons by adding all the products in the last column of the table.
b. What is the average number of comparisons if the search is guaranteed to be successful ($\alpha = 1$)?
c. What is the average number of comparisons if the search is guaranteed to be unsuccessful ($\alpha = 0$)?
d. What is the average number of comparisons if the search is successful half of the time ($\alpha = 0.5$)?

14. Repeat Part *a* of the previous exercise, but now assume that we are not equally likely to search for each value in the array. We could arrange the *n* items in the array such that the ones we are more likely to search for occur first. Suppose that we search for the first item one half of the time, the second item one quarter of the time, the third item one eighth of the time, and so on. We will search for the last two items $1/2^{n-1}$ of the time. Revise the table in the previous exercise accordingly.

Projects

1. When an object does not occur in an array, a sequential search for it must examine the entire array. If the array is sorted, you can improve the search by using the approach described in Exercise 2. A **jump search** is an attempt to reduce the number of comparisons even further.

Instead of examining the *n* objects in the array *a* sequentially, you look at the elements $a[j]$, $a[2j]$, $a[3j]$, and so on, for some positive $j < n$. If the target *t* is less than one of these objects, you need to search only the portion of the array between the current object and the previous object. For example, if *t* is less than $a[3j]$ but is greater than $a[2j]$, you search the elements $a[2j + 1]$, $a[2j + 2]$, . . ., $a[3j - 1]$ by using the method in Exercise 2. What should you do when $t > a[k * j]$, but $(k + 1) * j > n$?

Devise an algorithm for performing a jump search. Then, using $\lceil \sqrt{n} \rceil$ as the value of *j*, implement the jump search.

2. An **interpolation search** assumes that the data in an array is sorted and uniformly distributed. Whereas a binary search always looks at the middle item in an array, an interpolation search looks where the sought-for item is more likely to occur. For example, if you searched your telephone book for Victoria Appleseed, you probably would look near its beginning rather than its middle. And if you discovered many Appleseeds, you would look near the last Appleseed.

Instead of looking at the element `a[mid]` of an array `a`, as the binary search would, an interpolation search examines `a[index]`, where

```
p = (desiredElement - a[first])/(a[last] - a[first])
index = first+⌈(last−first)×p⌉
```

Implement an interpolation search of an array. For particular arrays, compare the outcomes of an interpolation search and of a binary search. Consider arrays that have uniformly distributed entries and arrays that do not.

3. Suppose that you have numerical data stored in a two-dimensional array, such as the one in Figure 18-9. The data in each row and in each column is sorted in increasing order.

a. Devise an efficient search algorithm for an array of this type.
b. If the array has *m* rows and *n* columns, what is the Big Oh performance of your algorithm?
c. Implement and test your algorithm.

FIGURE 18-9 A two-dimensional array for Project 3

1	4	55	88
7	15	61	91
14	89	90	99

4. Consider an array `data` of *n* numerical values in sorted order and a list of numerical target values. Your goal is to compute the smallest range of array indices that contains all of the target values. If a target value is smaller than `data[0]`, the range should start with -1. If a target value is larger than `data[n - 1]`, the range should end with *n*.

   For example, given the array in Figure 18-10 and the target values (8, 2, 9, 17), the range is -1 to 5.

   a. Devise an efficient algorithm that solves this problem.
   b. If you have *n* data values in the array and *m* target values in the list, what is the Big Oh performance of your algorithm?
   c. Implement and test your algorithm.

FIGURE 18-10 An array for Project 4

5	8	10	13	15	20	22	26
0	1	2	3	4	5	6	7

5. One way to organize a collection of words is to use an array of sorted lists. The array contains one sorted list for each letter of the alphabet. To add a word to this data structure, you add it to the sorted list that corresponds to the word's first letter. Design an ADT for such a collection, including the operations `add` and `contains`. Define a Java interface for your ADT. Then implement your interface as a class and test it. Use a text file of words to populate your data structure.

Answers to Self-Test Questions

1.
```
public int contains(T anEntry)
{
   boolean found = false;
   int result = -1;
   for (int index = 0; !found && (index < numberOfEntries); index++)
   {
      if (anEntry.equals(list[index]))
      {
         found = true;
         result = index;
      } // end if
   } // end for

   return result;
} // end contains
```

2.
```
public static <T> boolean contains(AList<T> theList, T anEntry)
{
   boolean found = false;
   int length = theList.getLength();
   for (int position = 1; !found && (position <= length); position++)
   {
      if (anEntry.equals(theList.getEntry(position)))
         found = true;
   } // end for
   return found;
} // end contains
```

3. The object o is compared with o1, then o2, o3, o4, and o5.

4.
```
public static <T> boolean contains(AList<T> theList, T anEntry)
{
   return search(theList, 1, theList.getLength(), anEntry);
} // end contains

private static <T> boolean search(AList<T> theList, int first, int last,
                                  T desiredItem)
{
   boolean found;

   if (first > last)
      found = false;
   else if (desiredItem.equals(theList.getEntry(first)))
      found = true;
   else
      found = search(theList, first + 1, last, desiredItem);
   return found;
} // end search
```

5. Searching for 8 requires seven comparisons, as follows:
8 == 10?
8 < 10?
8 == 5?
8 < 5?
8 == 7?
8 < 7?
8 == 8?

Searching for 16 requires eight comparisons, as follows:
16 == 10?
16 < 10?
16 == 18?
16 < 18?
16 == 12?
16 < 12?
16 == 15?
16 < 15?

6. **a.** 12 and 4.
b. 12, 4, and 8.
c. 12, 20, and 14.

7.
```
public int contains(T anEntry)
{
   return binarySearch(0, numberOfEntries - 1, anEntry);
} // end contains

private int binarySearch(int first, int last, T desiredItem)
{
   int result;
   int mid = first + (last - first) / 2;

   if (first > last)
      result = -1;
   else if (desiredItem.equals(list[mid]))
      result = mid;
   else if (desiredItem.compareTo(list[mid]) < 0)
```

```
      result = binarySearch(first, mid - 1, desiredItem);
   else
      result = binarySearch(mid + 1, last, desiredItem);
   return result;
} // end binarySearch
```

8. In the second `else if`, change < to >.

9. 20 (log 1,000,000 rounded up).

This page intentionally left blank

Chapter 19

Dictionaries

Contents

Prerequisites

Chapter	12	Lists
Chapter	15	Iterators
Chapter	18	Searching

Objectives

After studying this chapter, you should be able to

- Describe the operations of the ADT dictionary
- Distinguish between a dictionary and a list
- Use a dictionary in a program

If you need to check the meaning of a word, you look it up in a dictionary. If you need a friend's address, you consult your address book. If you need someone's telephone number, you check your list of contacts on your cell phone, use a telephone directory, or search for it online.

Each of these examples involves a kind of dictionary. This chapter describes and uses an abstract data type that generalizes our everyday notion of a dictionary. Subsequent chapters will examine implementations of this ADT.

The previous examples—finding a word's definition, a friend's address, or someone's telephone number—are all examples of searching a dictionary. The previous chapter examined how to search a list. You will see that a dictionary provides a more powerful way to organize data that will be searched.

Specifications for the ADT Dictionary

19.1 The ADT **dictionary**—also called a **map**, **table**, or **associative array**—contains entries that each have two parts:

- A keyword—usually called a **search key**—such as an English word or a person's name
- A value—such as a definition, an address, or a telephone number—associated with that key

The search key enables you to locate the desired entry.

VideoNote
The ADT dictionary

Figure 19-1 illustrates an everyday English dictionary. Each entry has a word as the search key and the word's definition as the value associated with the key. In general, the search keys and values in an ADT dictionary are objects, as shown in Figure 19-2. Each search key is paired with a corresponding value.

The ADT dictionary organizes and identifies its entries by their search keys, rather than by another criterion such as position. Thus, you can retrieve or remove an entry from a dictionary given only the entry's search key. The fact that every entry in a dictionary has a search key distinguishes the dictionary from other ADTs such as a list. Although you certainly could put an entry that has a search key in a list, a list's data is organized by position, not by search key.

FIGURE 19-1 An English dictionary

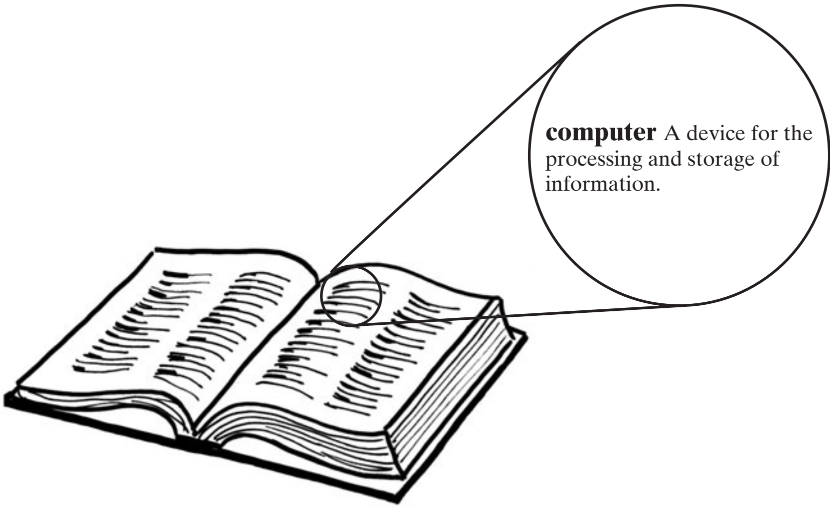
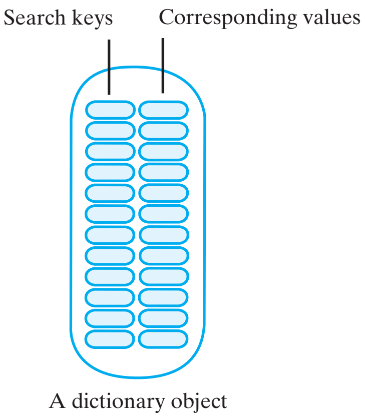

computer A device for the processing and storage of information.

FIGURE 19-2 An instance of an ADT dictionary has search keys paired with corresponding values



Some dictionaries have distinct search keys, but others allow two or more entries to have the same search key. For example, a dictionary of student records organized by student identification numbers has distinct search keys, since those numbers are unique. On the other hand, an English-language dictionary has duplicate search keys, since it often has several meanings for a word. For example, my dictionary has three entries for the word “book”: One is a noun, one is a verb, and one is an adjective.

Printed versions of a natural-language dictionary, a telephone directory, a library catalog, and a thesaurus all have entries sorted by their search keys. These databases are dictionaries, but the ADT dictionary does not require sorted entries. Some dictionaries do sort their entries by search key, while other dictionaries have unsorted entries. Why do our examples of printed dictionaries sort their entries? This makes it easier for the reader to find a particular entry. In contrast, if you searched a computerized thesaurus for a word, you would not be aware of the order of its entries. Nor would you care, as long as you could retrieve a particular entry. Thus, whether a dictionary has sorted or unsorted search keys is more of an implementation detail than a necessary characteristic of the dictionary. But remember that the details of any implementation affect the efficiencies of the ADT operations in various ways.

19.2 The ADT dictionary has the same major operations—insert, delete, retrieve, search, and traverse—that are common to most databases, even if a particular implementation sorts its entries or allows duplicate search keys. In particular, these operations are

- Add a new entry to the dictionary, given a search key and associated value
- Remove an entry, given its associated search key
- Retrieve the value associated with a given search key
- See whether the dictionary contains a given search key
- Traverse all the search keys in the dictionary
- Traverse all the values in the dictionary

In addition, the ADT dictionary has the following basic operations that are often included in an ADT:

- Detect whether a dictionary is empty
- Get the number of entries in the dictionary
- Remove all entries from the dictionary

Note: The ADT dictionary contains entries that are key-value pairs organized by their search keys. You can add a new entry, and you can locate, retrieve, or remove an entry, given its search key. In addition, you can traverse a dictionary's search keys or values.

The following specifications define a set of operations for the ADT dictionary:

Abstract Data Type: Dictionary

Data

- A collection of pairs (*k*, *v*) of objects *k* and *v*, where *k* is the search key and *v* is the corresponding value
- The number of pairs in the collection

Operations

Pseudocode	UML	Description
`add(key, value)`	`+add(key : K, value : V) : void`	Task: Adds the pair (`key`, `value`) to the dictionary. Input: `key` is an object search key, `value` is an associated object. Output: None.
`remove(key)`	`+remove(key : K) : V`	Task: Removes from the dictionary the entry that corresponds to a given search key. Input: `key` is an object search key. Output: Returns either the value that was associated with the search key or `null` if no such object exists.
`getValue(key)`	`+getValue(key : K) : V`	Task: Retrieves from the dictionary the value that corresponds to a given search key. Input: `key` is an object search key. Output: Returns either the value associated with the search key or `null` if no such object exists.
`contains(key)`	`+contains(key : K) : boolean`	Task: Sees whether any entry in the dictionary has a given search key. Input: `key` is an object search key. Output: Returns true if an entry in the dictionary has `key` as its search key.
`getKeyIterator()`	`+getKeyIterator() : Iterator<K>`	Task: Creates an iterator that traverses all search keys in the dictionary. Input: None. Output: Returns an iterator that provides sequential access to the search keys in the dictionary.

`getValueIterator()`	`+getValueIterator() : Iterator<V>`	Task: Creates an iterator that traverses all values in the dictionary. Input: None. Output: Returns an iterator that provides sequential access to the values in the dictionary.
`isEmpty()`	`+isEmpty() : boolean`	Task: Sees whether the dictionary is empty. Input: None. Output: Returns true if the dictionary is empty.
`getSize()`	`+getSize() : integer`	Task: Gets the size of the dictionary. Input: None. Output: Returns the number of entries (key-value pairs) currently in the dictionary.
`clear()`	`+clear() : void`	Task: Removes all entries from the dictionary. Input: None. Output: None.

19.3 **Refining the specifications.** Even though all dictionaries can have this common set of operations, you do need to refine some of the specifications according to whether a dictionary's search keys are distinct:

- **Distinct search keys.** The method `add` can ensure that the search keys in a dictionary are distinct. If `key` is already in the dictionary, the operation `add(key, value)` could either refuse to add another key-value entry or change the existing value associated with `key` to `value`. In the latter case, the method could return the old replaced value instead of not having an output, as indicated earlier.

  Regardless of how `add` guarantees distinct search keys, the remaining methods can have simpler implementations than if duplicate search keys are allowed. For example, the methods `remove` and `getValue` will either find the one value associated with a given search key or discover that no such entry exists.
- **Duplicate search keys.** If the method `add` adds every given key-value entry to a dictionary, the methods `remove` and `getValue` must deal with multiple entries that have the same search key. Which entry should be removed or returned? The method `remove` could either remove the first value it finds or remove all values associated with the given search key. If `getValue` returns an object, it could return the first value it finds. Or you could modify `getValue` to return a list of values, for example.

  Another possibility is to have a secondary search key that is used only when several entries have the same primary search key. For example, if you call directory assistance for a common name like John Smith, you most certainly will be asked for John's address.

For simplicity, we will assume distinct search keys and consider duplicate search keys in the exercises and projects at the end of this chapter.

A Java Interface

19.4 Listing 19-1 contains a Java interface for the ADT dictionary that specifies distinct search keys. The add method replaces the value associated with any search key that is already in the dictionary.

Like the interfaces for the ADTs list and sorted list, this interface specifies the data type of its entries generically. Since the search keys can have a data type that differs from the type of the associated values, we use two formal type parameters, K and V. K represents the data type of the search keys, and V is the type of the associated values.

LISTING 19-1 An interface for the ADT dictionary

```
import java.util.Iterator;
/**
    An interface for a dictionary with distinct search keys.
    @author Frank M. Carrano
*/
public interface DictionaryInterface<K, V>
{
   /** Adds a new entry to this dictionary. If the given search
       key already exists in the dictionary, replaces the
       corresponding value.
       @param key    an object search key of the new entry
       @param value  an object associated with the search key
       @return either null if the new entry was added to the dictionary
               or the value that was associated with key if that value
               was replaced */
   public V add(K key, V value);

   /** Removes a specific entry from this dictionary.
       @param key  an object search key of the entry to be removed
       @return either the value that was associated with the search key
               or null if no such object exists */
   public V remove(K key);

   /** Retrieves from this dictionary the value associated with a given
       search key.
       @param key  an object search key of the entry to be retrieved
       @return either the value that is associated with the search key
               or null if no such object exists */
   public V getValue(K key);

   /** Sees whether a specific entry is in this dictionary.
       @param key  an object search key of the desired entry
       @return true if key is associated with an entry in the
               dictionary */
   public boolean contains(K key);

   /** Creates an iterator that traverses all search keys in this
       dictionary.
       @return an iterator that provides sequential access to the search
               keys in the dictionary */
   public Iterator<K> getKeyIterator();

   /** Creates an iterator that traverses all values in this dictionary.
       @return an iterator that provides sequential access to the values
               in this dictionary */
   public Iterator<V> getValueIterator();

   /** Sees whether this dictionary is empty.
       @return true if the dictionary is empty */
   public boolean isEmpty();

   /** Gets the size of this dictionary.
       @return the number of entries (key-value pairs) currently
               in the dictionary */
   public int getSize();

   /** Removes all entries from this dictionary. */
   public void clear();
} // end DictionaryInterface
```

19.5 Let's see how to create an instance of a class `Dictionary` that implements `DictionaryInterface`. This dictionary will contain data about the students at your school. Assume that student numbers are the search keys and that we have the class `Student` to represent the student data. The following statement creates the instance `dataBase`:

```
DictionaryInterface<String, Student> dataBase =
    new Dictionary<String, Student>();
```

`String` corresponds to the parameter `K` in `DictionaryInterface`, so each occurrence of `K` in the interface is replaced by `String`. Similarly, `Student` replaces every occurrence of `V` in the interface. The same correspondence occurs between these actual types and the generic types of the class `Dictionary`.

We will examine several examples of dictionaries in more detail later in this chapter.

Iterators

19.6 The methods `getKeyIterator` and `getValueIterator` each return an iterator that conforms to the interface `java.util.Iterator` that we discussed in Chapter 15. You can create iterators for the dictionary `dataBase` that we instantiated in the previous segment by writing

```
Iterator<String> keyIterator = dataBase.getKeyIterator();
Iterator<Student> valueIterator = dataBase.getValueIterator();
```

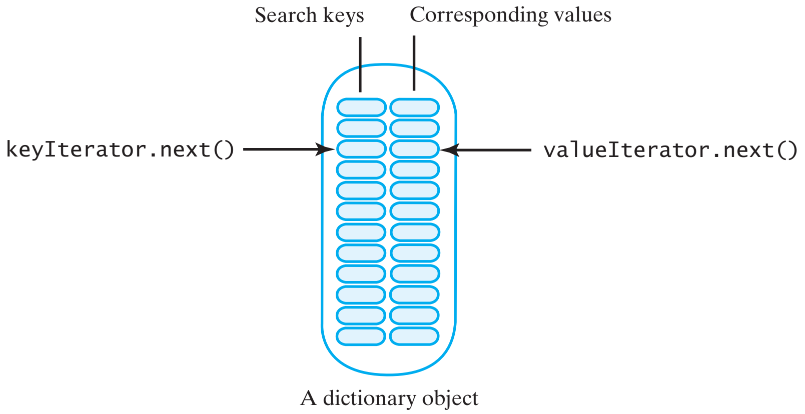
Recall that `Iterator` specifies a generic type in its definition. Here we have defined an iterator for the `String` search keys and another for the `Student` values.

You can use each of these iterators either separately or together. That is, you can traverse

- All of the search keys in a dictionary without traversing the values
- All of the values without traversing the search keys
- All the search keys and all the values in parallel

In the last case, the i^{th} search key returned by `keyIterator` corresponds to the i^{th} dictionary value returned by `valueIterator`, as Figure 19-3 illustrates. Clearly, the two iterations have the same length, since the number of search keys in a dictionary must be the same as the number of values.

FIGURE 19-3 Two iterators that traverse a dictionary's keys and values in parallel



The following loop displays each entry in the dictionary as a key-value pair:

```
while (keyIterator.hasNext())
   System.out.println(keyIterator.next() + ", " + valueIterator.next());
```

For a sorted dictionary, `keyIterator` traverses the search keys in sorted order. For an unsorted dictionary, this traversal order is not specified. The examples in the next section demonstrate these iterators in several contexts.

Note: An iteration of a dictionary's values corresponds to an iteration of its search keys. That is, the i^{th} value in one iteration is associated in the dictionary with the i^{th} search key in the second iteration.

SELF-TEST

Question 1 If the class `Dictionary` implements `DictionaryInterface`, write a Java statement that creates an empty dictionary `myDictionary`. This dictionary will contain the names and telephone numbers of your friends. Assume that the names are the search keys, and you have the class `Name` to represent them. Let each telephone number be a string.

Question 2 Write a Java statement that adds your name and telephone number to the dictionary that you created in Question 1.

Question 3 Write Java statements that display either Britney Storm's telephone number, if she is in the dictionary described in Question 1, or an error message if she is not.

Using the ADT Dictionary

The three examples in this section demonstrate how to use the ADT dictionary in a program. We begin by creating a telephone directory.

VideoNote
Using the ADT dictionary

A Problem Solved: A Directory of Telephone Numbers

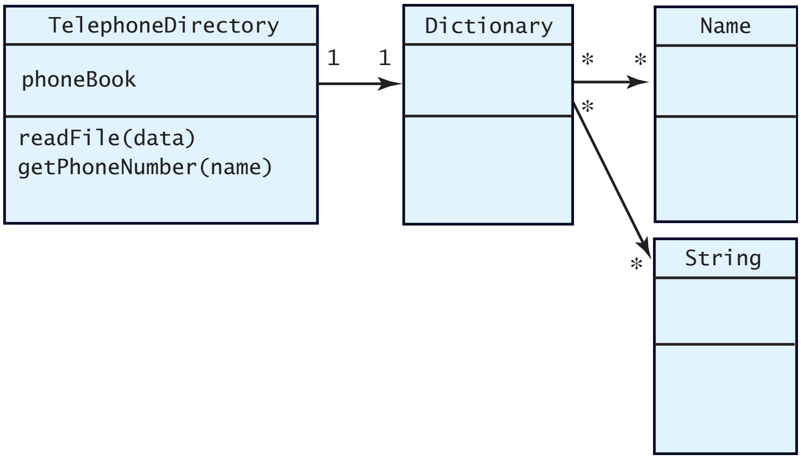
A telephone directory contains the names and telephone numbers of the people who live in a given geographical region. Implement software that defines such a directory.

19.7 The most frequent operation performed on a telephone directory is the retrieval of a telephone number, given a person's name. Thus, using the ADT dictionary to represent a telephone directory is a good choice. Clearly, the name should be the search key, and the telephone number should be the corresponding value. Often, but not always, retrieval is more efficient when the dictionary is sorted. Additionally, a sorted dictionary would make it easier to create a printed directory with entries alphabetized by name. To simplify this example, we assume that the directory will contain distinct names with no duplicates.

A major task, at least initially, is to create the directory from the available names and telephone numbers. Having this data in a text file will make this task convenient. After the telephone directory is created, operations on the directory, such as adding an entry, removing an entry, or changing a telephone number, will be used less often than searching for a given name. Traversing the directory is important to create either a hard copy or a text file of the data, but this operation too is not done frequently. As we noted in Chapter 4, you should choose an implementation of an ADT based on the efficiency of its expected use.

19.8 **Design and use of the class `TelephoneDirectory`.** Our next step is to design a class to represent the telephone directory. A sorted dictionary will represent the data, which consists of name-number pairs. Each person's name can be an instance of the class `Name` that we first encountered in Appendix B, and the telephone number can be a string without embedded blanks. Figure 19-4 shows a class diagram for our design. The class `TelephoneDirectory` contains an instance `phoneBook` of a dictionary. The class has the method `readFile`, which reads the data from the file and adds it to `phoneBook`. It also has the method `getPhoneNumber` to retrieve a telephone number, given a name. For simplicity, we are ignoring any other operations mentioned in the previous segment.

FIGURE 19-4 A class diagram for a telephone directory



Before we implement the class `TelephoneDirectory`, let's consider its use. A client would create an instance of `TelephoneDirectory` and read the data file by invoking the method `readFile`.

The first two highlighted lines in the main method of Listing 19-2 perform these steps. Given the name data.txt of the text file, main creates a scanner for the file and passes it to readFile. Note the exceptions that might occur in creating the scanner. If you need more information about either exceptions or files, consult Appendices E and F, respectively.

After the file is read, main interacts with the user via the private method getName. Each name read from the user is passed to the method getPhoneNumber, where it will be the key in a search of the telephone directory. Notice how getName uses Scanner to both read the user's input and then process it.

LISTING 19-2 A client of the class TelephoneDirectory

```
import java.util.Scanner;
import java.io.File;
import java.io.FileNotFoundException;
import java.io.IOException;

public class Driver
{
   private static final Name INPUT_ERROR = new Name("error", "error");
   private static final Name QUIT = new Name("quit", "quit");

   public static void main(String[] args)
   {
      TelephoneDirectory directory = new TelephoneDirectory();
      String fileName = "data.txt"; // or file name could be read

      try
      {
         Scanner data = new Scanner(new File(fileName));
         directory.readFile(data);
      }
      catch (FileNotFoundException e)
      {
         System.out.println("File not found: " + e.getMessage());
      }
      catch (IOException e)
      {
         System.out.println("I/O error " + e.getMessage());
      }

      Name nextName = getName();    // get name for search from user
      while (!nextName.equals(QUIT))
      {
         if (nextName.equals(INPUT_ERROR))
            System.out.println("Error in entering name. Try again.");
         else
         {
            String phoneNumber = directory.getPhoneNumber(nextName);
            if (phoneNumber == null)
```

```
               System.out.println(nextName + " is not in the directory.");
            else
               System.out.println("The phone number for " + nextName +
                                  " is " + phoneNumber);
         } // end if

         nextName = getName();
      } // end while
      System.out.println("Bye!");
   } // end main

   // Returns either the name read from user, INPUT_ERROR, or QUIT.
   private static Name getName()
   {
      Name result = null;
      Scanner keyboard = new Scanner(System.in);

      System.out.print("Enter first name and last name, " +
                       "or quit to end: ");
      String line = keyboard.nextLine();

      if (line.trim().toLowerCase().equals("quit"))
         result = QUIT;
      else
      {
         String firstName = null;
         String lastName = null;
         Scanner scan = new Scanner(line);

         if (scan.hasNext())
         {
            firstName = scan.next();
            if (scan.hasNext())
               lastName = scan.next();
            else
               result = INPUT_ERROR;
         }
         else
            result = INPUT_ERROR;

         if (result == null)
            result = new Name(firstName, lastName);
      } // end if

      return result;
   } // end getName
} // end Driver
```

Output

```
Enter first name and last name or quit to end: Maria Lopez
The phone number for Maria Lopez is 401-555-1234
Enter first name and last name or quit to end: Hunter
Error in entering name. Try again.
Enter first name and last name or quit to end: Hunter Smith
Hunter Smith is not in the directory.
Enter first name and last name or quit to end: quit
Bye!
```

19.9 **Beginning the implementation.** The class `TelephoneDirectory` begins as shown in Listing 19-3. We assume that the class `SortedDictionary` implements a sorted version of the ADT dictionary having distinct search keys. A sorted dictionary requires its search keys to belong to a class that implements the interface `Comparable`. We assume that `Name` does so.

LISTING 19-3 An outline of the class `TelephoneDirectory`

```
import java.util.Scanner;
public class TelephoneDirectory
{
   private DictionaryInterface<Name, String> phoneBook;

   public TelephoneDirectory()
   {
      phoneBook = new SortedDictionary<Name, String>();
   } // end default constructor
   /** Reads a text file of names and telephone numbers.
       @param data  a text scanner for the text file of data */
   public void readFile(Scanner data)
   {
      . . . < See Segment 19.10. >
   } // end readFile

   /** Gets the phone number of a given person. */
   public String getPhoneNumber(Name personName)
   {
      . . . < See Segment 19.11. >
   } // end getPhoneNumber
} // end TelephoneDirectory
```

19.10 To implement the method `readFile`, you need to know what the data file looks like. Suppose that each line in the file contains three strings—a first name, a last name, and a telephone number—separated by blanks. Thus, a typical line might appear as

Suzanne Nouveaux   401-555-1234

The method readFile must read each of these strings. Recall that the main method in Listing 19-2 in Segment 19.8 creates a scanner for the file and passes it to readFile. Using Scanner's method next, readFile can read each string in a line of data and assign them, respectively, to the variables firstName, lastName, and phoneNumber. The following Java statements, then, will add the desired entry to the dictionary phoneBook:

```
Name fullName = new Name(firstName, lastName);
phoneBook.add(fullName, phoneNumber);
```

We assume that the text file contains distinct names.

Here is the definition of readFile:

```
/** Reads a text file of names and telephone numbers.
    @param data  a text scanner for the text file of data */
public void readFile(Scanner data)
{
   while (data.hasNext())
   {
      String firstName   = data.next();
      String lastName    = data.next();
      String phoneNumber = data.next();

      Name fullName = new Name(firstName, lastName);
      phoneBook.add(fullName, phoneNumber);
   } // end while

   data.close();
} // end readFile
```

Using Scanner's methods hasNext and next, we extract each name and telephone number as strings from the text file. Then, using the two statements we examined earlier, we create a Name object and add it and the telephone number to the dictionary.

Programming Tip: java.util.Scanner

The class Scanner enables you to break a string into substrings, or **tokens**, that are separated by characters called **delimiters**. By default, white-space characters are the delimiters. You pass to Scanner's constructor either the string to be **parsed** or a text file represented as an instance of the class java.io.File.

The following methods in the class Scanner enable you to extract the tokens from any string:

```
public String next();
public boolean hasNext();
```

Appendix A discusses Scanner in more detail, beginning at Segment A.81.

SELF-TEST

Question 4 Although the statement

```
directory.readFile(data);
```

is inside a try block near the beginning of the method main in Listing 19-2, it need not be. Explain its present location, why it can appear outside of a try block, and what you can do to move it.

19.11 **A method that searches.** The class TelephoneDirectory has a method to find a person's telephone number. This method needs the person's name, and the user must supply it. If we assume that the client will interact with the user and provide the desired name to the method—as does the client in Listing 19-2—we could define the method as follows:

```
public String getPhoneNumber(Name personName)
{
   return phoneBook.getValue(personName);
} // end getPhoneNumber
```

The method either returns a string that contains the desired telephone number or returns `null` if the number is not found.

We could define a similar method instead of or in addition to the previous method, as follows:

```
public String getPhoneNumber(String firstName, String lastName)
{
   Name fullName = new Name(firstName, lastName);
   return phoneBook.getValue(fullName);
} // end getPhoneNumber
```

Additional methods to add or remove a person or to change a person's telephone number are straightforward and are left as exercises.

Question 5 Implement a method for the class `TelephoneDirectory` that removes an entry from the directory. Given the person's name, the method should return either the person's telephone number or `null` if the person is not in the directory.

Question 6 Implement a method for the class `TelephoneDirectory` that changes a person's telephone number. Given the person's name, the method should return either the person's old telephone number or `null` if the person was not in the directory but has been added to it.

A Problem Solved: The Frequency of Words

Some word processors provide a count of the number of times each word occurs in a document. Create a class `FrequencyCounter` that provides this capability.

19.12 This class is somewhat like the one in the previous example, so we will omit some of the design details. Basically, the class needs to count each occurrence of a word as it reads the document from a text file. It then needs to display the results. For example, if the text file contains

row, row, row your boat

the desired output would be

```
boat 1
row  3
your 1
```

The class will have a constructor and the methods `readFile` and `display`. As in the previous example, `readFile` will read the input text from a file. Then `display` will write the output. Listing 19-4 shows a client of `FrequencyCounter`. It is similar to the beginning of the client in the previous example.

LISTING 19-4 A client of the class `FrequencyCounter`

```
import java.util.Scanner;
import java.io.File;
import java.io.FileNotFoundException;
import java.io.IOException;

public class Driver
{
   public static void main(String[] args)
   {
      FrequencyCounter wordCounter = new FrequencyCounter();
      String fileName = "Data.txt"; // or file name could be read

      try
      {
         Scanner data = new Scanner(new File(fileName));
         wordCounter.readFile(data);
      }
      catch (FileNotFoundException e)
      {
         System.out.println("File not found: " + e.getMessage());
      }
      catch (IOException e)
      {
         System.out.println("I/O error " + e.getMessage());
      }
      wordCounter.display();
      System.out.println("Bye!");
   } // end main
} // end Driver
```

Output

```
   boat 1
   row  3
   your 1
```

19.13 Is the ADT dictionary the right one to use for this problem? A word and its frequency of occurrence in the document form a pair that is suitable as an entry in a dictionary. If we want to know a given word's frequency, the word should be the search key. Also, the words in the dictionary must be distinct, and if they are sorted, we can display them in alphabetical order. Thus, a sorted dictionary with distinct search keys is an appropriate choice for this problem. As in the previous example, we assume `SortedDictionary` is such an implementation.

The dictionary will be a data field of a new class `FrequencyCounter`, which will begin much like the class `TelephoneDirectory` in the previous example. Let's call the dictionary for this example `wordTable`. Since the value portion of any dictionary entry is an object, we use the wrapper class `Integer` to represent each frequency. Thus, our class can begin as shown in Listing 19-5.

LISTING 19-5 An outline of the class `FrequencyCounter`

```
import java.util.Iterator;
import java.util.Scanner;
```

```
public class FrequencyCounter
{
   private DictionaryInterface<String, Integer> wordTable;

   public FrequencyCounter()
   {
      wordTable = new SortedDictionary<String, Integer>();
   } // end default constructor

   /** Reads a text file of words and counts their frequencies
       of occurrence.
       @param data  a text scanner for the text file of data */
   public void readFile(Scanner data)
   {
       . . . < See Segment 19.16. >
   } // end readFile

   /** Displays words and their frequencies of occurrence. */
   public void display()
   {
       . . . < See Segment 19.17. >
   } // end display
} // end FrequencyCounter
```

19.14 **Creating the dictionary.** Now let's look at the method `readFile`, which creates the dictionary from the text file. We invoke this method like we did earlier in Listing 19-4 of Segment 19.12. That is, the client passes to `readFile` a `Scanner` object associated with the text file. The method then can process the text file by using the `Scanner` methods `hasNext` and `next`, in the same way that `readFile` processed the file in Segment 19.10.

After extracting the next word from the file, `readFile` checks whether the word is in the dictionary. If it is not, we add it with an associated value of 1. That is, this word has occurred once so far. However, if the word is in the dictionary already, we retrieve its associated value—its count—increment it, and store it back into the dictionary. To avoid issues of case, `readFile` can change all the words it reads to lowercase.

19.15 **Delimiters.** The programming tip at the end of Segment 19.10 mentioned that, by default, `Scanner` uses white-space characters as delimiters. But as in the example given in Segment 19.12, our data can contain punctuation, so those characters must also be delimiters. You can use the `Scanner` method `useDelimiter` to specify the delimiters. You represent them by using the notation shown in Figure A-6 of Appendix A. (See Segment A.83.) You then pass a string of the delimiters to `useDelimiter`.

The simplest way to specify white space and punctuation as delimiters is to use the notation `\W`, since it represents any character other than a letter, digit, or underscore. We then write `useDelimiter`'s argument as `"\\W+"`. Remember that we must duplicate the backslash to distinguish the notation from an escape character. The plus sign means *one or more occurrences of*. Thus, the statement

```
dataFile.useDelimiter("\\W+");
```

sets the delimiters to one or more occurrences of punctuation, white-space characters, and some other characters that will not occur in our data.

Programming Tip: When using a Scanner object to process text, any character can be a delimiter if it does not occur in any desired token. You create a string of these delimiters using a special notation and give it to the Scanner method useDelimiter. Consult Segment A.82 of Appendix A for more details.

19.16 The following implementation of the method readFile reflects the previous discussion:

```
/** Reads a text file of words and counts their frequencies
    of occurrence.
    @param data  a text scanner for the text file of data */
public void readFile(Scanner data)
{
   data.useDelimiter("\\W+");
   while (data.hasNext())
   {
      String nextWord = data.next();
      nextWord = nextWord.toLowerCase();
      Integer frequency = wordTable.getValue(nextWord);

      if (frequency == null)
      { // add new word to table
         wordTable.add(nextWord, new Integer(1));
      }
      else
      { // increment count of existing word; replace wordTable entry
        frequency++;
        wordTable.add(nextWord, frequency);
      } // end if
   } // end while

   data.close();
} // end readFile
```

Question 7 The previous method readFile does not call contains to see whether a word is already in the dictionary, but instead calls getValue. Why did we do this?

19.17 **Displaying the dictionary.** Now that we have created the dictionary, we need to display the results. An iteration of the search keys will produce the words in alphabetical order. A parallel iteration of the values provides the corresponding frequencies. The following method is a possible solution for this task:

```
public void display()
{
   Iterator<String> keyIterator = wordTable.getKeyIterator();
   Iterator<Integer> valueIterator = wordTable.getValueIterator();

   while (keyIterator.hasNext())
   {
      System.out.println(keyIterator.next() + " " +
                         valueIterator.next());
   } // end while
} // end display
```

Question 8 Implement a second method display for the class FrequencyCounter that displays only words that occur with a frequency given as the method's sole parameter.

A Problem Solved: A Concordance of Words

An *index* provides a way to locate the occurrence of certain words within a larger document. For example, the index to this book is an alphabetical listing of words paired with the page numbers on which the words occur. For this problem, we will create a simpler kind of index—called a *concordance*—to all the words in a text file. Instead of page numbers, a concordance provides the line numbers that contain a particular word.

19.18 Let's begin by looking at an example of a concordance. Suppose that a text file contains only these lines:

> Learning without thought is labor lost;
>
> thought without learning is perilous.

The following concordance of all the words in the file indicates the line numbers in which the words occur:

is	1 2
labor	1
learning	1 2
lost	1
perilous	2
thought	1 2
without	1 2

Although a word can appear in several lines of the file, it appears only once in the concordance. Like the previous word-frequency example, this feature of the concordance suggests that we use a dictionary whose search keys are the words in the concordance. But unlike the word-frequency example, the value associated with each of these words is a list of line numbers. Since the line numbers are sorted, we could use the ADT sorted list. However, by processing the lines in the file in order, we can add the line numbers to the end of an ordinary unsorted list and achieve a sorted order.

19.19 A class `Concordance` to represent the concordance and the class `FrequencyCounter` from the previous example are quite similar in their design and implementation. In fact, the use of these classes is virtually identical. By replacing `FrequencyCounter` with `Concordance` in Listing 19-4 of Segment 19.12, you will have a client for `Concordance`.

Listing 19-6 contains an outline of the class `Concordance`. Note the similarities to the outline of `FrequencyCounter` given in Listing 19-5 of Segment 19.13. The major difference, other than the implementations of the methods, is the data type of the value of each dictionary entry. Since the value is a list of `Integer` objects, and since we will want to traverse each list to display the line numbers, we give the value a data type of `ListWithIteratorInterface<Integer>`. Segment 15.17 of Chapter 15 defined this interface as having the method `getIterator` as well as the methods of `ListInterface`.

LISTING 19-6 An outline of the class `Concordance`

```
import java.util.Iterator;
import java.util.Scanner;
public class Concordance
{
   private DictionaryInterface<String, ListWithIteratorInterface<Integer>>
           wordTable;
   public Concordance()
   {
      wordTable =
          new SortedDictionary<String, ListWithIteratorInterface<Integer>>();
   } // end default constructor

   /** Reads a text file of words and creates a concordance.
       @param data  a text scanner for the text file of data */
   public void readFile(Scanner data)
   {
      . . . < See Segment 19.20. >
   } // end readFile

   /** Displays words and the lines in which they occur. */
   public void display()
   {
      . . . < See Segment 19.21. >
   } // end display
} // end Concordance
```

19.20 **The method `readFile`.** The method `readFile` reads the text file and uses the dictionary `wordTable` to create the concordance. Since we must record the line number of each word, we read the file a line at a time. We process all the words in a line before moving on to the next line. Thus, the following definition of `readFile` contains two loops that are nested. The outer loop uses the scanner passed to the method as an argument to read lines from the file. The inner loop uses another scanner to extract the words from a line as soon as it is read. The class `LinkedListWithIterator` from Segment 15.19 of Chapter 15 is used to form each list of line numbers.

```
public void readFile(Scanner data)
{
   int lineNumber = 1;

   while (data.hasNext())
   {
      String line = data.nextLine();
      line = line.toLowerCase();

      Scanner lineProcessor = new Scanner(line);
      lineProcessor.useDelimiter("\\W+");
      while (lineProcessor.hasNext())
      {
         String nextWord = lineProcessor.next();
         ListWithIteratorInterface<Integer> lineList =
                                            wordTable.getValue(nextWord);

         if (lineList == null)
         { // create new list for new word; add list and word to index
             lineList = new LinkedListWithIterator<Integer>();
             wordTable.add(nextWord, lineList);
         } // end if

         // add line number to end of list so list is sorted
         lineList.add(lineNumber);
      }  // end while
```

```
         lineNumber++;
      } // end while

      data.close();
   } // end readFile
```

The most interesting part of this method is the list of line numbers as the value associated with a search key. Since we have chosen a linked implementation of the list, we need to be concerned with the efficiency of adding to the end of the list. If the underlying chain of nodes has only a reference to the first node—as is true of `LinkedListWithIterator`—each such addition requires a traversal to reach the end of the chain. Choosing a list implementation that maintains a reference to the last node in the chain would make the addition to the end of the list quite efficient. We discussed such tail references in Chapter 11. We should make this adjustment to our class of lists for this application.

19.21 **The method `display`.** Earlier, we chose a list implementation that included an iterator so that the following method `display` could display the line numbers in the concordance efficiently. Notice that we use the dictionary iterators, just as we did in the analogous method `display` given in Segment 19.17 for the previous example. But here each value is a list with its own iterator, which we use to traverse the list's line numbers.

```
public void display()
{
   Iterator<String> keyIterator = wordTable.getKeyIterator();
   Iterator<ListWithIteratorInterface<Integer>> valueIterator =
                                       wordTable.getValueIterator();

   while (keyIterator.hasNext())
   {
      // display the word
      System.out.print(keyIterator.next() + " ");

      // get line numbers and iterator
      ListWithIteratorInterface<Integer> lineList = valueIterator.next();
      Iterator<Integer> listIterator = lineList.getIterator();

      // display line numbers
      while (listIterator.hasNext())
      {
         System.out.print(listIterator.next() + " ");
      } // end while

      System.out.println();
   } // end while
} // end display
```

Question 9 Write a method `getLineNumbers` for the class `Concordance` that returns a list of the numbers of the lines that contain a given word.

Java Class Library: The Interface `Map`

19.22 The standard package `java.util` contains the interface `Map<K, V>` that is similar to our interface for the ADT dictionary. The following method headers are for a selection of methods in `Map` that are like the ones you have seen in this chapter. We have highlighted where they differ from our methods.

```
public V put(K key, V value);
public V remove (Object key);
public V get(Object key);
public boolean containsKey(Object key);
public boolean containsValue(Object value);
public Set<K> keySet();
public Collection<V> values();
public boolean isEmpty();
public int size();
public void clear();
```

Notice the differences in the names of the methods. Map uses the method names put, get, containsKey, and size instead of our names add, getValue, contains, and getSize. Map also has the additional method containsValue that finds out whether a dictionary contains a given value.

Instead of our methods getKeyIterator and getValueIterator that return iterators to a dictionary's keys and values, respectively, Map specifies the method keySet, which returns a set of keys, and the method values, which returns a collection of values. The Java Class Library contains the interfaces Set and Collection, and each of these interfaces has a method iterator that returns an iterator to the values in the corresponding ADT.

Duplicate search keys are not permitted in a dictionary that conforms to the Map interface. Each key must correspond to only one value. Also, some of Map's methods use Object as the data type of the search key, whereas we use the more specific generic data type K.

Chapter Summary

- The entries in the ADT dictionary each contain two parts: a search key and a value associated with that key. The dictionary identifies its entries by their search keys.
- Dictionaries can organize their search keys in either sorted or unsorted order. The search keys can be either distinct or duplicate.
- You can add an entry to a dictionary given its search key and value. You can retrieve or remove an entry given only its search key. By using an iterator, you can traverse all the keys or all the values in a dictionary.
- An English dictionary, a directory of telephone numbers, an address book, and a library catalog are common examples of dictionaries.
- The Java Class Library contains the interface Map, which is similar to our DictionaryInterface.

Programming Tips

- The class Scanner enables you to break a string into substrings, or tokens, that are separated by characters called delimiters. By default, white-space characters are the delimiters. You pass to Scanner's constructor either the string to be parsed or a text file represented as an instance of the class java.io.File.
- The following methods in the class Scanner enable you to extract the tokens from any string:

  ```
  public String next();
  public boolean hasNext();
  ```

  Appendix A discusses Scanner in more detail beginning at Segment A.81.
- When using a Scanner object to process text, any character can be a delimiter, if it does not occur in any desired token. You create a string of these delimiters using a special notation and give it to the Scanner method useDelimiter. Consult Segment A.82 of Appendix A for more details.

Exercises

1. How does a dictionary differ from a sorted list?

2. Implement a method for the class `TelephoneDirectory`—described in Segment 19.9—that adds an entry to the directory, given the person's name and telephone number. The method should return true if the entry was added. If the person is already in the directory, the method should replace the person's telephone number and return false.

3. Implement a method for the telephone directory problem of Segment 19.9 to display everyone's name and telephone number.

4. In the telephone directory problem of Segment 19.9, the case of the letters in a name affects the name's order in the dictionary. What steps can you take so that case variations in the input file do not affect this order?

5. In the telephone directory problem of Segment 19.9, suppose that the text file of names and telephone numbers is sorted by name.

   a. What impact would this aspect of the file have on the efficiency of the method `readFile` for various implementations of the dictionary?
   b. Would it matter whether the file was in reverse alphabetical order?

6. A *reverse directory* allows one to search for the name corresponding to a given telephone number. Modify the class `TelephoneDirectory` of Segment 19.9 to give it this capability. Use a second dictionary as the reverse directory. Add a query method and modify the method `readFile` accordingly.

7. Draw a class diagram for the class `FrequencyCounter`, as outlined in Segment 19.13, that is analogous to the diagram in Figure 19-4 of Segment 19.8.

8. The word-frequency problem of Segment 19.12 finds the frequency with which each distinct word occurs within some given text. Describe the changes that you could make to the class `FrequencyCounter` if you wanted to list the words that occur for each frequency.

9. Repeat Exercise 7 for the class `Concordance`, as outlined in Segment 19.19.

10. In the concordance problem of Segment 19.18, if a word occurs more than once in a single line, the number of that line appears more than once in the concordance. Revise the Java code given in Segment 19.20 so that the line numbers associated with a given word are distinct.

11. Design an ADT that stores the side effects of various drugs. Each drug should have a list of associated side effects. Provide a method that returns the side effects of a given drug. Then use a dictionary to implement the class `DrugSideEffects`.

12. Consider a look-up service for the television shows on a given date. A file contains information about these shows. Each show's data appears on two lines. The first line gives the name of the station, the channel, the start time, the stop time, the name of the show, and a rating. These entries are separated by tildes (~), and the times are in 24-hour notation (for example, 1 p. m. is 13:00). The second line briefly describes the show.

    Implement a method with the header

```
public void readFile(Scanner data)
```

to read the file into a dictionary that will be searched. Decide what data should be the search key and what should be the associated value. Design any classes needed for the key and the value.

13. The ADT dictionary that we discussed in this chapter assumes distinct search keys. Revise the specifications of the dictionary to remove this restriction. Consider each of the following possibilities:

a. The method `add` adds an entry whose search key is already in the dictionary but whose value is not. The `remove` method deletes all entries with a given search key. The method `getValue` retrieves all values associated with a given search key.

b. The methods behave as Part *a* describes, but a secondary search key enables `remove` and `getValue` to delete or retrieve a single entry.

Projects

1. To simplify the telephone directory problem of Segment 19.7, we assumed that the text file contained distinct names. Remove this assumption, with and without a secondary search key. (See Exercise 13.)

2. Discovering the authorship of certain famous pieces of literature is an interesting problem. Comparisons are made between pieces whose authorship is disputed and those of known authorship. One approach is to compare the frequency of pairs of letters. There are 26 x 26 different pairs of letters. Not all of them will appear in a piece of writing. For example, "qz" is unlikely to appear, while "th" is likely to appear often. Design a program, similar to the frequency counter of Segments 19.12 through 19.17, that counts all the pairs of letters that appear in a given piece of text.

3. A compiler must examine tokens in a program and decide whether or not they are reserved words or identifiers defined by the user. Design a program that reads a Java program and makes a list of all the identifiers. To do this, you should make use of two dictionaries. The first dictionary should hold all the Java reserved words. The second dictionary should hold all the identifiers that you find. Whenever you encounter a token, you first should search the dictionary of reserved words. If the token is not a reserved word, you then should search the dictionary of identifiers. If the token is not in either dictionary, you should add it to the dictionary of identifiers.

4. Suppose that we want to implement the ADT set. Recall from Project 1 of Chapter 1 that a set is an unordered collection of objects where duplicates are not allowed. The operations that a set should support are

- Add a given object to the set
- Remove a given object from the set
- See whether the set contains a given object
- Clear all objects from the set
- Get the number of objects in the set
- Return an iterator to the set
- Return a set that combines the items in two sets (the union)
- Return a set of those items that occur in both of two sets (the intersection)

Define a class `Set` that uses a dictionary internally to implement these operations.

5. Suppose that we want to help physicians to diagnose illnesses. A physician observes a patient's symptoms and considers the illnesses that could be associated with those symptoms. Design and implement a class `PhysiciansHelper` that provides a list of those illnesses.

`PhysiciansHelper` should contain a dictionary of illnesses and symptoms. A method should read a file of illnesses with their symptoms into the dictionary. Each line in the file will contain the name of an illness followed by a colon and a comma-separated list of symptoms. For example, one line could be

head cold: nasal stuffiness, sneezing, runny nose

`PhysiciansHelper` should maintain a list of symptoms for the current patient. A method should add a symptom to this list and return a list of illnesses that is associated with those symptoms. Another method should remove a given symptom from the list, and a method should clear the patient symptom list.

6. Write a program that plays the game tic-tac-toe. Represent the game board by an array of nine values. Each location in the array contains either an X, an O, or a blank. The total number of possible board configurations is 3^9, or approximately 20,000. Associated with every possible configuration is a best move.

   Generate all possible board configurations, and let them be search keys in a dictionary. For each search key, let the next best move be its associated value. Once you have created the dictionary, use it to decide the moves for a computer-based player in a game of tic-tac-toe.

7. A *picture dictionary* is a collection of images, each of which is identified by a descriptive word. Form a picture dictionary from the data in external files, which you can create from royalty-free images found online. Design and implement a user interface that provides search and display functions.

Answers to Self-Test Questions

1.
```
DictionaryInterface<Name, String> myDictionary = new Dictionary<Name, String>();
```

2.
```
myDictionary.add(new Name("Joe", "Java"), "555-1234");
```

3.
```
Name britney = new Name("Britney", "Storm");
if (myDictionary.contains(britney))
   System.out.println("Britney's phone number is " + myDictionary.getValue(britney));
else
   System.out.println("Britney is not in the dictionary");
```

   or

```
String phoneNumber = myDictionary.getValue(new Name("Britney", "Storm"));
if (phoneNumber == null)
   System.out.println("Britney is not in the dictionary");
else
   System.out.println("Britney's phone number is " + phoneNumber);
```

4. The `Scanner` methods `hasNext` and `next` that `readFile` calls throw only runtime exceptions, which need not be caught. So although the call to `readFile` can be outside of a `try` block, it is inside the `try` block because its argument—the `Scanner` object `data`—is local to the `try` block. By declaring `data` outside of the `try` block, you could move the call to `readFile` after the last `catch` block.

5.
```
public String remove(Name personName)
{
   return phoneBook.remove(personName);
} // end remove
```

6.
```
public String changePhoneNumber(Name personName, String newPhoneNumber)
{
   return phoneBook.add(personName, newPhoneNumber);
} // end changePhoneNumber
```

7. We called `getValue` instead of `contains` to simplify the logic. If we called `contains` and found that the current word was already in the dictionary, we would need to call `getValue` to get its frequency. But we can use the result of `getValue` to see whether the word is in the dictionary.

8.

```
/** Displays only the words that occur with a given frequency.
    @param frequency  an integer count of the desired frequency. */
public void display(int frequency)
{
   Iterator<String> keyIterator = wordTable.getKeyIterator();
   Iterator<Integer> valueIterator = wordTable.getValueIterator();

   System.out.println("Words that occur " + frequency + " times:");
   boolean atLeastOneWord = false;
   while (keyIterator.hasNext())
   {
      String word = keyIterator.next();
      Integer count = valueIterator.next();

      if (count.equals(frequency))
      {
         atLeastOneWord = true;
         System.out.println(word);
      } // end if
   } // end while

   if (atLeastOneWord == false)
      System.out.println("(There are none.)");
} // end display
```

9.

```
public ListWithIteratorInterface<Integer> getLineNumbers(String word)
{
   return wordTable.getValue(word);
} // end getLineNumbers
```

This page intentionally left blank

Chapter 20

Dictionary Implementations

Contents

Prerequisites

Chapter	3	A Bag Implementation That Links Data
Chapter	4	The Efficiency of Algorithms
Chapter	12	Lists
Chapter	13	List Implementations That Use Arrays
Chapter	14	A List Implementation That Links Data
Chapter	15	Iterators
Chapter	18	Searching
Chapter	19	Dictionaries

Objectives

After studying this chapter, you should be able to

- Implement the ADT dictionary by using either an array, a vector, or a chain of linked nodes

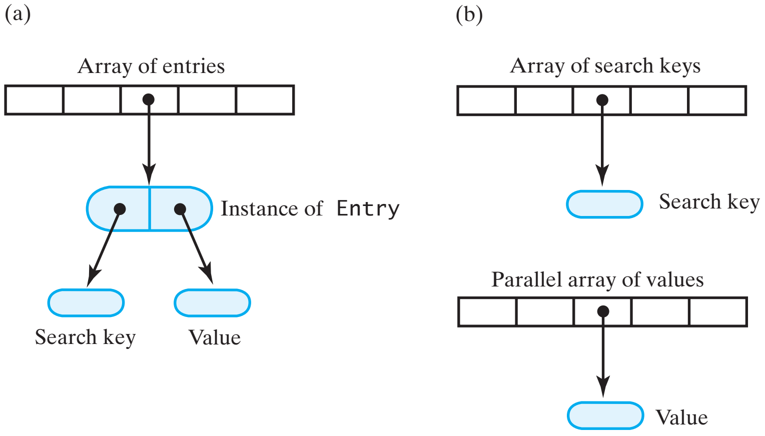
The implementations of the ADT dictionary that we present in this chapter employ techniques like the ones we used to implement the ADT list. We will store the dictionary's entries in either an array, an instance of `Vector`, or a chain of linked nodes. In doing so, we will consider both sorted and unsorted dictionaries with distinct search keys. Later chapters will present more-sophisticated implementations of the ADT dictionary.

Array-Based Implementations

VideoNote
Array-based dictionaries

20.1 The ability to resize an array, as introduced in Segment 2.32 of Chapter 2, means that an array can provide as much storage as necessary for the entries in a dictionary. Remember that each entry consists of two parts—a search key and a value. You can encapsulate the two parts into an object, as Figure 20-1a illustrates. With this approach, you define a class `Entry` to represent the entries. A second, less attractive approach uses two arrays, as shown in Figure 20-1b. One array represents the search keys and a second, **parallel array** represents the corresponding values. We will discuss the first approach and leave the exploration of the second as an exercise. At that time, you will see that parallel arrays can be awkward to manage.

FIGURE 20-1 Two possible ways to use arrays to represent the entries in a dictionary: (a) an array of objects that encapsulate each search key and corresponding value; (b) parallel arrays of search keys and values



Question 1 Figure 20-1 shows two ways to represent an array-based dictionary. How do the memory requirements for the two representations compare?

An Unsorted Array-Based Dictionary

20.2 **Beginning the implementation.** Our implementation uses one array, as pictured in Figure 20-1a, to represent the dictionary. Each entry in the dictionary, and therefore the array, is an instance of a class `Entry` that we must define. We can make this class either public, part of a package, or private and internal to the dictionary class. We chose the latter approach in defining the private class `Entry` shown in Listing 20-1.

The outer class `ArrayDictionary` begins with its data fields and constructors stated in terms of type parameters `K` and `V`. These parameters represent the data types of the search keys and their associated values, respectively.

LISTING 20-1 The class `ArrayDictionary` and its private inner class `Entry`

```
import java.util.Arrays;
import java.util.Iterator;
import java.util.NoSuchElementException;
```

```
/**
    A class that implements a dictionary by using an array.
    @author Frank M. Carrano
*/
public class ArrayDictionary<K, V> implements DictionaryInterface<K, V>
{
   private Entry<K, V>[] dictionary; // array of unsorted entries
   private int numberOfEntries;
   private final static int DEFAULT_CAPACITY = 25;

   public ArrayDictionary()
   {
      this(DEFAULT_CAPACITY); // call next constructor
   } // end default constructor

   public ArrayDictionary(int initialCapacity)
   {
      // the cast is safe because the new array contains null entries
      @SuppressWarnings("unchecked")
      Entry<K, V>[] tempDictionary =
                              (Entry<K, V>[])new Entry[initialCapacity];
      dictionary = tempDictionary;
      numberOfEntries = 0;
   } // end constructor

   < Implementations of methods in DictionaryInterface >
   . . .

   private class Entry<S, T>
   {
      private S key;
      private T value;

      private Entry(S searchKey, T dataValue)
      {
         key = searchKey;
         value = dataValue;
      } // end constructor

      private S getKey()
      {
         return key;
      } // end getKey

      private T getValue()
      {
         return value;
      } // end getValue

      private void setValue(T newValue)
      {
         value = newValue;
      } // end setValue
   } // end Entry
} // end ArrayDictionary
```

Notice that the inner class `Entry` has no method `setKey` to set or change the search key. Even though `setValue` can be useful in the implementation of `add`, you never need to change the search key. Without `setKey`, a default constructor would be useless, so none is defined.

Note: Compiler warning

The constructor for `ArrayDictionary`, as shown in Listing 20-1, allocates memory for the array `dictionary` as follows:

```
dictionary = new Entry[initialCapacity];
```

The compiler sees an array whose elements have type `Entry` assigned to an array whose elements are of type `Entry<K, V>`. It therefore warns us of an unchecked conversion. An attempt to cast the new array to `Entry<K, V>[]` results in a similar warning. In either event, all should be well despite the compiler's concern. Thus, we suppress the warning as we have done in the past.

20.3 **Some private methods.** One problem with array-based implementations of an ADT is the finite size of the array. To avoid a full dictionary, we double the array's size as necessary, just as we did in earlier chapters. We will use a private method as we did then. Its specification is as follows:

```
// Doubles the size of the array of entries if it is full.
private void ensureCapacity()
```

Adding, removing, or retrieving an entry requires a sequential search, since the search keys are not sorted. A sequential search must look at all the search keys in the array to conclude that an entry is not present in the dictionary. Implementing this search as the following private method will simplify the definitions of these three dictionary operations:

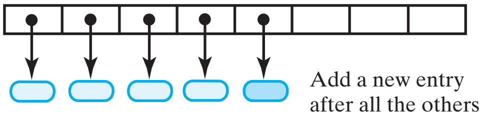
```
// Returns the index of the entry that contains key or
// returns numberOfEntries if no such entry exists.
private int locateIndex(K key)
```

20.4 **Adding an entry.** Another potential problem with array-based implementations is the shifting of array entries that often occurs. When a dictionary's search keys are unsorted, however, we can add or remove an entry without shifting other entries. When adding a new key-value entry, we can insert it after the last entry in the array and not move any other entry, as Figure 20-2 shows. In this case, `add` returns `null`. However, if the search key was in the dictionary already, we replace its corresponding value with the new value and return the original value. The following algorithm performs these steps:

```
Algorithm add(key, value)
//  Adds a new key-value entry to the dictionary and returns null. If key already exists
// in the dictionary, returns the corresponding value and replaces it with value.
```

```
result = null
Search the array for an entry containing key
if (an entry containing key is found in the array)
{
    result = value currently associated with key
    Replace key's associated value with value
}
else // insert new entry
{
    if (array is full)
       Double size of array
    Insert a new entry containing key and value after the last entry in the array
    Increment the size of the dictionary
}
return result
```

FIGURE 20-2 Adding a new entry to an unsorted array-based dictionary



20.5 **The method** add**.** The following implementation of the method add invokes the private methods specified in Segment 20.3:

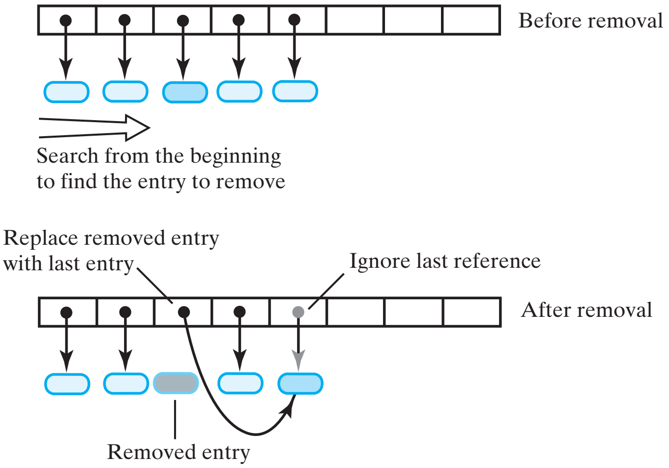
```
public V add(K key, V value)
{
   V result = null;
   int keyIndex = locateIndex(key);

   if (keyIndex < numberOfEntries)
   {
      // key found; return and replace old value
      result = dictionary[keyIndex].getValue();
      dictionary[keyIndex].setValue(value);
   }
   else
   {
      ensureCapacity();
      dictionary[numberOfEntries] = new Entry<K, V>(key, value);
      numberOfEntries++;
   } // end if

   return result;
} // end add
```

To search an unsorted array, locateIndex has the following definition:

```
private int locateIndex(K key)
{
   int index = 0;
   while ( (index < numberOfEntries) &&
           !key.equals(dictionary[index].getKey()) )
      index++;

   return index;
} // end locateIndex
```

20.6 **Removing an entry.** To remove an entry from an unsorted array-based dictionary, we first locate the entry and then replace it with the last entry in the dictionary, as Figure 20-3 illustrates. Thus, we can fill the "hole" in the array without shifting the other entries. Since the size of the dictionary is reduced by 1, the extra reference remaining after the current entries will be ignored.

FIGURE 20-3 Removing an entry from an unsorted array-based dictionary



The following algorithm describes the remove operation:

```
Algorithm remove(key)
// Removes an entry from the dictionary, given its search key, and returns its value.
// If no such entry exists in the dictionary, returns null.

result = null
Search the array for an entry containing key

if (an entry containing key is found in the array)
{
    result = value currently associated with key
    Replace the entry with the last entry in the array
    Decrement the size of the dictionary
}
// else result is null

return result
```

This logic can be implemented as follows:

```
public V remove(K key)
{
   V result = null;
   int keyIndex = locateIndex(key);

   if (keyIndex < numberOfEntries)
   {
      result = dictionary[keyIndex].getValue();
      dictionary[keyIndex] = dictionary[numberOfEntries - 1];
      numberOfEntries--;
   } // end if
```

```
      return result;
   } // end remove
```

20.7 **The remaining methods.** We leave the rest of the dictionary implementation to you as an exercise, since it is not difficult once you have reached this point. Note that an iteration, or traversal, of the dictionary entries simply moves from location to location within the array. Since the search keys are not sorted, the order of the iteration is not specified. Whatever order is easy to implement is fine. Typically, you start with the first entry in the array and move sequentially through the remaining entries.

20.8 **Efficiency.** For this implementation, the worst-case efficiencies of the operations are as follows:

Addition	O(*n*)
Removal	O(*n*)
Retrieval	O(*n*)
Traversal	O(*n*)

Even though additions occur after the last entry in the array `dictionary` without shifting any data, the search necessary to prevent duplicate search keys in the dictionary makes the overall operation O(*n*). Deletions and retrievals use a similar search of the array, making them O(*n*) as well. Finally, traversing an array is an O(*n*) operation.

Realize that if you fill the array of dictionary entries, you must allocate a new, larger array and copy entries from the original array to the new array. This requirement adds overhead to any array-based implementation that the previous analysis does not reflect. In Java, the array entries are references to objects, so copying the array is fast. But for languages whose arrays contain objects instead of references, copying can be quite time-consuming. Ideally, you want to choose a sufficiently large array, but not one that wastes space because it is overly large.

A Sorted Array-Based Dictionary

20.9 Some of the implementation for an unsorted dictionary, as shown in Segment 20.2, is independent of the order of the dictionary's entries, and so can be used for a sorted dictionary. However, the search keys must belong to a class that implements the interface `Comparable` so that we can order them. An outline for an implementation of a sorted dictionary appears in Listing 20-2. The notation `K extends Comparable<? super K>`, which was first introduced in Segment 8.2 of Chapter 8, defines the generic type `K`. It allows us to compare objects of type `K` with either objects of type `K` or objects of any superclass of `K`.

LISTING 20-2 An outline of the class `SortedArrayDictionary`

```
import java.util.Arrays;
import java.util.Iterator;
import java.util.NoSuchElementException;
/**
   A class that implements a dictionary by using a sorted array.
   @author Frank M. Carrano
*/
public class SortedArrayDictionary<K extends Comparable<? super K>, V>
             implements DictionaryInterface<K, V>
   < Data fields as shown in Listing 20.1 of Segment 20.2 >
   . . .

   < Constructors analogous to those in Listing 20.1 >
   . . .

   public V add(K key, V value)
   {
      . . . < See Segment 20.11. >
   } // end add

   < Implementations of other methods in DictionaryInterface >
   . . .

   < The private class Entry, as shown in Listing 20.1. >
} // end SortedArrayDictionary
```

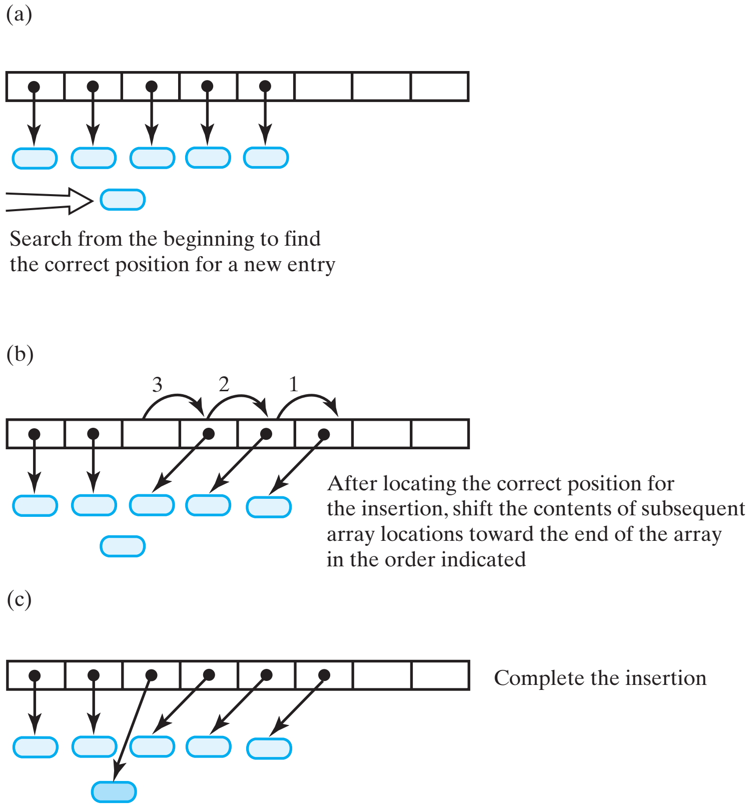
20.10 **Adding an entry.** When the dictionary's key-value entries are sorted by their search keys, adding a new entry requires a search of the array of entries to see where the new entry belongs. After you determine the correct position for the new entry, you must make room for it in the array. You do this by shifting subsequent array entries up by one position, beginning at the last entry, as Figure 20-4 shows. You then insert the new entry into the array so it is in its proper order by search key.

The following algorithm for adding an entry has similarities to the one given in Segment 20.4 for an unsorted dictionary:

```
Algorithm add(key, value)
// Adds a new key-value entry to the dictionary and returns null. If key already exists
// in the dictionary, returns the corresponding value and replaces it with value.

result = null
Search the array until you either find an entry containing key or locate the point where it
   should be
if (an entry containing key is found in the array)
{
   result = value currently associated with key
   Replace key's associated value with value
}
else // insert new entry
{
   if (array is full)
      Double size of array

   Make room in the array for a new entry at the index determined by the previous search
   Insert a new entry containing key and value into the vacated location of the array
   Increment the size of the dictionary
}
return result
```

Question 2 Describe how the previous algorithm differs from the one given in Segment 20.4 for an unsorted dictionary.

FIGURE 20-4 Adding an entry to a sorted array-based dictionary: (a) search; (b) make room; (c) insert



Search from the beginning to find the correct position for a new entry

After locating the correct position for the insertion, shift the contents of subsequent array locations toward the end of the array in the order indicated

Complete the insertion

20.11 **The method `add`.** We can implement this algorithm by using the private methods described in Segment 20.3, but we need to use a different implementation for `locateIndex`. When the dictionary is unsorted, `locateIndex` simply detects whether the dictionary contains a given search key. But here, `locateIndex` must also determine where in the array to make the insertion. Thus, we revise the method's specification as follows:

```
// Returns the index of either the entry that contains key or
// the location that should contain key, if no such entry exists.
private int locateIndex(K key)
```

The following additional method will also be helpful in the implementation:

```
// Makes room for a new entry at a given index by shifting
// array entries towards the end of the array.
private void makeRoom(int keyIndex)
```

Using these methods, we can implement the method add as follows:

```
public V add(K key, V value)
{
   V result = null;
   int keyIndex = locateIndex(key);
   if ( (keyIndex < numberOfEntries) &&
         key.equals(dictionary[keyIndex].getKey()) )
   {
      // key found; return and replace old value
      result = dictionary[keyIndex].getValue();
      dictionary[keyIndex].setValue(value);
   }
   else
   {
      ensureCapacity();
      makeRoom(keyIndex);
      dictionary[keyIndex] = new Entry<K, V>(key, value);
      numberOfEntries++;
   } // end if
   return result;
} // end add
```

20.12 **The method `locateIndex`.** Since the array is sorted, `locateIndex` can generally search it in less time than it could search an unsorted array. Recall from Segment 18.8 of Chapter 18 that a sequential search can detect when an entry is not in a sorted array without searching the entire array. Using that technique, we define the private method `locateIndex` as follows:

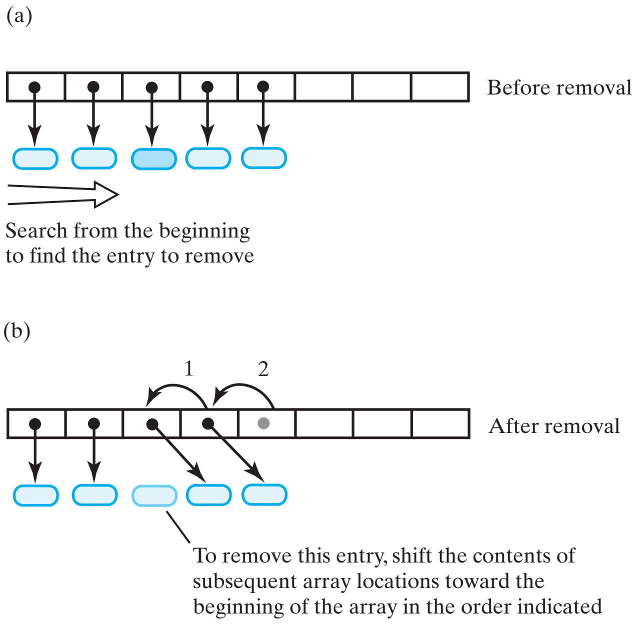
```
private int locateIndex(K key)
{
   // search until you either find an entry containing key or
   // pass the point where it should be
   int index = 0;
   while ( (index < numberOfEntries) &&
           key.compareTo(dictionary[index].getKey()) > 0 )
     index++;
   return index;
} // end locateIndex
```

The difference between this method and the one given in Segment 20.5 for an unsorted dictionary is highlighted.

Question 3 A binary search would be faster, in general, than the modified sequential search just given—particularly when the dictionary is large. Implement the private method `locateIndex` for a sorted dictionary using a binary search.

20.13 **Removing an entry.** To remove an entry from a sorted array-based dictionary, we first locate the entry by calling the method `locateIndex` that we used in the previous segment for the add method. Since the entries are sorted, we must maintain their order. Thus, any entries after the one to be removed must shift to the next lower position in the array. Figure 20-5 illustrates these two steps.

FIGURE 20-5 Removing an entry from a sorted array-based dictionary: (a) search; (b) shift entries


(a)
Before removal
Search from the beginning
to find the entry to remove
(b)
1
2
After removal
To remove this entry, shift the contents of
subsequent array locations toward the
beginning of the array in the order indicated

The following algorithm describes the remove operation:

```
Algorithm remove(key)
// Removes an entry from the dictionary, given its search key, and returns its value.
// If no such entry exists in the dictionary, returns null.

result = null
Search the array for an entry containing key

if (an entry containing key is found in the array)
{
    result = value currently associated with key
    Shift any entries that are after the located one to the next lower position in the array
    Decrement the size of the dictionary
}
return result
```

We leave the implementation of this algorithm as an exercise. Defining the following private method will be helpful:

```
// Removes an entry at a given index by shifting array
// entries toward the entry to be removed.
private void removeArrayEntry(int keyIndex)
```

20.14 **The remaining methods.** At the heart of the method `getValue`, which retrieves the value in an existing entry given its search key, is the method `locateIndex`, as described earlier. Since the array of entries is sorted, `locateIndex` can use a binary search, as Question 3 indicated.

An iteration, or traversal, of the entries in the dictionary starts with the first entry in the array and moves sequentially through the remaining entries. This part of the implementation can be the same as for an unsorted dictionary. But here, since the array is sorted, the iteration will traverse the dictionary in sorted search-key order.

We leave the completion of this implementation to you as an exercise.

20.15 **Efficiency.** When `locateIndex` uses a binary search in the sorted array-based implementation, the worst-case efficiencies of the dictionary operations are as follows:

Addition	O(n)
Removal	O(n)
Retrieval	O(log n)
Traversal	O(n)

This implementation is suitable for an application that creates a dictionary and then makes many retrievals. This point from Chapter 14 bears repeating here:

Programming Tip: When choosing an implementation for an ADT, you should consider the operations that your application requires. If you use a particular ADT operation frequently, you want its implementation to be efficient. Conversely, if you rarely use an operation, you can afford to use a class that has an inefficient implementation of that operation.

Programming Tip: Include comments in a class's implementation that advertise the efficiencies of its methods.

Question 4 When the sorted array-based implementation of a dictionary uses a binary search, its retrieval operation is O(log n). Since `add` and `remove` use a similar search, why are they not O(log n) as well?

Vector-Based Implementations

20.16 An implementation that uses an instance of Java's class `Vector` is similar in spirit to an array-based implementation. You can use one or two vectors, much like the one or two arrays pictured in Parts *a* and *b* of Figure 20-1. Since the underlying implementation of the class `Vector` is array based, the algorithms for the dictionary operations and their efficiencies are essentially the same whether you use an array or a vector.

With a vector, you do not need the private methods `ensureCapacity`, `makeRoom`, and `removeArrayEntry` that are in the array-based implementations. A vector accommodates the addition of a new entry—expanding as necessary—without any extra effort by us. A vector also shifts its entries if necessary when you either add or remove an entry. Lastly, a vector counts its entries, so you do not have to.

20.17 As an example of how to use a vector to implement a dictionary, consider the sorted implementation outlined in Listing 20-3. The type parameters `K` and `V` of `SortedVectorDictionary` are like those for `SortedArrayDictionary` shown in Listing 20-2 of Segment 20.9. The inner class `Entry` uses these type parameters, whereas the definition of `Entry` within `SortedArrayDictionary` (see Listing 20-1 of Segment 20.2) defines its own type parameters. Although we could use either definition here, this simpler one is fine because we are not allocating an array of `Entry` objects.

LISTING 20-3 An outline of the class `SortedVectorDictionary`

```
import java.util.Vector;
import java.util.Iterator;
import java.util.NoSuchElementException;
/**
    A class that implements a dictionary by using a sorted vector.
    @author Frank M. Carrano
*/

public class SortedVectorDictionary<K extends Comparable<? super K>, V>
             implements DictionaryInterface<K, V>
{
   private Vector<Entry> dictionary;

   public SortedVectorDictionary()
   {
      dictionary = new Vector<Entry>(); // doubles in size, as necessary
   } // end default constructor

   public SortedVectorDictionary(int initialCapacity)
   {
      dictionary = new Vector<Entry>(initialCapacity);
   } // end constructor

   < Implementations of methods in DictionaryInterface >

   < Private classes KeyIterator and ValueIterator  (See Segment 20.20) >

   private class Entry
   {
      private K key;
      private V value;

      private Entry(K searchKey, V dataValue)
      {
         key = searchKey;
         value = dataValue;
      } // end constructor

      private K getKey()
      {
         return key;
      } // end getKey
      private V getValue()
      {
         return value;
      } // end getValue

      private void setValue(V newValue)
      {
         value = newValue;
      } // end setValue
   } // end Entry
} // end SortedVectorDictionary
```

20.18 **The method add.** The implementation of the method add is similar to the one given earlier in Segment 20.11 for the sorted array-based implementation. This version is shorter, since much of the busy work is handled for us by Vector. You should compare this code with the code given earlier.

Assuming that we have already revised locateIndex, as given in Segment 20.12, to work with vectors, we call it with the statement

```
int keyIndex = locateIndex(key);
```

where key is the search key of the new entry. To see whether an entry having key as its search key is already in the dictionary, we check whether key is the same as the search key of the entry at keyIndex. Using Vector's method get, we reference this entry by writing

```
dictionary.get(keyIndex)
```

To get this entry's search key, we write

```
(dictionary.get(keyIndex)).getKey()
```

The pair of blue parentheses is optional. This expression is now the argument of the method equals that is invoked by key, the new entry's search key:

```
key.equals((dictionary.get(keyIndex)).getKey())
```

Ordinarily, we could simplify this expression by first assigning the argument of equals to a local variable, as follows:

```
Entry currentEntry = dictionary.get(keyIndex);
```

We then could write the expression as

```
key.equals(currentEntry.getKey())
```

But in our case, we also need to check the value of keyIndex. Thus, we are left with the choice of one long if clause or several awkwardly nested but shorter if statements. We settle for the following implementation of add:

```
public V add(K key, V value)
{
   V result = null;
   int keyIndex = locateIndex(key);

   if ( (keyIndex < dictionary.size()) &&
         key.equals( (dictionary.get(keyIndex)).getKey() ) )
   {
      // key found; return and replace old value
```

```
         Entry currentEntry = dictionary.get(keyIndex);
         result = currentEntry.getValue();
         currentEntry.setValue(value);
      }
      else // add new entry
      {
         Entry newEntry = new Entry(key, value);
         dictionary.add(keyIndex, newEntry);
      } // end if

     return result;
   } // end add
```

20.19 **The private method `locateIndex`.** To revise `locateIndex`, as given in Segment 20.12, to work with a vector instead of an array, we modify the boolean expression in the `while` statement. The resulting expression is similar to the boolean expression in the previous segment—it uses `Vector's` method `get`—but it invokes `compareTo` instead of `equals`. Thus, the revised method appears as follows:

```
private int locateIndex(K key)
{
   // search until you either find an entry containing key or
   // pass the point where it should be
   int numberOfEntries = dictionary.size();
   int index = 0;
   while ( (index < numberOfEntries) &&
            key.compareTo( (dictionary.get(index)).getKey() ) > 0 )
      index++;

   return index;
} // end locateIndex
```

The differences between this method and the array version are highlighted. In `SortedArrayDictionary`, `numberOfEntries` is a data field. Here it is a local variable set to `dictionary.size()`, the vector's size.

Implementations for the methods `remove` and `getValue` are similar to the implementation for `add`, and we leave them as exercises.

20.20 **An iterator for search keys.** Our goal in providing iterators for the dictionary is to give the client an easy way to traverse the search keys and their corresponding values. The interface for the dictionary given in Segment 19.4 of the previous chapter specifies two iterators, one for search keys and one for values. That chapter showed you how to use these iterators, and here we implement an iterator for the search keys.

Recall that the interface `java.util.Iterator` specifies the methods `hasNext`, `next`, and `remove`. We can define a private class `KeyIterator` that implements this interface and is internal to the class `SortedVectorDictionary`. We then implement the public method `getKeyIterator` within `SortedVectorDictionary` as follows:

```
public Iterator<K> getKeyIterator()
{
   return new KeyIterator();
} // end getKeyIterator
```

The implementation of the class `KeyIterator` is shown in Listing 20-4. `Vector`'s iterator `dictionary.iterator` traverses the entries in the vector `dictionary` and is the basis of this implementation. Each entry in the vector is an instance of `Entry`, so we use `Entry`'s method `getKey` to get each search key.

`Iterator`'s method `remove` is not relevant to the traversal of the search keys, so we do not support it. Instead we make `remove` throw an exception if it is invoked.

LISTING 20-4 `SortedVectorDictionary`'s private inner class `KeyIterator`

```
private class KeyIterator implements Iterator<K>
{
   private Iterator<Entry> traverser;

   private KeyIterator()
   {
      traverser = dictionary.iterator();
   } // end default constructor

   public boolean hasNext()
   {
      return traverser.hasNext();
   } // end hasNext

   public K next()
   {
      Entry nextEntry = traverser.next();
      return nextEntry.getKey();
   } // end next

   public void remove()
   {
      throw new UnsupportedOperationException();
   } // end remove
} // end KeyIterator
```

The method `getValueIterator` has an analogous implementation: You define another inner class similar to `KeyIterator`. We leave the details as an exercise.

Note: Using `ArrayList` instead of `Vector`

Since `ArrayList` and `Vector` each implement the interface `java.util.List`, and since we were careful to use only methods in this interface, you could replace `Vector` with `ArrayList` in the implementation of `SortedVectorDictionary`.

Linked Implementations

20.21 The last implementations of the ADT dictionary that we will consider in this chapter store the dictionary's entries in a chain of linked nodes. As presented in Chapters 3 and 14, for example, a chain can provide as much storage as necessary for the entries. You can encapsulate the two parts of an entry into an object, as Figure 20-6a illustrates, just as you did for an array or a vector. If you choose this option, your dictionary class can use the classes `Node` from Segment 3.25 and `Entry` from Segment 20.17.

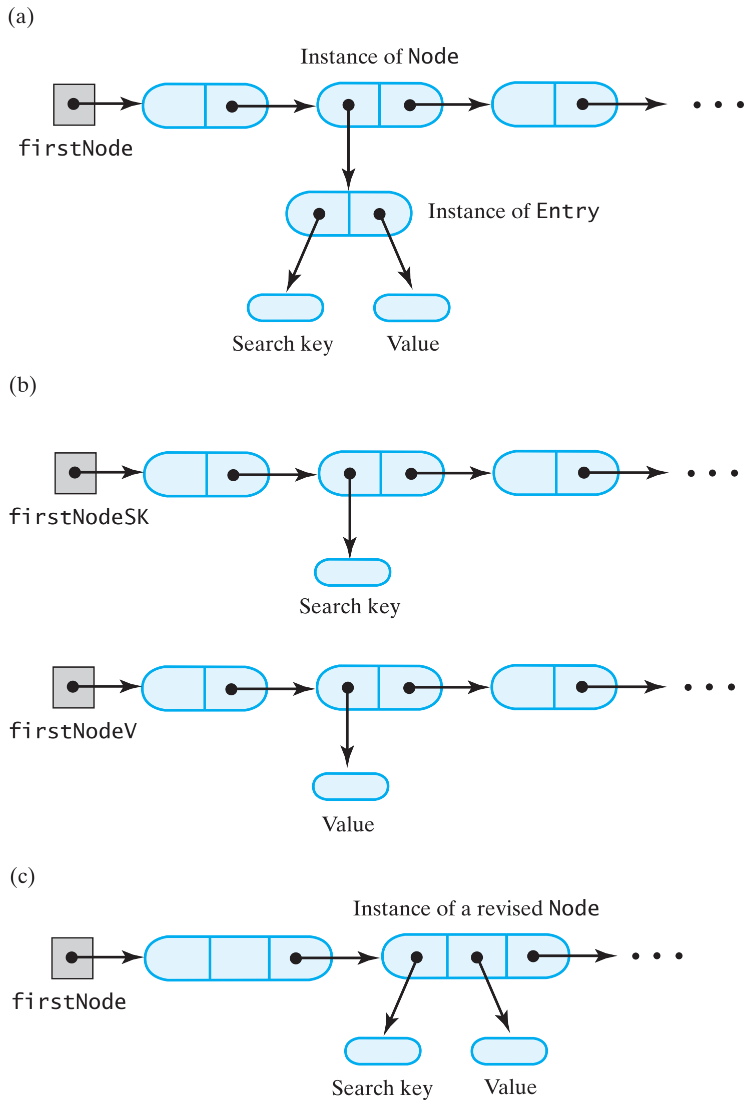
Another option does not use the class `Entry`. You could use two chains, as in Figure 20-6b, but a simpler approach is to revise the definition of a node to include both parts of the entry, as Figure 20-6c illustrates. The private inner class `Node`, defined within the dictionary class, would then contain the data fields.

VideoNote
Linked-chain dictionaries

```
    private K key;
    private V value;
    private Node next;
```

The generic types K and V are defined by the outer class. In addition to constructors, the class Node would contain the methods getKey, getValue, setValue, getNextNode, and setNextNode. Since changing the search key is not necessary and, in fact, could destroy the order of a sorted dictionary, no setKey method is provided.

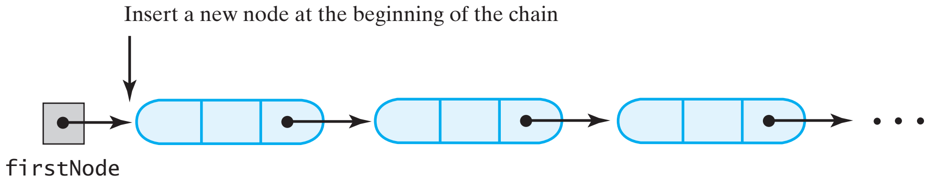
FIGURE 20-6 Three possible ways to use linked nodes to represent the entries in a dictionary: (a) a chain of nodes that each reference an entry object; (b) parallel chains of search keys and values; (c) a chain of nodes that each reference a search key and a value



An Unsorted Linked Dictionary

20.22 Since the entries in an unsorted dictionary are in no particular order, you add a new entry in the most efficient manner. When the entries are in a linked chain, such as the one in Figure 20-6c, the fastest addition is at the beginning of the chain, as Figure 20-7 shows. (If the class also maintains a tail reference to the last node of the chain, adding an entry after the last node would be equally fast.) While this aspect of an addition is O(1), preventing duplicate search keys would require a sequential search from the beginning of the chain.

FIGURE 20-7 Adding to an unsorted linked dictionary



Just as you would for an array, you would have to look at all the search keys in the chain to learn that a particular entry was not present. Removing or retrieving an entry uses a similar search. A traversal of either the search keys or the values involves the entire chain. Thus, for this implementation, the worst-case efficiencies of the operations are as follows:

Addition	O(*n*)
Removal	O(*n*)
Retrieval	O(*n*)
Traversal	O(*n*)

Question 5 To remove an entry from an unsorted array-based dictionary, we replaced the removed entry with the last entry in the array (see Segment 20.6). Should we use the same strategy to remove an entry from an unsorted linked dictionary? Explain.

A Sorted Linked Dictionary

20.23 **Adding an entry.** When the nodes in a chain are sorted by their search keys, adding a new entry to the dictionary requires a sequential search of the chain from its beginning to determine the correct location for the new node. Since the search keys are sorted, you can detect that a desired search key does not exist in the chain as soon as you pass the node that should have contained it. That is, you do not have to look at the entire chain, as you would if the search keys were unsorted. Segments 18.8 and 18.22 in Chapter 18 describe this variation of a sequential search.

The following algorithm adds a new entry to a sorted linked dictionary:

```
Algorithm add(key, value)
// Adds a new key-value entry to the dictionary and returns null. If key already exists
// in the dictionary, returns the corresponding value and replaces it with value.

result = null
Search the chain until either you find a node containing key or you pass the point where
    it should be
if (a node containing key is found in the chain)
{
    result = value currently associated with key
    Replace key's associated value with value
}
else
{
    Allocate a new node containing key and value
    Increment the size of the dictionary
    if (the chain is empty or the new entry belongs at the beginning of the chain)
        Add the new node to the beginning of the chain
    else
        Insert the new node before the last node that was examined during the search
}
return result
```

20.24 Listing 20-5 shows the beginning of the class `SortedLinkedDictionary` and the implementation of the method `add`. Implementations for the methods `remove` and `getValue` are similar to the implementation for `add`, but are a bit simpler. We leave them as exercises.

LISTING 20-5 The class `SortedLinkedDictionary`

```
import java.util.Iterator;
import java.util.NoSuchElementException;
/**
    A class that implements a dictionary by using a sorted linked chain.
    @author Frank M. Carrano
*/
public class SortedLinkedDictionary<K extends Comparable<? super K>, V>
             implements DictionaryInterface<K, V>
{
   private Node firstNode;   // reference to first node of chain
   private int  numberOfEntries;

   public SortedLinkedDictionary()
   {
      firstNode = null;
      numberOfEntries = 0;
   } // end default constructor

   public V add(K key, V value)
   {
      V result = null;

      // search chain until you either find a node containing key
      // or pass the point where it should be
      Node currentNode = firstNode;
      Node nodeBefore = null;
      while ( (currentNode != null) &&
              key.compareTo(currentNode.getKey()) > 0 )
      {
         nodeBefore = currentNode;
         currentNode = currentNode.getNextNode();
      } // end while

      if ( (currentNode != null) && key.equals(currentNode.getKey()) )
      {
         result = currentNode.getValue();
         currentNode.setValue(value); // replace value
      }
      else
      {
         Node newNode = new Node(key, value); // create new node
         numberOfEntries++; // increase length for both cases
         if (nodeBefore == null)
         { // add at beginning (includes empty chain)
            newNode.setNextNode(firstNode);
            firstNode = newNode;
         }
         else // add elsewhere in non-empty chain
         {
            newNode.setNextNode(currentNode); // currentNode is after new
                                              // node
            nodeBefore.setNextNode(newNode); // nodeBefore is before new
                                             // node
         } // end if
      } // end if

      return result;
   } // end add

   < Implementations of the other methods in DictionaryInterface >
   . . .

   < Private classes KeyIterator and ValueIterator  (See Segment 20.26) >
   . . .

   < The private class Node >
   . . .
} // end SortedLinkedDictionary
```

20.25 **Iterators.** As we mentioned in Segment 20.20, iterators provide the client with an easy way to traverse a dictionary's search keys and their corresponding values. The public methods `getKeyIterator` and `getValueIterator` have the same implementations here as they do in `SortedVectorDictionary`. The private inner classes `KeyIterator` and `ValueIterator`, however, differ. Each has a data field `nextNode` to mark an iteration's place in the chain as the traversal progresses. Listing 20-6 shows the private class

KeyIterator. ValueIterator has a similar definition. Both of these classes are like the inner class IteratorForLinkedList that appears in Segments 15.19 through 15.23 of Chapter 15.

LISTING 20-6 SortedLinkedDictionary's private inner class KeyIterator

```
private class KeyIterator implements Iterator<K>
{
   private Node nextNode; // node containing next entry in iteration

   private KeyIterator()
   {
      nextNode = firstNode;
   } // end default constructor

   public boolean hasNext()
   {
      return nextNode != null;
   } // end hasNext

   public K next()
   {
      K result;
      if (hasNext())
      {
         result = nextNode.getKey();
         nextNode = nextNode.getNextNode();
      }
      else
         throw new NoSuchElementException();

      return result;
   } // end next

   public void remove()
   {
      throw new UnsupportedOperationException();
   } // end remove
} // end KeyIterator
```

20.26 **Efficiency.** Like adding an entry, removing or retrieving an entry requires a sequential search of the chain. Traversal of a sorted chain proceeds just as it would for an unsorted chain. Thus, the worst-case efficiencies of the dictionary operations for a sorted linked implementation are as follows:

Addition	O(n)
Removal	O(n)
Retrieval	O(n)
Traversal	O(n)

The addition or removal of an entry is an O(n) operation regardless of whether you use an array, a vector, or a chain to implement a dictionary. Realize, however, that an array requires you to shift its elements, whereas a linked chain does not. Also, the preceding linked implementation does not require a good estimate of the dictionary's ultimate size. When you use an array that is too small, you can expand it by copying its entries to a new, larger array, but this takes time. If you use an array that is larger than necessary, you waste space. The same is true if you use a vector, but neither of these situations occur with a linked implementation.

Chapter Summary

- You can implement a dictionary by using either an array, a vector, or a chain of linked nodes. A linked implementation does not require a good estimate of the dictionary's ultimate size. When you use an array that is too small, you need to copy its entries to a new, larger array. If you use an array that is larger than necessary, you waste space. The same is true if you use a vector, but neither of these situations occur with a linked implementation.
- The worst-case efficiencies of the dictionary operations for array-based and linked implementations are as follows:

	Array-Based		Linked	
	Unsorted	Sorted	Unsorted	Sorted
Addition	O(n)	O(n)	O(n)	O(n)
Removal	O(n)	O(n)	O(n)	O(n)
Retrieval	O(n)	O(log n)	O(n)	O(n)
Traversal	O(n)	O(n)	O(n)	O(n)

- For a sorted or unsorted dictionary, the addition or removal of an entry is an O(n) operation regardless of whether you use an array, a vector, or a chain to implement it. Realize, however, that an array or vector requires the shifting of its entries, whereas a linked chain does not.
- Using either an array or a vector to implement a sorted dictionary allows for an efficient retrieval operation because you can use a binary search.
- To implement the method `getKeyIterator` or `getValueIterator`, define a private inner class for the dictionary class. This private class should implement the interface `java.util.Iterator`.

Programming Tip

- When choosing an implementation for an ADT, you should consider the operations that your application requires. If you use a particular ADT operation frequently, you want its implementation to be efficient. Conversely, if you rarely use an operation, you can afford to use a class that has an inefficient implementation of that operation.
- Include comments in a class's implementation that advertise the efficiencies of its methods.

Exercises

1. Begin an array-based implementation of the ADT dictionary according to the data structure illustrated in Figure 20-1b. Declare the data fields, define the constructors, and define the method `add` for unsorted data. Use arrays that you can resize during execution.

2. Begin two linked implementations of the ADT dictionary according to the two data structures illustrated in Parts *a* and *b* of Figure 20-6. Declare the data fields, define the constructors, and define the method `add` for unsorted data.

3. In the sorted, vector-based implementation of a dictionary, replace the sequential search performed by the method `locateIndex` with a binary search. (See Segment 20.19.)

4. For a sorted linked implementation of a dictionary, write iterative versions of the methods `remove` and `getValue`.

5. For a sorted linked implementation of a dictionary, write recursive versions of the methods `add`, `remove`, and `getValue`.

6. Segment 20.25 defines the class `KeyIterator`. An instance of this class is an iterator that traverses the search keys in the dictionary. In a similar fashion, define a class `ValueIterator` to provide a way to traverse the dictionary's values.

7. Define an iterator for the ADT dictionary that returns entries containing both a search key and a value. Describe the class of these entries. Implement a method `getEntryIterator` that returns such an iterator.

8. Consider adding operations to the ADT dictionary to form the union and intersection of two given dictionaries. Each operation returns a new dictionary. The union should combine the entries in both dictionaries into a third dictionary. The intersection should be a dictionary of the entries common to both of the two dictionaries.

   Within each given dictionary, search keys are not repeated. However, an entry in one dictionary could have the same search key as an entry in the second dictionary. Propose and discuss ways to specify these two operations for this case.

9. Implement the union and intersection operations that Exercise 8 describes for an unsorted array-based dictionary.

10. Repeat Exercise 9 for a sorted array-based dictionary.

11. Repeat Exercise 9 for a sorted linked dictionary.

Projects

1. Implement an unsorted array-based dictionary. Allow the array to expand as necessary during execution.

2. Repeat the previous project, but maintain the search keys in sorted order.

3. Implement an unsorted dictionary by using an instance of `Vector` or `ArrayList`.

4. Repeat the previous project, but maintain the search keys in sorted order.

5. Implement an unsorted dictionary by using a chain of linked nodes.

6. Implement a sorted dictionary by using a chain of linked nodes.

7. In this chapter, the ADT dictionary has distinct search keys. Implement a dictionary that removes this restriction. Choose one of the following possibilities:

   - The method `add` adds an entry whose search key is already in the dictionary but whose value is not. The method `remove` deletes all occurrences of the search key. The method `getValue` retrieves all values associated with a search key.
   - The methods behave as just described, but a secondary search key enables `remove` and `getValue` to delete or retrieve a single entry.

8. Segment 19.7 of the previous chapter began a discussion of a telephone directory. Use your dictionary implementation from Project 7 in a revision of the telephone directory that allows duplicate names.

9. Figure 20-1b illustrates how you can use parallel arrays to represent the entries in a dictionary. Implement the ADT dictionary by using this approach.

10. Revise the class `Entry` given in Listing 20-1 of Segment 20.2 as a top-level class that implements the interface `Comparable`. You compare two `Entry` objects by comparing their search keys. Using this class and an implementation of the ADT sorted list, write an implementation for a sorted dictionary. The classes, including `Entry`, should belong to the same package.

11. Revise the class `Entry`, which is in Listing 20-1, to make it public. Implement a sorted dictionary by using an array of `Entry` objects.

12. Repeat the previous project, but instead use a chain of linked nodes that each reference an instance of `Entry`.

13. Implement the interface `DictionaryInterface<String, String>` to create a class of glossaries. A *glossary* is a dictionary of specialized terms and their corresponding definitions. Represent the glossary as an array of 26 sorted lists, one list for each letter of the alphabet. Each entry—which consists of a term and its definition—in a glossary is stored in the sorted list corresponding to the term's first letter. Thoroughly test your class using a text file of terms and definitions as data for your glossary.

Answers to Self-Test Questions

1. The memory requirements for the search keys and the values are the same for each representation, so let's ignore them. The memory requirement for the representation shown in Figure 20-1a uses three references for each entry in the dictionary: one in the array and two in the `Entry` object. The parallel arrays in Figure 20-1b require only two references for each dictionary entry. Thus, for *n* entries in the dictionary, the representation in Part *a* requires 3*n* references, but the representation in Part *b* requires only 2*n* references. However, if each array has a length of *m*, where *m* is greater than *n*, Part *a* has *m* – *n* unused locations and Part *b* has twice that number.

2. The initial search determines the insertion point when the dictionary is sorted, whereas the insertion point for an unsorted dictionary is always right after the last entry in the array. Insertion into a sorted dictionary generally requires shifting other entries in the array. No shifting is necessary for an unsorted dictionary.

3.
```
private int locateIndex(K key)
{
   return binarySearch(0, numberOfEntries - 1, key);
} // end locateIndex

private int binarySearch(int first, int last, K key)
{
   int result;

   if (first > last)
      result = first;
   else
   {
      int mid = first + (last - first) / 2;
      K midKey = dictionary[mid].getKey();

     if (key.equals(midKey))
        result = mid;
     else if (key.compareTo(midKey) < 0)
        result = binarySearch(first, mid - 1, key);
     else
        result = binarySearch(mid + 1, last, key);
   } // end if

   return result;
} // end binarySearch
```

4. Typically, `add` must shift array entries to make room for a new entry, and `remove` must shift array entries to avoid a vacancy within the array. These shifts of data are O(n) operations in the worst case. The best case occurs when the addition or removal is at the end of the array. These operations are O(1).

5. No. Replacing the entry to be removed with the last entry in a chain would require a traversal of the chain. We would need references to both the last node and the next-to-last node so that we could delete the last node. Although we can ignore the last entry in an array, we should shorten the chain by setting the link portion of the next-to-last node to `null`. Note that having a tail reference does not eliminate the need for a traversal, since we need but do not have a reference to the next-to-last node. The strategy for an unsorted array-based dictionary avoids shifting any of the other entries. No shifting is needed in a linked implementation. After locating the node to delete, you simply adjust either the head reference or the reference in the preceding node.

This page intentionally left blank

Chapter 21

Introducing Hashing

Contents

Prerequisites

Chapter	19	Dictionaries
Chapter	20	Dictionary Implementations

Objectives

After studying this chapter, you should be able to

- Describe the basic idea of hashing
- Describe the purpose of a hash table, a hash function, and a perfect hash function
- Explain why you should override the method `hashCode` for objects used as search keys
- Describe how a hash function compresses a hash code into an index to the hash table
- Describe collisions and explain why they occur
- Describe open addressing as a method to resolve collisions
- Describe linear probing, quadratic probing, and double hashing as particular open addressing schemes
- Describe algorithms for the dictionary operations `getValue`, `add`, and `remove` when open addressing resolves collisions
- Describe separate chaining as a method to resolve collisions
- Describe algorithms for the dictionary operations `getValue`, `add`, and `remove` when separate chaining resolves collisions
- Describe clustering and the problems it causes

Because searching databases is such a widespread application of computers, the dictionary is an important abstract data type. The implementations that we discussed in the previous chapter are fine for certain applications, but for others they are inadequate. For example, if locating data is critical, even an O(log n) search can be too slow. Such is the case for the emergency telephone (911) system. If you call 911 from a land line, your telephone number is the key in a search of a dictionary of street addresses. Obviously, you want this search to find your location immediately!

This chapter introduces a technique called hashing that ideally can result in O(1) search times. We will complete our exploration of this topic in the next chapter. Hashing can be an excellent choice for implementing a dictionary when searching is the primary task. But as good as hashing can be, it is not always appropriate. For example, hashing cannot provide a traversal of the search keys in sorted order. Later in this book we will consider other implementations of the ADT dictionary.

What Is Hashing?

21.1 **A place for everything; everything in its place.** Do you spend time looking for your keys in the morning? Or do you know exactly where they are? Some of us spend too much time sequentially searching our unsorted possessions. Others have a special place for things and know just where to find each one.

VideoNote
Hashing

An array can provide a place for a dictionary's entries. Admittedly, arrays have their disadvantages, but you can access any entry in an array directly if you know its index. No other array entry need be involved. **Hashing** is a technique that determines this index using only an entry's search key, without searching. The array itself is called a **hash table**.

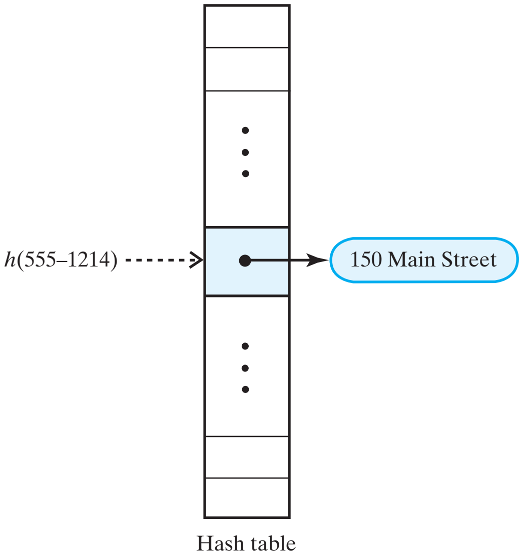
A **hash function** takes a search key and produces the integer index of an element in the hash table. This array element is where you would either store or look for the search key's associated value. For example, the 911 emergency system can take your telephone number, convert it to a suitable integer i, and store a reference to your street address in the array element `a[i]`. We say that the telephone number—that is, the search key—**maps**, or **hashes**, to the index `i`. This index is called a **hash index.** Sometimes we will say that the search key maps, or hashes, into the table location at the index `i`.

21.2 **Ideal hashing.** Consider an emergency system for a small town where everyone's telephone number begins with 555. Let the hash function h convert a telephone number to its last four digits. For example,

$$h(555\text{-}1214) = 1214$$

If `hashTable` is the hash table, we would place a reference to the street address associated with this telephone number in `hashTable[1214]`, as Figure 21-1 illustrates. If the cost of evaluating the hash function is low, adding an entry to the array `hashTable` is an O(1) operation.

FIGURE 21-1 A hash function indexes its hash table



To later find the street address associated with the number 555-1214, we once again compute *h*(555-1214) and use the result to index `hashTable`. Thus, from `hashTable[1214]`, we get the desired street address. This operation also is O(1). Notice that we did not search the array `hashTable`.

21.3 Let's summarize what we know so far by writing simple algorithms for the dictionary operations that add or retrieve entries:

```
Algorithm add(key, value)
index = h(key)
hashTable[index] = value

Algorithm getValue(key)
index = h(key)
return hashTable[index]
```

Will these algorithms always work? We can make them work if we know all the possible search keys. In this example, the search keys range from 555-0000 to 555-9999, so the hash function will produce indices from 0 to 9999. If the array `hashTable` has 10,000 elements, each telephone number will correspond to one unique element in `hashTable`. That element references the appropriate street address. This scenario describes the ideal case for hashing, and the hash function here is a **perfect hash function**.

Note: A perfect hash function maps each search key into a different integer that is suitable as an index to the hash table.

21.4 **Typical hashing.** Because we need a database of all street addresses in the previous example, we must have one entry in the hash table for each telephone number. Our perfect hash function needs a hash table this large because it produces 10,000 different indices between 0 and 9999 from the 10,000 possible search keys. This hash table is always full if every telephone number in the 555 exchange is assigned. Although a full hash table is quite reasonable for this application, most hash tables are not full and can even be **sparse**—that is, have only a few of their elements actually in use.

For example, if our small town required only 700 telephone numbers, most of the 10,000-location hash table would be unused. We would waste most of the space allocated to the hash table. If the 700 numbers were not sequential, we would need a different hash function if we wanted to use a smaller hash table.

We might develop this hash function as follows. Given a nonnegative integer i and a hash table with n locations, the value of i modulo n ranges from 0 to $n - 1$. Since i is nonnegative, i modulo n is the integer remainder after dividing i by n. This value is a valid index for the hash table. So, a hash function h for a telephone number could have the following algorithm:

```
Algorithm getHashIndex(phoneNumber)
// Returns an index to an array of tableSize locations.

i = last four digits of phoneNumber
return i % tableSize
```

This hash function—like typical hash functions—performs two steps:

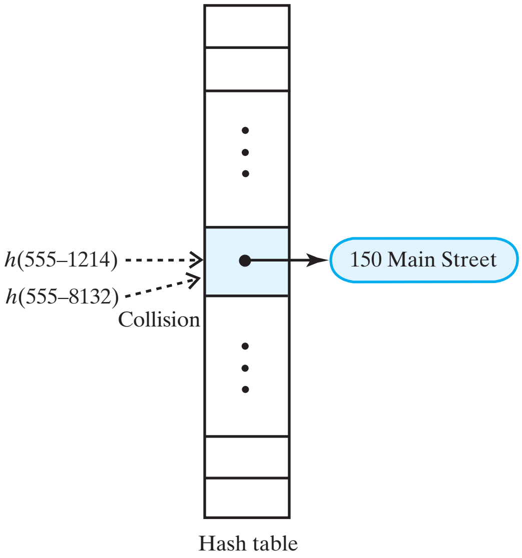
1. Convert the search key to an integer called the **hash code**.
2. **Compress** the hash code into the range of indices for the hash table.

Often the search key is not an integer, and frequently it is a string. So, a hash function first converts the key to an integer hash code. Next, it transforms that integer into one that is suitable as an index to the particular hash table.

The hash function that the algorithm `getHashIndex` describes is not a perfect hash function when `tableSize` is less than 10,000. Since 10,000 telephone numbers map into `tableSize` indices, some telephone numbers will map into the same index. We call such an occurrence a **collision**. For example, if `tableSize` is 101, `getHashIndex("555-1214")` and `getHashIndex("555-8132")` each map into 52. If we have already stored the street address for 555-1214 in `hashTable[52]`, as Figure 21-2 shows, what will we do with the address for 555-8132? Handling such collisions is called **collision resolution**. Before we look at collision resolution, we explore hash functions a bit further.

Note: Typical hash functions are not perfect, because they can allow more than one search key to map into a single index, causing a collision in the hash table.

FIGURE 21-2 A collision caused by the hash function *h*



Hash Functions

21.5 **General characteristics.** Any function can be a hash function if it produces an integer that is suitable as an array index. But not every such function is a *good* hash function. Our previous discussions suggest that a good hash function should

- Minimize collisions
- Be fast to compute

Recall that a typical hash function first converts a search key to an integer hash code. The hash function then compresses the hash code into an integer that is suitable as an index to the particular hash table.

Note: To reduce the chance of a collision, choose a hash function that distributes entries uniformly throughout the hash table.

First, consider how to convert a search key to an `int`. Realize that a search key can be either a primitive type or an instance of a class.

Computing Hash Codes

21.6 **The hash code for a class type.** Java's base class `Object` has a method `hashCode` that returns an integer hash code. Since every class is a subclass of `Object`, all classes inherit this method. But unless a class overrides `hashCode`, the method will return an `int` value based on the invoking object's memory address. This default hash code usually is not appropriate for hashing, because equal but distinct objects will have different hash codes. To be useful as a dictionary implementation, hashing must map equal objects into the same location in a hash table. Thus, a class should define its own version of `hashCode` that adheres to the following guidelines.

Note: Guidelines for the method `hashCode`

- If a class overrides the method `equals`, it should override `hashCode`.
- If the method `equals` considers two objects equal, `hashCode` must return the same value for both objects.
- If you call an object's `hashCode` more than once during the execution of a program, and if the object's data remains the same during this time, `hashCode` must return the same hash code.
- An object's hash code during one execution of a program can differ from its hash code during another execution of the same program.

A perfect hash function would require that unequal objects have distinct hash codes. In general, however, unequal objects might have the same hash codes. Since duplicate hash codes lead to collisions, you want to avoid this situation when possible.

21.7 **A hash code for a string.** Search keys are often strings, so generating a good hash code from a string is important. Typically, you begin by assigning an integer to each character in the string. For example, you could assign the integers 1 through 26 to the letters "A" through "Z" and the integers 27 through 52 to the letters "a" through "z." However, using a character's Unicode integer is more common and actually easier to do.

Suppose that the search keys for a telephone directory are names such as *Brett*, *Carol*, *Gail*, and *Josh*. You can compute hash codes for these names in several ways. For example, you could take the Unicode value of the first letter in each name and get distinct hash codes. But if several names begin with the same letter, their hash codes will be the same if you use this scheme. Since the letters that occur in any one position of a name do not occur with equal probability, a hash function that uses any particular letter will not distribute the names uniformly throughout the hash table.

Suppose that you sum the Unicode values for each letter in the search key. In an application where two different search keys never contain the same letters, this approach can work. But if your search keys are airport codes, for example, *DUB* and *BUD* would have the same hash code. This approach also can restrict the range of the hash codes, since the Unicode values for letters lie between 65 and 122. Thus, three-letter words would map into values between 195 and 366 under this plan.

Note: Real-world data is not uniformly distributed.

21.8 **A better hash code for a string.** A better approach to generating a hash code for a string involves multiplying the Unicode value of each character by a factor based on the character's position within the string. The hash code is then the sum of these products. Specifically, if the string *s* has *n* characters, let u_i be the Unicode value for the i^{th} character in *s* (*i* is zero for the first character). Then the hash code can have the form

$$u_0 g^{n-1} + u_1 g^{n-2} + \ldots + u_{n-2} g + u_{n-1}$$

for some positive constant g. This expression is a polynomial in g. To minimize the number of arithmetic operations, write the polynomial in the following algebraically equivalent form:

$$(\ldots((u_0\, g + u_1)\, g + u_2)\, g + \ldots + u_{n-2})\, g + u_{n-1}$$

This way of evaluating a polynomial is called *Horner's method.*

The following Java statements perform this computation for the string `s` and the `int` constant `g`:

```
int hash = 0;
int n = s.length();
for (int i = 0; i < n; i++)
   hash = g * hash + s.charAt(i);
```

The i^{th} character of the string is `s.charAt(i)`. Adding this character to the product `g * hash` actually adds the character's Unicode value. An explicit cast of `s.charAt(i)` to `int` is not necessary and would not affect the result.

This computation can cause an overflow, particularly for long strings. Java ignores these overflows and, for an appropriate choice of `g`, the result will be a reasonable hash code. Current implementations of the method `hashCode` in Java's class `String` use this computation with 31 as the value of `g`. Realize, however, that the overflows can produce a negative result. You can deal with that when you compress the hash code into an appropriate index for the hash table.

SELF-TEST

Question 1 Calculate the hash code for the string *Java* when *g* is 31. Compare your result with the value of the expression `"Java".hashCode()`.

21.9 **The hash code for a primitive type.** This segment contains Java operations that might be unfamiliar to you. However, they are not essential to the rest of this chapter.

If the search key's data type is `int`, you can use the key itself as the hash code. If the search key is an instance of either `byte`, `short`, or `char`, you can cast it to an `int` to get a hash code. Thus, casting to an `int` is one way to generate a hash code.

For other primitive types, you manipulate their internal binary representations. If the search key is an integer of type `long`, it contains 64 bits. An `int` has 32 bits. Simply casting the 64-bit search key to an `int`—or performing a modulo 2^{32}—would lose its first 32 bits. As a result, all keys that differ in only their first 32 bits will have the same hash code and collide. For this reason, ignoring part of a search key can be a problem.

Note: Derive the hash code from the *entire* search key. Do not ignore part of it.

Instead of ignoring a part of a `long` search key, divide it into several pieces. Then combine the pieces by using either addition or a bit-wise boolean operation such as **exclusive or**. This process is called **folding**.

For example, let's divide a `long` search key into two 32-bit halves. To get the left half, we can shift the search key to the right by a certain number of bits, or places. For example, if we shift the 8-bit binary number 10101100 to the right by 4 bits, we will get 00001010. We have isolated the number's left half and discarded its right half. If we now combine 00001010 with the original value and ignore the left half of the result, we will effectively have combined the left and right halves of the original key.

Now let's see how to do this in Java. The expression `key >> 32` shifts the 64-bit `key` to the right by 32 bits, in effect eliminating its right half. Java's exclusive-or operator is `^` and has the following effect on 1-bit quantities:

0 ^ 0 is 0
1 ^ 1 is 0
0 ^ 1 is 1
1 ^ 0 is 1

For two multibit quantities, the operator combines pairs of corresponding bits. So

1100 ^ 1010 is 0110

Thus, the expression `key ^ (key >> 32)` uses an exclusive-or operation to combine the halves of a 64-bit `key`. Although the result has 64 bits, the rightmost 32 bits contain the combined halves of `key`. We discard the leftmost 32 bits by casting the result to an `int`. Thus, the necessary computation is

```
(int)(key ^ (key >> 32))
```

We can perform a similar computation for a search key of type `double`. Since `key` is a real value, we cannot use it in the previous expression. Instead, we must get `key`'s bit pattern by calling `Double.doubleToLongBits(key)`. Thus, the following statements produce the desired hash code:

```
long bits = Double.doubleToLongBits(key);
int hashCode = (int)(bits ^ (bits >> 32));
```

Why not simply cast the search key from `double` to `int`? Since the search key is a real value, casting it to an `int` will simply give us the integral portion of the value. For example, if the key's value is 32.98, casting it to `int` results in the integer 32. While we could use 32 as the hash code, all search keys that have 32 as their integer portion also would have a hash code of 32. Unless you know that your real values have distinct integral portions, casting them to `int` values can cause many collisions.

The hash code of a search key of type `float` can be simply its 32 bits. You get these by calling `Float.floatToIntBits(key)`.

These computations of hash codes for the primitives types are actually used by the corresponding wrapper classes in their implementations of the method `hashCode`.

Compressing a Hash Code into an Index for the Hash Table

21.10 The most common way to scale an integer so that it lies within a given range of values is to use Java's `%` operator. For a positive hash code *c* and a positive integer *n*, *c* `%` *n* divides *c* by *n* and takes the remainder as the result. This remainder lies between 0 and *n* - 1. Thus, *c* `%` *n* is ideal for the index of a hash table that has *n* locations.

So, *n* should equal the size of the hash table, but not any *n* will do. For example, if *n* is even, *c* `%` *n* has the same **parity** as *c*—that is, if *c* is even, *c* `%` *n* is even; if *c* is odd, *c* `%` *n* is odd. If the hash codes are biased toward either even or odd values (and note that hash codes based on memory addresses are typically even), the indices to the hash table will have the same bias. Instead of a uniform distribution of indices, you will leave out the indices of many table locations if *n* is even. Thus, *n*—the size of the hash table—always should be an odd number.

When *n* is a **prime number**—one that is divisible only by 1 and itself—*c* `%` *n* provides values that are distributed throughout the index range 0 through *n* - 1. Prime numbers—with the exception of 2—are odd.

Note: The size of a hash table should be a prime number n greater than 2. Then, if you compress a positive hash code c into an index for the table by using c % n, the indices will be distributed uniformly between 0 and n - 1.

One final detail remains. You saw earlier that the method `hashCode` might return a negative integer, so you need to be a bit careful. If c is negative, c % n lies between 1 – n and 0. A zero result is fine, but if c % n is negative, add n to it so that it lies between 1 and n – 1.

21.11 We now can implement a hash function for the ADT dictionary. The following method computes the hash index for a given search key whose data type is the generic object type `K`. The data field `hashTable` is the array that serves as the hash table. Realize that `hashTable.length` is the size of the array, not the number of current entries in the hash table. We assume that this size is a prime number and that the method `hashCode` returns a hash code consistent with the previous discussion.

```
private int getHashIndex(K key)
{
   int hashIndex = key.hashCode() % hashTable.length;
   if (hashIndex < 0)
      hashIndex = hashIndex + hashTable.length;

   return hashIndex;
} // end getHashIndex
```

SELF-TEST

Question 2 Question 1 in Segment 21.8 asked you to compute the hash code for the string *Java*. Use that value to calculate what `getHashIndex("Java")` returns when the length of the hash table is 101.

Question 3 What one-character string, when passed to `getHashIndex`, will cause the method to return the same value as in the previous question?

Resolving Collisions

21.12 When adding to a dictionary, if your hash function maps a search key into a location in the hash table that is already in use, you need to find another spot for the search key's value. You have two fundamental choices:

- Use another location in the hash table
- Change the structure of the hash table so that each array location can represent more than one value

VideoNote
Resolving collisions

Finding an unused, or open, location in the hash table is called **open addressing**. This choice sounds simple, but it can lead to several complications. Changing the structure of the hash table is not as difficult as it might sound and can be a better choice for resolving collisions than using an open addressing scheme. We will examine both approaches, beginning with several variations of open addressing.

Open Addressing with Linear Probing

21.13 When a collision occurs during the addition of an entry to a hash table, an open addressing scheme locates an alternate location in the hash table that is available, or open. You then use this location to reference the new entry.

Locating an open location in the hash table is called **probing**, and various probing techniques are possible. With **linear probing**, if a collision occurs at `hashTable[k]`, we see whether `hashTable[k + 1]` is available. If not, we look at `hashTable[k + 2]`, and so on. The table locations that we consider in this search make up the **probe sequence**. If a probe sequence reaches the end of the hash table, it continues at the beginning of the table. Thus, we treat the hash table as if it were circular: The first location in the table comes immediately after the last location.

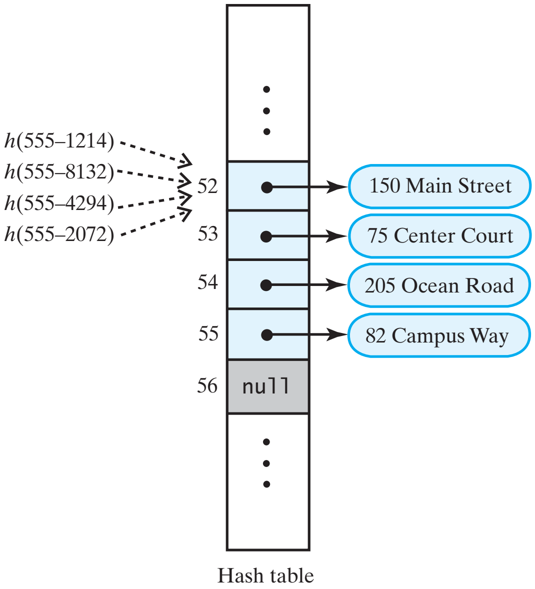
Note: **Linear probing** resolves a collision during hashing by examining consecutive locations in the hash table—beginning at the original hash index—to find the next available one.

21.14 **Additions that collide.** Recall the example illustrated in Figure 21-2. The search keys 555-1214 and 555-8132 both mapped into the index 52. Suppose that 555-4294 and 555-2072 also map into that same index, and we make the following additions to an empty dictionary `addressBook`:

```
addressBook.add("555-1214", "150 Main Street");
addressBook.add("555-8132", "75 Center Court");
addressBook.add("555-4294", "205 Ocean Road");
addressBook.add("555-2072", "82 Campus Way");
```

The first addition would use `hashTable[52]`. The second addition would find `hashTable[52]` occupied, and so it would probe ahead and use `hashTable[53]`. The third addition would find both `hashTable[52]` and `hashTable[53]` occupied, and so it would probe ahead and use `hashTable[54]`. Finally, the fourth addition would probe the locations at indices 52, 53, and 54 before using `hashTable[55]` for the addition. Figure 21-3 shows the result of these additions to the hash table.

FIGURE 21-3 The effect of linear probing after adding four entries whose search keys hash to the same index



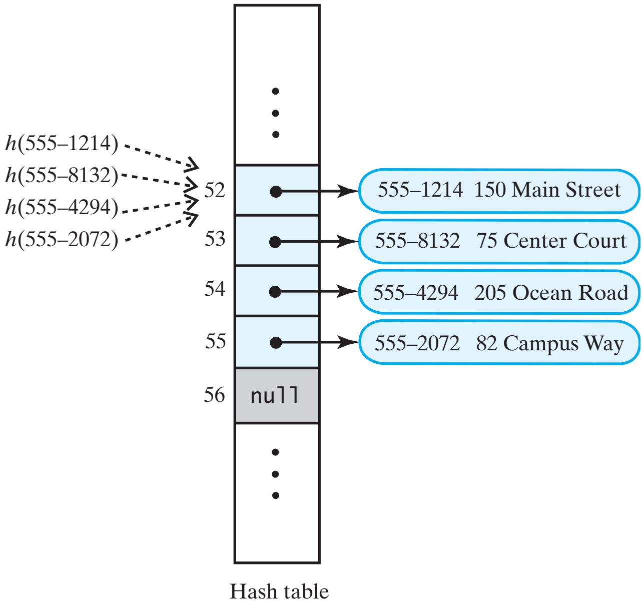
Note: Linear probing can examine every location in a hash table. As a result, this type of probing ensures the success of the `add` operation as long as the hash table is not full.

21.15 **Retrievals.** Now that we've used linear probing to resolve collisions while adding our four entries, how do we retrieve the street address associated with the last search key we added, 555-2072? That is, if the statement

```
String streetAddress = addressBook.getValue("555-2072");
```

is executed, what will `getValue` do? Since `getHashIndex("555-2072")` is 52, `getValue` will search consecutive locations in the array beginning at `hashTable[52]` until it finds the street address associated with the search key 555-2072. But wait! How can we tell which street address is the right one? We can't, unless we package a search key with its value. Segment 20.2 of Chapter 20 provided a class `Entry` that we could use for this purpose. Figure 21-4 shows the hash table given in Figure 21-3 after we make this revision.

FIGURE 21-4 A revision of the hash table shown in Figure 21-3 when linear probing resolves collisions; each entry contains a search key and its associated value



Now the search for 555-2072 can follow the same probe sequence that was used to add this search key and its value to the hash table. This fact will be useful later when we assess the efficiency of hashing.

Note: A successful search for an entry that corresponds to a given search key follows the same probe sequence used to add the entry to the hash table.

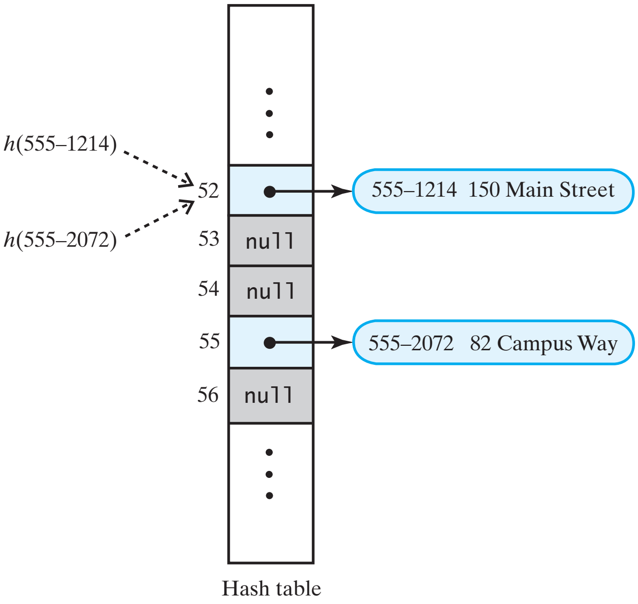
What happens if the search key is not in the hash table? The search of the probe sequence would encounter a `null` location, indicating an unsuccessful search. But before we can reach this conclusion, we need to know what the `remove` method does, because it has the potential to adversely affect subsequent retrievals.

21.16 **Removals.** Suppose that after the four additions illustrated in Figure 21-4, we removed two entries by executing the following code:

```
addressBook.remove("555-8132");
addressBook.remove("555-4294");
```

The simplest way to remove an entry from an array location is to place `null` in the location. Figure 21-5 shows the hash table after `remove` places `null` into `hashTable[53]` and `hashTable[54]`. But now an attempt to find the search key 555-2072 will terminate unsuccessfully at `hashTable[53]`. Although a location in the hash table that was never used should end a search, a location that had been used and is now available again for use should not.

FIGURE 21-5 A hash table if `remove` used `null` to remove entries



Thus, we need to distinguish among three kinds of locations in the hash table:

- Occupied—the location references an entry in the dictionary
- Empty—the location contains `null` and always has
- Available—the location's entry was removed from the dictionary

Accordingly, the method `remove` should not place `null` into the hash table, but instead should encode the location as available. The search during a retrieval should then continue if it encounters an available location and should stop only if it is successful or reaches a `null` location. A search during a removal behaves in the same way.

Question 4 Suggest ways to implement the three states of a location in a hash table. Should this state be a responsibility of the location or of the dictionary entry that it references?

21.17 **Reusing locations in the hash table during an addition.** Recall the hash table pictured in Figure 21-4. The entry whose search key is 555-2072 mapped into `hashTable[52]` but was added to the hash table at `hashTable[55]` due to collisions. Figure 21-6a shows this hash table again, but in a simpler form. The four occupied locations constitute a probe sequence; the other locations contain `null`. Since the search key 555-2072 maps into the first location of the probe sequence but actually occurs in the fourth location, a brief sequential search will find it.

Now let's try removing the middle two entries of the probe sequence, as Figure 21-6b shows. A search for 555-2072 starts at the beginning of the probe sequence, must continue beyond the removed entries, and stops successfully at the last location in the probe sequence. If 555-2072 does not occur in this last location, the search will end unsuccessfully at the next location, since it contains `null`. Figure 21-6c illustrates these searches.

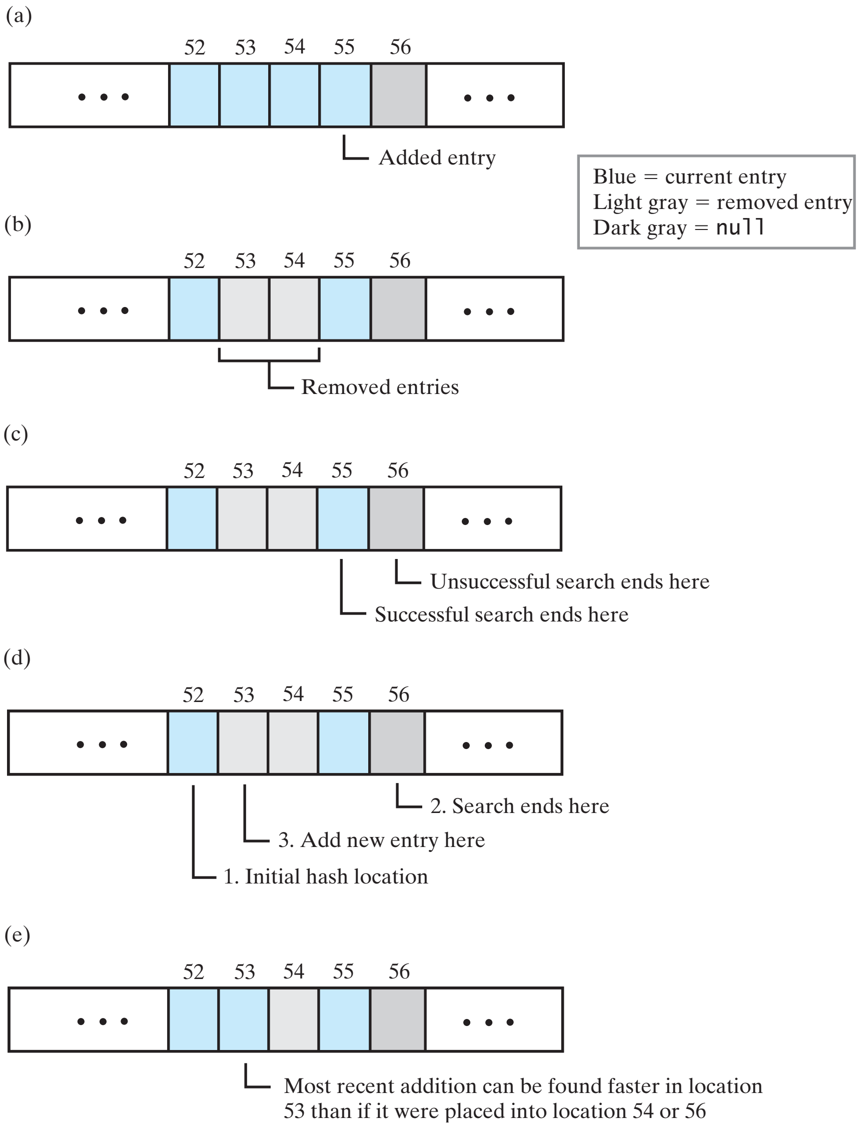
Finally, consider what happens when we add an entry that maps into this probe sequence. For example, the search key 555-1062 maps into `hashTable[52]`. The `add` operation first must see whether this search key is in the hash table already. To do so, it searches the probe sequence. It has to search the entire probe sequence and reach a `null` location to discover that 555-1062 is not in the table. Figure 21-6d shows that this search ends at `hashTable[56]`. Should `add` place the new entry in this location? It could, but that would fill the hash table faster than if `add` reused a location that is presently in the available state. Two such locations are at the indices 53 and 54. We should place the new entry at `hashTable[53]`—that is, closest to the beginning of the probe sequence—so we can find it more quickly later. Figure 21-6e illustrates the hash table after this addition.

Note: Searches that dictionary operations require when open addressing resolves collisions

- To retrieve an entry, `getValue(key)` searches the probe sequence for `key`. It examines entries that are present and ignores locations that are in the available state. The search stops when either `key` is found or `null` is reached.
- The operation `remove(key)` searches the probe sequence using the same logic as a retrieval. If it finds `key`, it marks the location as available.
- The operation `add(key, value)` searches the probe sequence using logic like that of a retrieval, but it also notes the index of the first location encountered that is either in the available state or contains `null`. The operation uses this location for a new entry if `key` is not found.

21.18 **Clustering.** Collisions that are resolved with linear probing cause groups of consecutive locations in the hash table to be occupied. Each group is called a **cluster**, and the phenomenon is known as **primary clustering**. Each cluster is actually a probe sequence that you must search when adding, removing, or retrieving a table entry. When few collisions occur, probe sequences remain short and can be searched rapidly. But during an addition, any collision within a cluster increases the size of the cluster. Bigger clusters mean longer search times following a collision. As the clusters grow in size, they can merge into even larger clusters, compounding the problem. This occurrence can place many entries in one part of the hash table while another part is relatively empty.

FIGURE 21-6 A linear probe sequence (a) after adding an entry; (b) after removing two entries; (c) after a search; (d) during the search while adding an entry; (e) after an addition to a formerly occupied location

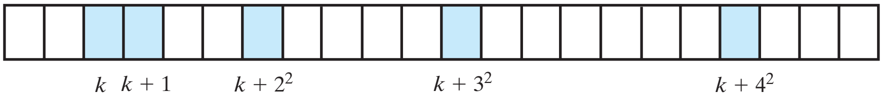


Note: Linear probing is apt to cause primary clustering. Each cluster is a group of consecutive and occupied locations in the hash table. During an addition, any collision with any location within a cluster causes the cluster to get larger.

Open Addressing with Quadratic Probing

21.19 You can avoid primary clustering by changing the probe sequence that you use to resolve a collision. As we discussed in the previous section, if a given search key hashes to index k, linear probing looks at the consecutive locations beginning at index k. **Quadratic probing**, on the other hand, considers the locations at indices $k + j^2$ for $j \geq 0$—that is, it uses the indices k, $k + 1$, $k + 4$, $k + 9$, and so on. As before, if the probe sequence reaches the end of the hash table, it wraps around to the beginning of the table. This open addressing scheme separates the entries in the probe sequence, after the first two. In fact, this separation increases as the sequence grows in length. Figure 21-7 highlights the locations in a hash table that form one such probe sequence of five entries.

FIGURE 21-7 A probe sequence of length five using quadratic probing



Except for the change in probe sequence, quadratic probing is like linear probing. It uses the three states that Segment 21.16 describes: occupied, empty, and available. Additionally, it reuses table locations in the available state, as described in Segment 21.17.

Although quadratic probing avoids primary clustering, entries that collide with an existing table entry use the same probe sequence, thereby increasing its length. This phenomenon—called **secondary clustering**—is usually not a serious problem, but it increases search times.

An advantage of linear probing is that it can reach every location in the hash table. As we mentioned earlier, this property is important since it guarantees the success of the `add` operation when the hash table is not full. Quadratic probing can also guarantee a successful `add` operation, as long as the hash table is at most half full and its size is a prime number. (See Exercise 8 at the end of this chapter.)

Quadratic probing requires more effort to compute the indices for the probe sequence than does linear probing. Exercise 2 at the end of this chapter shows how to compute these indices without multiplications or divisions.

Note: Quadratic probing

- Resolves a collision during hashing by examining locations in the hash table at the original hash index plus j^2, for $j \geq 0$
- Reaches half of the locations in the hash table if the size of the table is a prime number
- Avoids primary clustering but can lead to secondary clustering

Open Addressing with Double Hashing

21.20 Beginning at the original hash index k, both linear probing and quadratic probing add increments to k to define a probe sequence. These increments—1 for linear probing and j^2 for quadratic probing—are independent of the search key. **Double hashing** uses a second hash function to compute these increments in a key-dependent way. In this way, double hashing avoids both primary and secondary clustering.

Double hashing, like other open addressing schemes, should produce a probe sequence that reaches the entire table. Such will be the case if the size of the hash table is a prime number. (See Exercise 9 at the end of this chapter.) The second hash function must be different from the original hash function and must never have a zero value, since zero is not an appropriate increment.

21.21 **Example.** For example, consider the following pair of hash functions for a hash table whose size is 7:

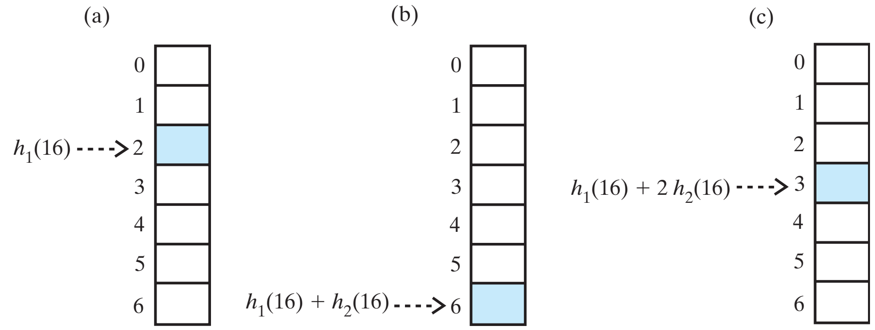
$h_1(key) = key \text{ modulo } 7$
$h_2(key) = 5 - key \text{ modulo } 5$

This hash table is unusually small, but it allows us to study the behavior of the probe sequence. For a search key of 16, we have

$h_1(16) = 2$
$h_2(16) = 4$

The probe sequence begins at 2 and probes locations at increments of 4, as Figure 21-8 illustrates. Remember that when probing reaches the end of the table, it continues at the table's beginning. The table locations in the probe sequence then have the following indices: 2, 6, 3, 0, 4, 1, 5, 2, …. This sequence reaches all locations in the table and then repeats itself. Notice that the table size, 7, is a prime number.

FIGURE 21-8 The first three locations in a probe sequence generated by double hashing for the search key 16



What happens if we change the size of the table to 6 and use the hash functions

$h_1(key) = key \text{ modulo } 6$
$h_2(key) = 5 - key \text{ modulo } 5$

For a search key of 16, we have

$h_1(16) = 4$
$h_2(16) = 4$

The probe sequence begins at 4 and probes locations at increments of 4. The sequence's indices are then 4, 2, 0, 4, 2, 0, …. The probe sequence does not reach all table locations before it begins to repeat. Notice that the table size, 6, is not prime.

Note: **Double hashing**

- Resolves a collision during hashing by examining locations in the hash table at the original hash index plus an increment defined by a second hash function. The second hash function should
  - ■ Differ from the first hash function
  - ■ Depend on the search key
  - ■ Have a nonzero value
- Reaches every location in the hash table, if the size of the table is a prime number
- Avoids both primary clustering and secondary clustering

Question 5 What size hash table should you use with double hashing when the hash functions are

$h_1(key) = key$ modulo 13
$h_2(key) = 7 - key$ modulo 7

Why?

Question 6 What probe sequence is defined by the hash functions given in the previous question when the search key is 16?

A Potential Problem with Open Addressing

21.22 The previous three open addressing schemes for collision resolution assume that each table location is in one of three states: occupied, empty, or available. Recall that only empty locations contain `null`. Frequent additions and removals can cause every location in the hash table to reference either a current entry or a former entry. That is, a hash table might have no location that contains `null`, regardless of how many or how few entries are actually in the dictionary. If this happens, our approach to searching a probe sequence will not work. Instead, every unsuccessful search can end only after considering every location in the hash table. Also, detecting the end of the search will be somewhat more involved and costly than simply looking for `null`.

You should safeguard your implementation against this failure. Increasing the size of the hash table (see Segment 22.7 in the next chapter) can correct the problem, if you act in time. Separate chaining—which we consider next—does not have this problem.

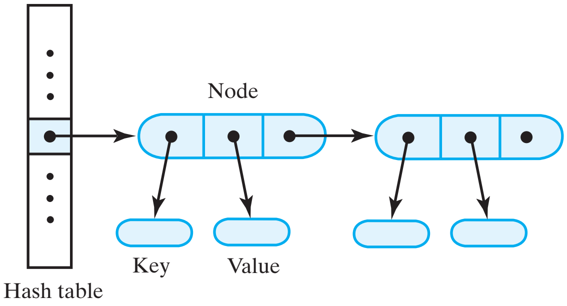
Note: Open addressing can be simplified when an application does not require removals. Such is the case, for example, when a compiler builds a symbol table. It can use a dictionary that does not permit removals. Locations in the hash table will either reference a dictionary entry or contain `null`. Defining the three states given in Segment 21.16 is not needed. Project 1 at the end of this chapter asks you to develop this implementation.

Separate Chaining

21.23 A second general approach to collision resolution alters the structure of the hash table so that each location can represent more than one value. Such a location is called a **bucket**. Anytime a new search key maps into a particular location, you simply place the key and its associated value in the bucket, much as we did with open addressing. To find a value, you hash the search key, locate the bucket, and look through the key-value pairs in it. In all likelihood, the bucket contains few values, so this mini-search will be fast. When you remove an entry, you find it in its bucket and delete it. Thus, the entry no longer exists in the hash table.

What can you use to represent a bucket? A list, a sorted list, a chain of linked nodes, an array, or a vector are some possibilities with which you are familiar. Anything that involves an array or vector will cause a substantial memory overhead, since each location in the hash table will have a fixed amount of memory allocated to it. Much of this memory will be unused. Either a linked implementation of a list or a chain of linked nodes is a reasonable choice for a bucket, since memory is allocated to the bucket only as needed. Figure 21-9 illustrates a hash table with linked chains as buckets. In this arrangement, each location in the hash table is a head reference to a chain of linked nodes that make up the bucket. Each node contains references to a search key, to the key's associated value, and to the next node in the chain. Notice that a node must reference the search key so that you can locate it later when you search the chain. Resolving collisions by using buckets that are linked chains is called **separate chaining**.

FIGURE 21-9 A hash table for use with separate chaining; each bucket is a chain of linked nodes



21.24 If your dictionary allows duplicate search keys, adding a new entry to the beginning of the appropriate chain is fastest, as Figure 21-10a indicates. However, if you want distinct search keys, adding a new entry requires you to search a chain for the search key. If you do not find it, you will be at the end of the chain, where you can add the new entry. Figure 21-10b illustrates this case. But since you have to search the chain anyway, you could maintain the chain in sorted order by search key, as Figure 21-10c shows. Subsequent searches would then be a little faster. As you will see, however, typical chains are short, so this refinement might not be worth the effort.

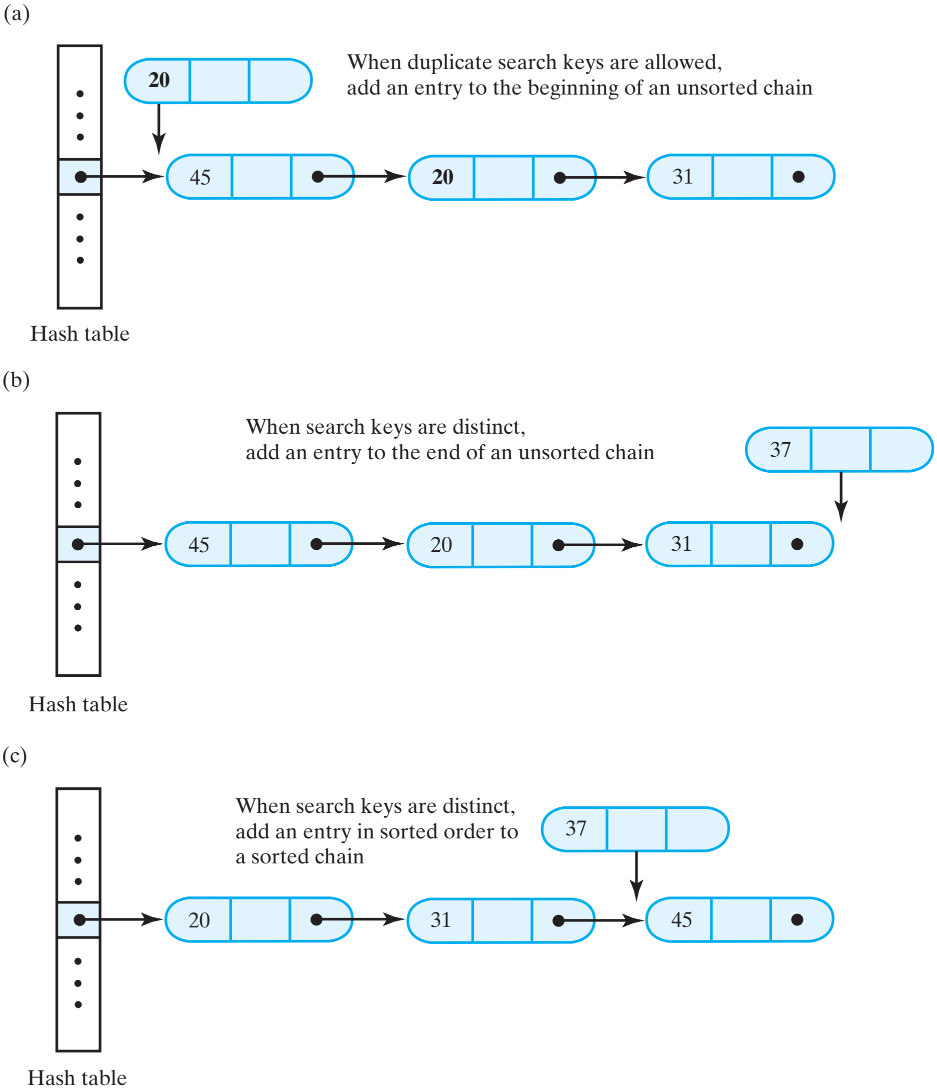
? SELF-TEST

Question 7 Consider search keys that are distinct integers. If the hash function is

$h(key) = key$ modulo 5

and separate chaining resolves collisions, where in the hash table do the following search keys appear after being added? 4, 6, 20, 14, 31, 29

FIGURE 21-10 Where to insert a new entry into a linked bucket when the integer search keys are (a) unsorted and possibly duplicate; (b) unsorted and distinct; (c) sorted and distinct


(a)
20
When duplicate search keys are allowed,
add an entry to the beginning of an unsorted chain
45
20
31
Hash table
(b)
When search keys are distinct,
add an entry to the end of an unsorted chain
37
45
20
31
Hash table
(c)
When search keys are distinct,
add an entry in sorted order to
a sorted chain
37
20
31
45
Hash table

21.25 With distinct search keys and unsorted chains, the algorithms for the dictionary's `add`, `remove`, and `getValue` methods are as follows:

```
Algorithm add(key, value)
index = getHashIndex(key)
if (hashTable[index] == null)
{
   hashTable[index] = new Node(key, value)
```

```
      numberOfEntries++
      return null
   }
   else
   {
      Search the chain that begins at hashTable[index] for a node that contains key
      if (key is found)
      { // assume currentNode references the node that contains key
         oldValue = currentNode.getValue()
         currentNode.setValue(value)
         return oldValue
      }
      else // add new node to end of chain
      { // assume nodeBefore references the last node
         newNode = new Node(key, value)
         nodeBefore.setNextNode(newNode)
         numberOfEntries++
         return null
      }

   }

   Algorithm remove(key)
   index = getHashIndex(key)
   Search the chain that begins at hashTable[index] for a node that contains key
   if (key is found)
   {
      Remove the node that contains key from the chain
      numberOfEntries--
      return value in removed node
   }
   else
      return null

   Algorithm getValue(key)
   index = getHashIndex(key)
   Search the chain that begins at hashTable[index] for a node that contains key
   if (key is found)
      return value in found node
   else
      return null
```

All three operations search a chain of nodes. Each chain should contain only a few entries, if the hash table is sufficiently large and the hash function distributes the entries uniformly throughout the table. Thus, these operations should be time efficient. For a dictionary of *n* entries, the operations certainly are faster than O(*n*). In the worst case, however, all entries are in one chain, so the efficiency degenerates to O(*n*). We will discuss the efficiency of hashing in more detail in the next chapter.

Note: **Separate chaining** provides an efficient and simple way to resolve collisions. Because the structure of the hash table is altered, however, separate chaining requires more memory than open addressing.

SELF-TEST

Question 8 With distinct search keys and separate chaining with sorted chains, write an algorithm for the dictionary's `add` method.

Question 9 Can you define an iteration of a dictionary's search keys in sorted order when you use hashing in its implementation? Explain.

Chapter Summary

- Hashing is a dictionary implementation that stores entries into an array called the hash table. A hash function transforms an entry's search key into the index of the array location that will contain the entry.
- All classes have a method `hashCode` that returns an integer hash code. If a class's instances are to be search keys, you should override `hashCode` to produce suitable hash codes. A hash code should depend on the entire search key.
- A hash function uses `hashCode` to compute a hash code from a search key and then compresses that hash code into an index to the hash table. A typical way to compress the hash code c is to compute c modulo n, where n is a prime number and the size of the hash table. This computation produces an index whose magnitude lies between 0 and $n - 1$.
- A perfect hash function maps each search key into a distinct location in the hash table. You can find such a function if you know all possible search keys. Using a perfect hash function makes possible O(1) implementations of the dictionary operations.
- With a typical hash function, more than one search key can map into the same location in the hash table. This occurrence is called a collision.
- Various methods are available to deal with collisions. Among them are open addressing and separate chaining.
- With open addressing, all entries that map into the same location are ultimately stored within the hash table. These entries are in a sequence of locations called the probe sequence. Several different versions of open addressing are common. Linear probing uses consecutive locations. Quadratic probing spaces the locations in a probe sequence at increasing increments. These increments are 1, 4, 9, and so on—that is, the squares of the integers 1, 2, 3, ... Double hashing uses a fixed increment that depends on the search key. A second hash function provides this increment.
- With open addressing, you remove an entry by placing it into a removed state. You do not set its table location to `null`, because that would terminate subsequent searches prematurely. You retrieve an entry by searching its probe sequence, ignoring removed entries, until you either find the desired entry or encounter `null`. You perform the same search when you add a new entry, but while searching, you note the first location—if any—that references a removed entry. You use this location for the added entry. If no such location exists, the addition extends the probe sequence by using the `null` location encountered after searching the entire sequence.
- A disadvantage of linear probing and quadratic probing is clustering. Clustering lengthens a probe sequence and so increases the time to search it. Double hashing avoids this problem.
- With separate chaining, the hash table is an array of buckets. All entries that map into the same array location are stored in the bucket that the location references. Each bucket can be a chain of linked nodes, for example. That is, each location in the hash table can reference the beginning of a chain.

- You can add new entries to a chain in sorted search-key order. Although sorted chains can improve search time somewhat, they usually are unnecessary, as typical chains are short. You add new entries to an unsorted chain either at the beginning, if duplicates are allowed, or at the end if not.
- With separate chaining, you retrieve or remove an entry by mapping its search key into a table location. You then search the bucket that the location references.
- An iteration of the entire hash table will not be in sorted order even if separate chaining with sorted buckets is used to resolve collisions.

Exercises

1. Define a hashCode method for the class Name, as given in Segment B.16 of Appendix B.

2. Quadratic probing uses the following indices to define a probe sequence:

   $(k + j^2)$ modulo n for $j \geq 0$

   where k is the hash index and n is the size of the hash table.

   a. If the hash table contains 17 locations and the hash index is 3, what are the first six indices of the array locations in the probe sequence that quadratic probing defines?

   b. You can compute the indices for the probe sequence more efficiently by using the recurrence relation

   $k_{i+1} = (k_i + 2i + 1)$ modulo n for $i \geq 0$ and $k_0 = k$

   Derive this recurrence relation.

   c. Demonstrate that you can replace the modulo operation in Part *b* with one comparison and an occasional subtraction.

3. Project 11 in Chapter 20 revised the class Entry, which is in Listing 20-1, to make it public, and then used it in an implementation of a dictionary. Consider a hash table of Entry objects. Without further changes to the definition of Entry, how could you indicate a removed entry?

4. Suppose that the size of your hash table is 31, that you use the hash code described in Segment 21.8, and that you use separate chaining to resolve collisions. List five different names that would hash to the same location in the table.

5. Assume the hash table and hash function described in Exercise 4, but use open addressing with linear probing to resolve collisions. List five different names that do not all hash to the same location in the table yet would nonetheless result in collisions and clustering.

6. Repeat Exercise 5, but instead use open addressing with quadratic probing to resolve collisions.

7. Give an example of a probe sequence produced by quadratic probing that does not reach the entire hash table, even if the size of the table is a prime number.

8. Demonstrate that quadratic probing will guarantee a successful addition, if the hash table is at most half full and its size is a prime number.

9. Demonstrate that double hashing will produce a probe sequence that reaches the entire table, if the size of the hash table is a prime number. *Hint*: Show that this is true if the increment and the table size are relatively prime. Then, if the table size is prime, all increments will be relatively prime to it.

10. Imagine that you alter the linear probing scheme of Segment 21.13 as follows. When a collision occurs at hashTable[k], you check hashTable[k + c], hashTable[k + 2 * c], hashTable[k + 3 * c], and so on, where c is a constant. Does this scheme eliminate primary clustering?

11. Consider data whose search key consists of three floating-point values (longitude, latitude, and altitude, for example). Suggest at least two possible hash functions for this data.

12. You have approximately 1000 thumbnail images that you want to store in a dictionary that uses hashing in its implementation. Each image is 20 pixels wide by 20 pixels high, and each pixel is one of 256 colors. Suggest some possible hash functions that you could use.

Projects

1. The note at the end of Segment 21.22 describes a dictionary that does not support a remove operation. Implement this dictionary by using open addressing with linear probing to resolve collisions.

2. Consider records for patients at a medical facility. Each record contains an integer identification for a patient and strings for the date, the reason for the visit, and the treatment prescribed. Design and implement the class `PatientRecord` so that it overrides the method `hashCode`. Write a main program that tests your new class.

3. Design a class `PatientDataBase` that stores instances of `PatientRecord`, as described in the previous project. The class should provide at least three query operations, as follows. Given a patient identification and date, the first operation should return the reason for the visit, and the second operation should return the treatment. The third query operation should return a list of dates, given a patient identification.

4. The following experiment compares the performance of linear probing and quadratic probing. You will need a list of 500 names or user names that can be obtained from your instructor or from a system administrator. Implement a hash table of size 1000, and use the hash code described in Segment 21.8. Count the number of collisions that occur for both linear probing and quadratic probing when 500 names are added to the table. Repeat the experiment for tables of size 950, 900, 850, 800, 750, 700, 650, and 600.

5. Design an experiment similar to the one in Project 4, but instead of comparing linear probing and quadratic probing, compare two different hash functions.

6. Write a program that uses hashing to guess which of two choices a user has made.[1] The following sample output demonstrates an interaction between computer and user:

```
Choose either A or B, and I will guess your choice.
Press Return when you are ready.
I guess that you chose A; am I right? no
Score: 0 correct, 1 incorrect

Choose either A or B, and I will guess your choice.
Press Return when you are ready.
I guess that you chose A; am I right? yes
Score: 1 correct, 1 incorrect

Choose either A or B, and I will guess your choice.
Press Return when you are ready.
I guess that you chose B; am I right? yes
Score: 2 correct, 1 incorrect

. . .
```

Initially, your program will make random guesses. After the user makes five choices, your program should begin to build a hash table that it can use to predict future choices. The last four user choices form a key to the hash table. The value stored in the table at the hash address represents how many times the user's next choice was A and how many times it was B. The program uses these counts to make its guess.

For example, if AAAB hashes to an object containing the counts 5 and 2, where 5 is the number of times the user has chosen A after having chosen A, A, A, and B, the program would predict A as the user's next choice. The specifics of how your program makes its prediction based on these counts are up to you.

[1] Based on an assignment idea by Raja Sooriamurthi, as given in SIGCSE's Nifty Assignments at `nifty.standford.edu`.

Answers to Self-Test Questions

1. 2301506. `"Java".hashCode()` has the same value.

2. 19

3. `"x"`

4. Since the implementation defines both the hash table and the dictionary entry, you have a choice as to where to add a field to indicate the state of a location in a hash table. You could add a field having three states to each table location, but you really need only a boolean field, since a `null` location is empty. If the field is true, the location is occupied; if it is false, it is available.

   Adding a similar data field to the dictionary entry instead of to the hash table leads to a cleaner implementation. As before, if the table location is `null`, it is empty. If the entry's field is true, the location is occupied; if it is false, it is available. Note that the implementation that appears in the next chapter uses this scheme.

5. 13. Since 13 is both prime and the modulo base in h_1, the probe sequence can reach all locations in the table before it repeats.

6. 3, 8, 0, 5, 10, 2, 7, 12, 4, 9, 1, 6, 11, 3, ...

7. `hashTable[0]` → 20
   `hashTable[1]` → 6 → 31
   `hashTable[2]` is `null`
   `hashTable[3]` is `null`
   `hashTable[4]` → 4 → 14 → 29

8. The `add` algorithm is the same as the one given in Segment 21.25, but it uses a different search of a chain. Regardless of whether the chain is sorted, you stop the search as soon as you find the desired search key. However, if the key is not in the chain, you have to search an entire unsorted chain to learn this. If the chain is sorted, you can stop the search when you reach the point where the key should have occurred if it were present.

9. No. Suppose that *a*, *b*, *c*, and *d* are search keys in sorted order in the hash table. With separate chaining, *b* and *d* might appear in one chain while the other keys appear in another. Traversing the chains in order will not visit the keys in sorted order. The same is true of open addressing when traversing the occupied array locations.

Chapter 22

Hashing as a Dictionary Implementation

Contents

Prerequisites

Chapter	4	The Efficiency of Algorithms
Chapter	13	List Implementations That Use Arrays
Chapter	14	A List Implementation That Links Data
Chapter	15	Iterators
Chapter	19	Dictionaries
Chapter	20	Dictionary Implementations
Chapter	21	Introducing Hashing

Objectives

After studying this chapter, you should be able to

- Describe the relative efficiencies of the various collision resolution techniques
- Describe a hash table’s load factor
- Describe rehashing and why it is necessary
- Use hashing to implement the ADT dictionary

The previous chapter described hashing as a technique for implementing a dictionary when searching is the primary task. We now study hashing's performance and examine the details of its implementation in Java.

The Efficiency of Hashing

22.1 As you saw in the previous chapter, implementations of the ADT dictionary depend on whether the dictionary requires distinct search keys. In this section, we consider only dictionaries with distinct search keys. Recall that the `add` method for such a dictionary must ensure that duplicate search keys do not occur.

VideoNote
Hashing efficiency

Each of the dictionary operations `getValue`, `remove`, and `add` searches the hash table for a given search key. The success or failure of a search for a given key directly affects the success or failure of the retrieval and removal operations. The successful addition of a new entry occurs after a search for a given key fails. An unsuccessful addition replaces the value of an existing entry instead of adding a new entry. This operation occurs after a successful search for a given key. Thus, we have the following observations about the time efficiency of these operations:

- A successful retrieval or removal has the same efficiency as a successful search
- An unsuccessful retrieval or removal has the same efficiency as an unsuccessful search
- A successful addition has the same efficiency as an unsuccessful search
- An unsuccessful addition has the same efficiency as a successful search

So it is sufficient to analyze the time efficiency of searching the hash table for a given search key.

Note: The successful retrieval of an entry searches the same chain or probe sequence that was searched when the entry was first added to the hash table. Thus, the cost of a successful retrieval of an entry is the same as the cost of inserting that entry.

The Load Factor

22.2 We began our discussion of hashing in the previous chapter with a perfect hash function that caused no collisions. If you can find a perfect hash function for your particular set of search keys, using it to implement the ADT dictionary will provide operations that are each O(1). Such an implementation is ideal. The good news is that finding a perfect hash function is quite feasible in certain situations. Unfortunately, using a perfect hash function is not always possible or practical. In those situations, collisions are likely to occur.

Resolving a collision takes time and thus causes the dictionary operations to be slower than an O(1) operation. As a hash table fills, collisions occur more often, decreasing performance even further. Since collision resolution takes considerably more time than evaluating the hash function, it is the prime contributor to the cost of hashing.

To help us express this cost, we define a measure of how full a hash table is. This measure—the **load factor** λ—is the ratio of the size of the dictionary to the size of the hash table. That is,

$$\lambda = \frac{\textit{Number of entries in the dictionary}}{\textit{Number of locations in the hash table}}$$

Notice that λ is zero when the dictionary—and hence the hash table—is empty. The maximum value of λ depends on the type of collision resolution you use. For open addressing schemes, λ's maximum value is 1 when the hash table is full. In that case, each entry in the dictionary uses one location in the hash table. Notice that the number of locations in the available state does not affect λ. For separate chaining, the number of entries in the dictionary can exceed the size of the hash table, so λ has no maximum value.

Note: The load factor

The load factor λ is a measure of the cost of collision resolution. It is the ratio of the number of entries in the dictionary to the size of the hash table. λ is never negative. For open addressing, λ does not exceed 1. For separate chaining, λ has no maximum value. As you will see, restricting the size of λ improves the performance of hashing.

SELF-TEST

Question 1 When λ is 0.5 with open addressing, how many locations in the hash table contain dictionary entries?

Question 2 With separate chaining, does λ indicate how many buckets in the hash table are not empty? Explain.

The Cost of Open Addressing

22.3 Recall that all open addressing schemes use one location in the hash table per entry in the dictionary. The dictionary operations `getValue`, `remove`, and `add` each require a search of the probe sequence indicated by both the search key and the collision resolution scheme in effect. Analyzing the efficiency of these searches is sufficient.

For each open addressing scheme that we considered earlier, we will state the number of comparisons necessary to locate a search key in the hash table. We will express these numbers in terms of the load factor λ. The derivations of these numbers are messy at best and in some cases difficult, so we omit them. Interpreting the results, however, is straightforward. Recall that for open addressing, λ ranges from 0, when the table is empty, to 1 when it is full.

22.4 **Linear probing.** When you use linear probing, more collisions will likely occur as the hash table fills. After a collision, you search a probe sequence that forms a cluster. If you add a new entry, the cluster grows in size. So you would expect the probe sequences to grow and, therefore, require longer search times. In fact, the average number of comparisons needed to search the probe sequence for a given search key is about

$$\frac{1}{2}\left\{1+\frac{1}{(1-\lambda)^2}\right\} \text{ for an unsuccessful search and}$$

$$\frac{1}{2}\left\{1+\frac{1}{(1-\lambda)}\right\} \text{ for a successful search}$$

After evaluating these expressions for a few values of λ, we get the results in Figure 22-1. As λ increases—that is, as the hash table fills—the number of comparisons for these searches increases. This result satisfies our initial intuition. For example, when the hash table is half full—that is, when λ is 0.5—an average unsuccessful search requires about 2.5 comparisons and an average successful search requires about 1.5 comparisons. As λ increases beyond 0.5, the number of comparisons for an unsuccessful search increases much more rapidly than for a successful search. Thus, performance degrades rapidly when the hash table is more than half full. Should this happen, you'd need to define a larger hash table, as we describe a bit later in this chapter in the section "Rehashing."

Note: The performance of hashing with linear probing degrades significantly as the load factor λ increases. To maintain reasonable efficiency, the hash table should be less than half full. That is, keep $\lambda < 0.5$.

FIGURE 22-1 The average number of comparisons required by a search of the hash table for given values of the load factor λ when using linear probing

λ	Unsuccessful Search	Successful Search
0.1	1.1	1.1
0.3	1.5	1.2
0.5	2.5	1.5
0.7	6.1	2.2
0.9	50.5	5.5

22.5 **Quadratic probing and double hashing.** Secondary clustering as a result of quadratic probing is not as serious as the primary clustering that occurs when you use linear probing. Here, the average number of comparisons needed to search the probe sequence for a given search key is about

$\frac{1}{(1-\lambda)}$ for an unsuccessful search and

$\frac{1}{\lambda}\log\left(\frac{1}{1-\lambda}\right)$ for a successful search

Figure 22-2 evaluates these expressions for the same values of λ that we used for linear probing. Notice that the number of comparisons for an unsuccessful search grows with λ more rapidly than for a successful search. Although, the degradation in performance as λ increases is not as severe as with linear probing, you still want $\lambda < 0.5$ to maintain efficiency.

Even though double hashing avoids the clustering of linear probing and quadratic probing, the estimate of its efficiency is the same as for quadratic probing.

Note: If you use quadratic probing or double hashing, the hash table should be less than half full. That is, λ should be less than 0.5.

FIGURE 22-2 The average number of comparisons required by a search of the hash table for given values of the load factor λ when using either quadratic probing or double hashing

λ	Unsuccessful Search	Successful Search
0.1	1.1	1.1
0.3	1.4	1.2
0.5	2.0	1.4
0.7	3.3	1.7
0.9	10.0	2.6

The Cost of Separate Chaining

22.6 With separate chaining as the collision resolution strategy, each entry in the hash table can reference a chain of linked nodes. The number of such chains, including empty ones, is then the size of the hash table. Thus, the load factor λ is the number of dictionary entries divided by the number of chains. That is, λ is the average number of dictionary entries per chain. Since this number is an average, we expect some chains to contain fewer than λ entries—or even none—and some to have more. We assume that the chains are not sorted and that the search keys in the dictionary are distinct.

The dictionary operations `getValue`, `remove`, and `add` each require a search of the chain indicated by the search key. As was the case for open addressing, analyzing the efficiency of these searches is sufficient. Again, we will state the number of comparisons necessary to locate a search key in the hash table in terms of the load factor λ.

An unsuccessful search of a hash table sometimes will encounter an empty chain, and so that operation is O(1) and would be the best case. But for the average case when the chains are not sorted, searching for an entry in the hash table without success examines λ nodes. In contrast, a successful search always inspects a chain that is not empty. In addition to seeing that the table location at the hash index is not `null`, an average successful search considers a chain of λ nodes and locates the desired entry after looking at $\lambda/2$ of them. Thus, the average number of comparisons during a search when separate chaining is used is about

λ	for an unsuccessful search
$1 + \lambda/2$	for a successful search

After evaluating these expressions for a few values of λ, we get the results in Figure 22-3. The number of comparisons for these searches increases only slightly as λ increases—that is, as the hash table fills. A typical upper bound for λ is 1, as smaller values do not provide significantly better performance. Notice the unusual result: Successful searches take more time than unsuccessful searches when $\lambda < 2$.

Remember that these results are for the average case. In the worst case, all search keys map into the same table location. Thus, all entries occur in the same chain of nodes. The worst-case search time, then, is O(n), where n is the number of entries.

Note: The average performance of hashing with separate chaining does not degrade significantly as the load factor λ increases. To maintain reasonable efficiency, you should keep $\lambda < 1$.

FIGURE 22-3 The average number of comparisons required by a search of the hash table for given values of the load factor λ when using separate chaining

λ	Unsuccessful Search	Successful Search
0.1	0.1	1.1
0.3	0.3	1.2
0.5	0.5	1.3
0.7	0.7	1.4
0.9	0.9	1.5
1.1	1.1	1.6
1.3	1.3	1.7
1.5	1.5	1.8
1.7	1.7	1.9
1.9	1.9	2.0
2.0	2.0	2.0

Note: Maintaining the performance of hashing

Collisions and their resolution typically cause the load factor λ to increase and the efficiency of the dictionary operations to decrease. To maintain efficiency, you should restrict the size of λ as follows:

$\lambda < 0.5$ for open addressing
$\lambda < 1.0$ for separate chaining

Should the load factor exceed these bounds, you must increase the size of the hash table, as the next section describes.

Rehashing

22.7 The previous section discussed the efficiency of hashing as a dictionary implementation when using various ways of resolving collisions. As you saw, to ensure an efficient implementation, you must not let the load factor λ get too large. You can readily compute λ and see whether it exceeds the upper limit for the particular collision resolution scheme, as given in the previous note.

So what do you do when λ reaches its limit? First, you can resize the array that serves as the hash table, as described in Chapter 2. Typically, you double the size of an ordinary array, but here you need to ensure that the array's size is a prime number. Expanding the array's size to a prime number that is at least twice its previous size is not too difficult.

Ordinarily, when you expand an array, the next step is to copy the contents of the original array into corresponding locations of the new array. This is not the case for a hash table, however. Since you have changed the size *n* of the hash table, the compression function *c* % *n* will compute different indices than it did for the original hash table. For example, if the hash table originally contained 101 locations, the function *c* % 101 compresses the hash code 505 to the index 0. The new hash table will contain 211 locations, since 211 is the smallest prime number greater than 2 times 101. But now *c* % 211 compresses 505 to the index 83. You cannot simply copy the location at index 0 from the original table to the location at index 0 in the new table. And you cannot copy it to the location at index 83 in the new table, because you also need to consider collisions.

After creating a new, larger hash table of an appropriate size, you use the dictionary method `add` to add each item in the original hash table to the new table. The method computes the hash index using the size of the new table and handles any collisions. This process of enlarging a hash table and computing new hash indices for its contents is called **rehashing**. You can see that increasing the size of a hash table requires considerably more work than increasing the size of an ordinary array. Rehashing is a task that you should not do often.

Note: Rehashing

When the load factor λ becomes too large, resize the hash table. To compute the table's new size, first double its present size and then increase the result to the next prime number. Use the method `add` to add the current entries in the dictionary to the new hash table.

SELF-TEST

Question 3 Consider a hash table of size 5. The function *c* `%` 5 places the hash codes 20, 6, 18, and 14 into locations at indices 0, 1, 3, and 4, respectively. Show the effects of rehashing on this hash table when linear probing resolves collisions.

Note: **Dynamic hashing** allows a hash table to grow or shrink in size without the expense of rehashing. This technique, which we will not cover, is particularly useful in database management environments when the database is stored in external files.

Comparing Schemes for Collision Resolution

22.8 In previous segments, you saw how the load factor λ affects the average number of comparisons required by a search of a hash table for various ways to resolve collisions. The graphs in Figure 22-4 illustrate this effect for various collision resolution schemes. When λ is less than 0.5, the average number of comparisons for a successful search is about the same regardless of the process used to resolve collisions. For unsuccessful searches, the three open addressing schemes have about the same efficiency when λ is less than 0.5. However, separate chaining is somewhat more efficient in this case.

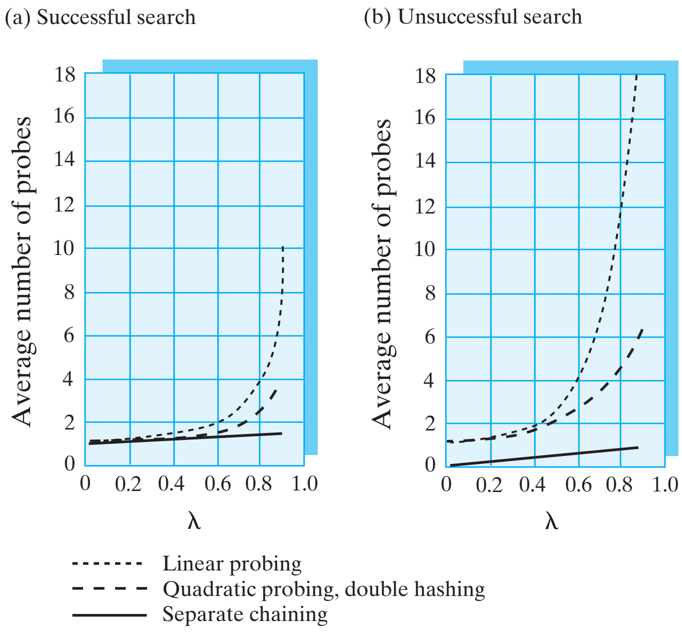
As λ exceeds 0.5, the efficiency of open addressing degrades rapidly, with linear probing the least efficient. Separate chaining, on the other hand, remains efficient for values of λ up to 1. In fact, the tabulated data in Figure 22-3 shows that its efficiency degrades only slightly for λ between 1 and 2.

Separate chaining certainly appears to be the fastest approach. But separate chaining can require more memory than open addressing, since each location in the hash table can reference a chain of linked nodes. On the other hand, the hash table itself can be smaller than when you use an open addressing scheme, since λ can be larger. Thus, space need not be a deciding factor in how you resolve collisions.

If all of these collision resolution schemes used hash tables of equal size, open addressing would be more likely to lead to rehashing than separate chaining would. To reduce the likelihood of rehashing, an open addressing strategy could use a large hash table. Remember that in Java, the table contains references that do not require much memory. So even though at least half of the table must remain unused, the actual space allocation would not be excessive.

Among open addressing schemes, double hashing is a good choice. It uses fewer comparisons than linear probing. Additionally, its probe sequence can reach the entire table, whereas quadratic probing cannot.

FIGURE 22-4 The average number of comparisons required by a search of the hash table versus the load factor λ for four collision resolution techniques when the search is (a) successful; (b) unsuccessful



A Dictionary Implementation That Uses Hashing

VideoNote
Implementing a dictionary

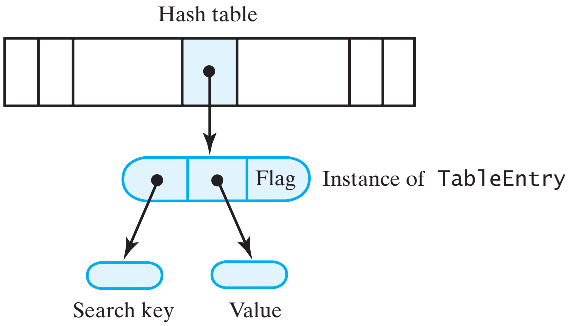
The efficiency of separate chaining makes it a desirable method for resolving collisions that occur during hashing. Because its implementation is relatively straightforward, we leave it to you to implement. Instead, we will implement the linear probing method of open addressing. Most of this dictionary implementation is independent of the particular open addressing technique that you use. Adapting it to use quadratic probing or double hashing involves few changes.

Entries in the Hash Table

22.9 Our hash table will be like the array in Figure 20-1a of Chapter 20 that we used to implement the dictionary. Each array location can reference an object that contains a search key and an associated value. The class `TableEntry` of these objects is similar to the class `Entry` that you saw in Segment 20.2.

However, with open addressing, each location in the hash table is in one of three states: occupied, empty, or available. (See Segment 21.16 of the previous chapter.) An empty location contains `null`. Rather than altering the structure of the hash table to indicate the other states, we make the entry objects indicate whether they are currently in the table or have been removed from it. Hence, we add another data field to the class of entry objects. This field is a boolean flag; it is true if the entry is in the dictionary or false if it has been removed. Figure 22-5 illustrates the hash table and one dictionary entry.

FIGURE 22-5 A hash table and one of its entry objects



Thus, we create the private class `TableEntry` and make it internal to the dictionary class. `TableEntry` begins as follows:

```
private class TableEntry<S, T>
{
   private S key;
   private T value;
   private boolean inTable; // true if entry is in hash table

   private TableEntry(S searchKey, T dataValue)
   {
      key = searchKey;
      value = dataValue;
      inTable = true;
   } // end constructor
   . . .
```

In addition to the methods `getKey`, `getValue`, and `setValue`, this class has the methods `isIn`, `isRemoved`, `setToIn`, and `setToRemoved` to either interrogate or set the value of the boolean flag `inTable`.

Note: A location in the hash table in the available state contains an entry in the removed state.

Data Fields and Constructors

22.10 If you do not use a perfect hash function, you must expect collisions. All open addressing methods for resolving collisions become less efficient as the hash table fills, so you need to increase the size of the table. As Segment 21.22 mentioned, increasing the size of the table can also ensure that it will contain a `null` entry—a necessity for ending the search of a probe sequence. Since our hash table is an array, we expand it and rehash the dictionary entries, as described in Segment 22.7. However, we modify the definition of the load factor λ by replacing the number of dictionary entries with the number of table locations in either the occupied or available state. This change increases λ so that rehashing occurs before the table loses its last `null` entry. Thus, the class begins as shown in Listing 22-1.

LISTING 22-1 An outline of the class `HashedDictionary`

```
import java.util.Iterator;
import java.util.NoSuchElementException;

/**
    A class that implements a dictionary by using hashing.
    @author Frank M. Carrano
*/
public class HashedDictionary<K, V>
             implements DictionaryInterface<K, V>
{
   private TableEntry<K, V>[] hashTable; // dictionary entries
   private int numberOfEntries;
   private int locationsUsed; // number of table locations not null
   private static final int DEFAULT_SIZE = 101;       // must be prime
   private static final double MAX_LOAD_FACTOR = 0.5; // fraction of
                                        // hash table that can be filled

   public HashedDictionary()
   {
      this(DEFAULT_SIZE); // call next constructor
   } // end default constructor

   public HashedDictionary(int tableSize)
   {
      int primeSize = getNextPrime(tableSize);

      hashTable = new TableEntry[primeSize];
      numberOfEntries = 0;
      locationsUsed = 0;
   } // end constructor

   < Implementations of methods in DictionaryInterface >
   . . .

   < Implementations of private methods >
   . . .

   private class TableEntry<S, T>
   {
      < See Segment 22.9 >
   } // end TableEntry
} // end HashedDictionary
```

The field `locationsUsed` counts the number of locations in the hash table that are in either the occupied or available state and is incremented each time an entry is added to the dictionary.

In contrast, the field numberOfEntries counts the number of entries currently in the dictionary. Thus, it is incremented when an entry is added to the dictionary but decremented when an entry is removed.

Each constructor allocates an array for the hash table. The second constructor lets the client specify a minimum size for the hash table. To ensure that the table's size is prime and at least as big as the client wants, this constructor calls the private method getNextPrime to find the first prime number that is greater than or equal to a given integer. The default constructor invokes the second constructor, giving it a predetermined size.

Note: To implement getNextPrime(anInteger), first see whether anInteger is even. If it is, it cannot be prime, so add 1 to make it odd. Then use a private method isPrime to find the first prime number among the parameter anInteger and subsequent odd integers.

To implement isPrime, note that 2 and 3 are prime but 1 and even integers are not. An odd integer 5 or greater is prime if it is not divisible by every odd integer up to its square root.

The Methods getValue, remove, and add

We now consider next the major operations of the dictionary: getValue, remove, and add. The note at the end of Segment 21.17 in the previous chapter summarized just what these operations need to do.

22.11 **The method getValue.** We begin with an algorithm for the retrieval method getValue:

```
Algorithm getValue(key)
// Returns the value associated with the given search key, if it is in the dictionary.
// Otherwise, returns null.

index = getHashIndex(key)
Search the probe sequence that begins at hashTable[index] for key
if (key is found)
   return value in found entry
else
   return null
```

In addition to the private method getHashIndex, which you saw in Segment 21.11, this algorithm suggests another private method that searches the probe sequence. We name this method locate and specify it informally as follows:

locate(index, key)
Follows the probe sequence that begins at index (key's hash index) and returns either the index of the entry containing key or -1, if no such entry exists.

We'll implement locate in Segment 22.13.

The method getValue then has the following implementation:

```
public V getValue(K key)
{
   V result = null;

   int index = getHashIndex(key);
   index = locate(index, key);

   if (index != -1)
      result = hashTable[index].getValue(); // key found; get value
   // else key not found; return null

   return result;
} // end getValue
```

22.12 **The method `remove`.** Removing an entry from the hash table, like retrieving an entry, involves locating the search key. If found, the entry is flagged as removed. The following pseudocode describes the necessary steps for this operation:

```
Algorithm remove(key)
// Removes a specific entry from the dictionary, given its search key.
// Returns either the value that was associated with the search key or null if no such object
// exists.

index = getHashIndex(key)
Search the probe sequence that begins at hashTable[index] for key
if (key is found)
{
   Flag entry as removed
   numberOfEntries--
   return value in removed entry
}
else
   return null
```

As the following implementation shows, you call the private method `locate` to locate the desired entry. If you find it, you change its state to removed and return its value. Otherwise, you return `null`.

```
public V remove(K key)
{
   V removedValue = null;

   int index = getHashIndex(key);
   index = locate(index, key);

   if (index != -1)
   { // key found; flag entry as removed and return its value
      removedValue = hashTable[index].getValue();
      hashTable[index].setToRemoved();
      numberOfEntries--;
   } // end if

   // else key not found; return null

   return removedValue;
} // end remove
```

22.13 **The private method `locate`.** Before we look at `add`, let's implement the method `locate` that both `getValue` and `remove` invoke. The method looks for the given search key along the probe sequence that begins at `hashTable[index]`, where `index` is the key's hash index. Recall that the search must ignore entries that are in the removed state. The search continues until it locates either `key` or `null`.

To follow the probe sequence, `locate` must implement a particular open addressing scheme to resolve collisions. For simplicity, we will implement linear probing. The following algorithm summarizes our approach:

```
Algorithm locate(index, key)
// Returns either the index of the entry containing key or -1 if no such entry is found.

while (key is not found and hashTable[index] is not null)
{
   if (hashTable[index] references an entry that is in the dictionary and contains key)
      Exit loop
   else
      index = next probe index
}

if (key is found)
   return index
else
   return -1
```

The implementation of `locate` now follows from this pseudocode:

```
private int locate(int index, K key)
{
   boolean found = false;

   while ( !found && (hashTable[index] != null) )
   {
      if ( hashTable[index].isIn() &&
           key.equals(hashTable[index].getKey()) )
         found = true; // key found
      else // follow probe sequence
         index = (index + 1) % hashTable.length; // linear probing
   } // end while
   // Assertion: either key or null is found at hashTable[index]

   int result = -1;
   if (found)
      result = index;

   return result;
} // end locate
```

You can change from linear probing to another open addressing scheme for collision resolution by replacing the highlighted assignment statement in the previous method.

22.14 **The method add.** We begin with the algorithm for adding a new entry:

```
Algorithm add(key, value)
// Adds a new key-value entry to the dictionary. If key is already in the dictionary,
// returns its corresponding value and replaces it in the dictionary with value.

if (hash table is too full)
   rehash()

index = getHashIndex(key)
Check for collision and resolve it (this step can alter index)
if (key is not found)
{ // add entry to hash table
   hashTable[index] = new TableEntry(key, value)
   numberOfEntries++
   locationsUsed++
   return null
}
else // search key is in table; return and replace entry's value
{
   oldValue = hashTable[index].getValue()
   hashTable[index].setValue(value)
   return oldValue
}
```

This algorithm suggests that you write several more private methods. We specify them informally as follows:

isHashTableTooFull()
Returns true if the hash table's load factor is greater than or equal to MAX_LOAD_FACTOR. Here we define the load factor as the ratio of locationsUsed to hashTable.length.

rehash()
Expands the hash table to a size that is both prime and at least double its current size, and then adds the current entries in the dictionary to the new hash table.

probe(index, key)
Detects whether key collides with hashTable[index] and resolves it by following a probe sequence. Returns the index of either an available location along the probe sequence or the entry containing key. This index is always legal, since the probe sequence stays within the hash table.

Using these private methods, we implement the method add as follows:

```
public V add(K key, V value)
{
   V oldValue; // value to return

   if (isHashTableTooFull())
      rehash();

   int index = getHashIndex(key);
   index = probe(index, key); // check for and resolve collision

   // Assertion: index is within legal range for hashTable
   assert (index >= 0) && (index < hashTable.length);

   if ( (hashTable[index] == null) || hashTable[index].isRemoved())
   { // key not found, so insert new entry
      hashTable[index] = new TableEntry<K,V>(key, value);
      numberOfEntries++;
      locationsUsed++;
      oldValue = null;
   }
   else
   { // key found; get old value for return and then replace it
      oldValue = hashTable[index].getValue();
      hashTable[index].setValue(value);
   } // end if

   return oldValue;
} // end add
```

22.15 **The private method probe.** The method probe(key, index) is similar to the method locate in that it looks for key along the probe sequence that begins at hashTable[index]. The search ignores entries that are in the removed state and continues until it locates either key or null. During this search, the method records the index of the first location, if any, that references an entry that has been removed from the table. This additional task is what distinguishes probe from locate. Thus, probe returns the index of a table location that either references an entry containing key or is available for an addition to the table.

Notice that probe returns the index of the removed entry that it *first* encounters along the probe sequence. Since add will insert a new entry into this location, a subsequent search for this entry will encounter it sooner than if add had inserted it in a location further along the probe sequence.

The following pseudocode summarizes the logic of `probe`:

```
Algorithm probe(index, key)
// Searches the probe sequence that begins at index. Returns either the index of the entry
// containing key or the index of an available location in the hash table.

while (key is not found and hashTable[index] is not null)
{
   if (hashTable[index] references an entry in the dictionary)
   {
      if (the entry in hashTable[index] contains key)
         Exit loop
      else
         index = next probe index
   }
   else // hashTable[index] references a removed entry
   {
      if (this is the first removed entry encountered)
         removedStateIndex = index

      index = next probe index
   }
}
if (key is found or a removed entry was not encountered)
   return index
else
   return removedStateIndex // index of first entry removed
```

The following method implements this algorithm:

```java
private int probe(int index, K key)
{
   boolean found = false;
   int removedStateIndex = -1; // index of first location in
                               // removed state
   while ( !found && (hashTable[index] != null) )
   {
      if (hashTable[index].isIn())
      {
         if (key.equals(hashTable[index].getKey()))
            found = true; // key found
         else // follow probe sequence
            index = (index + 1) % hashTable.length; // linear probing
      }
      else // skip entries that were removed
      {
         // save index of first location in removed state
         if (removedStateIndex == -1)
            removedStateIndex = index;

         index = (index + 1) % hashTable.length; // linear probing
      } // end if
   } // end while
   // Assertion: either key or null is found at hashTable[index]

   if (found || (removedStateIndex == -1) )
      return index;             // index of either key or null
   else
      return removedStateIndex; // index of an available location
} // end probe
```

The methods probe and locate are so similar that you can omit locate and use probe instead. To do so, you must change the implementations of remove and getValue slightly. The following question asks you to make this change.

Question 4 What changes to the methods remove and getValue are necessary so they can call probe instead of locate?

22.16 **The private method rehash.** Recall that the method rehash expands the hash table to a size that is both prime and at least double its current size. Since the hash function depends on the size of the table, you cannot copy entries from the old array and put them into the same positions in the new array. You need to apply the revised hash function to each entry to determine its proper position in the new table. But doing so can lead to collisions that need to be resolved. Thus, you should use the method add to add the existing entries to the new and larger hash table. Since add increments the data field numberOfEntries, you must remember to set this field to zero before adding the entries.

The method has the following implementation:

```java
private void rehash()
{
   TableEntry<K, V>[] oldTable = hashTable;
   int oldSize = hashTable.length;
   int newSize = getNextPrime(oldSize + oldSize);
   hashTable = new TableEntry[newSize]; // increase size of array

   numberOfEntries = 0; // reset number of dictionary entries, since
                        // it will be incremented by add during rehash
   locationsUsed = 0;

   // rehash dictionary entries from old array to the new and bigger
   // array; skip both null locations and removed entries
   for (int index = 0; index < oldSize; index++)
   {
      if ( (oldTable[index] != null) && oldTable[index].isIn() )
         add(oldTable[index].getKey(), oldTable[index].getValue());
   } // end for
} // end rehash
```

As we traverse the old hash table, notice that we skip both the null locations and the entries that have been removed from the dictionary but are still in the hash table.

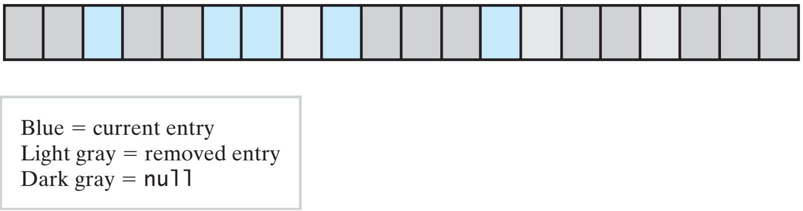
This method does not retain the instances of TableEntry that were in the old hash table. Instead, it uses an entry's key and value to create a new entry. You can avoid this reallocation of entries; Exercise 4 at the end of this chapter asks you to investigate this possibility.

Question 5 When the method add calls rehash, rehash calls add. But when rehash calls add, does add call rehash? Explain.

Iterators

22.17 Finally, we provide iterators for the dictionary, much as we did in Chapter 20. For example, we can implement an internal class KeyIterator to define an iteration of the search keys. The iteration must traverse the hash table, ignoring cells that either contain null or reference removed entries. Figure 22-6 shows a sample hash table. Cells in blue reference the dictionary entries, light gray cells reference removed entries, and dark gray cells contain null. As we traverse this table, we skip cells that are gray. The only real concern in this implementation is detecting when the iteration ends—that is, when the method `hasNext` should return false. The occurrence of a gray cell and the size of the hash table are not the proper criteria for this determination. Instead, you simply count backward from `currentSize` each time the method `next` returns the next search key.

FIGURE 22-6 A hash table containing dictionary entries, removed entries, and `null` values



The implementation of `KeyIterator` follows. A class that defines an iteration of values would have a similar implementation.

```
private class KeyIterator implements Iterator<K>
{
   private int currentIndex; // current position in hash table
   private int numberLeft;   // number of entries left in iteration

   private KeyIterator()
   {
      currentIndex = 0;
      numberLeft = numberOfEntries;
   } // end default constructor

   public boolean hasNext()
   {
      return numberLeft > 0;
   } // end hasNext

   public K next()
   {
      K result = null;

      if (hasNext())
      {
        // find index of next entry
        while ( (hashTable[currentIndex] == null) ||
                 hashTable[currentIndex].isRemoved() )
        {
           currentIndex++;
        } // end while

        result = hashTable[currentIndex].getKey();
        numberLeft--;
        currentIndex++;
      }
      else
         throw new NoSuchElementException();

      return result;
   } // end next
```

```
      public void remove()
      {
         throw new UnsupportedOperationException();
      } // end remove
   } // end KeyIterator
```

Note: Hashing as an implementation of the ADT dictionary does not provide the ability to sort its entries. Such an implementation is not suitable for any application that requires a sorted iteration of the entries.

Java Class Library: The Class `HashMap`

22.18 The standard package `java.util` contains the class `HashMap<K, V>`. This class implements the interface `java.util.Map` that we mentioned in Segment 19.22. Recall that this interface is similar to our `DictionaryInterface`. `HashMap` assumes that the search-key objects belong to a class that overrides the methods `hashCode` and `equals`.

The hash table is a collection of buckets, where each bucket can contain several entries. As you know, a hash table's load factor λ is a measure of how full the table is. The constructors for `HashMap` enable you to specify the initial number of buckets and the maximum load factor λ_{max}. These constructors are as follows:

`public HashMap()`
Creates an empty hash table with a default initial size of 16 and a default maximum load factor of 0.75.

`public HashMap(int initialSize)`
Creates an empty hash table with a given initial size and a default maximum load factor of 0.75.

`public HashMap(int initialSize, float maxLoadFactor)`
Creates an empty hash table with a given initial size and a given maximum load factor.

`public HashMap(Map<? extends K,? extends V> table)`
Creates a hash table with the same entries as `table`.

The authors of `HashMap` chose a default maximum load factor of 0.75 to provide a balance between time and memory requirements. Even though higher values of the load factor permit smaller hash tables, they cause higher search times, which in turn reduce the efficiency of the `get`, `put`, and `remove` methods.

When the number of entries in the hash table exceeds λ_{max} times the number of buckets, the size of the hash table is increased by using rehashing. But rehashing takes time. You can avoid rehashing if you choose

$$\textit{Number of buckets} > \frac{\textit{Maximum number of entries in the dictionary}}{\lambda_{max}}$$

Of course, too large a hash table wastes space.

Java Class Library: The Class `HashSet`

22.19 The package `java.util` of the Java Class Library also contains the class `HashSet<T>`. This class implements the interface `java.util.Set` that we presented in Segment 1.21 of Chapter 1. Recall that a set is a collection that does not contain duplicate entries, but otherwise is similar to a bag. `HashSet` uses an instance of the class `HashMap`, as introduced in the previous segment, to contain the entries in a set.

Among the constructors defined in `HashSet` are the following:

`public HashSet()`
Creates an empty set having a default initial capacity of 16. The underlying instance of `HashMap` uses a load factor of 0.75.

`public HashSet(int initialCapacity)`
Creates an empty set having the given initial capacity. The underlying instance of `HashMap` uses a load factor of 0.75.

`public HashSet(int initialCapacity, float loadFactor)`
Creates an empty set having the given initial capacity. The underlying instance of `HashMap` uses the specified load factor.

Chapter Summary

- Hashing is efficient as long as the ratio of dictionary size to hash-table size remains small. This ratio is called the load factor. The load factor should be less than 1 for separate chaining and less than 0.5 for open addressing. If the load factor exceeds these bounds, you must rehash the table.
- Rehashing is the process that increases the size of a hash table to a prime number that is greater than twice the table's current size. Since the hash function depends on the table size, you cannot simply copy entries from the old table to the new one. Instead, you use the method `add` to add all current entries to the new table.
- Separate chaining, as compared to open addressing, provides faster dictionary operations on average, can use a smaller hash table, and needs rehashing less frequently. If both approaches have the same size array for a hash table, separate chaining uses more memory due to its linked chains.
- Hashing as a dictionary implementation does not support operations that involve sorted search keys. For example, you cannot easily traverse the keys in sorted order, find keys that lie within a given range, or identify the largest or smallest search key.
- The package `java.util` contains both the class `HashMap<K, V>`, which implements the interface `Map<K, V>`, and the class `HashSet<T>`, which implements the interface `Set<T>`.

Exercises

1. Suppose that you use open addressing to resolve collisions. Now imagine that your hash table is getting full. To avoid the bad performance that results from a nearly full hash table, you should create a new, larger hash table.

   a. What steps should you take to move all of your entries to this new table?
   b. What happens to the hash function?

2. To guarantee that the average number of probes is less than or equal to 4, what is the maximum load factor that a hash table can have if it uses

   a. Linear probing
   b. Double hashing
   c. Separate chaining

3. Revise the method `add` given in Segment 22.14 when duplicate search keys are allowed in the dictionary.

4. The method `rehash` does not retain the instances of `TableEntry` that were in the old hash table. It could if the method `add` had an entry as its parameter instead of the search key and value. Write such a method as an additional but private `add` method, and then revise `rehash` so it retains the instances of `TableEntry` that were in the old hash table.

5. Imagine a collection of names that are instances of the class `Name`, as modified in Exercise 1 of Chapter 21. For each name, imagine a string that represents a nickname. Suppose that each nickname is a search key, and you plan to add nickname-name pairs to a dictionary that is an instance of the class `HashMap`, as described in Segment 22.18.

   a. Suppose that you plan to add 1000 entries to this dictionary. Create an instance of the class `HashMap` that can accommodate the 1000 entries without rehashing.

   b. Write statements that add four nickname-name pairs to your dictionary. Then write statements that retrieve and display the name that corresponds to a nickname of your choice.

6. Can you use a hash table to implement a priority queue? Explain.

Projects

1. Implement the ADT dictionary by using hashing and separate chaining. Use a chain of linked nodes as each bucket. The dictionary's entries should have distinct search keys.

2. Repeat Project 1, but use the ADT list for each bucket instead of a chain of linked nodes. What implementation of the list would be reasonable?

3. Implement the class `PatientDataBase`, that you designed in Project 3 of the previous chapter. Use a hash table to store the patient records. Write a main program that demonstrates and tests this class.

4. Even though two implementations of a hash table may require the same average number of comparisons, their distributions may be different. The following experiment will examine this possibility for linear probing and double hashing. You will need two disjoint lists of names: one with at least 1000 names and the other with at least 10,000 names.

   a. For both of the collision resolution schemes linear probing and double hashing, determine the load factor that results in an average of 1.5 comparisons for an unsuccessful search of a hash table holding 100 objects. From the load factor, determine the size of the table required.

   b. Create two hash tables of the appropriate size and two corresponding empty lists, which will hold counts. Use linear probing for one table and double hashing for the other. Inside a loop that iterates 1000 times, do the following:

   - Clear the hash tables.
   - Randomly choose 100 names from the list of 1000 and insert them into the tables.
   - Randomly choose 100 names from the list of 10,000 and search the tables for each name. (Each search will be unsuccessful because the two lists have no names in common.)
   - Count the number of comparisons made in each table for the 100 searches and record the count in the list corresponding to the table.

   After the iteration is complete, each list should contain 1000 values. Each of these values is the total number of comparisons required to search for 100 names. Compute and display the average and standard deviation of each list. We expect the average number of comparisons for both hash tables to be equal to 150 (1.5 times 100).

5. Modify the previous project as follows:

   - Let the user enter the desired average number of comparisons.
   - Display a histogram of the results. A histogram shows the frequency of data values in given intervals of the same length. Use the floor of the average number of comparisons as the interval length.

6. Chapter 1 defined a set as a bag that does not permit duplicate entries. Define a class of sets that uses a hash table to store a set's entries.

Answers to Self-Test Questions

1. Half of them.

2. No. Even when λ is large, all entries could be in one bucket.

3. After expanding the table to 11 locations (11 is the prime number larger than twice the current table size of 5), the function *c* % 11 places 20 at index 9, 6 at index 6, and 14 at index 3. 18 causes a collision at index 9, so we probe ahead and place it at index 10.

4. In each method, replace the statements

```
index = locate(index, key);
if (index != -1)
```

with

```
index = probe(index, key);
if ((hashTable[index] != null) && (hashTable[index].isIn()))
```

5. No. The table size has increased, so rehashing is not necessary.

This page intentionally left blank

Chapter 23

Trees

Contents

Prerequisites

Chapter	5	Stacks
Chapter	7	Recursion
Chapter	14	A List Implementation That Links Data
Chapter	15	Iterators
Chapter	18	Searching

Objectives

After studying this chapter, you should be able to

- Describe binary trees and general trees, using standard terminology
- Traverse a tree in one of four ways: preorder, postorder, inorder, or level order
- Give examples of binary trees, including expression trees, decision trees, binary search trees, and heaps
- Give examples of general trees, including parse trees and game trees

As a plant, a tree is well known. As a way to organize data, the tree is more familiar than you might think. A family tree or a chart of players in a tournament are two common examples of a tree. A tree provides a hierarchical organization in which data items have ancestors and descendants. The organization is richer and more varied than any you have seen previously.

This chapter explores the ADT tree in its two forms—binary and general—and provides several examples of how such trees are used.

Tree Concepts

23.1 The data organizations that you have seen so far have placed data in a linear order. Objects in a stack, queue, list, or dictionary appear one after the other. As useful as these organizations are, you often must categorize data into groups and subgroups. Such a classification is **hierarchical**, or **nonlinear**, since the data items appear at various levels within the organization.

We begin by looking at several familiar examples of hierarchical data. Each example will be illustrated by a diagram that represents a tree.

Hierarchical Organizations

23.2 **Example: Family trees.** Your relatives can be arranged hierarchically in more than one way. Figure 23-1 shows Carole's children and grandchildren. Her son Brett has one daughter, Susan. Carole's daughter, Jennifer, has two children—Jared and Jamie.

Using a different arrangement, Figure 23-2 shows Jared's parents and grandparents. Jared's father is John and his mother is Jennifer. John's father and mother are James and Mary; Jennifer's parents are Robert and Carole.

FIGURE 23-1 Carole's children and grandchildren

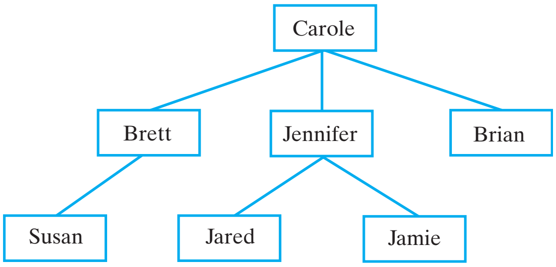
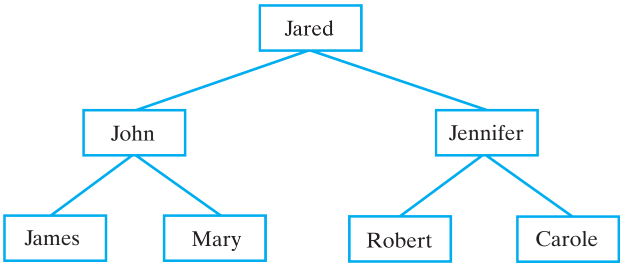
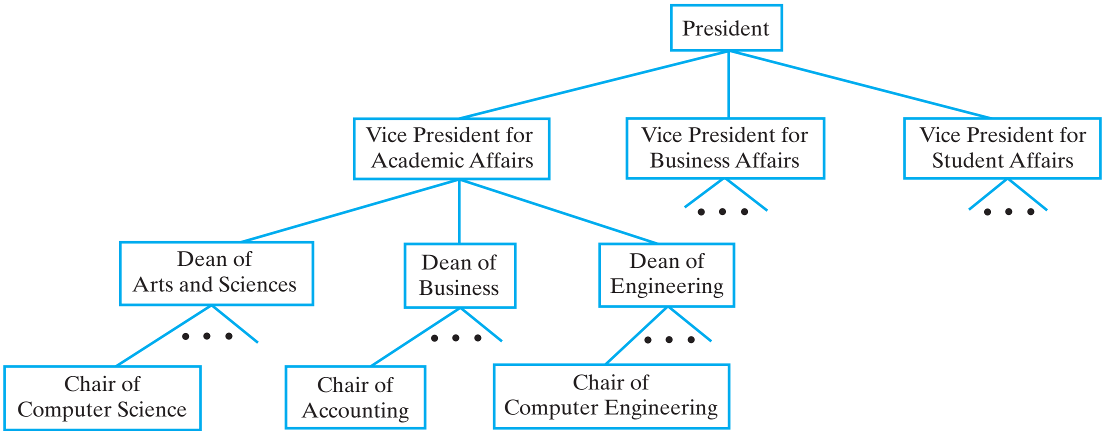


FIGURE 23-2 Jared's parents and grandparents



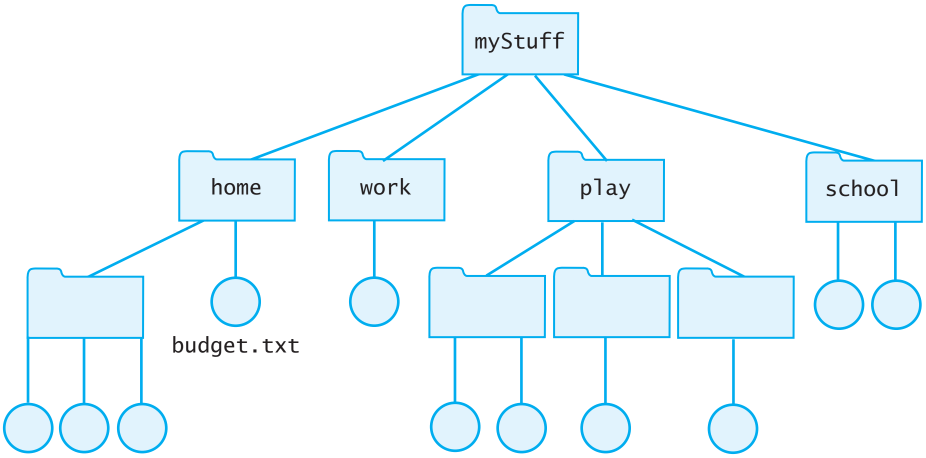
23.3 **Example: A university's organization.** Corporations, schools, churches, and governments all organize their staff hierarchically. For example, Figure 23-3 shows a portion of the administrative structure of a typical university. All offices ultimately report to the president. Immediately beneath the president are three vice presidents. The Vice President for Academic Affairs, for example, oversees the deans of the colleges. The deans in turn supervise the chairs of the various academic departments, such as computer science and accounting.

FIGURE 23-3 A portion of a university's administrative structure


President
Vice President for Academic Affairs
Vice President for Business Affairs
Vice President for Student Affairs
Dean of Arts and Sciences
Dean of Business
Dean of Engineering
Chair of Computer Science
Chair of Accounting
Chair of Computer Engineering

23.4 **Example: File directories.** Typically, you organize the files on your computer into folders, or directories. Each folder contains several other folders and/or files. Figure 23-4 shows the organization of the folders and files on Paul's computer. This organization is hierarchical. That is, all of Paul's files are organized within folders that are ultimately within the folder `myStuff`. For example, to look at his budget, Paul would start with the folder `myStuff`, find the folder `home`, and finally locate the file `budget.txt`.

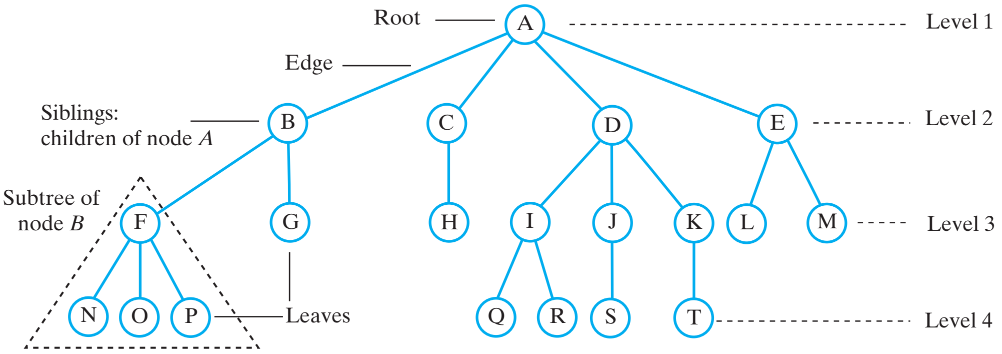
FIGURE 23-4 Computer files organized into folders



Tree Terminology

23.5 Each of the previous figures is an example of a tree. A **tree** is a set of **nodes** connected by **edges** that indicate the relationships among the nodes. The nodes are arranged in **levels** that indicate the nodes' hierarchy. At the top level is a single node called the **root**. Figure 23-5 shows a tree that, except for the names of the nodes, is identical to the tree in Figure 23-4. In Figure 23-4, the root of the tree is the folder `myStuff`; in Figure 23-5, the root is node *A*.

FIGURE 23-5 A tree equivalent to the tree in Figure 23-4



The nodes at each successive level of a tree are the **children** of the nodes at the previous level. A node that has children is the **parent** of those children. In Figure 23-5, node *A* is the parent of nodes *B*, *C*, *D*, and *E*. Since these children have the same parent, they are called **siblings**. They also are the **descendants** of node *A*, and node *A* is their **ancestor**. Furthermore, node *P* is a descendant of *A*, and *A* is an ancestor of *P*. Notice that node *P* has no children. Such a node is called a **leaf**. A node that is not a leaf—that is, one that has children—is called either an **interior node** or a **nonleaf**. Such a node is also a parent.

Note: Trees

While the roots of most plants are firmly in the ground, the root of an ADT tree is at the tree's top; it is the origin of a hierarchical organization. Each node can have children. A node with children is a parent; a node without children is a leaf. The root is the only node that has no parent; all other nodes have one parent each.

SELF-TEST

Question 1 Consider the tree in Figure 23-5.

a. Which nodes are the leaves?
b. Which nodes are the siblings of node *K*?
c. Which nodes are the children of node *B*?
d. Which nodes are the descendants of node *B*?
e. Which nodes are the ancestors of node *N*?
f. Which nodes are parents?

23.6 In general, each node in a tree can have an arbitrary number of children. We sometimes call such a tree a **general tree**. If each node has no more than *n* children, the tree is called an ***n*-ary tree**. Realize that not every general tree is an *n*-ary tree. If each node has at most two children, the tree is called a **binary tree**. The tree in Figure 23-2 is a binary tree, but the trees in the other previous figures are general trees.

Note: Can a tree be empty?

We allow any of our trees to be empty. Some people allow empty binary trees but require that general trees contain at least one node. While the reasons for doing so are quite valid, we will avoid confusion here by not making this subtle distinction between binary and general trees.

Any node and its descendants form a **subtree** of the original tree. A **subtree of a node** is a tree rooted at a child of that node. For example, one subtree of node *B* in Figure 23-5 is the tree rooted at *F*. A **subtree of a tree** is a subtree of the tree's root.

SELF-TEST **Question 2** This book has a hierarchical organization that you can represent by using a tree. Sketch a portion of this tree and indicate whether it is a general tree or a binary tree.

23.7 The **height** of a tree is the number of levels in the tree. We number the levels in a tree beginning with the root at level 1. The tree in Figure 23-5 has four levels, and so its height is 4. The height of a one-node tree is 1, and the height of an empty tree is 0.

We can express the height of a nonempty tree recursively by considering its subtrees:

Height of tree *T* = 1 + height of the tallest subtree of *T*

The root of the tree in Figure 23-5 has four subtrees of heights 3, 2, 3, and 2. Since the tallest of these subtrees has height 3, the tree has height 4.

We can reach any node in a tree by following a **path** that begins at the root and goes from node to node along the edges that join them. The path between the root and any other node is unique. The **length of a path** is the number of edges that compose it. For example, in Figure 23-5, the path that passes through the nodes *A*, *B*, *F*, and *N* has length 3. No other path from the root to a leaf is longer than this particular path. This tree has height 4, which is 1 more than the length of this longest path. In general, the height of a tree is 1 more than the length of the longest of the paths between its root and its leaves. Alternatively, the height of a tree is the number of nodes along the longest path between the root and a leaf.

Note: The path between a tree's root and any other node is unique.

Note: The **height of a tree** is the number of levels in the tree. The height also equals the number of nodes along the longest path between the root and a leaf.

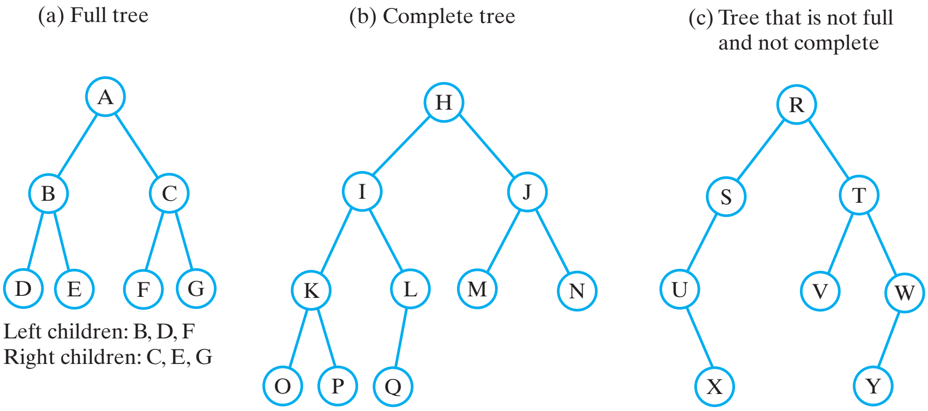
Note: Alternate definitions of height and level

Some people define both the height of a tree and its levels to be 1 less than those we will use in this book. For example, a one-node tree would have height 0 instead of 1. Also, the root of a tree would be at level 0 instead of 1.

SELF-TEST **Question 3** What are the heights of the trees in Figures 23-1, 23-2, and 23-4?

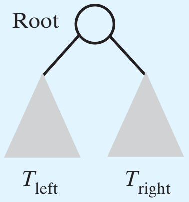
23.8 **Binary trees.** As we mentioned earlier, each node in a binary tree has at most two children. They are called the **left child** and the **right child**. For example, each tree in Figure 23-6 is a binary tree. In Figure 23-6a, nodes *B*, *D*, and *F* are left children, and nodes *C*, *E*, and *G* are right children. The root of this binary tree has two subtrees. The **left subtree** is rooted at *B* and the **right subtree** is rooted at *C*. Thus, the left subtree of a binary tree is the left subtree of its root; likewise for the right subtree.

FIGURE 23-6 Three binary trees



Every subtree in a binary tree is also a binary tree. In fact, we can think of a binary tree recursively, as follows:

Note: A binary tree is either empty or has the following form:



where T_{left} and T_{right} are binary trees.

When a binary tree of height *h* has all of its leaves at level *h* and every nonleaf (parent) has exactly two children, the tree is said to be **full**. Figure 23-6a shows a full binary tree. If all levels of a binary tree but the last contain as many nodes as possible, and the nodes on the last level are filled in from left to right—as in Figure 23-6b—the tree is **complete**. The binary tree in Figure 23-6c is neither full nor complete. In this case, a node can have a left child but no right child (for example, node *S*), or a right child but no left child (for example, node *U*).

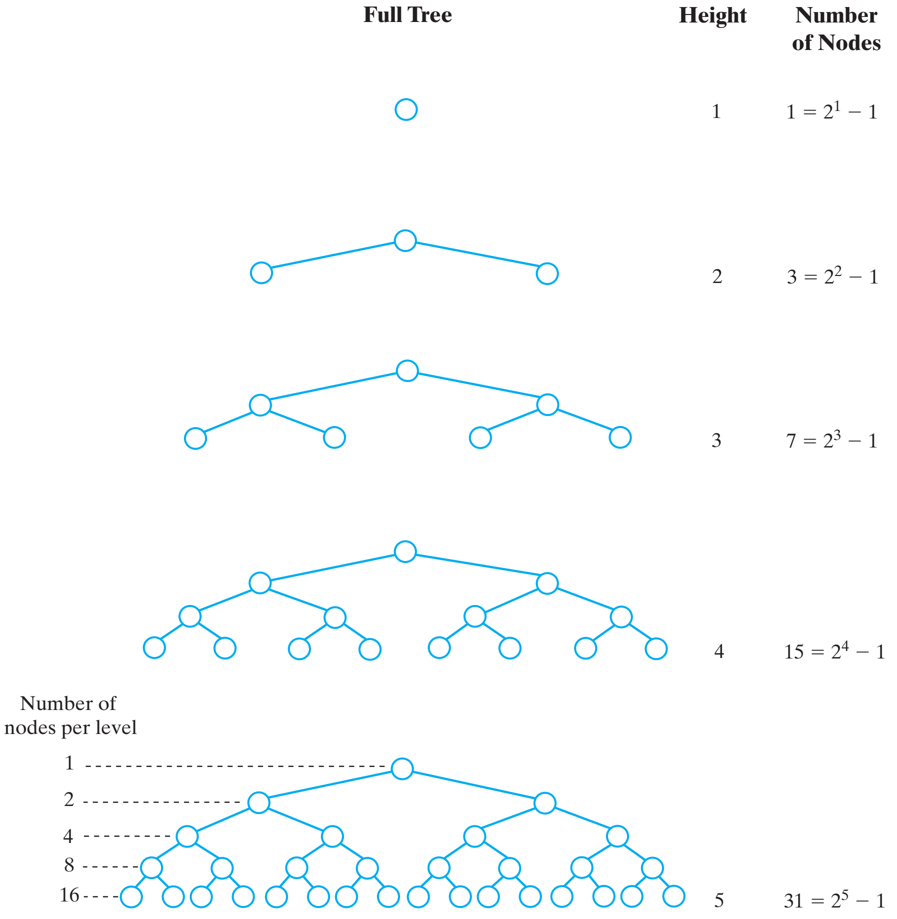
Note: All leaves in a full binary tree are on the same level and every nonleaf has exactly two children. A complete binary tree is full to its next-to-last level, and its leaves on the last level are filled from left to right. Binary trees are used extensively, and these special trees will be important to our later discussions.

23.9 **The height of full or complete trees.** In later chapters, the height of trees that are either full or complete will be important in our discussions of efficiency. Figure 23-7 shows some full trees that get progressively taller. We can compute the number of nodes that each tree contains as a function of its height. Beginning at the root of the tallest tree in the figure, we can see that the number of nodes at each level doubles as we move toward the leaves. The total number of nodes in this tallest tree is 1 + 2 + 4 + 8 + 16, or, 31. In general, the number of nodes in a full binary tree is

$$\sum_{i=0}^{h-1} 2^i$$

where h is the tree's height. This sum is equal to $2^h - 1$. You can convince yourself that this result is true by examining Figure 23-7, and you can prove it as an exercise by using mathematical induction.

FIGURE 23-7 The number of nodes in a full binary tree as a function of the tree's height



Now, if n is the number of nodes in a full tree, we have the following results:

$$n = 2^h - 1$$
$$2^h = n + 1$$
$$h = \log_2 (n + 1)$$

That is, the height of a full tree that has n nodes is $\log_2 (n + 1)$.

We leave it to you as an exercise to prove that the height of a complete tree having n nodes is $\log_2 (n + 1)$ rounded up.

Note: The height of a binary tree with n nodes that is either complete or full is $\log_2 (n + 1)$ rounded up.

Programming Tip: To compute $\log_2 x$ in Java, first observe that $\log_a x = \log_b x / \log_b a$. In Java, `Math.log(x)` returns the natural logarithm of `x`. So `Math.log(x) / Math.log(2.0)` computes the base 2 logarithm of `x`.

SELF-TEST

Question 4 Show that the relationship between a tree's height and its number of nodes is true for the binary trees in Parts *a* and *b* of Figure 23-6.

Question 5 How many nodes are in a full binary tree of height 6?

Question 6 What is the height of a complete tree that contains 14 nodes?

Traversals of a Tree

23.10 Until now, we treated the contents of the nodes in a tree simply as labels for identification. Because the tree is an ADT, however, its nodes contain data that we process. We now consider the nodes in this way.

VideoNote
The ADT Tree

Traversing the items in a data collection is a common operation that we have seen in previous chapters. In those cases, data was arranged linearly, so the sequence of the items in the traversal was clear. Such is not the case for a tree.

In defining a traversal, or iteration, of a tree, we must **visit**, or process, each data item exactly once. However, the order in which we visit items is not unique. We can choose an order suitable to our application. Because traversals of a binary tree are somewhat easier to understand than traversals of a general tree, we begin there. To simplify our discussion, we will use the phrase "visit a node" to mean "process the data within a node."

Note: "Visiting a node" means "processing the data within a node." It is an action that we perform during a traversal of a tree. A traversal can pass through a node without visiting it at that moment.

Traversals of a Binary Tree

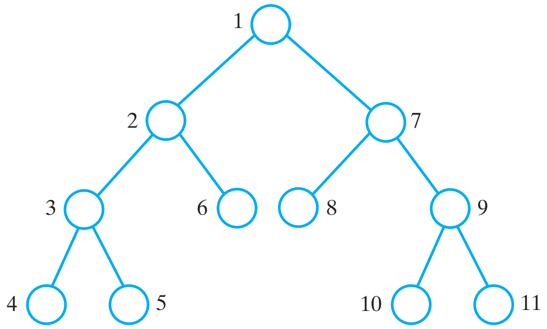
23.11 We know that the subtrees of the root of a binary tree are themselves binary trees. Using this recursive nature of a binary tree in the definition of its traversal is natural. To visit all the nodes in a binary tree, we must

Visit the root
Visit all the nodes in the root's left subtree
Visit all the nodes in the root's right subtree

Visiting the nodes in the left subtree before visiting those in the right subtree is simply a convention. Whether we visit the root before, between, or after visiting these two subtrees, however, defines three common orders for a traversal.

In a **preorder traversal**, we visit the root *before* we visit the root's subtrees. We then visit all the nodes in the root's left subtree before we visit the nodes in the right subtree. Figure 23-8 numbers the nodes in a binary tree in the order in which a preorder traversal visits them. After first visiting the root, we visit the nodes in the root's left subtree. Since this subtree is a binary tree, visiting its nodes in preorder means that we visit its root before visiting its left subtree. The traversal continues in this recursive manner until all nodes are visited.

FIGURE 23-8 The visitation order of a preorder traversal

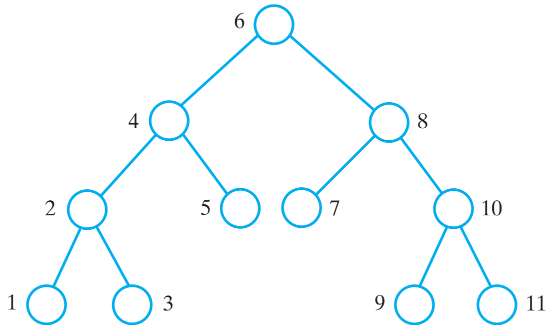


23.12 An **inorder traversal** visits the root of a binary tree *between* visiting the nodes in the root's subtrees. In particular, it visits nodes in the following order:

Visit all the nodes in the root's left subtree
Visit the root
Visit all the nodes in the root's right subtree

Figure 23-9 numbers the nodes in a binary tree in the order in which an inorder traversal visits them. Recursively visiting the nodes in the left subtree results in visiting the leftmost leaf first. We visit that leaf's parent next and then the parent's right child. We visit the tree's root after we have visited all of the nodes in the root's left subtree. Finally, we visit the nodes in the root's right subtree in this recursive manner.

FIGURE 23-9 The visitation order of an inorder traversal

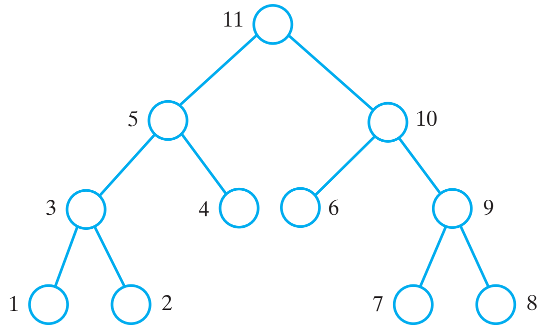


23.13 A **postorder traversal** visits the root of a binary tree *after* visiting the nodes in the root's subtrees. In particular, it visits nodes in the following order:

Visit all the nodes in the root's left subtree
Visit all the nodes in the root's right subtree
Visit the root

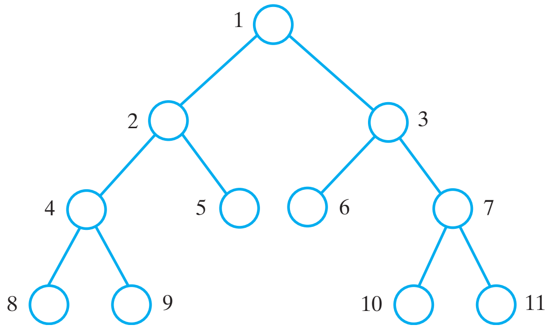
Figure 23-10 numbers the nodes in a binary tree in the order in which a postorder traversal visits them. Recursively visiting the nodes in the left subtree results in visiting the leftmost leaf first. We then visit that leaf's sibling and then their parent. After visiting all the nodes in the root's left subtree, we visit the nodes in the root's right subtree in this recursive manner. Finally we visit the root.

FIGURE 23-10 The visitation order of a postorder traversal



23.14 A **level-order traversal**—the last traversal we will consider—begins at the root and visits nodes one level at a time. Within a level, it visits nodes from left to right. Figure 23-11 numbers the nodes in a binary tree in the order in which a level-order traversal visits them.

FIGURE 23-11 The visitation order of a level-order traversal



The level-order traversal is an example of a **breadth-first traversal**. It follows a path that explores an entire level before moving to the next level. The preorder traversal is an example of a **depth-first traversal**. This kind of traversal fully explores one subtree before exploring another. That is, the traversal follows a path that descends the levels of a tree as deeply as possible until it reaches a leaf.

Note: Traversals of a binary tree

A preorder traversal visits the root of a binary tree before visiting the nodes in its two subtrees.
An inorder traversal visits the root between visiting the nodes in its two subtrees.
A postorder traversal visits the root after visiting the nodes in its two subtrees.
A level-order traversal visits nodes from left to right within each level of the tree, beginning with the root.

SELF-TEST

Question 7 Suppose that visiting a node means simply displaying the data in the node. What are the results of each of the following traversals of the binary tree in Figure 23-2? Preorder, postorder, inorder, and level order.

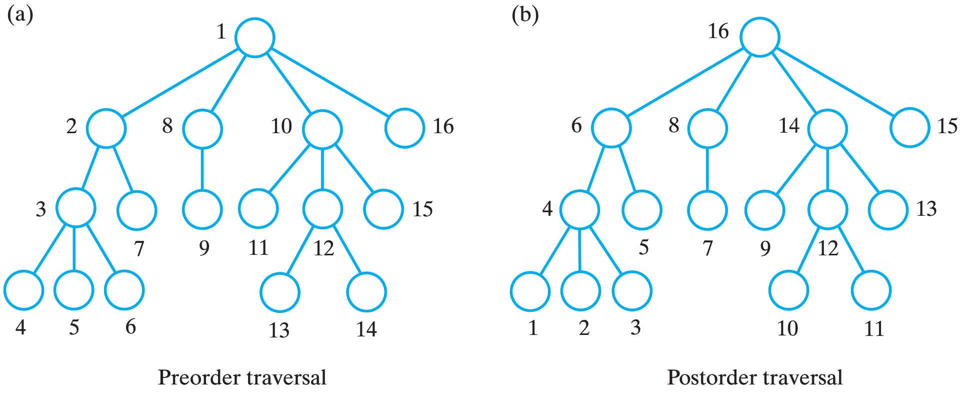
Traversals of a General Tree

23.15 A general tree has traversals that are in level order, preorder, and postorder. An inorder traversal is not well defined for a general tree.

A level-order traversal visits nodes level by level, beginning at the root. This traversal is just like a level-order traversal of a binary tree, except that nodes in a general tree can have more than two children each.

A preorder traversal visits the root and then visits the nodes in each of the root's subtrees. A postorder traversal first visits the nodes in each of the root's subtrees and then visits the root last. Figure 23-12 gives an example of a preorder traversal and a postorder traversal for a general tree.

FIGURE 23-12 The visitation order of two traversals of a general tree: (a) preorder; (b) postorder



SELF-TEST

Question 8 In what order will a level-order traversal visit the nodes of the tree in Figure 23-12?

Java Interfaces for Trees

Trees come in many shapes and have varied applications. Writing one Java interface for an ADT tree that satisfies every use would be an unwieldy task. Instead we will write several interfaces that we can combine as needed for a particular application. We will include these interfaces in a package that also will contain the classes that implement them. In this way, the package can contain implementation details, such as a class of nodes, that we want to hide from the trees' clients. The next chapter will examine these implementations.

Interfaces for All Trees

23.16 **Fundamental operations.** We begin with an interface that specifies operations common to all trees. The interface in Listing 23-1 uses the generic type `T` as the type of data in the nodes of the tree.

LISTING 23-1 An interface of methods common to all trees

```
package TreePackage;
public interface TreeInterface<T>
{
   public T getRootData();
   public int getHeight();
   public int getNumberOfNodes();
   public boolean isEmpty();
   public void clear();
} // end TreeInterface
```

This interface is quite basic. It does not include operations to add or remove nodes, as even the specification of these operations depends on the kind of tree. We also did not include traversal operations in this interface, since not every application uses them. Instead we will provide a separate interface for traversals.

23.17 **Traversals.** One way to traverse a tree is to use an iterator that has the methods `hasNext` and `next`, as given in the interface `java.util.Iterator`. As in previous chapters, we can define a method that returns such an iterator. Since we can have several kinds of traversals, a tree class could have several methods that each return a different kind of iterator. Listing 23-2 defines an interface for these methods. A tree class can implement this interface and define as many of the methods as are needed.

LISTING 23-2 An interface of traversal methods for a tree

```
package TreePackage;
import java.util.Iterator;
public interface TreeIteratorInterface<T>
{
   public Iterator<T> getPreorderIterator();
   public Iterator<T> getPostorderIterator();
   public Iterator<T> getInorderIterator();
   public Iterator<T> getLevelOrderIterator();
} // end TreeIteratorInterface
```

An Interface for Binary Trees

23.18 Many applications of trees in fact use binary trees. We could use a Java class of general trees for such an application, but using a special class of binary trees is more convenient and efficient. Because binary trees occur so frequently, developing special Java classes for them is worthwhile.

We can define an interface for a basic binary tree by adding methods to those already in the interfaces TreeInterface and TreeIteratorInterface. Since a Java interface can extend more than one interface, we can write the interface shown in Listing 23-3 for a class of binary trees.

LISTING 23-3 An interface for a binary tree

```
package TreePackage;
public interface BinaryTreeInterface<T> extends TreeInterface<T>,
                                                TreeIteratorInterface<T>
{
   /** Sets this binary tree to a new one-node binary tree.
       @param rootData   an object that is the data in the new tree's root
   */
   public void setTree(T rootData);

   /** Sets this binary tree to a new binary tree.
       @param rootData   an object that is the data in the new tree's root
       @param leftTree   the left subtree of the new tree
       @param rightTree  the right subtree of the new tree */
   public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
                                   BinaryTreeInterface<T> rightTree);
} // end BinaryTreeInterface
```

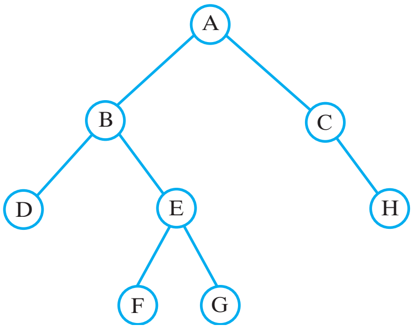
The two setTree methods transform an existing binary tree object into a new tree composed of given arguments. The first method forms a one-node tree from a given data object. The second method forms a tree whose root node contains a given data object and has as its subtrees the two given binary trees. A class that implements this interface certainly could have constructors that perform the same tasks as these two methods. However, since an interface cannot contain constructors, we have no way to force an implementor to provide them.

23.19 **Example.** Suppose that the class BinaryTree implements the interface BinaryTreeInterface. To construct the binary tree in Figure 23-13, we first represent each of its leaves as a one-node tree. Notice that each node in this tree contains a one-letter string. Moving up the tree from its leaves, we use setTree to form larger and larger subtrees until we have the desired tree. Here are some Java statements that build the tree and then display some of its characteristics:

```
// represent each leaf as a one-node tree
BinaryTreeInterface<String> dTree = new BinaryTree<String>();
dTree.setTree("D");
BinaryTreeInterface<String> fTree = new BinaryTree<String>();
fTree.setTree("F");
BinaryTreeInterface<String> gTree = new BinaryTree<String>();
gTree.setTree("G");
BinaryTreeInterface<String> hTree = new BinaryTree<String>();
hTree.setTree("H");
BinaryTreeInterface<String> emptyTree = new BinaryTree<String>();

// form larger subtrees
BinaryTreeInterface<String> eTree = new BinaryTree<String>();
eTree.setTree("E", fTree, gTree); // subtree rooted at E
```

```
BinaryTreeInterface<String> bTree = new BinaryTree<String>();
bTree.setTree("B", dTree, eTree); // subtree rooted at B

BinaryTreeInterface<String> cTree = new BinaryTree<String>();
cTree.setTree("C", emptyTree, hTree); // subtree rooted at C

BinaryTreeInterface<String> aTree = new BinaryTree<String>();
aTree.setTree("A", bTree, cTree); // desired tree rooted at A

// display root, height, number of nodes
System.out.println("Root of tree contains " + aTree.getRootData());
System.out.println("Height of tree is " + aTree.getHeight());
System.out.println("Tree has " + aTree.getNumberOfNodes() + " nodes");

// display nodes in preorder
System.out.println("A preorder traversal visits nodes in this order:");
Iterator<String> preorder = aTree.getPreorderIterator();
while (preorder.hasNext())
   System.out.print(preorder.next() + " ");
System.out.println();
```

FIGURE 23-13 A binary tree whose nodes contain one-letter strings



Examples of Binary Trees

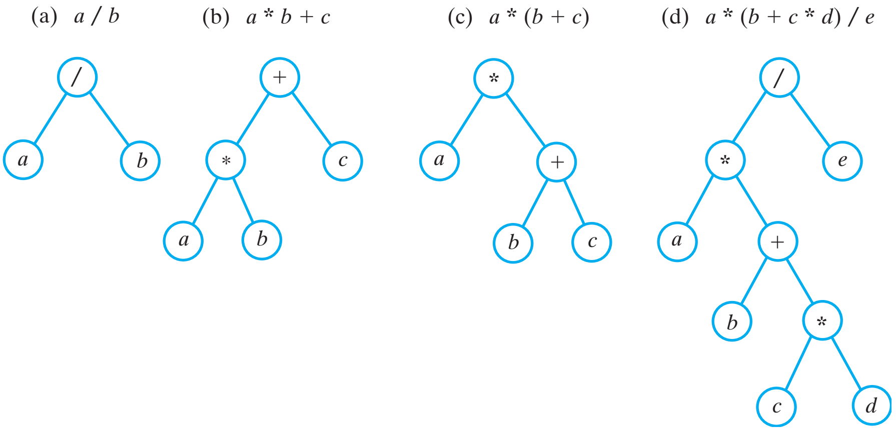
We now look at some examples that use trees to organize data, leaving details of the implementations for the next chapter. Our first example includes a demonstration of some of the traversals introduced earlier in this chapter.

Expression Trees

23.20 We can use a binary tree to represent an algebraic expression whose operators are binary. Recall from Segment 5.5 in Chapter 5 that a binary operator has two operands. For example, we can represent the expression *a* / *b* as the binary tree in Figure 23-14a. The root of the tree contains the operator / and the root's children contain the operands for the operator. Notice that the order of the children matches the order of the operands. Such a binary tree is called an **expression tree.** Figure 23-14 also contains other examples of expression trees. Notice that any parentheses in an expression do not appear in its tree. The tree in fact captures the order of the expression's operations without the need for parentheses.

VideoNote
Using the ADT tree

FIGURE 23-14 Expression trees for four algebraic expressions



(a) a / b (b) $a * b + c$ (c) $a * (b + c)$ (d) $a * (b + c * d) / e$

23.21 Segment 5.5 mentioned that we can write an algebraic expression in several ways. The expressions that we normally write, in which each binary operator appears between its two operands, are called infix expressions. A prefix expression places each operator before its two operands, and a postfix expression places each operator after its two operands. Various traversals of an expression tree are related to these forms of an expression.

An inorder traversal of an expression tree visits the variables and operators in the tree in the order in which they appear in the original infix expression. If we were to write each node's contents when we visited it, we would get the infix expression, but without any parentheses.

A preorder traversal produces a prefix expression that is equivalent to the original infix expression. For example, a preorder traversal of the tree in Figure 23-14b visits nodes in this order: + * a b c. This result is the prefix form of the infix expression $a * b + c$. Recall that, like an expression tree, a prefix expression never contains parentheses.

A postorder traversal produces a postfix expression that is equivalent to the original expression. A postfix expression also has no parentheses, so the traversal produces the correct result. For example, a postorder traversal of the tree in Figure 23-14b visits nodes in the following order: a b * c +. This result is the postfix form of the infix expression $a * b + c$.

Question 9 Write an expression tree for each of these algebraic expressions.

a. $a + b * c$
b. $(a + b) * c$

Question 10 In what order are nodes visited by a preorder, inorder, and postorder traversal of the trees in Parts *a*, *c*, and *d* of Figure 23-14?

Question 11 Which trees, if any, in Figure 23-14 are full? Which are complete?

23.22 **Evaluating an algebraic expression.** Since an expression tree represents the order of an expression's operations, we can use it to evaluate the expression. The root of an expression tree is always an operator whose operands are represented by the root's left and right subtrees. If we can evaluate the subexpressions that these subtrees represent, we can evaluate the entire expression. Notice that such is the case for each expression tree in Figure 23-14, if we know the values of the variables.

A postorder traversal of an expression tree visits the root's left subtree, then the root's right subtree, and finally the root. If during the visits to the subtrees we evaluate their expressions, we can combine the results with the operator in the root and get the value of the original expression. Thus, the value of an expression tree is given by the following recursive algorithm:

```
Algorithm evaluate(expressionTree)
if (expressionTree is empty)
   return 0
else
{
   firstOperand = evaluate(left subtree of expressionTree)
   secondOperand = evaluate(right subtree of expressionTree)
   operator = the root of expressionTree
   return the result of the operation operator and its operands firstOperand
          and secondOperand
}
```

We will implement an expression tree in the next chapter.

Question 12 What value does the previous algorithm return for the expression tree in Figure 23-14b? Assume that *a* is 3, *b* is 4, and *c* is 5.

Decision Trees

23.23 **Example: Expert systems.** An **expert system** helps its users solve problems or make decisions. Such a program might help you pick a major or apply for financial aid. It reaches a conclusion based upon your answers to a series of questions.

A **decision tree** can be the basis of an expert system. Each parent (nonleaf) in a decision tree is a question that has a finite number of responses. For example, we might use questions whose answers are true or false, yes or no, or multiple choice. Each possible answer to the question corresponds to a child of that node. Each child might be an additional question or a conclusion. Nodes that are conclusions would have no children, and so they would be leaves.

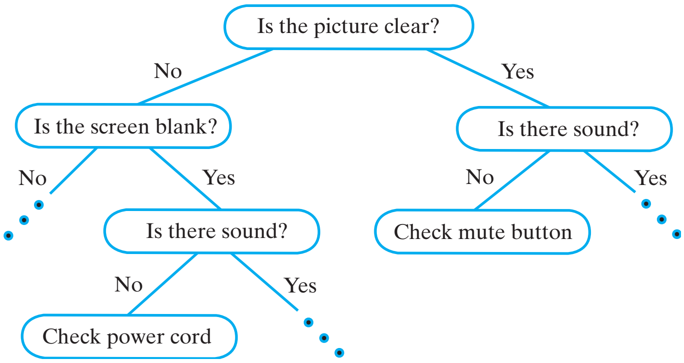
In general, a decision tree is an *n*-ary tree so that it can accommodate multiple-choice questions. Often, however, a decision tree is a binary tree. For example, the decision tree in Figure 23-15 shows part of a binary tree of yes-or-no questions that diagnose a problem with a television. To use this decision tree, we first would display the question in the root. According to the user's answer, we would move to the appropriate child and display its contents. Thus, we move along a path in a decision tree from the root to a leaf according to responses made by the user. At each nonleaf, we display a question. When we reach a leaf, we provide a conclusion. Notice that each node in a binary decision tree either has two children or is a leaf.

A decision tree provides operations that move us along a path through the tree and access the current node. Listing 23-4 contains a possible Java interface for a binary decision tree.

LISTING 23-4 An interface for a binary decision tree

```java
package TreePackage;
public interface DecisionTreeInterface<T> extends BinaryTreeInterface<T>
{
   /** Gets the data in the current node.
       @return the data object in the current node, or
               null if the current node is null */
   public T getCurrentData();
   /** Sets the data in the current node.
       Precondition: The current node is not null.
       @param newData  the new data object */
   public void setCurrentData(T newData);

   /** Sets the data in the children of the current node,
       creating them if they do not exist.
       Precondition: The current node is not null.
       @param answerForNo   the new data object for the left child
       @param answerForYes  the new data object for the right child */
   public void setAnswers(T answerForNo, T answerForYes);

   /** Sees whether the current node contains an answer.
       @return true if the current node is a leaf, or
               false if it is a nonleaf */
   public boolean isAnswer();

   /** Sets the current node to its left child.
       If the child does not exist, sets the current node to null.
       Precondition: The current node is not null. */
   public void advanceToNo();

   /** Sets the current node to its right child.
       If the child does not exist, sets the current node to null.
       Precondition: The current node is not null. */
   public void advanceToYes();

   /** Makes the root of the tree the current node.*/
   public void reset();
} // end DecisionTreeInterface
```

FIGURE 23-15 A portion of a binary decision tree



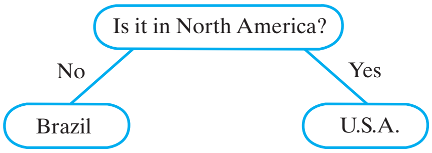
23.24 **Example: Guessing game.** In a guessing game, you think of something and I have to guess what it is by asking you questions that have a yes or no answer. Suppose that a program asks the questions for me. This program uses a binary decision tree that grows as the game progresses. Instead of creating the tree before it is used, the program acquires facts from the user and adds them to the decision tree. Thus, the program learns by playing the game and becomes more proficient over time.

To simplify the problem, let's restrict your choice of things. For example, suppose that you think of a country. The program could begin with the simple three-node tree pictured in Figure 23-16. With this tree, the program asks the question in the root and makes one of two guesses. Depending on the answer to the question. Here is one possible exchange between the program and the user (user replies are blue):

```
Is it in North America?
Yes
My guess is U. S. A. Am I right?
Yes
I win.
Play again?
```

The program has guessed correctly; the tree remains unchanged.

FIGURE 23-16 An initial decision tree for a guessing game



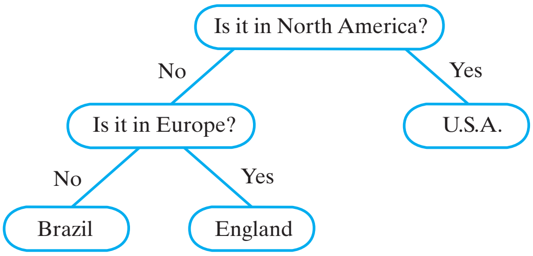
23.25 **Augmenting the tree in the guessing game.** Suppose the user is thinking of something else. The exchange might go like this:

```
Is it in North America?
No
My guess is Brazil. Am I right?
No
I give up; what are you thinking of?
England
Give me a question whose answer is yes for England and no for Brazil.
Is it in Europe?
Play again?
```

With this new information, we augment the tree, as in Figure 23-17. We replace the contents of the leaf that contained the wrong answer—Brazil in this case—with the new question provided by the user. We then give the leaf two children. One child contains the guess that was in the former leaf (Brazil), and the other contains the user's answer (England) as a new guess. The program now can distinguish between Brazil and England.

23.26 **A class for the guessing game.** We demonstrate some of the methods declared in the interface `DecisionTreeInterface` by implementing part of a class `GuessingGame`. This class, as shown in Listing 23-5, begins with a decision tree as a data field and a constructor that creates an initial tree. The tree has one yes-or-no question as its root and two guesses as children, one guess for each possible answer to the question. We assume that `DecisionTree` will have the constructors that we used here.

FIGURE 23-17 The decision tree for a guessing game after acquiring another fact



LISTING 23-5 The class `GuessingGame`

```
import TreePackage.DecisionTreeInterface;
import TreePackage.DecisionTree;
public class GuessingGame
{
   private DecisionTreeInterface<String> tree;

   public GuessingGame(String question, String noAnswer, String yesAnswer)
   {
      DecisionTree<String> no = new DecisionTree<String>(noAnswer);
      DecisionTree<String> yes = new DecisionTree<String>(yesAnswer);
      tree = new DecisionTree<String>(question, no, yes);
   } // end default constructor

   public void play()
   {
      tree.reset();
      while (!tree.isAnswer())
      {
         // ask current question
         System.out.println(tree.getCurrentData());

         if (Client.isUserResponseYes())
            tree.advanceToYes();
         else
           tree.advanceToNo();
      } // end while
      assert tree.isAnswer(); // Assertion: leaf is reached

      // make guess
      System.out.println("My guess is " + tree.getCurrentData() +
                         ". Am I right?");
      if (Client.isUserResponseYes())
         System.out.println("I win.");
      else
        learn();
   } // end play

   private void learn()
   {
      < Implementation left as a project in the next chapter. >
      . . .
   } // end learn
} // end GuessingGame
```

The public method `play` uses methods of `DecisionTree` to maintain the tree. Since the game requires user interaction, we assume that the client of `GuessingGame` provides methods that communicate with the user. In particular, we assume that a class `Client` has a static method `isUserResponseYes` that returns true if the user responds "yes" to a question.

The private method `learn` asks the user for a question that distinguishes between two guesses. Using this information, the method adds nodes to the decision tree, as described earlier in Segment 23.25. The next chapter will give you the tools to implement this method.

Question 13 Why should the method `learn` be private within the class `GuessingGame`?

Binary Search Trees

23.27 Earlier chapters have already discussed the importance of searching for data. Since we can traverse the nodes in any tree, searching a tree for a specific piece of data is certainly feasible. Doing so, however, can be as inefficient as performing a sequential search of an array. A **search tree**, on the other hand, organizes its data so that a search can be more efficient. In this chapter, we present the simplest kind of search tree, the binary search tree. Chapter 27 will look at other search trees.

A **binary search tree** is a binary tree whose nodes contain `Comparable` objects and are organized as follows:

Note: For each node in a binary search tree,

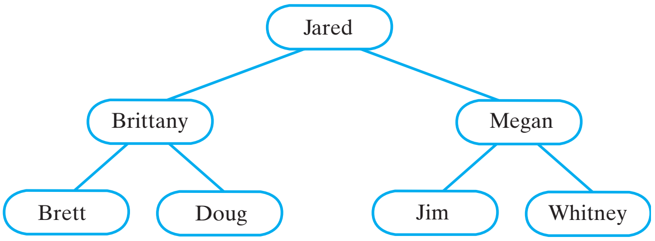
- The node's data is greater than all the data in the node's left subtree
- The node's data is less than all the data in the node's right subtree

For example, Figure 23-18 shows a binary search tree of names. As a string, *Jared* is greater than all the names in *Jared*'s left subtree but less than all names in *Jared*'s right subtree. These characteristics are true for every node in the tree, not only for the root. Notice that each of *Jared*'s subtrees is itself a binary search tree.

Note: Every node in a binary search tree is the root of a binary search tree.

The previous definition of a binary search tree implies that the tree's entries are distinct. We have imposed this restriction to make our discussion simpler, but we could revise our definition to allow duplicate entries. Chapter 25 considers this possibility.

FIGURE 23-18 A binary search tree of names



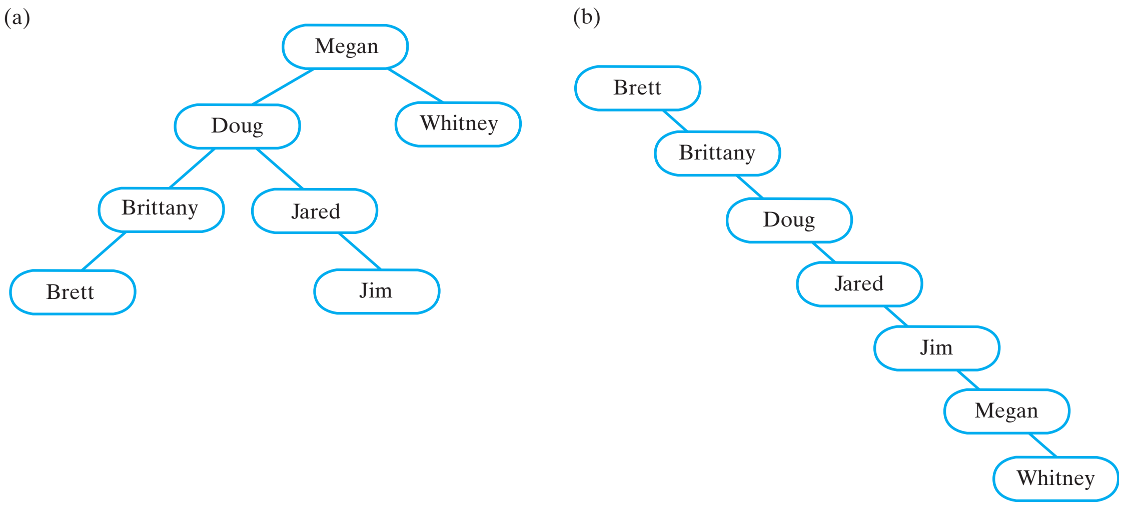
23.28 The configuration of a binary search tree is not unique. That is, we can form several different binary search trees from the same set of data. For example, Figure 23-19 shows two binary search trees containing the same names that are in Figure 23-18; other binary search trees are possible.

SELF-TEST

Question 14 How many different binary search trees can you form from the strings *a*, *b*, and *c*?

Question 15 What are the heights of the shortest and tallest trees that you formed in Question 14?

FIGURE 23-19 Two binary search trees containing the same data as the tree in Figure 23-18



23.29 **Searching a binary search tree.** The organization of the nodes in a binary search tree enables us to search the tree for a particular data object, given its search key. For example, suppose that we search the tree in Figure 23-18 for the string *Jim*. Beginning at the root of the tree, we compare *Jim* with *Jared*. Since the string *Jim* is greater than the string *Jared*, we search the right subtree of the root. Comparing *Jim* to *Megan*, we find that *Jim* is less than *Megan*. We search *Megan*'s left subtree next and find *Jim*.

To search for *Laura*, we would compare *Laura* with *Jared*, then with *Megan*, and then with *Jim*. Since *Laura* is greater than *Jim*, we would search *Jim*'s right subtree. But this subtree is empty, so we conclude that *Laura* does not occur in the tree.

We can express our search algorithm recursively: To search a binary search tree, we search one of its two subtrees. The search ends when either we find the item we seek or we encounter an empty subtree. We can formalize this search by writing the following pseudocode:

```
Algorithm bstSearch(binarySearchTree, desiredObject)
// Searches a binary search tree for a given object.
// Returns true if the object is found.

if (binarySearchTree is empty)
   return false
else if (desiredObject == object in the root of binarySearchTree)
   return true
else if (desiredObject < object in the root of binarySearchTree)
   return bstSearch(left subtree of binarySearchTree, desiredObject)
else
   return bstSearch(right subtree of binarySearchTree, desiredObject)
```

This algorithm is somewhat like a binary search of an array. Here we search one of two subtrees; a binary search searches one half of an array. You will see how to implement this algorithm in Chapter 25.

If you think that you could implement the ADT dictionary by using a binary search tree, you would be right. Chapter 25 will show you how.

23.30 **The efficiency of a search.** The algorithm `bstSearch` examines nodes along a path through a binary search tree, beginning at the tree's root. The path ends at either the node that contains the desired object or some other node that is a leaf. In the previous segment, the search for *Jim* in Figure 23-18 examined the three nodes containing *Jared*, *Megan*, and *Jim*. In general, the number of comparisons that a successful search requires is the number of nodes along the path from the root to the node that contains the desired item.

Searching for *Jim* in Figure 23-19a requires four comparisons; searching Figure 23-19b for *Jim* requires five comparisons. Both trees in Figure 23-19 are taller than the tree in Figure 23-18. As you can see, the height of a tree directly affects the length of the longest path from the root to a leaf and hence affects the efficiency of a worst-case search. Thus, searching a binary search tree of height h is $O(h)$.

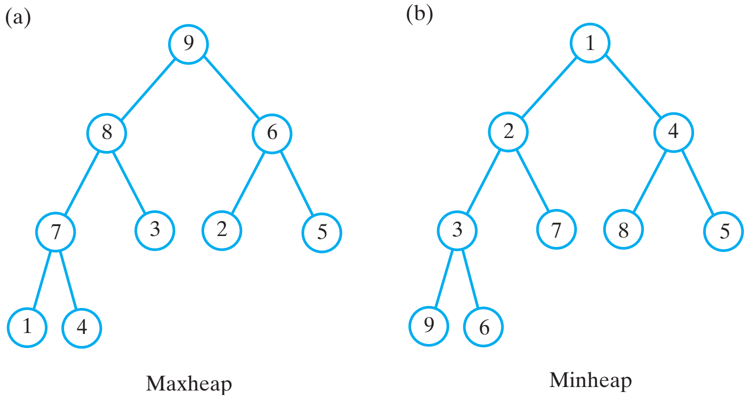
Note that the tree in Figure 23-19b is as tall as a tree containing seven nodes can be. A search of this tree has the performance of a sequential search of either a sorted array or a sorted linked chain. Each of these searches has an efficiency of $O(n)$.

To make searching a binary search tree as efficient as possible, the tree must be as short as possible. The tree in Figure 23-18 is full and is the shortest possible binary search tree that we can form with this data. As you will see in Chapter 25, inserting or deleting nodes can change the shape of a binary search tree. Thus, such operations can decrease the time efficiency of a search. Chapter 27 will show you strategies for maintaining the search's efficiency.

Heaps

23.31 **Definitions.** A **heap** is a complete binary tree whose nodes contain `Comparable` objects and are organized as follows. Each node contains an object that is no smaller (or no larger) than the objects in its descendants. In a **maxheap**, the object in a node is greater than or equal to its descendant objects. In a **minheap**, the relation is less than or equal to. Figure 23-20 gives an example of a maxheap and a minheap. For simplicity, we use integers instead of objects in our illustrations.

FIGURE 23-20 (a) A maxheap and (b) a minheap that contain the same values



The root of a maxheap contains the largest object in the heap. Notice that the subtrees of any node in a maxheap are also maxheaps. Although we will focus on maxheaps, minheaps behave in an analogous fashion.

Note: A maxheap is a complete binary tree such that each node in the tree contains a `Comparable` object that is greater than or equal to the objects in the node's descendants.

23.32 **Operations.** In addition to typical ADT operations such as `add`, `isEmpty`, `getSize`, and `clear`, a heap has operations that retrieve and remove the object in its root. This object is either the largest or the smallest object in the heap, depending on whether we have a maxheap or a minheap. This characteristic enables us to use a heap to implement the ADT priority queue, as you will see in the next segment.

The Java interface in Listing 23-6 specifies operations for a maxheap.

LISTING 23-6 An interface for a maxheap

```
public interface MaxHeapInterface<T extends Comparable<? super T>>
{
   /** Adds a new entry to this heap.
       @param newEntry  an object to be added */
   public void add(T newEntry);

   /** Removes and returns the largest item in this heap.
       @return either the largest object in the heap or,
               if the heap is empty before the operation, null */
  public T removeMax();

   /** Retrieves the largest item in this heap.
       @return either the largest object in the heap or,
               if the heap is empty, null */
  public T getMax();
```

```
   /** Detects whether this heap is empty.
       @return true if the heap is empty, else returns false */
   public boolean isEmpty();

   /** Gets the size of this heap.
       @return the number of entries currently in the heap */
   public int getSize();

   /** Removes all entries from this heap. */
   public void clear();
} // end MaxHeapInterface
```

If you place items into a maxheap and then remove them, you will get the items in descending order. Thus, we can use a heap to sort an array, as you will see in Chapter 26.

Question 16 Does a maxheap that contains a given set of objects have a unique root? Justify your answer by using the maxheap in Figure 23-20a as an example.

Question 17 Is a maxheap that contains a given set of objects unique? Justify your answer by using the maxheap in Figure 23-20a as an example.

23.33 **Priority queues.** We can use a heap to implement the ADT priority queue. Assuming that the class `MaxHeap` implements `MaxHeapInterface`, a class that implements the priority queue as an adapter class begins as given in Listing 23-7. Recall that we defined `PriorityQueueInterface` in Segment 10.19 of Chapter 10.

LISTING 23-7 The beginning of the class `PriorityQueue`

```
public class PriorityQueue<T extends Comparable<? super T>>
             implements PriorityQueueInterface<T>
{
   private MaxHeapInterface<T> pq;

   public PriorityQueue()
   {
      pq = new MaxHeap<T>();
   } // end default constructor

   public void add(T newEntry)
   {
      pq.add(newEntry);
   } // end add
   < Implementations of remove, peek, isEmpty, getSize, and clear are here. >
   . . .
} // end PriorityQueue
```

Alternatively, the class `MaxHeap` could implement `PriorityQueueInterface`. We then could define a priority queue of strings, as follows:

```
PriorityQueueInterface<String> pq = new MaxHeap<String>();
```

Examples of General Trees

We conclude this chapter with two examples of general trees. A parse tree is useful in the construction of a compiler; a game tree is a generalization of the decision tree that Segment 23.23 described.

Parse Trees

23.34 Segment 7.44 in Chapter 7 gave the following rules to describe strings that are valid algebraic expressions:

- An algebraic expression is either a term or two terms separated by a + or - operator.
- A term is either a factor or two factors separated by a * or / operator.
- A factor is either a variable or an algebraic expression enclosed in parentheses.
- A variable is a single letter.

These rules form a **grammar** for algebraic expressions, much like the grammar that describes the English language. In fact, every programming language has a grammar.

Typically, computer scientists use a notation to write the rules of a grammar. For example, the rules just given for algebraic expressions could appear as follows, where the symbol | means "or":

<*expression*> ::= <*term*> | <*term*> + <*term*> | <*term*> - <*term*>
<*term*> ::= <*factor*> | <*factor*> * <*factor*> | <*factor*> / <*factor*>
<*factor*> ::= <*variable*> | (<*expression*>)
<*variable*> ::= a | b | ... | z | A | B ... | Z

To see whether a string is a valid algebraic expression—that is, to check its syntax—we must see whether we can derive the string from <*expression*> by applying these rules. If we can, the derivation can be given as a **parse tree** with <*expression*> as its root and the variables and operators of the algebraic expression as its leaves. A parse tree for the expression a * (b + c) is shown in Figure 23-21. Beginning at the tree's root, we see that an expression is a term. A term is the product of two factors. The first factor is a variable, in particular, a. The second factor is an expression enclosed in parentheses. That expression is the sum of two terms. Each of those terms is a factor; each of those factors is a variable. The first variable is b; the second is c. Since we are able to form this parse tree, the string a * (b + c) is a valid algebraic expression.

A parse tree must be a general tree so that it can accommodate any expression. In fact, we are not restricted to algebraic expressions. We can use a parse tree to check the validity of any string according to any grammar. Since programming languages have grammars, compilers use parse trees both to check the syntax of a program and to produce executable code.

Question 18 Draw a parse tree for the algebraic expression a * b + c.

Game Trees

23.35 For a two-person game such as tic-tac-toe, we can use a general decision tree to represent the possible moves in any situation. Such a decision tree is called a **game tree**. If a given node in the tree represents the state of the game after one player has made a move, the node's children represent the states possible after the second player makes a move. Figure 23-22 shows a portion of a game tree for tic-tac-toe.

We can use a game tree like the one shown in the figure in a program that plays tic-tac-toe. We could create the tree ahead of time or have the program build the tree as it plays. In either case, the program could ensure that poor moves do not remain in the tree. In this way, the program could use a game tree to improve its play.

FIGURE 23-21 A parse tree for the algebraic expression $a * (b + c)$

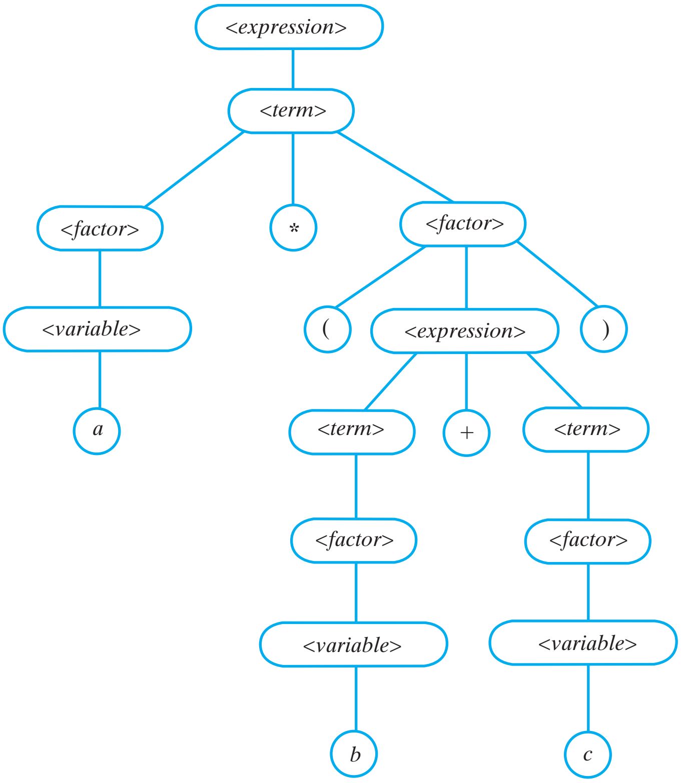
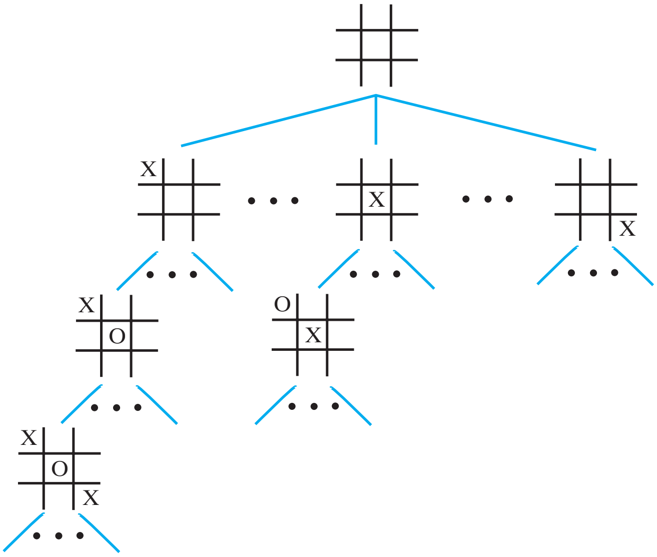


FIGURE 23-22 A portion of a game tree for tic-tac-toe



Chapter Summary

- A tree is a set of nodes connected by edges that indicate the relationships among the nodes. The nodes are arranged in levels that denote their hierarchy. At the top level is a single node called the root.
- At each successive level of a tree are nodes that are the children of the nodes at the previous level. A node with no children is called a leaf. A node that has children is the parent of those children. The root is the only node with no parent. All other nodes have one parent each.
- A node in a binary tree has at most two children. In an n-ary tree, a node can have up to n children. In a general tree, a node can have any number of children.
- The height of a tree is the number of levels in the tree. The height also equals the number of nodes along the longest path between the root and a leaf.
- All leaves in a full binary tree are on the same level, and every nonleaf has exactly two children.
- A full tree of height h has $2^h - 1$ nodes, which is as many as it can contain.
- A complete binary tree is full to its next-to-last level. Its leaves on the last level are filled from left to right.
- The height of a binary tree with n nodes that is either complete or full is $\log_2 (n + 1)$ rounded up.
- You can traverse the nodes in a tree by visiting each node exactly once. Several traversal orders are possible. A level-order traversal begins at the root and visits nodes from left to right, one level at a time. In a preorder traversal, you visit the root before you visit nodes in the root's subtrees. In a postorder traversal, you visit the root after you visit the root's subtrees. For a binary tree, an inorder traversal visits the nodes in the left subtree, then the root, and finally the nodes in the right subtree. For a general tree, an inorder traversal is not well defined.
- An expression tree is a binary tree that represents an algebraic expression whose operators are binary. The operands of the expression appear in the tree's leaves. Any parentheses in an expression do not appear in the tree. You can use an expression tree to evaluate an algebraic expression.
- A decision tree contains a question in each nonleaf. Each child of the nonleaf corresponds to one possible response to the question. Within each of these children is either an additional question or a conclusion. Nodes that are conclusions have no children, and so they are leaves. You can use a decision tree to create an expert system.
- A binary search tree is a binary tree whose nodes contain `Comparable` objects that are organized as follows:
  - The data in a node is greater than the data in the node's left subtree.
  - The data in a node is less than the data in the node's right subtree.
- A search of a binary search tree can be as fast as O(log n) or as slow as O(n). The performance of the search depends on the shape of the tree.
- A heap is a complete binary tree whose nodes contain `Comparable` objects. The data in each node is no smaller (or no larger) than the data in the node's descendants.
- You can use a heap to implement a priority queue.
- Certain rules form a grammar that describes an algebraic expression. A parse tree is a general tree that pictures how these rules apply to a specific expression. You can use a parse tree to check the syntax of a given expression.
- A game tree is a general decision tree that contains the possible moves for a game such as tic-tac-toe.

Exercises

1. In Chapter 7, Figure 7-10a shows the recursive computation of the term F_6 in the Fibonacci sequence. Recall that this sequence is defined as follows:

   $F_0 = 1, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ when $n \geq 2$

   The root of the tree is the value for F_6. The children of F_6 are F_5 and F_4, the two values necessary to compute F_6. Notice that the leaves of the tree contain the base-case values F_0 and F_1.

   Using Figure 7-10a as an example, draw a binary tree that represents the recursive calls in the algorithm `mergeSort`, as given in Segment 9.3 of Chapter 9. Assume an array of 20 entries.

2. What is the height of the shortest binary tree that contains 21 nodes? Is this tree full?

3. Consider a binary tree that has three levels.

   a. What is the maximum number of nodes in this tree?
   b. What is the maximum number of leaves in this tree?
   c. Answer the previous two questions for a binary tree that has 10 levels.

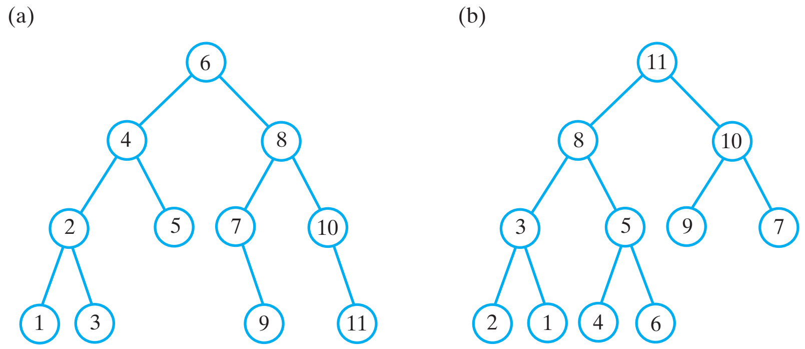
4. Write a recursive algorithm that counts the nodes in a binary tree.

5. Suppose that you draw a binary tree so that no two nodes align vertically. Demonstrate that a vertical line moving from left to right across the tree crosses the nodes in the same order in which an inorder traversal visits nodes.

6. Consider a traversal of a binary tree. Suppose that visiting a node means to simply display the data in the node. What are the results of each of the following traversals of the tree in Figure 23-23a?

   a. Preorder
   b. Postorder
   c. Inorder
   d. Level order

FIGURE 23-23 Two trees for Exercises 6, 7, and 8



7. Repeat Exercise 6, but instead traverse the tree in Figure 23-23b.

8. The two trees in Figure 23-23 contain integer data.

   a. Is the tree in Part *a* a binary search tree? Why or why not?
   b. Is the tree in Part *b* a maxheap? Why or why not?

9. Draw the shortest possible binary search tree from the following strings: *Ann*, *Ben*, *Chad*, *Drew*, *Ella*, *Jenn*, *Jess*, *Kip*, *Luis*, *Pat*, *Rico*, *Scott*, *Tracy*, *Zak*. Is your tree unique?

10. Suppose we know that the preorder traversal of a binary search tree is

6 2 1 4 3 7 10 9 11

What is the postorder traversal of the tree?

11. Draw a maxheap from the strings given in Exercise 9. Is your maxheap unique?

12. Can a binary search tree ever be a maxheap? Explain.

13. Prove that the sum

$$\sum_{i=0}^{h-1} 2^i$$

is equal to $2^h - 1$. Use mathematical induction.

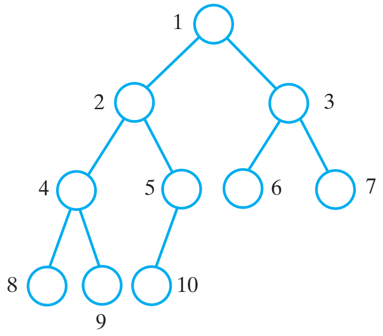
14. At most, how many nodes can a binary tree have at level n? Use induction to prove your answer.

15. Prove that the height of a complete tree having n nodes is $\log_2 (n + 1)$ rounded up.

16. Suppose that you number the nodes of a complete binary tree in the order in which a level-order traversal would visit them. The tree's root would then be node 1. Figure 23-24 shows an example of such a tree. What number, in terms of i, is node i's

a. Sibling, if any
b. Left child, if any
c. Right child, if any
d. Parent, if any

FIGURE 23-24 A complete binary tree with its nodes numbered in level order (Exercise 16)



17. Consider a full n-ary tree of height h. Its leaves are all on the last level. During the traversal of such a tree,

a. What fraction of the time would be spent at a leaf node?
b. What fraction of the time would be spent at nodes in the top half of the tree (nodes at levels 1 through $h/2$)?
c. Compare the fractions in Parts a and b for $n = 2$, 10, and 100.

18. Suppose that you have *n* values to put into an empty binary search tree.

- **a.** In how many different orders can you add the *n* values to the tree? This is not the same as the number of possible binary search trees for *n* values. Explain why.
- **b.** Figure 23-19b shows a binary search tree that effectively acts like a sorted list. In how many different orders can you add the *n* values to the tree such that every parent has only one child? Such a tree has worst-case performance.
- **c.** What is the probability that a randomly constructed binary search tree has worst-case performance? *Hint*: Compute the fraction of the total number of possible orders that results in the worst case.

19. Draw an expression tree for the algebraic expression $(a + b) * (c - d)$.

20. What value does the algorithm given in Segment 23.22 return for the expression tree in Figure 23-14c? Assume that *a* is 3, *b* is 4, and *c* is 5.

21. Draw a parse tree for each of the following algebraic expressions:

- **a.** $a + b * c$
- **b.** $(a + b) * (c - d)$

Projects

1. Draw a class diagram for the guessing game described in Segments 23.24 through 23.26.

For each of the following projects, assume that you have a class that implements `BinaryTreeInterface`*, given in Segment 23.18. The next chapter will discuss such implementations.*

2. Write Java code like the code in Segment 23.19 that creates a binary tree whose eight nodes contain the strings *A*, *B*, . . ., *H*, such that the inorder traversal of the tree visits the nodes in alphabetical order. Write one version that creates a full tree and one version that creates a tree of maximum height. The inorder traversals of both trees should produce the same result.

3. Given an array `wordList` of 15 strings in any order, write Java code that creates a full binary tree whose inorder traversal returns the strings in alphabetical order. *Hint*: Sort the list of strings and then use the eighth string as the root.

4. Design an algorithm that produces a binary expression tree from a given postfix expression. You can assume that the postfix expression is a string that has only binary operators and one-letter operands.

5. Repeat the previous project, but begin with an infix expression instead of a postfix expression.

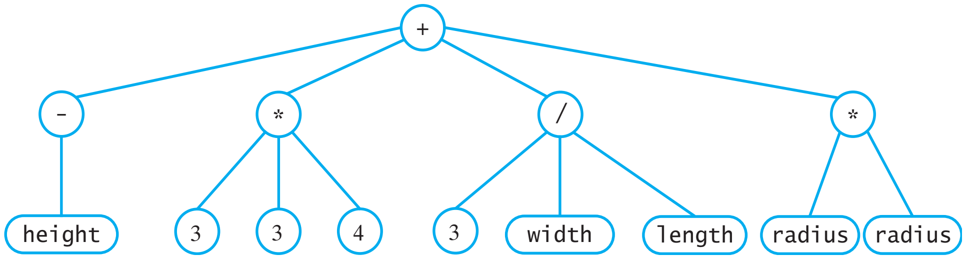
6. Develop an interface `GeneralTreeInterface` for a general tree.

7. Given a class `GeneralTree` that implements the `GeneralTreeInterface` from Project 6, implement a program that will read a fully parenthesized Lisp expression, as described in Projects 6 and 7 of Chapter 5, and create an expression tree. For example, the expression

```
(+ (- height)
   (* 3 3 4)
   (/ 3 width length)
   (* radius radius)
)
```

has the expression tree shown in Figure 23-25.

FIGURE 23-25 An expression tree for Project 7



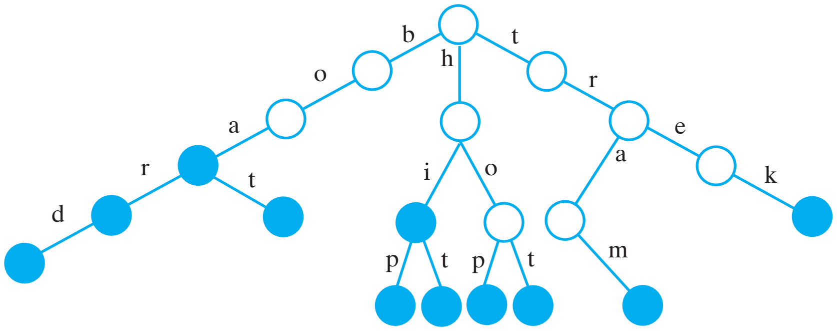
8. Design and create a spelling checker that has at least the following methods:

- `void add(String word)`—Adds a word to a spelling checker's collection of correctly spelled words
- `boolean check(String word)`—Returns true if the given word is spelled correctly

Store the collection of correctly spelled words in a 26-ary tree. Each node in this tree has a child corresponding to a letter in the alphabet. Each node also indicates whether the word represented by the path between the root and the node is spelled correctly. For example, the tree shown in Figure 23-26 depicts this indication as a filled-in node. This tree stores the words "boa," "boar," "boat," "board," "hi," "hip," "hit," "hop," "hot," "trek," and "tram."

To check whether a given word is spelled correctly, you begin at the tree's root and follow the reference associated with the first letter in the word. If the reference is `null`, the word is not in the tree. Otherwise, you follow the reference associated with the second letter in the word, and so on. If you finally arrive at a node, you check whether it indicates a correctly spelled word. For example, the tree in Figure 23-26 indicates that "t," "tr," and "tre" are spelling mistakes, but "trek" is spelled correctly.

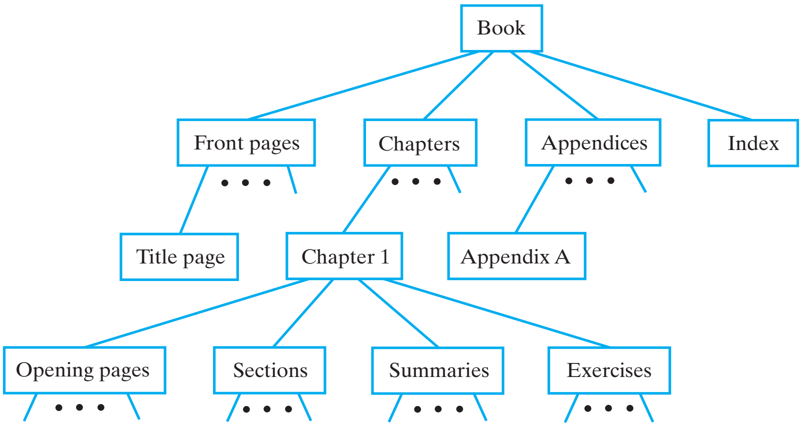
FIGURE 23-26 A general tree for Project 8



Answers to Self-Test Questions

1. **a.** N, O, P, G, H, Q, R, S, T, L, M.
b. I, J.
c. F, G.
d. F, G, N, O, P.
e. F, B, A.
f. A, B, C, D, E, F, I, J, K

2. A tree that represents the organization of this book is a general tree, such as the following:



3. 3, 3, and 4, respectively.

4. **a.** For the tree in Figure 23-6a, n is 7 and h is 3. Since the tree is full and $7 = 2^3 - 1$, the relationship $n = 2^h - 1$ is true. Also, $3 = \log_2 (7 + 1)$, so the relationship $h = \log_2 (n + 1)$ is true.

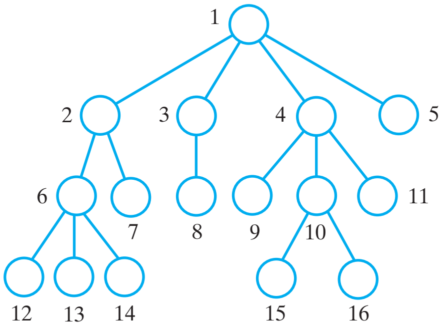
b. For the tree in Figure 23-6b, n is 10 and h is 4. The tree is complete, and $\log_2 (10 + 1)$ is approximately 3.5, which rounded up is 4. Thus, the relationship $h = \log_2 (n + 1)$ rounded up is true.

5. $2^6 - 1$, or 63.

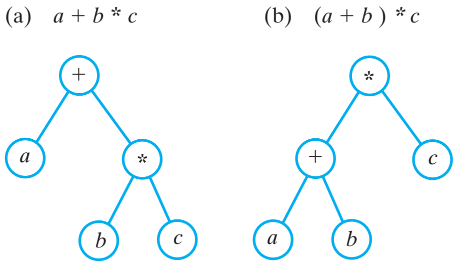
6. $\log_2 (14 + 1)$ is approximately 3.9. When rounded up, we get 4

7. Preorder: Jared, John, James, Mary, Jennifer, Robert, Carole
Postorder: James, Mary, John, Robert, Carole, Jennifer, Jared
Inorder: James, John, Mary, Jared, Robert, Jennifer, Carole
Level order: Jared, John, Jennifer, James, Mary, Robert, Carole

8.



9.



10. Figure 23-14a
Preorder: / *a b*
Inorder: *a* / *b*
Postorder: *a b* /

Figure 23-14c
Preorder: * *a* + *b c*
Inorder: *a* * *b* + *c*
Postorder: *a b c* + *

Figure 23-14d
Preorder: / * *a* + *b* * *c d e*
Inorder: *a* * *b* + *c* * *d* / *e*
Postorder: *a b c d* * + * *e* /

11. The tree in Figure 23-14a is full; the tree in Figure 23-14b is complete.

12. 17.

13. The method `learn` augments the tree under conditions that the class `GuessingGame` must control. It would be inappropriate for a client to invoke this method.

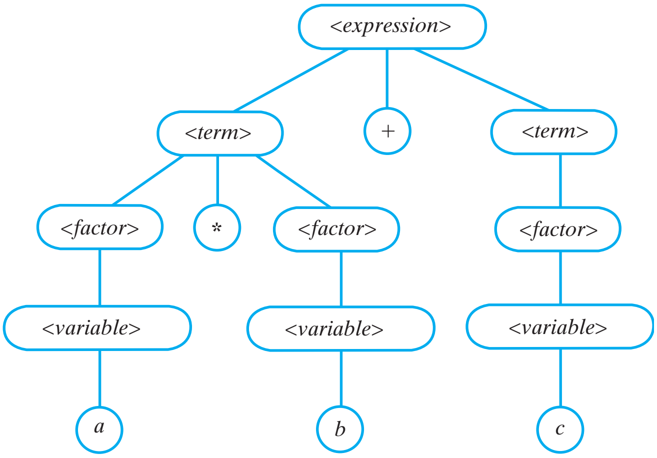
14. 5.

15. The shortest tree has height 2; the tallest tree has height 3.

16. The root of a maxheap contains the object with the largest value. If this object is unique in the set of objects, the root is unique. If another object has the same value, it would be a child of the root. In Figure 23-20a, only 9 can be the root.

17. No. The order of siblings is not specified in a heap, so several different heaps can contain the same data. For example, in Figure 23-20a, you could exchange 2 and 5, or you could exchange the root's two subtrees, and still have a maxheap.

18.



Chapter 24

Tree Implementations

Contents

Prerequisites

Appendix C	Creating Classes from Other Classes
Appendix E	Exception handling
Chapter 5	Stacks
Chapter 10	Queues
Chapter 14	A List Implementation That Links Data
Chapter 23	Trees

Objectives

After studying this chapter, you should be able to

- Describe the necessary operations on a node within a binary tree
- Implement a class of nodes for a binary tree
- Implement a class of binary trees
- Implement an expression tree by extending the class of binary trees
- Describe the necessary operations on a node within a general tree
- Use a binary tree to represent a general tree

The most common implementation of a tree uses a linked structure. Nodes, analogous to the nodes we used in a linked chain, represent each element in the tree. Each node can reference its children, which are other nodes in the tree. This chapter emphasizes binary trees, although it concludes with a brief discussion of general trees. We do not cover binary search trees here, as the entire next chapter is devoted to them.

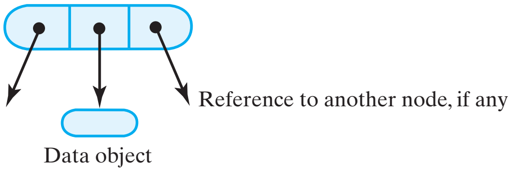
Although we could use either an array or a vector to implement a tree, we will not do so in this chapter. These implementations are attractive only when the tree is complete. In such cases, the link between a parent and child is not stored explicitly, so the data structure is simpler than if the tree is not complete. In Chapter 26, we will encounter a use for a complete tree, so we will postpone until then any other implementation of the tree.

The Nodes in a Binary Tree

24.1 The elements in a tree are called nodes, as are the Java objects in a linked chain. We will use similar objects to represent a tree's nodes and call them nodes as well. The distinction between a node in a tree that you draw and the Java node that represents it usually is not essential.

A node object that represents a node in a tree references both data and the node's children. We could define one class of nodes for all trees, regardless of how many children a node has. But such a class would not be convenient or efficient for a node in a binary tree, since it has at most two children. Figure 24-1 illustrates a node for a binary tree. It contains a reference to a data object and references to its left child and right child, which are other nodes in the tree. Either reference to a child could be `null`. If both of them are `null`, the node is a leaf node.

FIGURE 24-1 A node in a binary tree



Although the nodes in a linked chain belong to a private class `Node` that is internal to classes such as `LinkedStack` and `LList`, our class of tree nodes will not be internal to the class of binary trees. Since any class that extends our fundamental class of binary trees might need to manipulate nodes, we will define our class of tree nodes outside of our binary tree class. But we will not make this class of nodes public. Instead, we will give it package access within a package that contains the classes of the various trees and their interfaces. In this way the node remains an implementation detail that is not available to any of the tree's clients.

Note: A node object in a linked chain references another node in the chain. Although we can process the chain recursively, a node does not reference a chain. Likewise, a node object in a binary tree references other nodes in the tree. Although we often think of a binary tree recursively, as Segment 23.8 describes, a tree node does not reference another tree.

An Interface for a Node

24.2 Listing 24-1 contains a Java interface for a class of nodes suitable for a binary tree. We place the interface in the package `TreePackage` and omit its access modifier. Without this modifier, the interface is accessible only by classes within `TreePackage`.

LISTING 24-1 An interface for the nodes in a binary tree

```java
package TreePackage;
interface BinaryNodeInterface<T>
{
   /** Retrieves the data portion of this node.
       @return the object in the data portion of the node */
   public T getData();

   /** Sets the data portion of this node.
       @param newData  the data object */
   public void setData(T newData);

   /** Retrieves the left child of this node.
       @return the node that is this node's left child */
   public BinaryNodeInterface<T> getLeftChild();

   /** Retrieves the right child of this node.
       @return the node that is this node's right child */
   public BinaryNodeInterface<T> getRightChild();

   /** Sets this node's left child to a given node.
       @param leftChild  a node that will be the left child */
   public void setLeftChild(BinaryNodeInterface<T> leftChild);

   /** Sets this node's right child to a given node.
       @param rightChild  a node that will be the right child */
   public void setRightChild(BinaryNodeInterface<T> rightChild);

   /** Detects whether this node has a left child.
       @return true if the node has a left child */
   public boolean hasLeftChild();

   /** Detects whether this node has a right child.
       @return true if the node has a right child */
   public boolean hasRightChild();

   /** Detects whether this node is a leaf.
       @return true if the node is a leaf */
   public boolean isLeaf();

   /** Counts the nodes in the subtree rooted at this node.
       @return the number of nodes in the subtree rooted at this node */
   public int getNumberOfNodes();

   /** Computes the height of the subtree rooted at this node.
       @return the height of the subtree rooted at this node */
   public int getHeight();
   /** Copies the subtree rooted at this node.
       @return the root of a copy of the subtree rooted at this node */
   public BinaryNodeInterface<T> copy();
} // end BinaryNodeInterface
```

These nodes have more responsibilities than the nodes in a linked chain. Soon you will see how the last three methods in this interface simplify the implementation of the binary tree. But first we will implement this interface as the class BinaryNode.

An Implementation of BinaryNode

24.3 Since we want to hide the node from clients of the binary tree, we place BinaryNode within TreePackage, along with BinaryNodeInterface, and omit its access modifier. We present a portion of the implementation of BinaryNode in Listing 24-1.

LISTING 24-2 The class BinaryNode

```
package TreePackage;
class BinaryNode<T> implements BinaryNodeInterface<T>
{
   private T data;
   private BinaryNode<T> left;
   private BinaryNode<T> right;

   public BinaryNode()
   {
      this(null); // call next constructor
   } // end default constructor

   public BinaryNode(T dataPortion)
   {
      this(dataPortion, null, null); // call next constructor
   } // end constructor

   public BinaryNode(T dataPortion, BinaryNode<T> leftChild,
                                    BinaryNode<T> rightChild)
   {
      data = dataPortion;
      left = leftChild;
      right = rightChild;
   } // end constructor

   public T getData()
   {
      return data;
   } // end getData
   public void setData(T newData)
   {
      data = newData;
   } // end setData

   public BinaryNodeInterface<T> getLeftChild()
   {
      return left;
   } // end getLeftChild

   public void setLeftChild(BinaryNodeInterface<T> leftChild)
   {
      left = (BinaryNode<T>)leftChild;
   } // end setLeftChild

   public boolean hasLeftChild()
   {
      return left != null;
   } // end hasLeftChild

   public boolean isLeaf()
   {
      return (left == null) && (right == null);
   } // end isLeaf

      < Implementations of getRightChild, setRightChild, and hasRightChild are
         analogous to their left-child counterparts. >

      < Implementation of copy appears in Segment 24.6. >

      < Implementations of getHeight and getNumberOfNodes appear in Segment 24.11. >
   . . .
} // end BinaryNode
```

Note: Typically, the class that represents a node in a tree is a detail that you hide from the client. Omitting its access modifier and placing it within a package of classes that implement trees makes it available only to other classes in the package.

An Implementation of the ADT Binary Tree

The previous chapter described several variations of a binary tree. The expression tree and decision tree, for example, each include operations that augment the basic operations of a binary tree. We will define a class of binary trees that can be the superclass of other classes like the class of expression trees.

Creating a Basic Binary Tree

24.4 Recall from Segment 23.18 of the previous chapter the following interface for a class of binary trees:

```
public interface BinaryTreeInterface<T>
       extends TreeInterface<T>, TreeIteratorInterface<T>
{
   public void setTree(T rootData);

   public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
                                   BinaryTreeInterface<T> rightTree);
} // end BinaryTreeInterface
```

VideoNote
Creating a binary tree

Recall that TreeInterface in Segment 23.16 specifies basic operations—getRootData, getHeight, getNumberOfNodes, isEmpty, and clear—common to all trees, and TreeIteratorInterface in Segment 23.17 specifies operations for traversals of a tree. These three interfaces are in our package TreePackage.

We begin our implementation of a binary tree with constructors and the setTree methods, as given in Listing 24-3. The private method privateSetTree has parameters of type BinaryTree, whereas the public setTree that the interface specifies has parameters of type BinaryTreeInterface. We use this private method in the implementation of setTree to simplify the casts from BinaryTreeInterface to BinaryTree.

The third constructor—which has parameters of type BinaryTree—also calls privateSetTree. If it called setTree, we would declare setTree as a final method so that no subclass could override it and thereby change the effect of the constructor. Note as well that we could have named the private method setTree instead of privateSetTree.

LISTING 24-3 A first draft of the class BinaryTree

```
package TreePackage;
import java.util.Iterator;
import java.util.NoSuchElementException;
import StackAndQueuePackage.*; // needed by tree iterators
/**
    A class that implements the ADT binary tree.
    @author Frank M. Carrano.
*/
public class BinaryTree<T> implements BinaryTreeInterface<T>
{
   private BinaryNodeInterface<T> root;

   public BinaryTree()
   {
      root = null;
   } // end default constructor

   public BinaryTree(T rootData)
   {
      root = new BinaryNode<T>(rootData);
   } // end constructor
   public BinaryTree(T rootData, BinaryTree<T> leftTree,
                                 BinaryTree<T> rightTree)
   {
      privateSetTree(rootData, leftTree, rightTree);
   } // end constructor

   public void setTree(T rootData)
   {
      root = new BinaryNode<T>(rootData);
   } // end setTree

   public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
                                   BinaryTreeInterface<T> rightTree)
   {
      privateSetTree(rootData, (BinaryTree<T>)leftTree,
                               (BinaryTree<T>)rightTree);
   } // end setTree

   private void privateSetTree(T rootData, BinaryTree<T> leftTree,
                                           BinaryTree<T> rightTree)
   {
      < FIRST DRAFT - See Segments 24.5 - 24.8 for improvements. >
      root = new BinaryNode<T>(rootData);

      if (leftTree != null)
         root.setLeftChild(leftTree.root);

      if (rightTree != null)
         root.setRightChild(rightTree.root);
   } // end privateSetTree

   < Implementations of getRootData, getHeight, getNumberOfNodes, isEmpty, clear,
     and the methods specified in TreeIteratorInterface are here. >
   . . .
} // end BinaryTree
```

Programming Tip: No cast is needed when you pass an instance of `BinaryTree` to a method whose parameter has the type `BinaryTreeInterface`. The converse, however, requires a cast.

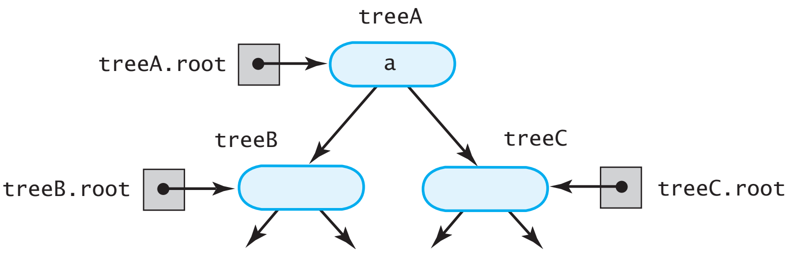
The Method `privateSetTree`

24.5 **A problem.** The implementation of `privateSetTree` just given is really not sufficient to handle all possible uses of the method. Suppose that the client defines three distinct instances of `BinaryTree`—`treeA`, `treeB`, and `treeC`—and executes the statement

```
treeA.setTree(a, treeB, treeC);
```

Since setTree calls privateSetTree, treeA shares nodes with treeB and treeC, as Figure 24-2 illustrates. If the client now changes treeB, for example, treeA also changes. This result generally is undesirable.

FIGURE 24-2 The binary tree treeA shares nodes with treeB and treeC



24.6 **One solution.** One solution is for privateSetTree to copy the nodes in treeB and treeC. Then treeA will be separate and distinct from treeB and treeC. Any subsequent changes to either treeB or treeC will not affect treeA. Let's explore this approach.

Since we are copying nodes, we use the method copy as specified in the interface BinaryNodeInterface. To copy a node, we actually must copy the subtree rooted at the node. Beginning with the node, we copy it and then copy the nodes in its left and right subtrees. Thus, we perform a preorder traversal of the subtree. For simplicity, we will not copy the data in the nodes.

We define the method copy in the class BinaryNode as follows:

```
public BinaryNodeInterface<T> copy()
{
   BinaryNode<T> newRoot = new BinaryNode<T>(data);

   if (left != null)
       newRoot.left = (BinaryNode<T>)left.copy();

   if (right != null)
       newRoot.right = (BinaryNode<T>)right.copy();

   return newRoot;
} // end copy
```

Now privateSetTree can invoke copy to copy the nodes from the two given subtrees:

```
private void privateSetTree(T rootData, BinaryTree<T> leftTree,
                                        BinaryTree<T> rightTree)
{
   root = new BinaryNode<T>(rootData);

   if ((leftTree != null) && !leftTree.isEmpty())
      root.setLeftChild(leftTree.root.copy());

   if ((rightTree != null) && !rightTree.isEmpty())
      root.setRightChild(rightTree.root.copy());
} // end privateSetTree
```

Since copying nodes is expensive, we could consider other implementations of privateSetTree. As you will see next, we must copy at least some nodes in certain situations.

? SELF-TEST

Question 1 In the previous method copy, are the casts to BinaryNode<T> necessary? Explain.

24.7 **Another approach, more problems.** Instead of always copying nodes, `privateSetTree` could behave as follows. Returning to our earlier example,

```
treeA.setTree(a, treeB, treeC);
```

`privateSetTree` first could link the root node of `treeA` to the root nodes of `treeB` and `treeC`. It then could set `treeB.root` and `treeC.root` to `null`. This approach solves the problem of a node appearing in more than one tree, but it makes the trees that the client passed as arguments empty. As a result, two other difficulties can occur.

Suppose that the client executes

```
treeA.setTree(a, treeA, treeB);
```

If `privateSetTree` makes the subtrees empty, `setTree` will destroy the new `treeA`!

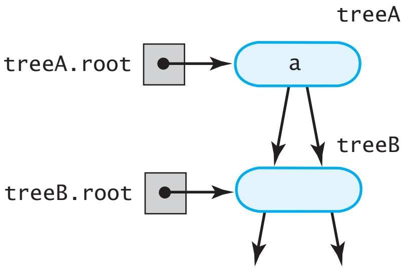
Another problem occurs if the client executes

```
treeA.setTree(a, treeB, treeB);
```

In this case, the left and right subtrees of `treeA`'s root will be identical, as Figure 24-3 illustrates. The solution to this dilemma is to copy the nodes of `treeB` so that the subtrees are distinct. Thus, the general case cannot avoid copying nodes, but such copying will be infrequent.

We now implement a solution to these difficulties.

FIGURE 24-3 `treeA` has identical subtrees



24.8 **The second solution.** To summarize, `privateSetTree` should take the following steps:

1. Create a root node *r* containing the given data.
2. If the left subtree exists and is not empty, attach its root node to *r* as a left child.
3. If the right subtree exists, is not empty, and is distinct from the left subtree, attach its root node to *r* as a right child. But if the right and left subtrees are the same, attach a copy of the right subtree to *r* instead.
4. If the left subtree exists and differs from the tree object used to call `privateSetTree`, set the subtree's data field `root` to `null`.
5. If the right subtree exists and differs from the tree object used to call `privateSetTree`, set the subtree's data field `root` to `null`.

An implementation of `privateSetTree` follows:

```
private void privateSetTree(T rootData, BinaryTree<T> leftTree,
                                        BinaryTree<T> rightTree)
{
   root = new BinaryNode<T>(rootData);
```

```
      if ((leftTree != null) && !leftTree.isEmpty())
         root.setLeftChild(leftTree.root);

      if ((rightTree != null) && !rightTree.isEmpty())
      {
         if (rightTree != leftTree)
            root.setRightChild(rightTree.root);
         else
            root.setRightChild(rightTree.root.copy());
      } // end if

      if ((leftTree != null) && (leftTree != this))
         leftTree.clear();

      if ((rightTree != null) && (rightTree != this))
         rightTree.clear();
   } // end privateSetTree
```

?
SELF-TEST

Question 2 At the end of the implementation of privateSetTree, can you set rightTree to null instead of invoking clear? Explain.

Accessor and Mutator Methods

24.9 The public methods getRootData, isEmpty, and clear are easy to implement. In addition to these methods, we define several protected methods—setRootData, setRootNode, and getRootNode—that will be useful in the implementation of a subclass. The implementations of these methods follow:

VideoNote
Binary tree operations

```
public T getRootData()
{
   T rootData = null;

   if (root != null)
      rootData = root.getData();

   return rootData;
} // end getRootData

public boolean isEmpty()
{
   return root == null;
} // end isEmpty

public void clear()
{
   root = null;
} // end clear

protected void setRootData(T rootData)
{
   root.setData(rootData);
} // end setRootData

protected void setRootNode(BinaryNodeInterface<T> rootNode)
{
   root = rootNode;
} // end setRootNode

protected BinaryNodeInterface<T> getRootNode()
{
   return root;
} // end getRootNode
```

Computing the Height and Counting Nodes

24.10 **Methods within `BinaryTree`.** The methods `getHeight` and `getNumberOfNodes` are more interesting than the methods given in the previous segment. Although we could perform the necessary computations within the class `BinaryTree`, performing them within the class `BinaryNode` is easier. Thus, the following methods of `BinaryTree` invoke analogous methods of `BinaryNode`:

```
public int getHeight()
{
   return root.getHeight();
} // end getHeight

public int getNumberOfNodes()
{
   return root.getNumberOfNodes();
} // end getNumberOfNodes
```

We now complete the methods `getHeight` and `getNumberOfNodes` within `BinaryNode`.

24.11 **Methods within `BinaryNode`.** Within `BinaryNode`, the method `getHeight` returns the height of the subtree rooted at the node used to invoke the method. Likewise, `getNumberOfNodes` returns the number of nodes within that same subtree.

The public method `getHeight` can call a private recursive method `getHeight` that has a node as its parameter. The height of the tree rooted at a node is 1—for the node itself—plus the height of the node's tallest subtree. Thus, we have the following implementation:

```
public int getHeight()
{
  return getHeight(this); // call private getHeight
} // end getHeight

private int getHeight(BinaryNode<T> node)
{
   int height = 0;

   if (node != null)
       height = 1 + Math.max(getHeight(node.left),
                             getHeight(node.right));

   return height;
} // end getHeight
```

We could implement `getNumberOfNodes` by using the same approach, but instead we will show you another way. The number of nodes in a tree rooted at a given node is 1—for the node itself—plus the number of nodes in both the left and right subtrees. Thus, we have the following recursive implementation:

```
public int getNumberOfNodes()
{
   int leftNumber = 0;
   int rightNumber = 0;

   if (left != null)
      leftNumber = left.getNumberOfNodes();

   if (right != null)
      rightNumber = right.getNumberOfNodes();

   return 1 + leftNumber + rightNumber;
} // end getNumberOfNodes
```

Traversals

24.12 **Traversing a binary tree recursively.** The previous chapter described four orders in which we could traverse all the nodes in a binary tree: inorder, preorder, postorder, and level order. An inorder traversal, for example, visits all nodes in the root's left subtree, then visits the root, and finally visits all nodes in the root's right subtree. Since an inorder traversal visits the nodes in the subtrees by using an inorder traversal, its description is recursive.

We could add a recursive method to the class `BinaryTree` to perform an inorder traversal. Such a method, however, must do something specific to or with the data in each node that it visits. For simplicity, we will display the data, even though a class that implements an ADT generally should not perform input or output.

For the method to process the subtrees recursively, it needs the root of a subtree as a parameter. To hide this detail from the client, we make the recursive method private and call it from a public method that has no parameters. Thus, we have the following result:

```
public void inorderTraverse()
{
   inorderTraverse(root);
} // end inorderTraverse

private void inorderTraverse(BinaryNodeInterface<T> node)
{
   if (node != null)
   {
      inorderTraverse(node.getLeftChild());
      System.out.println(node.getData());
      inorderTraverse(node.getRightChild());
   } // end if
} // end inorderTraverse
```

We could implement similar methods for preorder and postorder traversals.

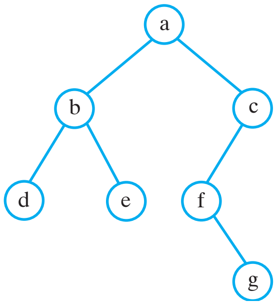
Question 3 Trace the method `inorderTraverse` with the binary tree in Figure 24-4. What data is displayed?

Question 4 Implement a recursive method `preorderTraverse` that displays the data in a binary tree in preorder.

Note: Generally, the methods in a class that implements an ADT should not perform input and output. We are doing so here to simplify the discussion that follows. However, instead of actually displaying the data in the tree, a method like `inorderTraverse` could return a string composed of the data. The client that uses the tree could display this string by using a statement such as

```
    System.out.println(myTree.inorderTraverse());
```

FIGURE 24-4 A binary tree



24.13 **Traversals that use an iterator.** A method such as `inorderTraverse` is not hard to implement, but this method only displays the data during the traversal. In addition, the entire traversal takes place once the method is invoked. To provide the client with more flexibility, we should define the traversals as iterators. In this way, the client can do more than simply display data during a visit and can control when each visit takes place.

Recall that Java's interface `Iterator` declares the methods `hasNext` and `next`. These methods enable a client to retrieve the data from the current node in the traversal at any time. That is, the client can retrieve a node's data, do something with it, perhaps do something else, and then retrieve the data in the next node in the iteration.

If we look at `BinaryTreeInterface` in Segment 24.4, we see that the class `BinaryTree` must implement the methods in the interface `TreeIteratorInterface`. For example, the method `getInorderIterator` can be implemented within `BinaryTree` as follows:

```
public Iterator<T> getInorderIterator()
{
   return new InorderIterator();
} // end getInorderIterator
```

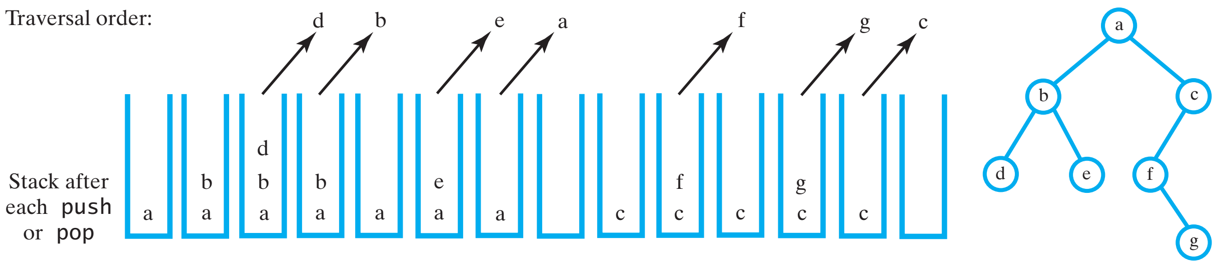
As we did in earlier chapters, we define the class `InorderIterator` as a private inner class of `BinaryTree`.

An iterator must be able to pause during a traversal. This suggests that we not use recursion in its implementation. Chapter 5 showed how to use a stack instead of recursion. That is what we will do here.

24.14 **An iterative version of `inorderTraverse`.** Before we define an iterator, let's consider an iterative version of the method `inorderTraverse`. This method will be a little easier to construct than the iterator, yet it will take similar steps.

Figure 24-5 shows the tree in Figure 24-4 and the result of using a stack to perform its inorder traversal. We begin by pushing the root, *a*, onto the stack. We then traverse to the left as far as possible, pushing each node onto the stack. We then pop the *d* from the stack and display it. Since *d* has no children, we pop the stack again and display *b*. Now *b* has a right child, *e*, which we push onto the stack. Since *e* has no children, we pop it from the stack and display it. The process continues until we have visited all the nodes—that is, until both the stack is empty and the current node is `null`.

FIGURE 24-5 Using a stack to perform an inorder traversal of a binary tree



Here is an iterative implementation of `inorderTraverse`:

```
public void inorderTraverse()
{
   StackInterface<BinaryNodeInterface<T>> nodeStack =
                                 new LinkedStack<BinaryNodeInterface<T>>();
   BinaryNodeInterface<T> currentNode = root;

   while (!nodeStack.isEmpty() || (currentNode != null))
   {
      // find leftmost node with no left child
      while (currentNode != null)
      {
         nodeStack.push(currentNode);
         currentNode = currentNode.getLeftChild();
      } // end while

      // visit leftmost node, then traverse its right subtree
      if (!nodeStack.isEmpty())
      {
         BinaryNodeInterface<T> nextNode = nodeStack.pop();
         assert nextNode != null;   // since nodeStack was not empty
                                    // before the pop
         System.out.println(nextNode.getData());
         currentNode = nextNode.getRightChild();
      } // end if
   } // end while
} // end inorderTraverse
```

Question 5 In the previous method, if you replace each occurrence of `BinaryNodeInterface` with `BinaryNode`, what other changes, if any, must you make?

24.15 **The private class `InorderIterator`.** Now let's implement an inorder traversal as an iterator. We distribute the logic of the previous method `inorderTraverse` among the iterator's constructor and the methods `hasNext` and `next`. The stack and the variable `currentNode` are data fields in the iterator class. The method `next` advances `currentNode`, adds to the stack as necessary, and eventually pops the stack to return the data in the node that is next in the iteration. Thus, the implementation of the private inner class `InorderIterator` appears as given in Listing 24-3.

LISTING 24-4 The private inner class `InorderIterator`

```java
private class InorderIterator implements Iterator<T>
{
   private StackInterface<BinaryNodeInterface<T>> nodeStack;
   private BinaryNodeInterface<T> currentNode;

   public InorderIterator()
   {
      nodeStack = new LinkedStack<BinaryNodeInterface<T>>();
      currentNode = root;
   } // end default constructor

   public boolean hasNext()
   {
      return !nodeStack.isEmpty() || (currentNode != null);
   } // end hasNext

   public T next()
   {
      BinaryNodeInterface<T> nextNode = null;

      // find leftmost node with no left child
      while (currentNode != null)
      {
         nodeStack.push(currentNode);
         currentNode = currentNode.getLeftChild();
      } // end while

      // get leftmost node, then move to its right subtree
      if (!nodeStack.isEmpty())
      {
         nextNode = nodeStack.pop();
         assert nextNode != null; // since nodeStack was not empty
                                  // before the pop
         currentNode = nextNode.getRightChild();
      }
      else
         throw new NoSuchElementException();

      return nextNode.getData();
   } // end next

   public void remove()
   {
      throw new UnsupportedOperationException();
   } // end remove
} // end InorderIterator
```

24.16 **Preorder, postorder, and level-order traversals.** Figure 24-6 shows the result of using a stack to perform a preorder traversal and a postorder traversal of the tree in Figure 24-4. A level-order traversal has logic similar to that of a preorder traversal, but we use a queue instead of a stack.

FIGURE 24-6 Using a stack to traverse a binary tree in (a) preorder; (b) postorder

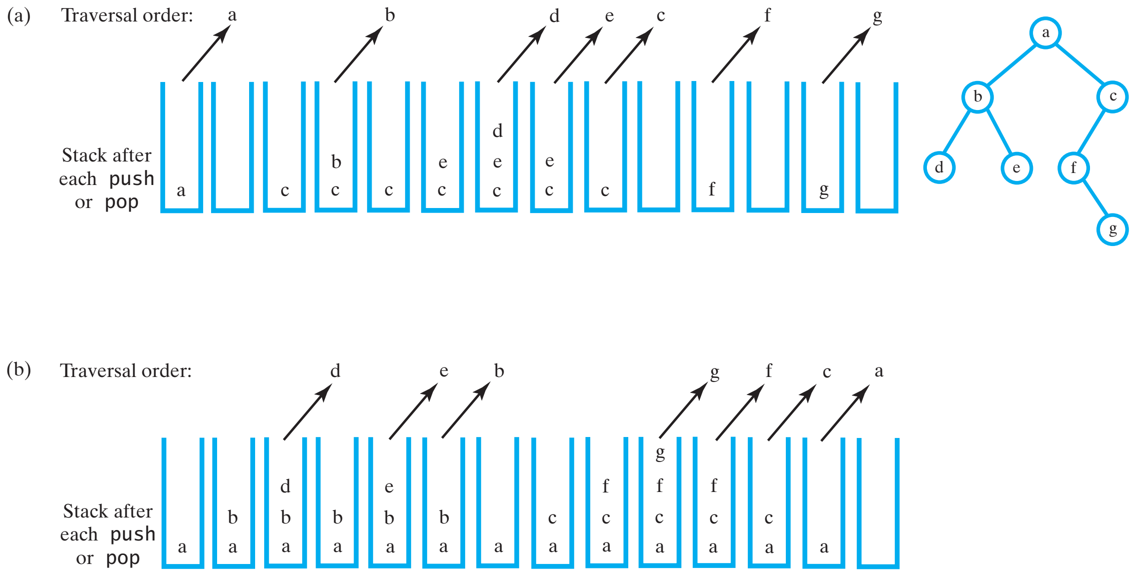
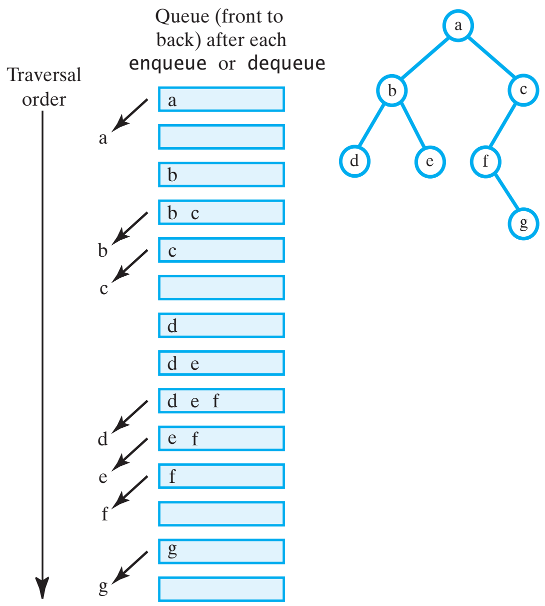

(a)
Traversal order: a b d e c f g
Stack after each push or pop
a
c
b c
c
e c
d e c
e c
c
f
g
a
b
c
d
e
f
g
(b)
Traversal order: d e b g f c a
Stack after each push or pop
a
b a
d b a
b a
e b a
b a
a
c a
f c a
g f c a
f c a
c a
a

Figure 24-7 shows the result of using a queue to perform a level-order traversal of the same tree. We leave the implementation of the necessary iterator classes for you as an exercise.

FIGURE 24-7 Using a queue to traverse a binary tree in level order



Programming Tip: An iterator object that has not traversed the entire binary tree can be adversely affected by changes to the tree.

Note: A complete traversal of an *n*-node binary tree is an O(*n*) operation, if visiting a node is O(1), for both recursive and iterative implementations.

An Implementation of an Expression Tree

24.17 In the previous chapter, you saw that an expression tree is a binary tree that represents an algebraic expression. Figure 23-14 provided some examples of these trees. By using the algorithm given in Segment 23.22, we can evaluate the expression in this type of tree.

We can define an interface for an expression tree by extending the interface for a binary tree and adding a declaration for the method `evaluate`, as shown in Listing 24-4. Since you can treat the components of an expression as strings, we assume that an expression tree contains strings as its data.

LISTING 24-5 An interface for an expression tree

```
package TreePackage;
public interface ExpressionTreeInterface
                 extends BinaryTreeInterface<String>
{
   /** Computes the value of the expression in this tree.
       @return the value of the expression */
   public double evaluate();
} // end ExpressionTreeInterface
```

24.18 An expression tree is a binary tree, so we can derive a class of expression trees from `BinaryTree`. We implement the method `evaluate` as a part of the derived class. A portion of the class `ExpressionTree` appears in Listing 24-5.

LISTING 24-6 The class `ExpressionTree`

```
package TreePackage;
public class ExpressionTree extends BinaryTree<String>
                            implements ExpressionTreeInterface
{
   public ExpressionTree()
   {
   } // end default constructor

   public double evaluate()
   {
      return evaluate(getRootNode());
   } // end evaluate
   private double evaluate(BinaryNodeInterface<String> rootNode)
   {
      double result;
      if (rootNode == null)
         result = 0;
      else if (rootNode.isLeaf())
      {
         String variable = rootNode.getData();
         result = getValueOf(variable);
      }
      else
      {
         double firstOperand = evaluate(rootNode.getLeftChild());
         double secondOperand = evaluate(rootNode.getRightChild());
         String operator = rootNode.getData();
         result = compute(operator, firstOperand, secondOperand);
       } // end if

       return result;
   } // end evaluate

   private double getValueOf(String variable)
   {
      . . .
   } // end getValueOf

   private double compute(String operator, double firstOperand,
                                           double secondOperand)
   {
      . . .
   } // end compute
} // end ExpressionTree
```

The public method `evaluate` calls a private method `evaluate` that is recursive. This private method calls the private methods `getValueOf` and `compute` as well as methods declared in `BinaryNodeInterface`. The method `getValueOf` returns the numeric value of a given variable in the expression, and `compute` returns the result of a given arithmetic operation and two given operands.

Notice how important the methods of the class `BinaryNode` are to the implementation of `evaluate`. For this reason, we do not want `BinaryNode` to be hidden within `BinaryTree`. Rather, it should be part of a package.

SELF-TEST

Question 6 Trace the method `evaluate` for the expression tree in Figure 23-14c of the previous chapter. What value is returned? Assume that *a* is 3, *b* is 4, and *c* is 5.

General Trees

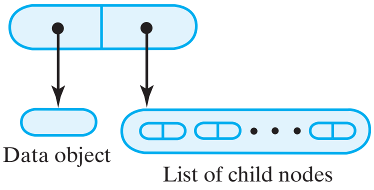
To wrap up our discussion of tree implementations, we will consider one way to represent a node for a general tree. Rather than developing an implementation of a general tree that uses this node, we will see that we can use a binary tree to represent a general tree.

A Node for a General Tree

24.19 Since a node in a binary tree can have only two children, it is reasonable for each node to contain two references to these children. In addition, the number of node operations that test for, set, or get each child is reasonable. But dealing with more children per node in this way quickly becomes unwieldy.

We can define a node for a general tree that accommodates any number of children by referencing an object, such as a list or a vector, that contains the children. For example, the node in Figure 24-8 contains two references. One reference is to the data object, and the other is to a list of child nodes. An iterator for the list enables us to access these children.

FIGURE 24-8 A node for a general tree



In the interface for a general node given in Listing 24-6, `getChildrenIterator` returns an iterator to the node's children. A separate operation adds a child to the node, assuming that the children are in no particular order. If the order of the children is important, the iterator could provide an operation to insert a new child at the current position within the iteration.

LISTING 24-7 An interface for a node in a general tree

```java
package TreePackage;
import java.util.Iterator;
interface GeneralNodeInterface<T>
{
   public T getData();
   public void setData(T newData);
   public boolean isLeaf();
   public Iterator<T> getChildrenIterator();
   public void addChild(GeneralNodeInterface<T> newChild);
} // end GeneralNodeInterface
```

Note: Algorithms for the operations on a general tree are more complex than those for a binary tree due to the flexible number of children per node in a general tree. For this reason, general trees are often represented by binary trees. The next section discusses how to transform a general tree into an equivalent binary tree.

Using a Binary Tree to Represent a General Tree

24.20 Instead of the implementation just suggested, we can use a binary tree to represent any general tree. For example, let's represent the general tree in Figure 24-9a as a binary tree. As an intermediate step, we connect the nodes with new edges, as follows. We give the root *A* one of its original children—*B* in this case—as a left child. We then draw an edge from *B* to its sibling *C* and from *C* to another sibling *D*, as Figure 24-9b shows. Likewise, we give each parent in the general tree one of its original children as a left child in the binary tree, and link these children by edges.

If we consider each node in Figure 24-9b that is to the right of its sibling as the right child of that sibling, we will have a binary tree that has an unorthodox form. We can move the nodes in the drawing without disconnecting them to get the familiar look of a binary tree, as Figure 24-9c shows.

24.21 **Traversals.** Let's examine the various traversals of the general tree in Figure 24-9a and compare them with traversals of the equivalent binary tree pictured in Figure 24-9c. The general tree has the following traversals:

Preorder:	A B E F C G H I D J
Postorder:	E F B G H I C J D A
Level order:	A B C D E F G H I J

The traversals of the binary tree are as follows:

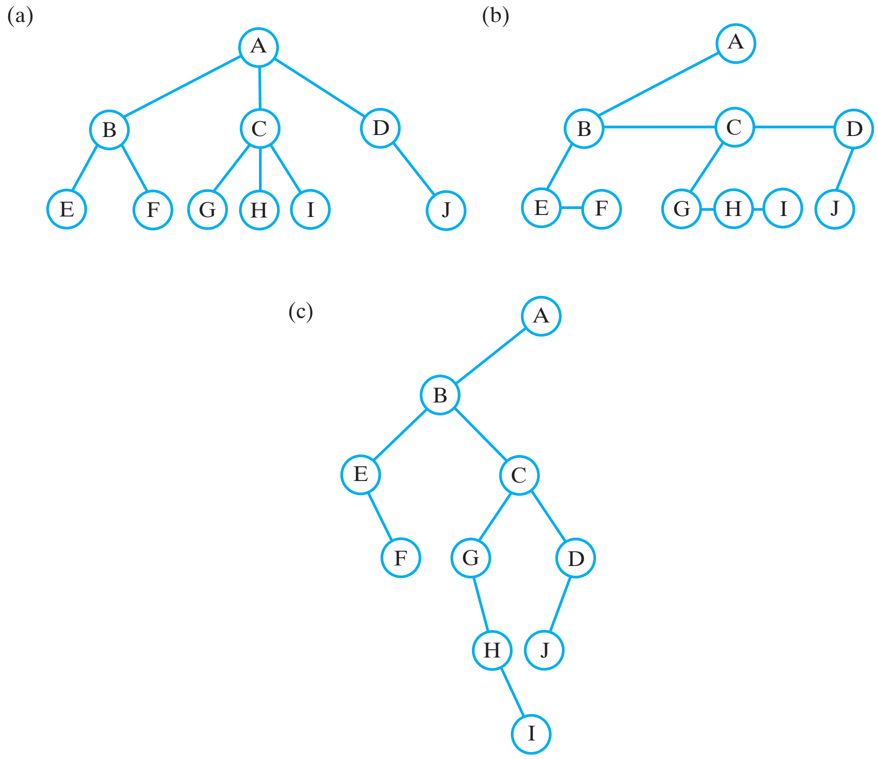
Preorder:	A B E F C G H I D J
Postorder:	F E I H G J D C B A
Level order:	A B E C F G D H J I
Inorder:	E F B G H I C J D A

The preorder traversals of the two trees are the same. The postorder traversal of the general tree is the same as the inorder traversal of the binary tree. We must invent a new kind of traversal of the binary tree to get the same results as a level-order traversal of the general tree. We leave that task to you as an exercise.

? SELF-TEST

Question 7 What binary tree can represent the general tree in Figure 23-1 of the previous chapter?

FIGURE 24-9 (a) A general tree; (b) an equivalent binary tree; (c) a more conventional view of the same binary tree



Chapter Summary

- A node in a binary tree is an object that references a data object and two child nodes in the tree.
- A basic class of binary trees contains methods common to all trees: `getRootData`, `getHeight`, `getNumberOfNodes`, `isEmpty`, `clear`, and various traversals. The basic class also has a method that sets the root and subtrees of an existing binary tree to given values.
- The implementation of `getHeight` and `getNumberOfNodes` is easier if the class of nodes has similar methods.
- Preorder, postorder, and inorder traversals have simple recursive implementations. But to implement a traversal as an iterator, you must use an iterative approach, since an iterator needs to be able to pause during the traversal. You use a stack for preorder, postorder, and inorder traversals; you use a queue for a level-order traversal.
- You can derive a particular binary tree, such as an expression tree, from the class of basic binary trees.
- A node in a general tree is an object that references its children and a data object. To accommodate any number of children, the node can reference a list or a vector, for example. An iterator can provide access to the children. In this way, the node contains only two references.
- Instead of creating a general node for a general tree, you can use a binary tree to represent a general tree.

Programming Tips

- No cast is needed when you pass an instance of `BinaryTree` to a method whose parameter has the type `BinaryTreeInterface`. The converse, however, requires a cast.
- An iterator object that has not traversed the entire binary tree can be adversely affected by changes to the tree.

Exercises

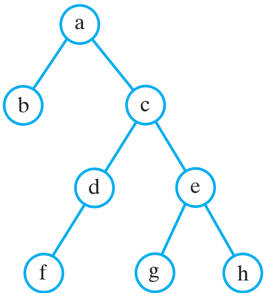
1. Implement `getHeight` in the class `BinaryNode`, using the approach that Segment 24.11 uses for `getNumberOfNodes`. That is, `getHeight` should not call a private method.

2. Implement `getNumberOfNodes` in the class `BinaryNode`, using the approach that Segment 24.11 uses for `getHeight`. That is, `getNumberOfNodes` should not call a private method.

3. In Segment 24.12, Question 4 asked you to implement a recursive preorder traversal of a binary tree. Implement a recursive method `postorderTraverse` that displays the data in a binary tree in postorder.

4. Trace the iterative method `inorderTraverse` given in Segment 24.14 with the binary tree in Figure 24-10. Show the contents of the stack after each `push` and `pop`.

FIGURE 24-10 A binary tree for Exercises 4, 5, 6, and 17



5. Show the contents of the stack after each `push` and `pop` during a preorder traversal of the binary tree in Figure 24-10. Repeat for a postorder traversal.

6. Show the contents of the queue after each `enqueue` and `dequeue` during a level-order traversal of the binary tree in Figure 24-10.

7. Suppose we want to create a method for the class `BinaryTree` that counts the number of times an object occurs in the tree. The header of the method could be as follows:

```
public int count(T anObject)
```

a. Write this method using a private recursive method of the same name.
b. Write the method using one of the iterators of the binary tree.
c. Compare the efficiencies of the previous two versions of the method.

8. Trace the method `evaluate` given in Segment 24.18 for the expression tree in Figure 23-14d of the previous chapter. What value is returned? Assume that *a* is 2, *b* is 4, *c* is 5, *d* is 6, and *e* is 4.

9. Write a recursive definition for the number of possible binary trees that have distinct shapes and contain *n* nodes.

10. What binary tree represents the general tree in each of the following figures from the previous chapter?

- **a.** Figure 23-5
- **b.** Figure 23-21

11. Given a general tree, consider an equivalent binary tree. Define a traversal of this binary tree that is equivalent to a level-order traversal of the general tree.

12. Knowing the preorder and inorder traversals of a binary tree will enable you to uniquely define the tree. The same is true for the postorder and inorder traversals.

- **a.** Draw the unique binary tree that has the following preorder and inorder traversals:

  Preorder: A, B, D, E, C, F, G, H
  Inorder: E, D, B, A, G, F, H, C

- **b.** Draw the unique binary tree that has the following postorder and inorder traversals:

  Postorder: B, D, F, G, E, C, A
  Inorder: B, A, D, C, F, E, G

13. Although you can uniquely construct a binary tree from either its preorder and inorder traversals or its postorder and inorder traversals, more than one binary tree can have the same preorder traversal and the same postorder traversal. Give an example of two different binary trees that have the same preorder and postorder traversals.

14. Suppose we want to create a method for the class `BinaryTree` that decides whether two trees have the same structure. The header of the method could be as follows:

```
public boolean isIsomorphic(BinaryTreeInterface<T> otherTree)
```

Write this method, using a private recursive method of the same name.

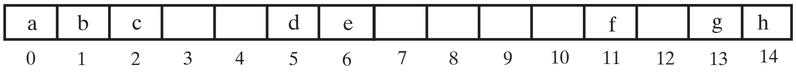
15. Consider two binary trees that have the same structure. A node in one tree can contain different data than the corresponding node in the other tree. Write code that uses a dictionary to map the objects in the first tree to the corresponding objects in the second tree.

16. Sometimes you need to move from a tree node to its parent. To do this, a binary tree node would need a reference to its parent. You then would be able to traverse a path from a leaf to the root. Redesign the node used in a binary tree so that each one has a reference to its parent as well as to its left child and right child. What methods will need to be changed?

17. Another way of representing a binary tree is to use an array. The items in the tree are assigned to locations in the array in a level-order fashion. For example, Figure 24-11 shows an array that represents the binary tree in Figure 24-10. Notice that gaps in the array correspond to missing nodes in the tree. The array is sufficiently large to represent any binary tree up to height 4.

- **a.** What are the indices of the children of the node stored at index *i*?
- **b.** What is the parent of the node stored at index *i*?
- **c.** What are the advantages and disadvantages of this representation?

FIGURE 24-11 An array for Exercise 17 that represents the binary tree in Figure 24-10



a	b	c			d	e					f		g	h
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

18. Add a method to the class `BinaryTree` that accepts as its argument a `BinaryTree` object and returns true if the argument tree is structurally identical to this binary tree. Two trees are structurally identical if their nodes correspond in value and position.

PROJECTS

1. Using the examples in Figures 24-6 and 24-7 to suggest algorithms, implement iterator classes for preorder, postorder, and level-order traversals of a binary tree.

2. Write a Java program that distinguishes among 10 different animals. The program should play a guessing game similar to the one described in Segment 23.24 of the previous chapter. The user thinks of one of the 10 animals, and the program asks a sequence of questions until it can guess the animal.

   Your program should learn from the user. If the program makes an incorrect guess, it asks the user to enter a new question that can distinguish between the correct animal and the program's incorrect guess. The decision tree should be updated with this new question.

3. Complete the implementation of an expression tree that was begun in Segment 24.18. Add two constructors that create an expression tree given either a postfix expression or an infix expression. Projects 4 and 5 in the previous chapter asked you to design algorithms for these two tasks. To simplify the method `getValueOf`, you can restrict the choice of variables and give them specific values.

4. Consider the redesigned node for a binary tree that Exercise 16 describes. Add an additional data field to the node to record the height of the subtree tree rooted at the node. Modify all the methods in the implementation of the binary tree so that the height field is updated anytime the structure of the tree changes.

5. Project 6 in the previous chapter asked you to develop an interface `GeneralTreeInterface` for a general tree. Using that interface as a base, develop an interface `GeneralSearchTreeInterface` for a general search tree. Then write the class `GeneralSearchTree` that implements `GeneralSearchTreeInterface`. Use a binary tree to represent the general tree, as described in Segment 24.20. Implement iterators for the preorder and postorder traversals. As an extra challenge, implement an iterator for the level-order traversal.

6. Some implementations of a binary tree do not use `null` to indicate the absence of a child. Instead, they use references to a single dummy node. The reference to an empty tree is to this same dummy node. Modify the implementation of `BinaryTree` in this way.

7. Implement the class `ArrayBinaryTree` that uses an array representation of the tree, as described in Exercise 17.

8. Create a dictionary for a glossary of terms, as described in Project 13 of Chapter 20. Instead of an array of sorted lists, use a 26-ary tree to represent the glossary. Figure 23-26 in Project 8 of the previous chapter illustrates such a tree, but one that is used for a spelling checker. To adapt that tree for this project, place a term's definition in the filled-in node instead of an indicator of the term's spelling.

9. *Huffman coding* is a technique that compresses the size of data. For example, an audio file in mp3 format is a compressed version of an original recording that, for most people, sounds like the original. This so-called lossless data compression is a result of Huffman coding.

   Although ASCII and Unicode use bit strings having a fixed length—8 and 16, respectively—to represent symbols, Huffman codes have variable lengths. These codes are based on the frequency of occurrence of a symbol in a given data set. Symbols that occur frequently have shorter codes than those occurring less frequently. A binary tree—called a **Huffman tree**—is used to generate these codes.

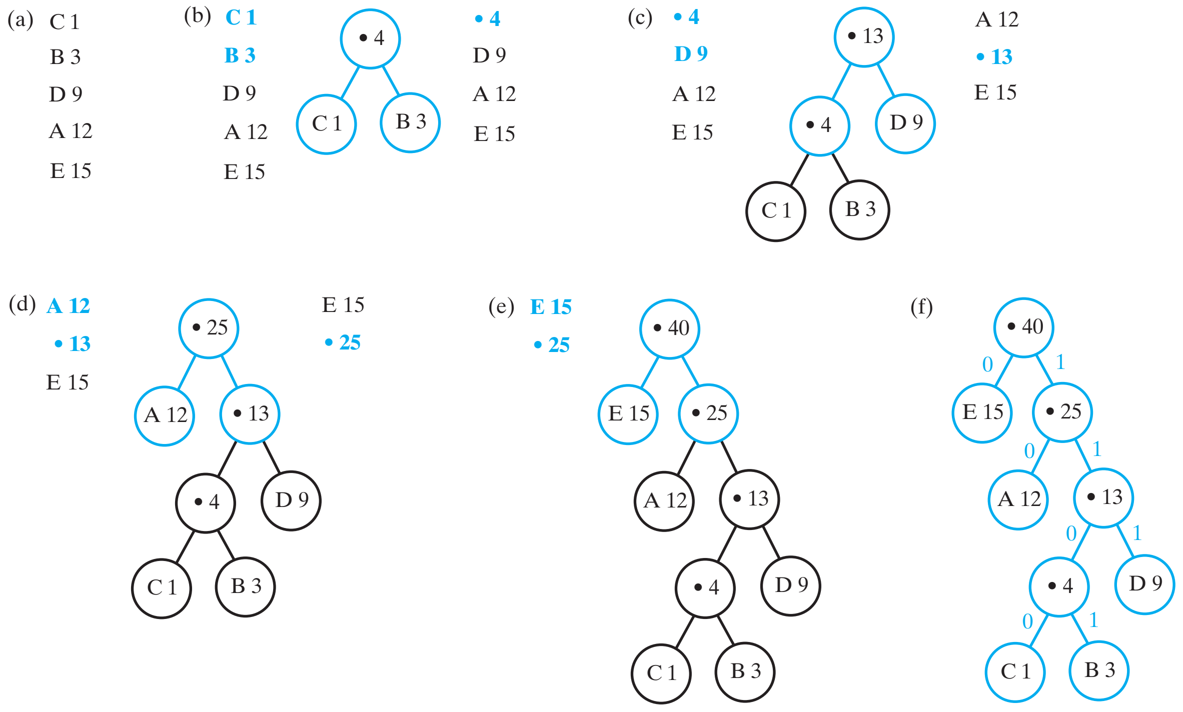
For example, let's encode some text composed only of the letters A through E. Suppose these letters occur with the following frequencies: A, 12 times; B, 3 times; C, 1 time; D, 9 times; and E, 15 times. We need to arrange these letters by their frequencies in increasing order. To do so, we associate each letter with its frequency of occurrence and add these pairs to a collection, such as a sorted list or a priority queue. The result is shown in Figure 24-12a. Now we remove the two entries having the lowest frequencies and make them leaves in a binary tree. The parent of these leaves is a node containing the sum of the frequencies in the leaves, as illustrated in Figure 24-12b. Since the parent contains only a frequency, the letter portion of the node is `null`, which is shown as • in the figure. We now add the contents of the parent to our list or queue, as shown to the right of the tree in Figure 24-12b.

When we remove the next two entries from the list, we create a new leaf containing D 9 and join it to the existing tree with a new root containing the sum of the frequencies in its two children. The result is given in Figure 24-12c. Note that we then place the contents of this new root in its correct order within the remaining data. Parts *d* and *e* of the figure show the remaining steps in this process.

Figure 24-12f shows the resulting binary tree with its left links and right links labeled with 0 and 1, respectively. To encode a character, you begin at a leaf and traverse the tree to its root, recording 0s and 1s in reverse order according to the left and right branches that you take. The Huffman codes for our example are as follows: A is 10, B is 1101, C is 1100, D is 111, and E is 0. To decode a Huffman code, you traverse the tree from its root to a leaf by taking a left branch for each 0 encountered and a right branch for each 1 encountered.

Write a program that reads a text file of alphabetic data, creates a Huffman tree, and uses the tree to compress the file. Your program should then take the compressed file and, using your tree, decode it.

FIGURE 24-12 The steps in creating a binary tree for Huffman coding



Answers to Self-Test Questions

1. Yes. The fields `left` and `right` of `BinaryNode` (see Segment 24.3) have `BinaryNode<T>` as their data type, but the return type of the method `copy` is `BinaryNodeInterface<T>`.

2. No. Setting `rightTree` to `null` affects only the local copy of the reference argument `rightTree`. An analogous comment applies to `leftTree`.

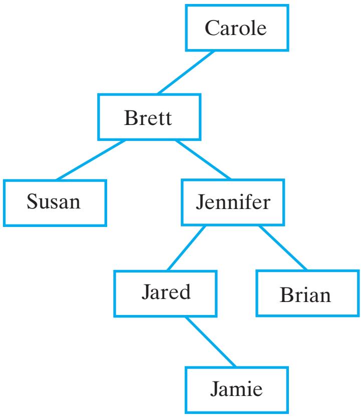
3. The data in the objects *d*, *b*, *e*, *a*, *f*, *g*, and *c* is displayed on separate lines.

4.
```
public void preorderTraverse()
{
   preorderTraverse(root);
} // end preorderTraverse

private void preorderTraverse(BinaryNodeInterface<T> node)
{
   if (node != null)
   {
      System.out.println(node.getData());
      preorderTraverse(node.getLeftChild());
      preorderTraverse(node.getRightChild());
   } // end if
} // end preorderTraverse
```

5. You must cast to `BinaryNode<T>` three times, as follows:
   - `BinaryNode<T> currentNode = (BinaryNode<T>)root;`
   - `currentNode = (BinaryNode<T>)currentNode.getLeftChild();`
   - `currentNode = (BinaryNode<T>)nextNode.getRightChild();`

6. 27.

7.



Chapter 25

A Binary Search Tree Implementation

Contents

Prerequisites

Appendix	C	Creating Classes from Other Classes
Chapter	7	Recursion
Chapter	19	Dictionaries
Chapter	23	Trees
Chapter	24	Tree Implementations

Objectives

After studying this chapter, you should be able to

- Decide whether a binary tree is a binary search tree
- Locate a given entry in a binary search tree using the fewest comparisons
- Traverse the entries in a binary search tree in sorted order
- Add a new entry to a binary search tree
- Remove an entry from a binary search tree
- Describe the efficiency of operations on a binary search tree
- Use a binary search tree to implement the ADT dictionary

Recall from Chapter 23 that a search tree stores data in a way that facilitates searching. In particular, we saw the binary search tree, which is both a binary tree and a search tree. The nature of a binary search tree enables us to search it by using a simple recursive algorithm. This algorithm is similar in spirit to a binary search of an array and can be just as efficient. However, the shape of a binary search tree affects the efficiency of this algorithm. Since we can create several different binary search trees from the same data, we want to pick the tree whose shape provides the most efficient search.

For a database that remains stable, a binary search tree provides a relatively simple way to achieve an efficient search. Most databases, however, change to remain current. Thus, we must add nodes to and remove nodes from the binary search tree. Unfortunately, these operations change the shape of the tree, often making a search less efficient.

This chapter implements the binary search tree and, in doing so, describes the algorithms for adding and removing entries. Chapter 27 looks at ways that a search tree can provide an efficient search despite additions and removals.

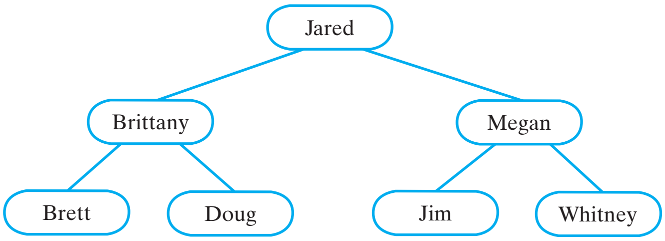
Getting Started

25.1 A **binary search tree** is a binary tree whose nodes contain `Comparable` objects and are organized as follows. For each node in the tree,

- The data in a node is greater than the data in the node's left subtree
- The data in a node is less than the data in the node's right subtree

Figure 25-1 shows the binary search tree that you saw in Chapter 23.

FIGURE 25-1 A binary search tree of names



Recall that a `Comparable` object belongs to a class that implements the interface `Comparable`. We use the class's method `compareTo` to compare such objects. The basis for this comparison varies from class to class, depending on the data fields `compareTo` examines.

An Interface for the Binary Search Tree

25.2 **The operations.** In addition to the common operations of a tree, as given in the interface `TreeInterface`, a binary search tree has basic database operations that search, retrieve, add, remove, and traverse its entries. We can design an interface for a binary search tree, as well as for other search trees that you will see in Chapter 27. Listing 25-1 provides such an interface.

LISTING 25-1 An interface for a search tree

```
package TreePackage;
import java.util.Iterator;
public interface SearchTreeInterface<T extends Comparable<? super T>>
       extends TreeInterface<T>
{
   /** Searches for a specific entry in this tree.
       @param entry  an object to be found
       @return true if the object was found in the tree */
   public boolean contains(T entry);

   /** Retrieves a specific entry in this tree.
       @param entry  an object to be found
       @return either the object that was found in the tree or
               null if no such object exists */
   public T getEntry(T entry);

   /** Adds a new entry to this tree.
       If the entry matches an object that exists in the tree
       already, replaces the object with the new entry.
       @param newEntry  an object to be added to the tree
       @return either null if newEntry was not in the tree already, or
               an existing entry that matched the parameter newEntry
               and has been replaced in the tree */
   public T add(T newEntry);

   /** Removes a specific entry from this tree.
       @param entry  an object to be removed
       @return either the object that was removed from the tree or
               null if no such object exists */
   public T remove(T entry);

   /** Creates an iterator that traverses all entries in this tree.
       @return an iterator that provides sequential and ordered access
               to the entries in the tree */
   public Iterator<T> getInorderIterator();
} // end SearchTreeInterface
```

25.3 **Understanding the specifications.** These specifications allow us to use a binary search tree in the implementation of the ADT dictionary, as you will see later in this chapter. The methods use return values instead of exceptions to indicate whether an operation has failed. The return value for a successful retrieve, add, or remove operation, however, might seem strange at first. For example, it appears that the retrieve operation, `getEntry`, returns the same entry it is given to find. In fact, `getEntry` returns an object that is in the tree and that matches the given entry according to the entry's `compareTo` method. Let's look at an example that adds entries and then retrieves them.

Imagine a class `Person` that has two strings as data fields representing the person's name and identification number. The class implements the `Comparable` interface, and so has a `compareTo` method. Suppose that `compareTo` bases its comparison only on the name field. Consider the following statements that create and add to a binary search tree:

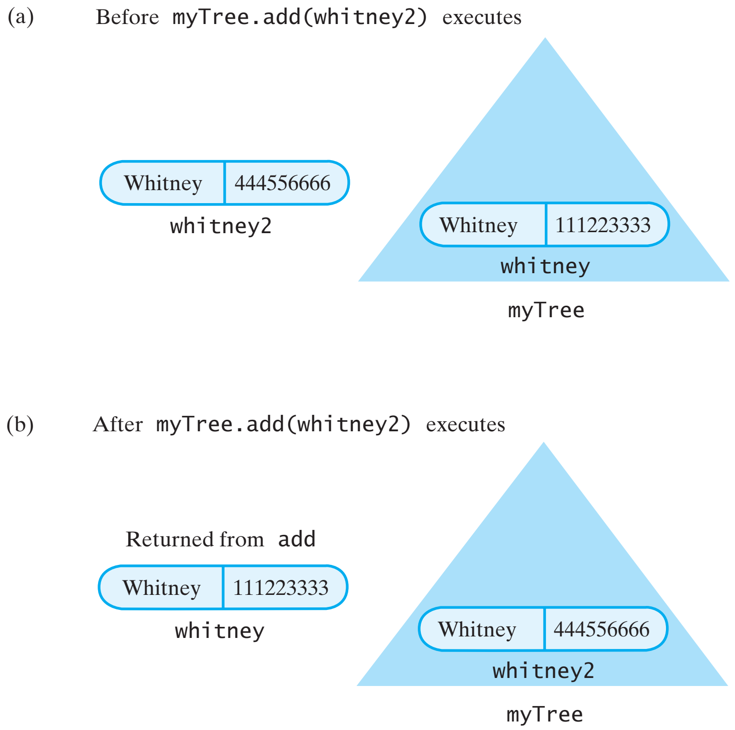
```
SearchTreeInterface<Person> myTree = new BinarySearchTree<Person>();
Person whitney = new Person("Whitney", "111223333");
Person returnValue = myTree.add(whitney);
```

Following the `add` operation, `returnValue` is `null`, since `whitney` was not in the tree already. Now suppose we try to add another *Whitney*, who has a different identification number:

```
Person whitney2 = new Person("Whitney", "444556666");
returnValue = myTree.add(whitney2);
```

Since `whitney` and `whitney2` have the same names, they are equal. That is, the expression `whitney.compareTo(whitney2)` is zero. Therefore, the `add` method will not add `whitney2` to the tree. Instead it replaces `whitney` with `whitney2` and returns `whitney`, the original object in the tree, as Figure 25-2 illustrates. We can think of this as a way to change the identification number of a person named *Whitney*.

FIGURE 25-2 Adding an entry that matches an entry already in a binary search tree



Now the statement

```
returnValue = myTree.getEntry(whitney);
```

sets `returnValue` to `whitney2`, since it is in the tree and matches `whitney`. Similarly,

```
returnValue = myTree.remove(whitney);
```

returns and removes `whitney2`.

Now imagine that the method `compareTo` uses both the name and identification fields of a `Person` object to make a comparison. Since `whitney` and `whitney2` would not be equal according to this `compareTo`, we could add both objects to the tree. Then `getEntry(whitney)` would return `whitney`, and `remove(whitney)` would remove and return `whitney`.

Duplicate Entries

25.4 To make our discussion a bit simpler, we insist that a binary search tree contain distinct entries. Notice that the `add` method, as specified in `SearchTreeInterface`, ensures that duplicates are never added to the tree. In practice, this restriction can be desirable for many applications, but sometimes it is not. By making a small change to our definition of a binary search tree, we can allow duplicate entries, that is, multiple entries that are equal according to `compareTo`.

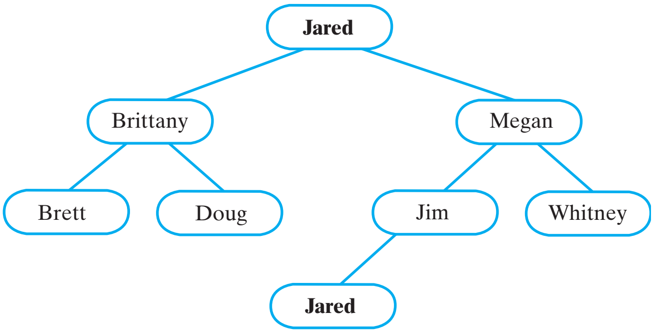
Figure 25-3 shows a binary search tree in which *Jared* occurs twice. If we are at the root of this tree and want to know whether *Jared* occurs again, it would help to know in which subtree we should look. Thus, if any entry *e* has a duplicate entry *d*, we arbitrarily require that *d* occur in the right subtree of *e*'s node. Accordingly, we modify our definition as follows:

For each node in a binary search tree,

- The data in a node is greater than the data in the node's left subtree
- The data in a node is less than *or equal to* the data in the node's right subtree

Notice that an inorder traversal of the tree in Figure 25-3 visits the duplicate entry *Jared* immediately after visiting the original *Jared*.

FIGURE 25-3 A binary search tree with duplicate entries



With duplicate entries permitted, the `add` method has less to do. But which entry will `getEntry` retrieve? Will the method `remove` delete the first occurrence of an entry or all occurrences? Exactly what happens is up to the class designer, but these questions should indicate the complications that duplicate entries cause. We will not consider duplicate entries any further, and will leave this issue to you as a programming project. (See Project 1 at the end of this chapter.)

Note: Duplicate entries

If you permit duplicate entries in a binary search tree, you can arbitrarily place the duplicate of an entry in the entry's right subtree. Once you choose the right subtree, you must be consistent. Project 2 at the end of this chapter suggests another strategy for handling duplicates.

SELF-TEST

Question 1 If you add a duplicate entry *Megan* to the binary search tree in Figure 25-3 as a leaf, where should you place the new node?

Beginning the Class Definition

VideoNote
Creating a binary search tree

25.5 **An outline of the class.** Let's begin the definition of a class of binary search trees. Since a binary search tree is a binary tree, we derive our new class from the class `BinaryTree` that we defined in the previous chapter. Thus, we begin our class as indicated in Listing 25-2. Note the call by the constructor to the protected method `setRootNode`, which the class inherits from `BinaryTree`. Segment 24.9 of the previous chapter contains a definition of `setRootNode`.

LISTING 25-2 An outline of the class `BinarySearchTree`

```
package TreePackage;
import java.util.Iterator;
public class BinarySearchTree<T extends Comparable<? super T>>
             extends BinaryTree<T>
             implements SearchTreeInterface<T>
{
   public BinarySearchTree()
   {
      super();
   } // end default constructor

   public BinarySearchTree(T rootEntry)
   {
      super();
      setRootNode(new BinaryNode<T>(rootEntry));
   } // end constructor

   public void setTree(T rootData) // disable setTree (see Segment 25.6)
   {
      throw new UnsupportedOperationException();
   } // end setTree

   public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
                                   BinaryTreeInterface<T> rightTree)
   {
      throw new UnsupportedOperationException();
   } // end setTree
   < Implementations of contains, getEntry, add, and remove are here. Their definitions appear
     in subsequent sections of this chapter. Other methods in SearchTreeInterface are inherited
     from BinaryTree. >
   . . .

} // end BinarySearchTree
```

25.6 **Disable `setTree`.** Before we go any further, consider the two `setTree` methods that our class inherits from `BinaryTree`. The client could use these methods to create a tree that, unfortunately, is not a binary search tree. This outcome would be impossible if the client used `SearchTreeInterface` to declare an instance of the tree. For example, if we wrote

```
SearchTreeInterface<String> dataSet = new BinarySearchTree<String>();
```

`dataSet` would not have either of the `setTree` methods, since they are not in `SearchTreeInterface`. But if we wrote

```
BinarySearchTree<String> dataSet = new BinarySearchTree<String>();
```

`dataSet` would have the `setTree` methods.

To prevent a client from using either version of `setTree`, we should override these two methods so that they throw an exception if called. Listing 25-2 shows definitions for these methods that do just that.

? SELF-TEST

Question 2 The second constructor in the class `BinarySearchTree` calls the method `setRootNode`. Is it possible to replace this call with the call `setRootData(rootEntry)`? Explain.

Question 3 Is it necessary to define the methods `isEmpty` and `clear` within the class `BinarySearchTree`? Explain.

Searching and Retrieving

25.7 **The search algorithm.** Segment 23.29 presented the following recursive algorithm to search a binary search tree:

```
Algorithm bstSearch(binarySearchTree, desiredObject)
// Searches a binary search tree for a given object.
// Returns true if the object is found.

if (binarySearchTree is empty)
   return false
else if (desiredObject == object in the root of binarySearchTree)
   return true
else if (desiredObject < object in the root of binarySearchTree)
   return bstSearch(left subtree of binarySearchTree, desiredObject)
else
   return bstSearch(right subtree of binarySearchTree, desiredObject)
```

This algorithm is the basis of the method `getEntry`.

Note: Searching a binary search tree is like performing a binary search of an array: You search one of two subtrees of the binary search tree instead of searching one of two halves of an array.

25.8 While it is convenient to express our recursive algorithm in terms of trees and subtrees, our implementation of a binary tree in the previous chapter suggests that we use root nodes instead. The root node of a tree or subtree provides a way for us to search or manipulate its descendant nodes.

The following algorithm is equivalent to the one just given, but describes our actual implementation more closely:

```
Algorithm bstSearch(binarySearchTreeRoot, desiredObject)
// Searches a binary search tree for a given object.
// Returns true if the object is found.

if (binarySearchTreeRoot is null)
   return false
else if (desiredObject == object in binarySearchTreeRoot)
   return true
else if (desiredObject < object in binarySearchTreeRoot)
   return bstSearch(left child of binarySearchTreeRoot, desiredObject)
else
   return bstSearch(right child of binarySearchTreeRoot, desiredObject)
```

We will continue to express subsequent algorithms in terms of trees and subtrees, but will use root nodes in our implementations without explicitly mentioning it.

25.9 **The method `getEntry`.** As is often the case with recursive algorithms, we implement the actual search as a private method `findEntry` that the public method `getEntry` invokes. Although the algorithm returns a boolean value, our implementation will return the located data object. Thus, we have the following methods:

```
public T getEntry(T entry)
{
   return findEntry(getRootNode(), entry);
} // end getEntry

private T findEntry(BinaryNodeInterface<T> rootNode, T entry)
{
   T result = null;

   if (rootNode != null)
   {
      T rootEntry = rootNode.getData();

      if (entry.equals(rootEntry))
         result = rootEntry;
      else if (entry.compareTo(rootEntry) < 0)
         result = findEntry(rootNode.getLeftChild(), entry);
      else
         result = findEntry(rootNode.getRightChild(), entry);
   } // end if

   return result;
} // end findEntry
```

We use the methods `compareTo` and `equals` to compare the given entry with the existing entries in the tree. Also, notice our use of methods from the class `BinaryNode`. We assume that we have at least package access to this class.

You can implement `getEntry` iteratively as well, with or without the use of a private method such as `findEntry`. We leave this implementation as an exercise.

SELF-TEST

Question 4 When `getEntry` calls `findEntry`, it passes `getRootNode()` as the first argument. This argument's data type is `BinaryNodeInterface<T>`, which corresponds to the type of the parameter `rootNode`. If you change `rootNode`'s type to `BinaryNode<T>`, what other changes, if any, must you make?

25.10 **The method `contains`.** The method `contains` can simply call `getEntry` to see whether a given entry is in the tree:

```
public boolean contains(T entry)
{
   return getEntry(entry) != null;
} // end contains
```

Traversing

25.11 `SearchTreeInterface` provides the method `getInorderIterator`, which returns an inorder iterator. Since our class is a subclass of `BinaryTree`, it inherits `getInorderIterator`. For a binary search tree, this iterator traverses the entries in ascending order, as defined by the entries' method `compareTo`.

SELF-TEST

Question 5 Under what circumstance will a client of `BinarySearchTree` be able to call the other methods in `TreeIteratorInterface`? Under what circumstance will such a client be unable to call these methods?

Adding an Entry

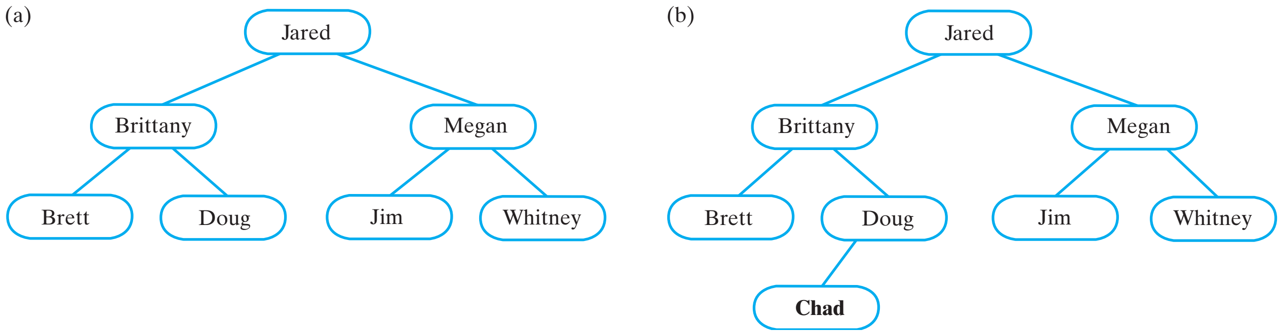
VideoNote
Binary search tree additions and removals

25.12 Adding entries to a binary search tree is an essential operation, since that is how we build one initially. So suppose that we have a binary search tree and we want to add a new entry to it. We cannot add it just anywhere in the tree, because we must retain the relationships among the nodes. That is, the tree must still be a binary search tree after the addition. Also, the method `getEntry` must be able to locate the new entry. For example, if we want to add the entry *Chad* to the tree in Figure 25-4a, we could not add the new node to *Jared*'s right subtree. Since *Chad* comes before *Jared*, *Chad* must be in *Jared*'s left subtree. Since *Brittany* is the root of this left subtree, we compare *Chad* with *Brittany* and find that *Chad* is larger. Thus, *Chad* belongs in *Brittany*'s right subtree. Continuing, we compare *Chad* with *Doug* and find that *Chad* belongs in *Doug*'s left subtree. But this subtree is empty. That is, *Doug* has no left child.

If we make *Chad* the left child of *Doug*, we will get the binary search tree in Figure 25-4b. Now `getEntry` will be able to locate *Chad* by making the same comparisons we just described. That is, `getEntry` will compare *Chad* with *Jared*, *Brittany*, and *Doug* before locating *Chad*. Notice that the new node is a leaf.

Note: Every addition to a binary search tree adds a new leaf to the tree.

FIGURE 25-4 (a) A binary search tree; (b) the same tree after adding *Chad*



SELF-TEST

Question 6 Add the names *Chris*, *Jason*, and *Kelley* to the binary search tree in Figure 25-4b.

Question 7 Add the name *Miguel* to the binary search tree in Figure 25-4a, and then add *Nancy*. Now go back to the original tree and add *Nancy* and then add *Miguel*. Does the order in which you add the two names affect the tree that results?

A Recursive Implementation

25.13 The method `add` has an elegant recursive implementation. Consider again the example given in the previous segment. If we want to add *Chad* to the binary search tree in Figure 25-4a, we take the following steps:

- To add *Chad* to the binary search tree whose root is *Jared*:

  Observe that *Chad* is less than *Jared*.
  Add *Chad* to *Jared*'s left subtree, whose root is *Brittany*.

- To add *Chad* to the binary search tree whose root is *Brittany*:

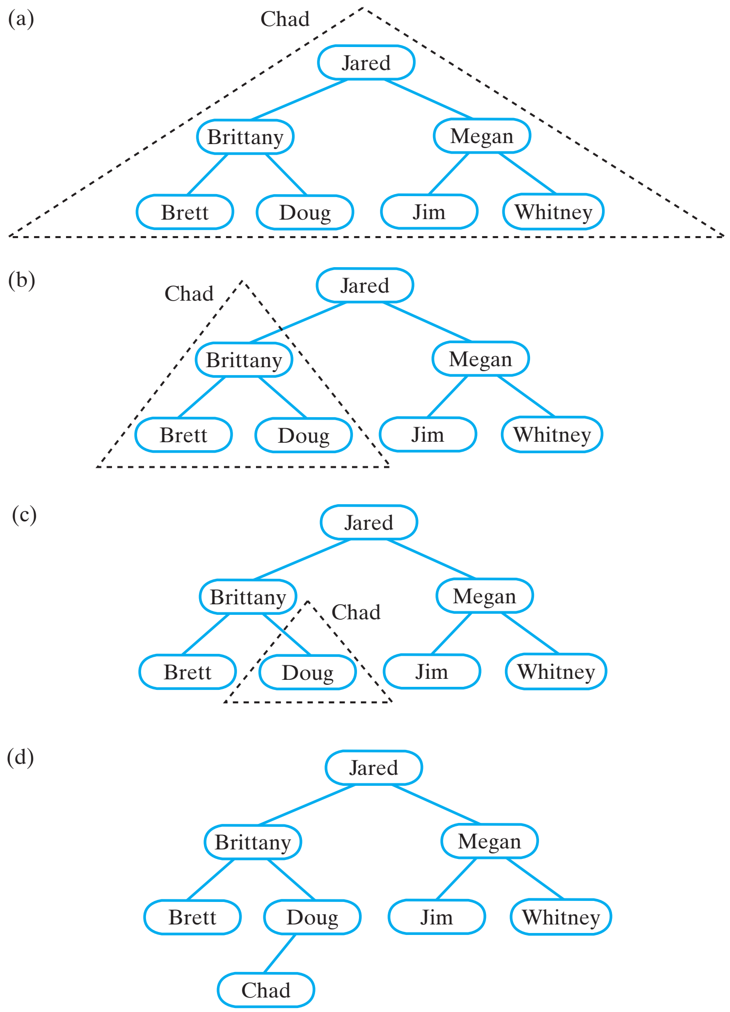
  Observe that *Chad* is greater than *Brittany*.
  Add *Chad* to *Brittany*'s right subtree, whose root is *Doug*.

- To add *Chad* to the binary search tree whose root is *Doug*:

  Observe that *Chad* is less than *Doug*.
  Since *Doug* has no left subtree, make *Chad* the left child of *Doug*.

We can see that adding an entry to the tree rooted at *Jared* depends upon adding to progressively smaller subtrees, as Figure 25-5 shows.

FIGURE 25-5 Recursively adding *Chad* to smaller subtrees of a binary search tree



25.14 **A recursive algorithm for adding a new entry.** The following recursive algorithm formalizes this approach, in accordance with the specifications of the method `add` in `SearchTreeInterface`. Recall that we decided to have only distinct entries in the binary search tree. If we try to add an entry to a tree that matches an entry already in the tree, we replace that entry with the new entry and return the old entry.

To simplify our algorithm, let's assume for the moment that the binary search tree is not empty:

```
Algorithm addEntry(binarySearchTree, newEntry)
// Adds a new entry to a binary search tree that is not empty.
// Returns null if newEntry did not exist already in the tree. Otherwise, returns the
// tree entry that matched and was replaced by newEntry.

result = null
if (newEntry matches the entry in the root of binarySearchTree)
{
   result = entry in the root
   Replace entry in the root with newEntry
}
else if (newEntry < entry in the root of binarySearchTree)
{
   if (the root of binarySearchTree has a left child)
      result = addEntry(left subtree of binarySearchTree, newEntry)
   else
     Give the root a left child containing newEntry
}
else // newEntry > entry in the root of binarySearchTree
{
   if (the root of binarySearchTree has a right child)
      result = addEntry(right subtree of binarySearchTree, newEntry)
   else
      Give the root a right child containing newEntry
}

return result
```

We can handle the addition to an empty binary search tree as a special case within another algorithm that invokes `addEntry`, as follows:

```
Algorithm add(binarySearchTree, newEntry)
// Adds a new entry to a binary search tree.
// Returns null if newEntry did not exist already in the tree. Otherwise, returns the
// tree entry that matched and was replaced by newEntry.

result = null
if (binarySearchTree is empty)
   Create a node containing newEntry and make it the root of binarySearchTree
else
   result = addEntry(binarySearchTree, newEntry)

return result;
```

25.15 **The private recursive method `addEntry`.** Recall the recursive search algorithm given in Segment 25.7. The public method `getEntry` in Segment 25.9 invokes a private recursive method `findEntry` that implements the search algorithm. We have a similar organization here. The public method `add` calls a private recursive method `addEntry`, if the tree is not empty. Like `findEntry`, `addEntry` has a node as a parameter that is initially the root node of the tree. When `addEntry` is called recursively, this parameter is either the left child or the right child of the current root.

Remember where we place a new node into a binary search tree. As Figures 25-4 and 25-5 illustrate, a new node always becomes a leaf in the tree. Now imagine the recursive calls to `addEntry` when adding *Chad* to the tree in Figure 25-5a. Eventually, the node containing *Doug* is passed to `addEntry` as its argument (Figure 25-5c). Since *Chad* is less than *Doug*, and *Doug*'s node has no left child, `addEntry` creates one containing *Chad* (Figure 25-5d).

The following implementation of addEntry closely follows the pseudocode given in Segment 25.14:

```
// Adds newEntry to the nonempty subtree rooted at rootNode.
private T addEntry(BinaryNodeInterface<T> rootNode, T newEntry)
{
   assert rootNode != null;
   T result = null;
   int comparison = newEntry.compareTo(rootNode.getData());

   if (comparison == 0)
   {
      result = rootNode.getData();
      rootNode.setData(newEntry);
   }
   else if (comparison < 0)
   {
      if (rootNode.hasLeftChild())
         result = addEntry(rootNode.getLeftChild(), newEntry);
      else
         rootNode.setLeftChild(new BinaryNode<T>(newEntry));
   }
   else
   {
      assert comparison > 0;

      if (rootNode.hasRightChild())
         result = addEntry(rootNode.getRightChild(), newEntry);
      else
         rootNode.setRightChild(new BinaryNode<T>(newEntry));
   } // end if

   return result;
} // end addEntry
```

We begin by comparing the new entry with the entry in the root. If the entries match, we replace and return the original entry in the root. If the comparison is “less than,” and the root has a left child, we pass that child to addEntry. Remember that when we are coding a recursive method such as addEntry, we assume that it works when we write the recursive call. Thus, addEntry places a new node containing newEntry into the root’s left subtree. If the root has no left child, we give it one containing the new entry. Analogous code handles the case when the new entry is greater than the entry in the root.

25.16 **The public method add.** The public method add not only invokes the recursive addEntry, it ensures that the tree it passes to addEntry is not empty. Accordingly, add deals with empty trees itself. The following implementation of add adheres to the algorithm given in Segment 25.14. Note the use of the protected methods setRootNode and getRootNode that are inherited from BinaryTree.

```
public T add(T newEntry)
{
   T result = null;

   if (isEmpty())
      setRootNode(new BinaryNode<T>(newEntry));
   else
      result = addEntry(getRootNode(), newEntry);

   return result;
} // end add
```

An Iterative Implementation

You can implement the method addEntry iteratively. We will mimic the logic of the recursive version of addEntry given earlier, so you can compare the two approaches. Exercise 12 at the end of this chapter suggests another iterative algorithm.

25.17 **An iterative algorithm for adding a new entry.** The following iterative algorithm adds a new entry to a binary search tree that is not empty:

```
Algorithm addEntry(binarySearchTree, newEntry)
// Adds a new entry to a binary search tree that is not empty.
// Returns null if newEntry did not exist already in the tree. Otherwise, returns the
// tree entry that matched and was replaced by newEntry.

result = null
currentNode = root node of binarySearchTree
found = false

while (found is false)
{
   if (newEntry matches the entry in currentNode)
   {
      found = true
      result = entry in currentNode
      Replace entry in currentNode with newEntry
   }
   else if (newEntry < entry in currentNode)
   {
      if (currentNode has a left child)
         currentNode = left child of currentNode
      else
      {
         found = true
         Give currentNode a left child containing newEntry
      }
   }
   else // newEntry > entry in currentNode
   {
      if (currentNode has a right child)
         currentNode = right child of currentNode
      else
      {
         found = true
         Give currentNode a right child containing newEntry
      }
   }
}

return result
```

The while loop tries to match the new entry with an existing entry in the tree. If the new entry is not in the tree already, the search for it ends at a node's null child reference. This is where a new node belongs. But if the new entry matches an entry in the tree, we return the existing entry and replace it in the tree with the new entry.

25.18 **An iterative implementation of the method addEntry.** The Java implementation of the previous algorithm closely follows the algorithm's logic. Note the use of the protected method getRootNode that is inherited from BinaryTree.

```
private T addEntry(T newEntry)
{
   BinaryNodeInterface<T> currentNode = getRootNode();
   assert currentNode != null;
   T result = null;
   boolean found = false;

   while (!found)
   {
      T currentEntry = currentNode.getData();
      int comparison = newEntry.compareTo(currentEntry);

      if (comparison == 0)
      {  // newEntry matches currentEntry;
         // return and replace currentEntry
         found = true;
         result = currentEntry;
         currentNode.setData(newEntry);
      }
      else if (comparison < 0)
      {
         if (currentNode.hasLeftChild())
            currentNode = currentNode.getLeftChild();
         else
         {
            found = true;
            currentNode.setLeftChild(new BinaryNode<T>(newEntry));
         } // end if
      }
      else
      {
         assert comparison > 0;

         if (currentNode.hasRightChild())
            currentNode = currentNode.getRightChild();
         else
         {
            found = true;
            currentNode.setRightChild(new BinaryNode<T>(newEntry));
         } // end if
      } // end if
   } // end while

   return result;
} // end addEntry
```

The method add that calls this iterative `addEntry` is like the one given in Segment 25.16, except for the actual invocation of `addEntry`. Since the iterative `addEntry` has one parameter instead of two, the invocation is `addEntry(newEntry)` instead of `addEntry(getRootNode(), newEntry)`.

Whether you use this iterative `addEntry` method, the one suggested in Exercise 12, or the recursive version given earlier depends in part on which approach is clearest to you. You'll spend less time debugging if you really understand your algorithm.

Removing an Entry

25.19 To remove an entry from a binary search tree, we pass a matching entry to the method `remove`. The desired entry is then removed from the tree and returned to the client. If no such entry exists, the method returns `null` and the tree remains unchanged.

Removing an entry is somewhat more involved than adding an entry, as the required logic depends upon how many children belong to the node containing the entry. We have three possibilities:

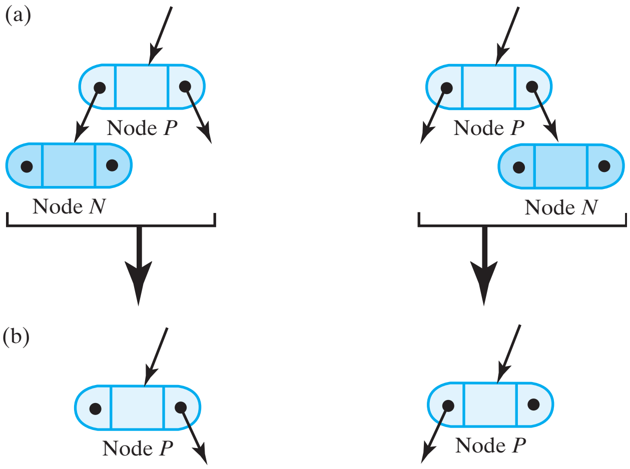
- The node has no children—it is a leaf
- The node has one child
- The node has two children

We now consider these three cases.

Removing an Entry Whose Node Is a Leaf

25.20 The simplest case in removing an entry is when the node is a leaf, that is, has no children. For example, suppose that node *N* contains the entry to be removed from the binary search tree. Figure 25-6a shows two possibilities for node *N*: It could be either the left child or the right child of its parent node *P*. Since *N* is a leaf, we can delete it by setting the appropriate child reference in node *P* to `null`. Figure 25-6b shows the result of this operation.

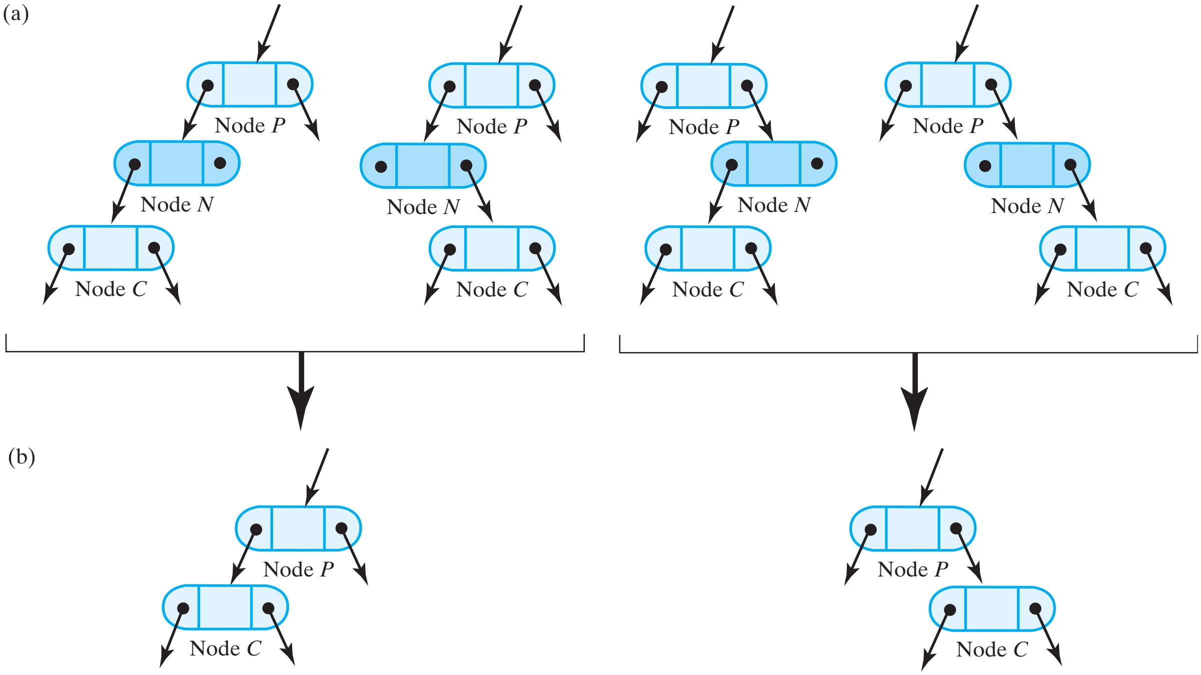
FIGURE 25-6 (a) Two possible configurations of a leaf node *N*; (b) the resulting two possible configurations after removing node *N*



Removing an Entry Whose Node Has One Child

25.21 Now imagine that the entry to be removed is in a node *N* that has exactly one child *C*. Figure 25-7a shows the four possibilities for node *N* and its parent *P*. To remove the entry in *N*, we remove *N* from the tree. We do this by making *C* a child of *P* instead of *N*. As Figure 25-7b shows, if *N* was a left child of *P*, we make *C* be the left child of *P*. Likewise, if *N* was a right child of *P*, we make *C* be the right child of *P*.

FIGURE 25-7 (a) Four possible configurations of a node *N* that has one child; (b) the resulting two possible configurations after removing node *N*

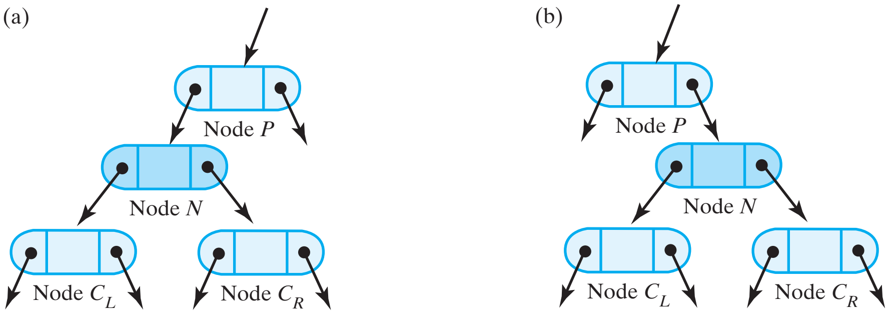


Removing an Entry Whose Node Has Two Children

25.22 The previous two cases are really not too difficult, conceptually or in practice. But this last case is a bit involved. Once again, suppose that the entry to be removed is in a node *N*, but now *N* has two children. Figure 25-8 shows two possible configurations for *N*. If we try to remove node *N*, we will leave its two children without a parent. Although node *P* could reference one of them, it hasn't room for both. Thus, removing node *N* is not an option.

We do not actually have to remove node *N* to remove its entry. Let's find a node *A* that is easy to remove—it would have no more than one child—and replace *N*'s entry with the entry now in *A*. We then can remove node *A* and still have the correct entries in the tree. But will the tree still be a binary search tree? Clearly, node *A* cannot be just any node; it must contain an entry in the tree that legally can be in node *N*.

FIGURE 25-8 Two possible configurations of a node *N* that has two children



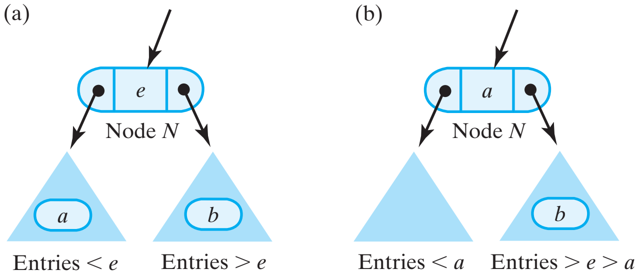
25.23 We know that the entries in the tree are distinct. Let *e* be the entry in node *N*. Since node *N* has two children, *e* is larger than the entry in *N*'s left child and smaller than the entry in *N*'s right child. Thus, *e* cannot be the smallest entry in the tree, nor can it be the largest. So, if we imagine the tree's entries in ascending order, we can write

... $a < e < b$ ...

Here, *a* is the entry that is immediately before *e*, and *b* is the one that is immediately after. An inorder traversal of the tree would visit these entries in this same order. Thus, *a* is called the **inorder predecessor** of *e,* and *b* is the **inorder successor** of *e*.

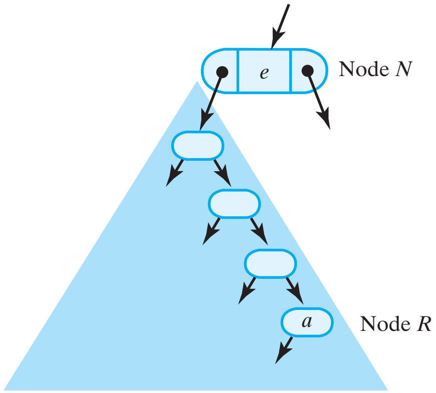
The entry *a* must occur in a node in *N*'s left subtree; *b* is in a node in *N*'s right subtree, as Figure 25-9a illustrates. Moreover, *a* is the largest entry in *N*'s left subtree, since *a* is the entry that is immediately before *e*. Suppose that we are able to delete the node that contains *a* and replace *e* with *a*, as Figure 25-9b shows. Now all of the remaining entries in *N*'s left subtree are less than *a*, as needed. All of the entries in *N*'s right subtree are greater than *e* and so are greater than *a*. Thus, we still have a binary search tree.

FIGURE 25-9 Node *N* and its subtrees: (a) the entry *a* is immediately before the entry *e*, and *b* is immediately after *e*; (b) after deleting the node that contained *a* and replacing *e* with *a*



25.24 **Locating the entry *a*.** The previous segment assumed that we could find the appropriate entry *a* and delete its node. So, let's locate the node that contains *a* and verify that it does not have two children. Consider again the original tree in Figure 25-9a. We already know that *a* must be in *N*'s left subtree, and that *a* is the largest entry in that subtree. To find an entry larger than the one in any given node, we look at the node's right child. Thus, *a* occurs in the subtree's rightmost node *R*, as Figure 25-10 illustrates. Node *R* cannot have a right child, because if it did, the child's entry would be greater than *a*. Therefore, node *R* has no more than one child and so can be removed from the tree easily.

FIGURE 25-10 The largest entry *a* in node *N*'s left subtree occurs in the subtree's rightmost node *R*



25.25 The following pseudocode summarizes this discussion:

```
Algorithm Delete the entry e from a node N that has two children
Find the rightmost node R in N's left subtree
Replace the entry in node N with the entry that is in node R
Delete node R
```

An alternate approach involves *b*, the entry that is immediately after *e* in sorted order. We have already noted that *b* occurs in *N*'s right subtree. It would have to be the smallest entry in that subtree, so it would occur in the leftmost node in the subtree. Thus, we have the following alternate pseudocode:

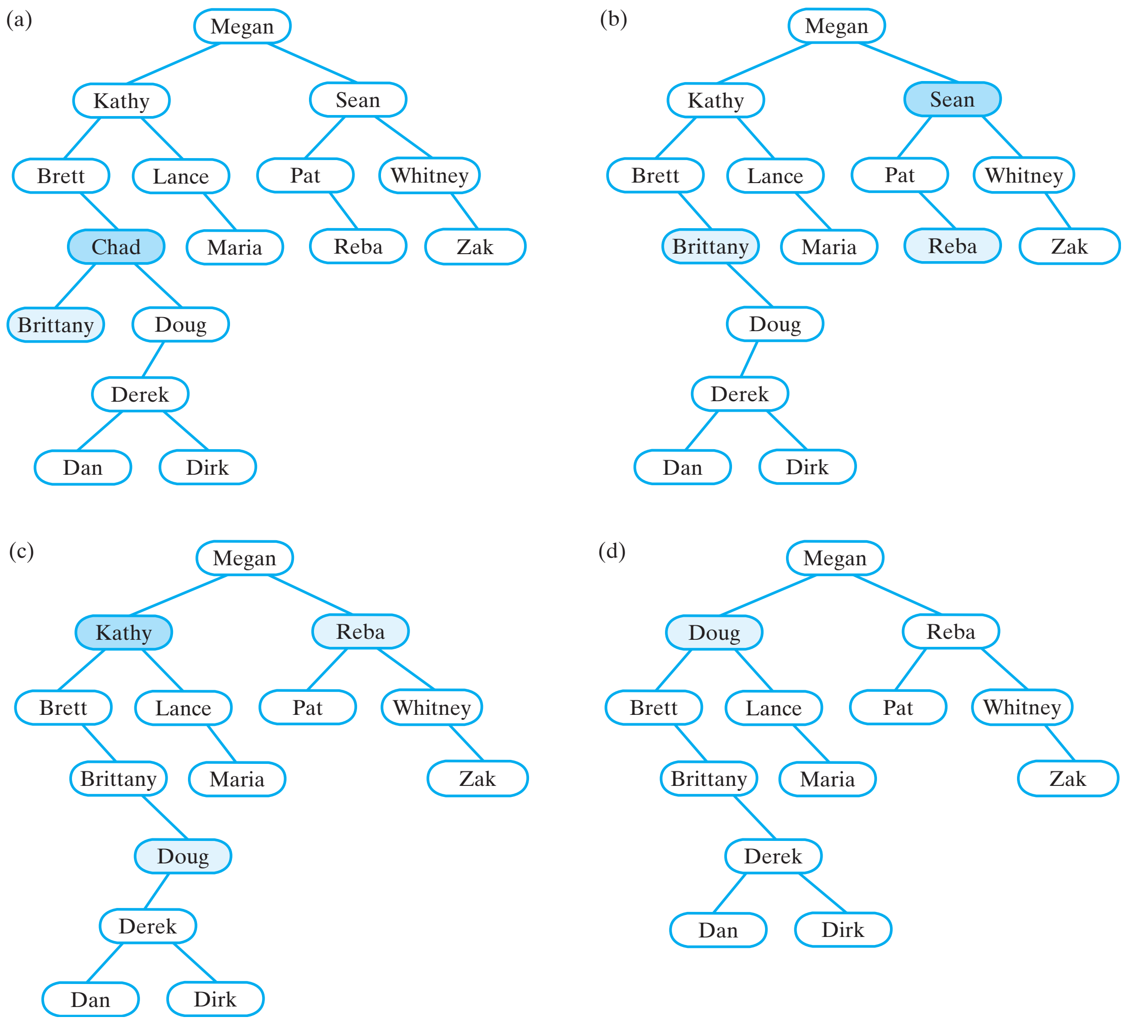
```
Algorithm Delete the entry e from a node N that has two children
Find the leftmost node L in N's right subtree
Replace the entry in node N with the entry that is in node L
Delete node L
```

Both approaches work equally well.

Note: To remove an entry whose node has two children, you first replace the entry with another whose node has no more than one child. You then remove the second node from the binary search tree.

25.26 **Example.** Figure 25-11 shows several consecutive removals from a binary search tree of names. The first algorithm given in the previous segment is used. To remove *Chad* from the tree in Figure 25-11a, we replace it with its inorder predecessor *Brittany*. We then remove the node that contained *Brittany* to get the tree in Figure 25-11b. To remove *Sean* from this new tree, we replace it with its inorder predecessor *Reba* and remove *Reba*'s original node. This gives us the tree in Figure 25-11c. Finally, to remove *Kathy* from this tree, we replace it with its inorder predecessor *Doug* and remove *Doug*'s original node, to get the tree in Figure 25-11d.

FIGURE 25-11 (a) A binary search tree; (b) after removing *Chad*; (c) after removing *Sean*; (d) after removing *Kathy*



Question 8 The second algorithm described in Segment 25.25 involves the inorder successor. Using this algorithm, remove *Sean* and *Chad* from the tree in Figure 25-11a.

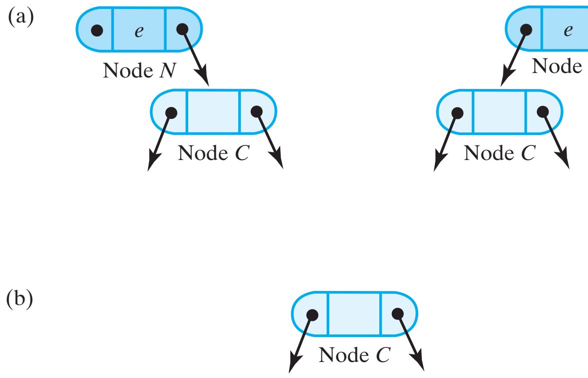
Question 9 Remove *Megan* from the tree in Figure 25-11a in two different ways.

Removing an Entry in the Root

25.27 Removing an entry that is in the root of the tree is a special case only if we actually remove the root node. That will occur when the root has at most one child. If the root has two children, the previous segment shows that we would replace the root's entry and delete a different node.

If the root is a leaf, the tree has only one node. Deleting it results in an empty tree. If the root has one child, as Figure 25-12 illustrates, the child is either a right child or a left child. In either case, we simply delete the root node by making the child node *C* the root of the tree.

FIGURE 25-12 (a) Two possible configurations of a root that has one child; (b) after removing the root



A Recursive Implementation

25.28 **The algorithm.** The entry to be removed from the tree is the one that matches the object passed to the method `remove` as its argument. The method returns the removed entry. The following recursive algorithm describes the method's logic at a high level:

```
Algorithm remove(binarySearchTree, entry)
oldEntry = null
if (binarySearchTree is not empty)
{
   if (entry matches the entry in the root of binarySearchTree)
   {
      oldEntry = entry in root
      removeFromRoot(root of binarySearchTree)
   }
   else if (entry < entry in root)
      oldEntry = remove(left subtree of binarySearchTree, entry)
   else   // entry > entry in root
      oldEntry = remove(right subtree of binarySearchTree, entry)
}
return oldEntry
```

The method `removeFromRoot` will remove the entry in the root of a given subtree based on how many children belong to the root.

25.29 **The public method `remove`.** We have several details to consider before implementing the previous algorithm. The public method `remove` should have only one parameter—`entry`—so just as the method `add` calls the private recursive method `addEntry`, `remove` will call a private recursive method `removeEntry`.

As we mentioned in Segment 25.8, we will pass the root of the tree, instead of the tree itself, to `removeEntry`. Since the method might remove the root node from the tree, we must be careful to always retain a reference to the tree's root. As a result, we make `removeEntry` return a reference to the root of the revised tree, which `remove` can save. But `removeEntry` must also give to `remove` the entry it removes. A solution is to pass another parameter—`oldEntry`—to `removeEntry` and have the method change its value to the removed entry. Thus, the header for `removeEntry` will be

```
private BinaryNodeInterface<T> removeEntry(BinaryNodeInterface<T> rootNode,
                                           T entry, ReturnObject oldEntry)
```

ReturnObject is an inner class that has a single data field and simple methods set and get to manipulate it. Initially, oldEntry's data field is null, since remove returns null when the entry is not found in the tree.

Thus, the public remove has the following implementation:

```
public T remove(T entry)
{
   ReturnObject oldEntry = new ReturnObject(null);
   BinaryNodeInterface<T> newRoot = removeEntry(getRootNode(), entry,
                                                oldEntry);
   setRootNode(newRoot);

   return oldEntry.get();
} // end remove
```

25.30 **The private method removeEntry.** Since remove handles the communication with removeEntry, most of the algorithm of Segment 25.28 is left for removeEntry. If the entry to be removed is in the root, removeEntry calls the yet-to-be-written method removeFromRoot to remove it. If the entry is in either of the root's subtrees, removeEntry calls itself recursively. The implementation of removeEntry follows:

```
// Removes an entry from the tree rooted at a given node.
// rootNode is a reference to the root of a tree.
// entry is the object to be removed.
// oldEntry is an object whose data field is null.
// Returns the root node of the resulting tree; if entry matches
//         an entry in the tree, oldEntry's data field is the entry
//         that was removed from the tree; otherwise it is null.
private BinaryNodeInterface<T> removeEntry(BinaryNodeInterface<T> rootNode,
                                           T entry, ReturnObject oldEntry)
{
   if (rootNode != null)
   {
      T rootData = rootNode.getData();
      int comparison = entry.compareTo(rootData);

      if (comparison == 0)       // entry == root entry
      {
         oldEntry.set(rootData);
         rootNode = removeFromRoot(rootNode);
      }
      else if (comparison < 0)   // entry < root entry
      {
         BinaryNodeInterface<T> leftChild = rootNode.getLeftChild();
         BinaryNodeInterface<T> subtreeRoot = removeEntry(leftChild,
                                                   entry, oldEntry);
         rootNode.setLeftChild(subtreeRoot);
      }
      else                       // entry > root entry
      {
         BinaryNodeInterface<T> rightChild = rootNode.getRightChild();
         rootNode.setRightChild(removeEntry(rightChild, entry, oldEntry));
      } // end if
   } // end if

   return rootNode;
} // end removeEntry
```

25.31 **The algorithm removeFromRoot.** The previous method removeEntry removes the entry in the root of a given subtree by calling the method removeFromRoot. In that method, we see whether the root node has zero, one, or two children and then proceed according to the discussion in Segments 25.20 through 25.27. If the given node has at most one child, we delete the node and its entry. To remove the entry in a node having two children, we must find the largest entry in the node's left subtree. We remove the node containing this largest entry. The largest entry then replaces the entry to be removed.

The following algorithm summarizes these steps:

```
Algorithm removeFromRoot(rootNode)
// Removes the entry in a given root node of a subtree.

if (rootNode has two children)
{
   largestNode = node with the largest entry in the left subtree of rootNode
   Replace the entry in rootNode with the entry in largestNode
   Remove largestNode from the tree
}
else if (rootNode has a right child)
   rootNode = rootNode's right child
else
   rootNode = rootNode's left child  // possibly null
// Assertion: if rootNode was a leaf, it is now null

return rootNode
```

25.32 **The private method `removeFromRoot`.** The implementation of the previous algorithm calls the private methods `findLargest` and `removeLargest`, which we will write shortly. Although `removeFromRoot` is not recursive, both `findLargest` and `removeLargest` are.

Given the root of a subtree, `removeFromRoot` returns the root of the subtree after a node is removed.

```
// Removes the entry in a given root node of a subtree.
// rootNode is the root node of the subtree.
// Returns the root node of the revised subtree.
private BinaryNodeInterface<T> removeFromRoot(BinaryNodeInterface<T> rootNode)
{
   // Case 1: rootNode has two children
   if (rootNode.hasLeftChild() && rootNode.hasRightChild())
   {
      // find node with largest entry in left subtree
      BinaryNodeInterface<T> leftSubtreeRoot = rootNode.getLeftChild();
      BinaryNodeInterface<T> largestNode = findLargest(leftSubtreeRoot);

      // replace entry in root
      rootNode.setData(largestNode.getData());

      // remove node with largest entry in left subtree
      rootNode.setLeftChild(removeLargest(leftSubtreeRoot));
   } // end if

   // Case 2: rootNode has at most one child
   else if (rootNode.hasRightChild())
      rootNode = rootNode.getRightChild();
   else
      rootNode = rootNode.getLeftChild();

   // Assertion: if rootNode was a leaf, it is now null

   return rootNode;
} // end removeEntry
```

25.33 **The private method `findLargest`.** The node with the largest entry will occur in the rightmost node of a binary search tree. Thus, as long as a node has a right child, we search the subtree rooted at that child. The following recursive method performs this search, given the tree:

```
// Finds the node containing the largest entry in a given tree.
// rootNode is the root node of the tree.
// Returns the node containing the largest entry in the tree.
```

```
private BinaryNodeInterface<T> findLargest(BinaryNodeInterface<T> rootNode)
{
   if (rootNode.hasRightChild())
      rootNode = findLargest(rootNode.getRightChild());

   return rootNode;
} // end findLargest
```

25.34 **The private method `removeLargest`.** To remove the node with the largest entry, we cannot simply call `findLargest` and then remove the returned node. We cannot remove a node from a tree knowing only its reference. We must have a reference to its parent as well. The following recursive method removes the node with the largest entry—that is, the rightmost node—but unfortunately it must repeat the search that `findLargest` just performed.

```
// Removes the node containing the largest entry in a given tree.
// rootNode is the root node of the tree.
// Returns the root node of the revised tree.
private BinaryNodeInterface<T> removeLargest(BinaryNodeInterface<T> rootNode)
{
   if (rootNode.hasRightChild())
   {
      BinaryNodeInterface<T> rightChild = rootNode.getRightChild();
      BinaryNodeInterface<T> root = removeLargest(rightChild);
      rootNode.setRightChild(root);
   }
   else
      rootNode = rootNode.getLeftChild();

   return rootNode;
} // end removeLargest
```

The method begins much like `findLargest`. To remove the rightmost node from the given tree, we remove the rightmost node from tree's right subtree. The recursive call returns the root of the revised subtree. This root must become the right child of the original tree's root.

When a tree's root has no right child, the left child is returned, effectively deleting the root. Notice that this recursive method does not explicitly keep track of the parent of the current right child. Rather, a reference to this parent is retained in the implicit stack of the recursion.

Note: The previous recursive approach to removing an entry from a binary search tree is typical. A language, such as Java, that uses only call-by-value to pass arguments tends to complicate this recursive implementation by forcing methods to return references to root nodes. You might find the following iterative approach somewhat easier to understand. Since it deletes the node containing the inorder predecessor without repeating the search for it, the iterative `remove` is more efficient than the recursive version.

An Iterative Implementation

25.35 **The algorithm.** Recall that the method `remove` is given an entry that matches the entry to be removed from the tree. So `remove`'s first step is to search the tree. We locate the node whose data matches the given entry, and we note the node's parent, if any. Whether we delete the node we've found or another one depends on how many children it has. Although Segment 25.19 listed three possibilities, we can collapse them into two cases:

1. The node has two children
2. The node has at most one child

In the second case, we delete the node itself. But if the node has two children, we delete another node that has at most one child. That is, we transform Case 1 into Case 2.

The following pseudocode describes what `remove` must do:

```
Algorithm remove(entry)
result = null
currentNode = node that contains a match for entry
parentNode = currentNode's parent

if (currentNode != null)   // that is, if entry is found
{
   result = currentNode's data (the entry to be removed from the tree)

   // Case 1
   if (currentNode has two children)
   {
      // get node to remove and its parent
      nodeToRemove = node containing entry's inorder predecessor; it has at most one child
      parentNode = nodeToRemove's parent

      Copy entry from nodeToRemove to currentNode
      currentNode = nodeToRemove
      // Assertion: currentNode is the node to be removed; it has at most one child
      // Assertion: Case 1 has been transformed to Case 2
   }

   // Case 2: currentNode has at most one child
   Delete currentNode from the tree
}

return result
```

25.36 **The public method `remove`.** We will implement the major steps of the previous algorithm as private methods that `remove` can call. The private method `findNode` locates the node that contains a match for the given entry. Since we need a reference to that node as well as one to its parent, we make `findNode` return a pair of nodes. To that end, we design a private class `NodePair` that has constructors and the accessor methods `getFirst` and `getSecond`. `NodePair` will be an inner class of our class `BinarySearchTree`.

The private method `getNodeToRemove` finds the node containing the inorder predecessor of the the entry in a given node. Since we also need that node's parent, the method returns a pair of nodes as an instance of the class `NodePair`.

Finally, the private method `removeNode` deletes a node that has at most one child. We give the method references to the node and its parent, if any.

Using these private methods, we can implement `remove`, as follows:

```
public T remove(T entry)
{
   T result = null;

   // locate node (and its parent) that contains a match for entry
   NodePair pair = findNode(entry);
   BinaryNodeInterface<T> currentNode = pair.getFirst();
   BinaryNodeInterface<T> parentNode = pair.getSecond();

   if (currentNode != null) // entry is found
   {
      result = currentNode.getData(); // get entry to be removed

      // Case 1: currentNode has two children
      if (currentNode.hasLeftChild() && currentNode.hasRightChild())
      {
         // replace entry in currentNode with the entry in another node
         // that has at most one child; that node can be deleted

         // get node to remove (contains inorder predecessor; has at
         // most one child) and its parent
         pair = getNodeToRemove(currentNode);
         BinaryNodeInterface<T> nodeToRemove = pair.getFirst();
         parentNode = pair.getSecond();

         // copy entry from nodeToRemove to currentNode
         currentNode.setData(nodeToRemove.getData());

         currentNode = nodeToRemove;
         // Assertion: currentNode is the node to be removed; it has at
         // most one child
         // Assertion: Case 1 has been transformed to Case 2
      } // end if

      // Case 2: currentNode has at most one child; delete it
      removeNode(currentNode, parentNode);
   } // end if

   return result;
} // end remove
```

25.37 **The private method `findNode`.** To find the node that contains a match for a given entry, we use the `compareTo` method within a loop to compare the given entry with the other entries in the tree. The method returns a pair of references to the desired node and its parent as an instance of the class `NodePair`. Thus, `findNode` has the following form:

```
private NodePair findNode(T entry)
{
   NodePair result = new NodePair();
   boolean found = false;
   . . .

   if (found)
      result = new NodePair(currentNode, parentNode);
      // found entry is currentNode.getData()

   return result;
} // end findNode
```

The details of the implementation of `findNode` are left as an exercise.

Question 10 Complete the implementation of the method `findNode`.

25.38 **The private method `getNodeToRemove`.** After `remove` locates the node that contains the entry to be removed from the tree, it proceeds according the number of the node's children. If the node has two children, `remove` must remove another node that has no more than one child. The private method `getNodeToRemove` finds this node. In particular, the method implements the first step of the pseudocode given in Segment 25.25:

Find the rightmost node R in N's left subtree

Here, node *N* is `currentNode` and node *R* is `rightChild`.

The details of this step are described by the following pseudocode:

```
// find the inorder predecessor by searching the left subtree; it will be the largest
// entry in the subtree, occurring in the node as far right as possible
leftSubtreeRoot = left child of currentNode
rightChild = leftSubtreeRoot
priorNode = currentNode

while (rightChild has a right child)
{
   priorNode = rightChild
   rightChild = right child of rightChild
}
// Assertion: rightChild is the node to be removed and has no more than one child
```

The following Java code implements `getNodeToRemove`:

```
private NodePair getNodeToRemove(BinaryNodeInterface<T> currentNode)
{
   // find node with largest entry in left subtree by
   // moving as far right in the subtree as possible
   BinaryNodeInterface<T> leftSubtreeRoot = currentNode.getLeftChild();
   BinaryNodeInterface<T> rightChild = leftSubtreeRoot;
   BinaryNodeInterface<T> priorNode = currentNode;

   while (rightChild.hasRightChild())
   {
      priorNode = rightChild;
      rightChild = rightChild.getRightChild();
   } // end while

   // rightChild contains the inorder predecessor and is the node to
   // remove; priorNode is its parent

   return new NodePair(rightChild, priorNode);
} // end getNodeToRemove
```

25.39 **The private method `removeNode`.** Our last method assumes that the node to remove—call it `nodeToRemove`—has at most one child. If `nodeToRemove` is not the root, `parentNode` is its parent.

The method begins by setting `childNode` to the child, if any, of `nodeToRemove`. If `nodeToRemove` is a leaf, `childNode` is set to `null`. Then the method removes `nodeToRemove`, accounting for the case when the node is the root as follows:

```
if (nodeToRemove is the root of the tree)
   Set the root of the tree to childNode
else
   Link parentNode to childNode, thereby deleting nodeToRemove
```

If we set the root of the tree to `childNode`, realize that we will correctly set the root to `null` if `nodeToRemove` is a leaf.

The implementation of `removeNode` follows:

```
private void removeNode(BinaryNodeInterface<T> nodeToRemove,
                        BinaryNodeInterface<T> parentNode)
{
   BinaryNodeInterface<T> childNode;

   if (nodeToRemove.hasLeftChild())
      childNode = nodeToRemove.getLeftChild();
   else
      childNode = nodeToRemove.getRightChild();
   // Assertion: if nodeToRemove is a leaf, childNode is null
   assert (nodeToRemove.isLeaf() && childNode == null) ||
          !nodeToRemove.isLeaf();

   if (nodeToRemove == getRootNode())
      setRootNode(childNode);
   else if (parentNode.getLeftChild() == nodeToRemove)
      parentNode.setLeftChild(childNode);
   else
     parentNode.setRightChild(childNode);
} // end removeNode
```

The Efficiency of Operations

25.40 Each of the operations `add`, `remove`, and `getEntry` requires a search that begins at the root of the tree. When adding an entry, the search ends at a leaf if the entry is not already in the tree; otherwise, the search can end sooner. When removing or retrieving an entry, the search ends at a leaf if it is unsuccessful; a successful search can end sooner. So in the worst case, these searches begin at the root and examine each node on a path that ends at a leaf. The longest path from the root to a leaf has a length that equals the height of the tree. Thus, the maximum number of comparisons that each operation requires is directly proportional to the height h of the tree. That is, the operations `add`, `remove`, and `getEntry` are O(h).

Recall that several different binary search trees can contain the same data. Figure 25-13 contains two such trees. Figure 25-13a is the shortest binary search tree that we can form from this data; Figure 25-13b is the tallest such tree.

The tallest tree has height n if it contains n nodes. In fact, this tree looks like a linked chain, and searching it is like searching a linked chain. It is an O(n) operation. Thus, `add`, `remove`, and `getEntry` for this tree are also O(n) operations.

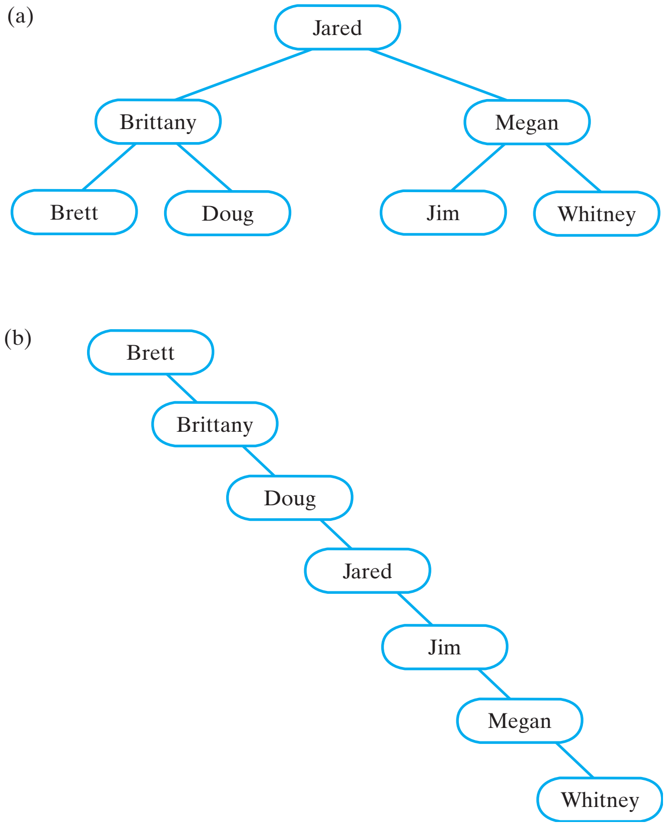
In contrast, the shortest tree is full. Searching this tree will be as efficient as possible. In Chapter 23, we saw that the height of a full tree containing n nodes is $\log_2 (n + 1)$. Thus, in the worst case, searching a full binary search tree is an O(log n) operation. So `add`, `remove`, and `getEntry` are O(log n) operations in this case.

? SELF-TEST

Question 11 Using Big Oh notation, what is the time complexity of the method `contains`?

Question 12 Using Big Oh notation, what is the time complexity of the method `isEmpty`?

FIGURE 25-13 Two binary search trees that contain the same data



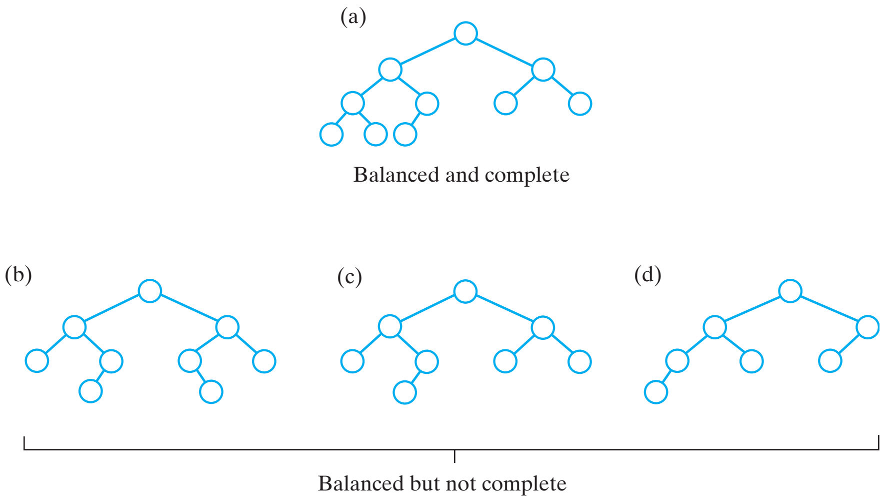
The Importance of Balance

25.41 We do not need a full binary search tree to get O(log n) performance from the addition, removal, and retrieval operations. For example, if we remove some of the leaves from a full tree, we will not change the performance of these operations. In particular, a complete tree will also give us O(log n) performance.

The notion of *balance* affects the performance of a particular search tree. In a **completely balanced** tree, the subtrees of each node have exactly the same height. The only completely balanced binary trees are full. Other trees are said to be **height balanced**, or simply **balanced**, if the subtrees of each node in the tree differ in height by no more than 1. A complete binary tree is height balanced, for example, but so are some trees that are not complete, as Figure 25-14 shows. Moreover, the concept of balance applies to all trees, not just binary trees or binary search trees.

It happens that the addition, removal, and retrieval operations of a binary search tree will have O(log *n*) performance if the tree is height balanced. Certainly when we create a binary search tree, we want it to be height balanced. Unfortunately, we can disturb the balance of a binary search tree by adding or removing entries, since these operations affect the shape of the tree.

FIGURE 25-14 Some binary trees that are height balanced



The Order in Which Nodes Are Added

25.42 If you answered Question 7 in Segment 25.12 correctly, you realized that the order in which you add entries to a binary search tree affects the shape of the tree. This observation is most important when you create a binary search tree by making additions to an initially empty tree.

For example, suppose that we want to create the full binary search tree in Figure 25-13a from a given set of data. Often such data sets are sorted, so it is reasonable to assume that we have the names in alphabetical order. Now imagine that we define an empty binary search tree and then add the names to it in the following order: *Brett*, *Brittany*, *Doug*, *Jared*, *Jim*, *Megan*, *Whitney*. Figure 25-13b shows the tree that results from these additions. It is as tall as possible and has the least efficient operations among the trees that we could build.

Note: If you add entries into an initially empty binary search tree, do not add them in sorted order.

25.43 In what order should we add the entries? *Jared* is the root of the tree in Figure 25-13a, so let's add *Jared* first. Next add *Brittany* and then *Brett* and *Doug*. Finally, add *Megan*, *Jim*, and *Whitney*. While it should be clear that by using this order we get the tree in Figure 25-13a, how do we determine the order ahead of time? Looking at our alphabetical set of names, notice that *Jared* is exactly in the middle. We add *Jared* first. *Brittany* is in the middle of the left half of the data set, so we add *Brittany* next. The halves that *Brittany* defines each contain only one name, so we add them next. We repeat this process with the names that occur after *Jared*—that is, the right half of the data set.

We shouldn't have to do this much work! In fact, if we add data to a binary search tree in random order, we can expect a tree whose operations are O(log *n*). It probably will not be the shortest tree we could create, but it will be close.

The operations of a binary search tree ensure that the tree remains a binary search tree. Unfortunately, they do not ensure that the tree remains balanced. Chapter 27 looks at search trees that are responsible for maintaining their balance, and hence their efficiency.

An Implementation of the ADT Dictionary

25.44 We can use the ideas developed thus far in this chapter to implement the ADT dictionary. Recall from Chapter 19 that a dictionary stores search keys and their associated values. For example, suppose that you want a dictionary of names and telephone numbers. In terms of the ADT dictionary, the name could be the search key and the telephone number could be the corresponding value. To retrieve a telephone number, we would provide a name, and the dictionary would return its value.

Here is the interface for a dictionary as given in Segment 19.4, but without the comments:

```
import java.util.Iterator;
public interface DictionaryInterface<K, V>
{
   public V add(K key, V value);
   public V remove(K key);
   public V getValue(K key);
   public boolean contains(K key);
   public Iterator<K> getKeyIterator();
   public Iterator<V> getValueIterator();
   public boolean isEmpty();
   public int getSize();
   public void clear();
} // end DictionaryInterface
```

Earlier in this book we saw several implementations of the ADT dictionary. A dictionary implementation that uses a balanced search tree to store its entries can be an attractive alternative to these implementations. As an example of such an implementation, we will use a binary search tree here, even though it might not remain balanced after additions or removals. Chapter 27 presents search trees that are always balanced and could be used instead to implement the dictionary.

25.45 **The data entries.** We need a class of data objects that will contain both a search key and an associated value. A class `Entry`—similar to the class we used in the array-based implementation of the ADT dictionary in Chapter 20—is suitable for our purpose. Here, we make the class `Comparable` by defining the method `compareTo`. This method compares two instances of `Entry` by comparing their search keys. Thus, the search keys of this dictionary must belong to a `Comparable` class.

The class `Entry` can be private and internal to the class `Dictionary`, as Listing 25-3 shows. This listing also shows `Dictionary`'s data field—a binary search tree—as well as the constructor that allocates the tree. Notice how `Entry` is used in both the declaration and allocation of the tree.

LISTING 25-3 An outline of an implementation of the ADT dictionary that uses a binary search tree

```
import TreePackage.SearchTreeInterface;
import TreePackage.BinarySearchTree;
import java.util.Iterator;
public class Dictionary<K extends Comparable<? super K>, V>
       implements DictionaryInterface<K, V>
{
   private SearchTreeInterface<Entry<K, V>> bst;

   public Dictionary()
   {
      bst = new BinarySearchTree<Entry<K, V>>();
   } // end default constructor

   < Methods that implement dictionary operations are here. >
   . . .

   private class Entry<S extends Comparable<? super S>, T>
           implements Comparable<Entry<S, T>>
   {
      private S key;
      private T value;

      private Entry(S searchKey, T dataValue)
      {
         key = searchKey;
         value = dataValue;
      } // end constructor

      public int compareTo(Entry<S, T> other)
      {
         return key.compareTo(other.key);
      } // end compareTo

   < The class Entry also defines the methods equals, getKey, getValue, and setValue;
     no setKey method is provided. >
   . . .
   } // end Entry
} // end Dictionary
```

25.46 **The `Dictionary` methods.** The method `add` encapsulates the given search key and value into an instance of `Entry` that it passes to `BinarySearchTree`'s `add` method. It then uses the entry that this method returns to form its own return value. `Dictionary`'s `add` method has the following implementation:

```
public V add(K key, V value)
{
   Entry<K, V> newEntry = new Entry<K, V>(key, value);
   Entry<K, V> returnedEntry = bst.add(newEntry);

   V result = null;
   if (returnedEntry != null)
      result = returnedEntry.getValue();

   return result;
} // end add
```

Both `remove` and `getValue` have implementations that are similar to `add`'s. Since these methods have only a search key as a parameter, the instances of `Entry` that they form encapsulate the key and a `null` value. For example, `remove` begins as

```
public V remove(K key)
{
   Entry<K, V> findEntry = new Entry<K, V>(key, null);
   Entry<K, V> returnedEntry = bst.remove(findEntry);
```

and ends just like the method `add`. The implementation of the method `getValue` is identical to that of `remove`, except that it calls `getEntry` from `BinarySearchTree` instead of `remove`.

We can implement the methods `getSize`, `isEmpty`, `contains`, and `clear` by calling appropriate methods of `BinarySearchTree`. We leave these to you as exercises.

SELF-TEST

Question 13 Implement each of the `Dictionary` methods `getSize`, `isEmpty`, `contains`, and `clear` by calling methods of `BinarySearchTree`.

Question 14 Write another implementation of the method `contains` by invoking `Dictionary`'s method `getValue`.

25.47 **The iterators.** `DictionaryInterface` specifies two methods that return iterators. The method `getKeyIterator` returns an iterator that accesses the search keys in sorted order; `getValueIterator` returns an iterator that provides the values belonging to these search keys.

For example, `getKeyIterator` has the following implementation:

```
public Iterator<K> getKeyIterator()
{
   return new KeyIterator();
} // end getKeyIterator
```

The class `KeyIterator` is internal to `Dictionary` and uses the method `getInorderIterator` from `BinarySearchTree`. It has the following implementation:

```
private class KeyIterator implements Iterator<K>
{
   Iterator<Entry<K, V>> localIterator;

   public KeyIterator()
   {
      localIterator = bst.getInorderIterator();
   } // end default constructor
```

```
      public boolean hasNext()
      {
         return localIterator.hasNext();
      } // end hasNext

      public K next()
      {
         Entry<K, V> nextEntry = localIterator.next();
         return nextEntry.getKey();
      } // end next

      public void remove()
      {
         throw new UnsupportedOperationException();
      } // end remove
   } // end KeyIterator
```

You implement `getValueIterator` in a similar manner.

25.48 **Comments.** This implementation of the ADT dictionary is as time efficient as the underlying search tree. When the binary search tree is balanced, the operations are O(log *n*). But a binary search tree can lose its balance, and so the efficiency of the dictionary operations can degrade to O(*n*) as entries are added or removed. A search tree that stays balanced, such as those you will see in Chapter 27, would provide a better implementation of the dictionary than the one shown here.

Also, notice that a binary search tree maintains the dictionary entries in sorted order by their search keys. As a result, `getKeyIterator` enables us to traverse the search keys in sorted order. In contrast, other dictionary implementations—hashing, for example—traverse the search keys in unsorted order.

Chapter Summary

- A binary search tree is a binary tree whose nodes contain `Comparable` objects. For each node in the tree,
  - The data in a node is greater than the data in the node's left subtree
  - The data in a node is less than (or equal to) the data in the node's right subtree
- A search tree has the operations `contains`, `getEntry`, `add`, `remove`, and `getInorderIterator`, in addition to the operations common to all trees.
- The class `BinarySearchTree` can be a subclass of `BinaryTree`, but it must disallow `setTree`. A client must create a binary search tree by using only the `add` method, to avoid changing the order of the nodes in the tree.
- The search algorithm to locate an entry in a binary search tree forms the basis of the methods `getEntry`, `add`, and `remove`. These methods each have reasonable iterative and recursive implementations.
- Each addition of an entry to a binary search tree adds a leaf to the tree. The new entry is placed where the search algorithm will find it.
- Removing an entry from a binary search tree depends on the number of children that belong to the node containing the entry. When the node is a leaf or has one child, you remove the node itself. The node's parent can adopt a solitary child when it exists. However, when the node *N* has two children, you replace the node's entry with another one *r* whose node is easy to remove. To maintain the order of the binary search tree, this entry *r* can be either the largest entry in *N*'s left subtree or the smallest entry in *N*'s right subtree. It follows that *r*'s node is either a leaf or a node with one child.

- In a completely balanced binary tree, the subtrees of each node have exactly the same height. Such trees must be full. Other binary trees are said to be height balanced if the subtrees of each node in the tree differ in height by no more than 1.
- The retrieve, add, and remove operations on a binary search tree can be as fast as O(log *n*) or as slow as O(*n*). The performance of the search depends on the shape of the tree. When the tree is height balanced, the operations on a binary search tree are O(log *n*).
- The order in which you add entries to a binary search tree affects the tree's shape and hence its balance. Random additions, as opposed to sorted ones, tend to result in a balanced tree.
- You can implement the ADT dictionary by using a binary search tree. Although the implementation is not difficult to write, its efficiency can suffer if additions and removals destroy the balance of the tree.

Exercises

1. Show the results of adding the following search keys to an initially empty binary search tree: 10, 5, 6, 13, 15, 8, 14, 7, 12, 4.

2. What ordering of the search keys 10, 5, 6, 13, 15, 8, 14, 7, 12, 4 would result in the most balanced tree if they were added to an initially empty binary search tree?

3. Give four different orderings of the search keys 10, 5, 6, 13, 15, 8, 14, 7, 12, 4 that would result in the least balanced tree if they were added to an initially empty binary search tree.

4. In Chapter 7, Figure 7-10a shows the recursive computation of the term F_6 in the Fibonacci sequence. Is this tree height balanced?

5. Implement the method `getEntry` iteratively.

6. Remove *Doug* from the binary search tree pictured in Figure 25-11a. Then remove *Chad* in two different ways.

7. Remove *Doug* from the binary search tree pictured in Figure 25-11d in two different ways.

8. Suppose that a node with two children contains an entry *e*, as Figure 25-9a illustrates. Show that you will have a binary search tree if you replace *e* with its inorder successor *b* and remove the node that contains *b*.

9. Why does an inorder traversal of a binary search tree visit the nodes in sorted search-key order? Use the definition of a binary search tree given in Segment 25.1.

10. Consider the full binary search tree pictured in Figure 25-13a. Now imagine that you traverse the tree and save its data in a file. If you then read the file and add the data to an initially empty binary search tree, what tree will you get if the traversal was

a. Preorder **b.** Inorder **c.** Level order **d.** Postorder

11. Imagine that you traverse a binary search tree and save its data in a file. If you then read the file and add the data to an initially empty binary search tree, what traversal should you use when writing the file so that the new tree is

a. As tall as possible
b. Identical to the original binary search tree

12. Segment 25.17 gave an iterative algorithm for the method `addEntry`. Implement the following alternate algorithm for this method:

```
Algorithm addEntry(binarySearchTree, newEntry)

result = null
currentNode = root node of binarySearchTree
parentNode = null

while (newEntry is not found and currentNode is not null)
{
   if (newEntry matches entry in currentNode)
   {
      result = entry in currentNode
      Replace entry in currentNode with newEntry
   }
   else if (newEntry < entry in currentNode)
   {
      parentNode = currentNode
      currentNode = the left child of currentNode
   }
   else // newEntry > entry in currentNode
   {
      parentNode = currentNode
      currentNode = the right child of currentNode
   }
}

if (newEntry is not found in the tree)
{
   Create a new node and place newEntry into it

   if (newEntry < entry in parentNode)
      Make the new node the left child of parentNode
   else
     Make the new node the right child of parentNode
}

return result
```

13. The methods `remove` and the recursive `removeEntry`, as described in Segments 25.28 through 25.30, use the inner class `ReturnObject`. In this way, `removeEntry` can convey to `remove` both the root of the revised tree and the entry it removed. Revise these methods to instead use a class `Pair<T1, T2>`, like the one given in Segment B.40 of Appendix B. `Pair` will need accessor methods for its fields. The method `removeEntry` can then return the root and the removed entry as a `Pair` object.

14. Segment 25.43 builds a balanced binary search tree from one particular group of search keys. Generalize this approach, and write a recursive algorithm that creates a balanced binary search tree from a sorted collection of *n* items.

15. Write an algorithm that returns the smallest search key in a binary search tree.

16. Beginning with Segment 25.23, you saw how to find the inorder predecessor or the inorder successor of a node with two children. Unfortunately, this approach will not work for a leaf node. For a node with one child, the technique will find either the predecessor or the successor, but not both. Discuss how the structure of a node might be modified so that the inorder predecessor or the inorder successor can be found for any node.

17. Write an algorithm that returns the second largest value in a binary search tree containing at least two nodes.

18. Why might a binary search tree work poorly as an implementation of a priority queue?

19. Consider a method for a binary search tree that decides whether the tree is height balanced, as Segment 25.41 describes. The header of the method could be as follows:

```
public boolean isBalanced()
```

Write this method for the class `BinarySearchTree`. It should call a private recursive method of the same name.

20. Write a static method that accepts as its argument a `BinaryTree` object and returns true if the argument tree is a binary search tree. Examine each node in the given tree only once.

21. Implement the method `toString` for the class `BinarySearchTree`. The method should return a string that, when displayed, shows the shape of the tree in two dimensions. Ignore the data in each node. For example a tree might appear as follows:

```
        o
       / \
      o   o
         / \
        o   o
```

22. Consider two empty binary search trees that allow duplicate entries with the same search key. To one of the trees, add *m* unique entries, each with a different search key. To the other, add each of these *m* entries *k* times for a total of $m \times k$ entries. Assuming that each entry is stored in a single node, compare the heights of the two trees. Discuss how the order in which entries are added to the second tree affects its height. Give the addition orders that lead to the tallest tree and the shortest tree.

23. Segment 25.4 describes a binary search tree that allows duplicates. You place the duplicate of an entry in the entry's right subtree.

   a. What is an advantage and a disadvantage of this scheme?
   b. Suppose that we change the definition of a binary search tree so that the duplicate of an entry can be in either the entry's right subtree or its left subtree. If we choose the subtree randomly, what is an advantage and a disadvantage of this scheme?

PROJECTS

1. Specify and implement a class of binary search trees in which duplicate entries are allowed. Place the duplicate of an entry in the entry's right subtree, as suggested in Segment 25.4. Provide a method that searches the tree for a given entry and returns the first one it finds. Also, provide a similar method that returns a list of all entries that match the given one.

2. Repeat the previous project, but instead randomly place the duplicate of an entry in the entry's left or right subtree. Thus, we modify the definition of a binary search tree as follows.

   For each node in a binary search tree,

   - The data in a node is greater than *or equal to* the data in the node's left subtree
   - The data in a node is less than *or equal to* the data in the node's right subtree

   Searching for a duplicate must allow a search of both subtrees.

3. Implement the ADT sorted list by using a binary search tree.

4. Devise an algorithm that uses a binary search tree to sort an array of objects. Such a sort is called **treesort**. Implement and test your algorithm. Discuss the time efficiency of your treesort in both the average and worst cases.

5. Implement a binary search tree that includes the following methods based on Exercises 15 and 16:

```
/** @return the entry with the smallest search key */
public T getMin();

/** @return the entry with the largest search key */
public T getMax();

/** @return either the inorder predecessor of entry, or
            entry if it's the smallest element in the tree, or
            null if entry is not in the tree */
public T getPredecessor(T entry);

/** @return either the inorder successor of entry, or
            entry if it's the largest element in the tree, or
            null if entry is not in the tree */
public T getSuccessor(T entry);
```

6. Implement the class `ArrayBinarySearchTree` that extends `ArrayBinaryTree`, described in Project 7 of Chapter 24.

7. Write Java code that creates a binary search tree from *n* random integer values and returns the height of the search tree. Run the code for $n = 2^h - 1$, where *h* ranges from 4 to 12. Compare the height of the randomly built search tree with *h*, the height of the shortest binary search tree.

8. Chapter 1 defined a set as a bag that does not permit duplicate entries. Define a class of sets that uses a binary search tree to store a set's entries.

9. Repeat Project 3 of Chapter 19, but use binary search trees to implement the two dictionaries. Write Java code that will create within the first dictionary a balanced binary search tree of the reserved words in the Java language. Why is it important that the search tree containing Java reserved words be balanced? Can you guarantee that the search tree of user-defined identifiers is also balanced?

10. Compare the performance of two binary search trees as more objects are added to them. Initially, one tree is balanced and the other is not.

First modify `BinarySearchTreeInterface` and `BinarySearchTree` so that the `add` method returns the number of comparisons used. Then write a program that uses the new version of `BinarySearchTree`, as follows. Create two empty binary search trees. Associate two variables with each tree. One variable sums the number of comparisons used in adding values to a tree, and the other sums the heights of a tree at certain times following the insertion of several values. Name these variables `comparisonSum1`, `comparisonSum2`, `heightSum1`, and `heightSum2`.

In a loop that executes 100 times, do the following:

- Add the values 1000, 2000, 3000, 4000, 5000, 6000, and 7000 to both trees. In the first tree, add them in increasing order. In the second, add them in an order that forms a complete tree. Your first tree will be unbalanced, while the second tree will be balanced.
- Generate 10 random values between 0 and 8000. Add these values to each tree in the same order. After each of these additions, update each tree's `comparisonSum` variable by the number of comparisons performed for the insertion.
- Add each tree's height to its `heightSum` variable.
- Clear the two trees.

After the loop ends, compute the average number of comparisons needed to insert values into each tree. (For each tree, divide its `comparisonSum` by 1000. Note that 1000 is 100—the number of iterations—multiplied by 10—the number of values inserted in one iteration.) Also compute the average height of each tree after the insertions. (Divide each `heightSum` variable by 100.) Display and record your results.

Run the program a second time, but instead add 100 random values between 0 and 8000 during each iteration of the loop. Run it a third time, but instead add 1000 random values. Discuss your results and draw a conclusion.

11. A ***k*d-tree**, or ***k*-dimensional tree**, is a binary tree that organizes points in *k*-dimensional space. Every node contains and represents a *k*-dimensional point. Every node *N* that is not a leaf corresponds to a hyperplane[1] that divides the space into two portions. Points to the left of the hyperplane are in *N*'s left subtree, and points to the right of the hyperplane are in *N*'s right subtree. The relationship between the *k*-dimensional space and a *k*d-tree enables you to use the tree to find all points within a given range—a *range search*—or to find the closest point to a given point—a *nearest-neighbor search*.

In this project, we will choose *k* to be 2 and consider two-dimensional space and a **2d-tree** whose nodes contain points in that space. In an attempt to avoid any confusion that the term "2d-tree" might cause, computer scientists commonly would describe the tree as a "2-dimensional *k*d-tree." We, however, will use the shorter name "2d-tree" here.

A 2d-tree generalizes a binary search tree in that it positions each node according to either the *x* or *y* coordinate of its data point. The coordinate choice depends on the level at which the node is inserted into the tree. The first point you insert into an empty tree is placed into a node that becomes the tree's root. If the next point to be inserted has an *x*-coordinate that is less than the *x*-coordinate of the point in the root, you place the new point into the left child of the root. Otherwise, you place it into the root's right child. Insertions at the next level—level 3—compare *y*-coordinates; insertions at level 4 compare *x*-coordinates, and so on.

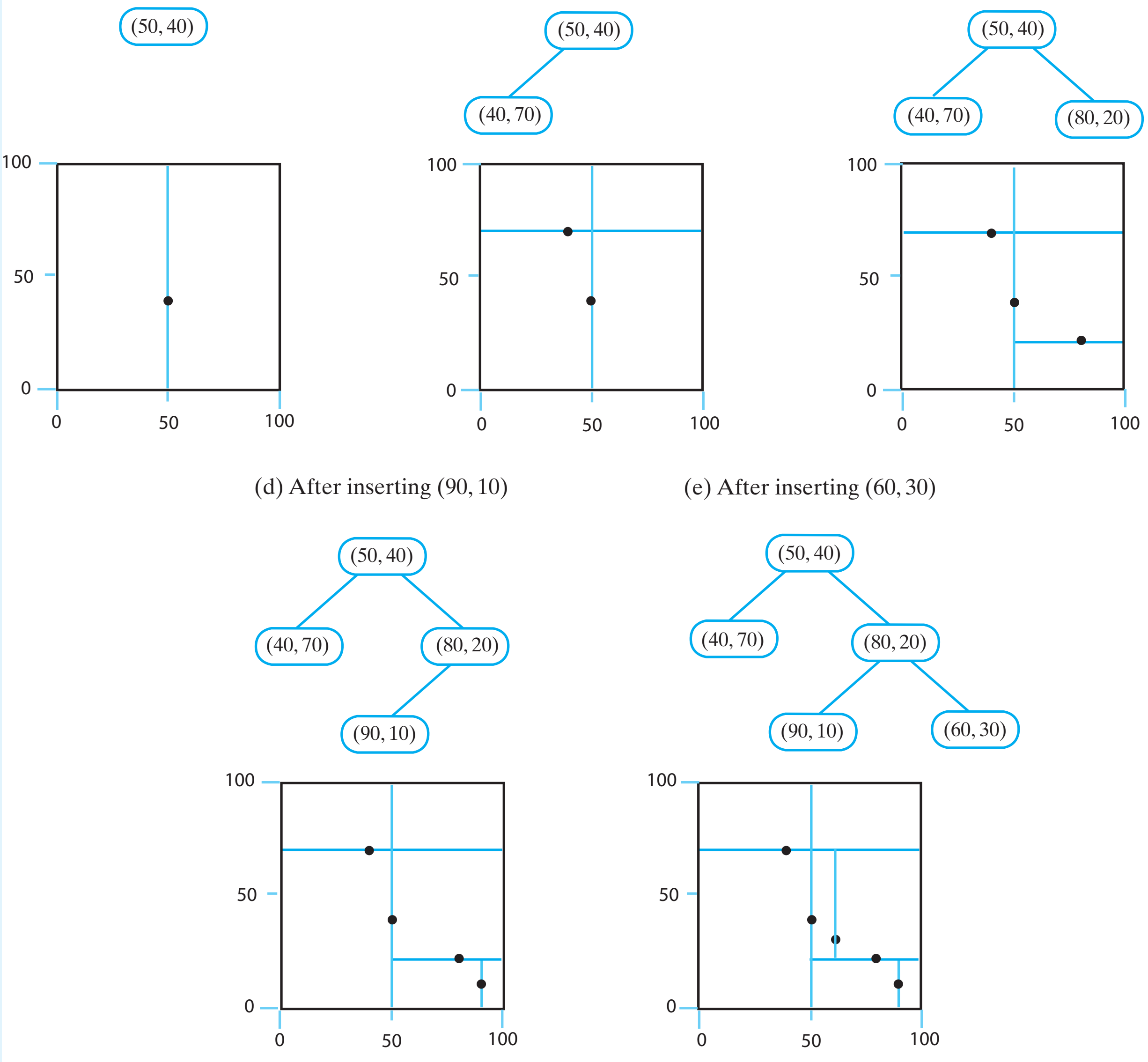
For example, let's insert the points (50, 40), (40, 70), (80, 20), (90, 10), and (60, 30) into an initially empty 2d-tree. Figure 25-15 traces the construction of this tree. Part *a* shows the root containing the first point, (50, 40). (For the moment, ignore the drawings beneath the trees.) To insert (40, 70) into a child of the root, you compare 40, the point's *x*-coordinate with 50, the *x*-coordinate of the point in the root. Since 40 is less than 50, the new point goes into the left child of the root, as Figure 25-15b shows. Similarly, since 80 is greater than 50, you place the next point, (80, 20), into the right child of the root (Figure 25-15c). To insert (90, 10), you begin at the tree's root and compare *x*-coordinates. Since 90 is greater than 50, you move to the root's right child and compare *y*-coordinates. We find that 10 is less than 20, so (90, 10) goes into the left child of the root's right child, as shown in Figure 25-15d. The final point, (60, 30), is positioned using similar steps to obtain the tree in Figure 25-15e.

The graphical significance of a 2d-tree is illustrated beneath the trees shown in Figure 25-15. We begin with a square that contains all of the points in the tree. For example, a 100 by 100 square, as shown in Figure 25-15a, will contain the five points in our example. By passing a vertical line through the *x*-coordinate of the point in the root, we divide the square into two regions. Any points in the root's left subtree will be to the left of this line, while points in the root's right subtree will lie to the right of the line. Figure 25-15b shows a horizontal line through the point (40, 70). Points in the left subtree of the node containing (40, 70) lie above this horizontal line and to the left of the vertical line; that is, they lie within the upper-left rectangle within the original square.

[1] A *hyperplane* in *k*-dimensional space is a $(k - 1)$-dimensional surface, described by a single linear equation in *k* variables, that divides the space into two regions. For example, in 2-dimensional space, a straight line divides the space and is described by a linear equation in the variables *x* and *y*. In 3-dimensional space, a plane divides the space and is described by a linear equation in the variables *x* , *y*, and *z*.

Implement a 2d-tree, providing at least a method to insert a new point and a method to test whether a given point is in the tree.

FIGURE 25-15 The steps in creating a 2d-tree for five given points



(d) After inserting (90, 10)

(e) After inserting (60, 30)

Answers to Self-Test Questions

1. As the left child of the node that contains *Whitney*.

2. No. The constructor first calls the default constructor of `BinaryTree`, which sets `root` to `null`. The method `setRootData` contains the call `root.setData(rootData)`, which would cause an exception.

3. No; `BinarySearchTree` inherits these methods from `BinaryTree`.

4. In `getEntry`'s call to `findEntry`, you would cast `getRootNode()` to `BinaryNode<T>`, as follows:

```
findEntry((BinaryNode<T>)getRootNode(), entry)
```

Within `findEntry`, the first recursive call to `findEntry` must be

```
findEntry((BinaryNode<T>)rootNode.getLeftChild(), entry)
```

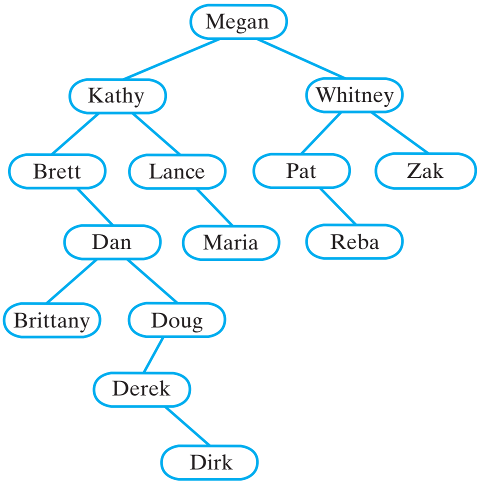
since the return type of `getLeftChild` is `BinaryNodeInterface<T>`. Analogous comments apply to the second recursive call and `getRightChild`.

5. The situation is like that described for `setTree` in Segment 25.6. `BinarySearchTree` inherits the methods declared in `TreeIteratorInterface` from `BinaryTree`. An object whose static type is `BinarySearchTree` can invoke these methods, but an object whose static type is `SearchTreeInterface` cannot.

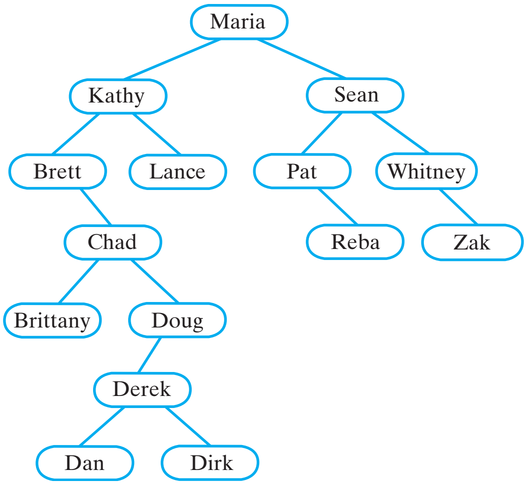
6. *Chris* is the right child of *Chad*. *Jason* is the left child of *Jim*. *Kelley* is the right child of *Jim*.

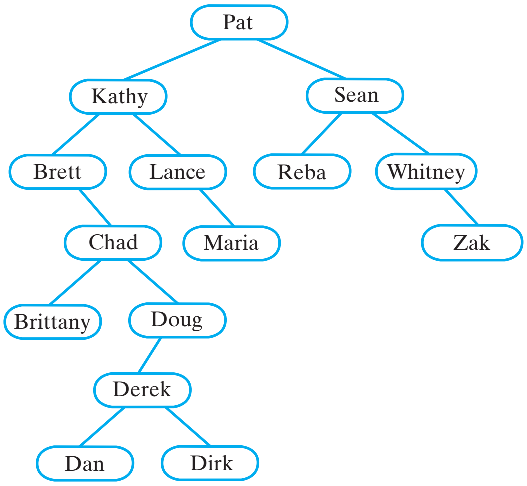
7. When you add *Miguel* first, *Miguel* is the left child of *Whitney*, and *Nancy* is the right child of *Miguel*. When you add *Nancy* first, *Nancy* is the left child of *Whitney*, and *Miguel* is the left child of *Nancy*. Thus, the order of the additions does affect the tree that results.

8.



9.





10.

```
private NodePair findNode(T entry)
{
   NodePair result = new NodePair();
   boolean found = false;

   BinaryNodeInterface<T> currentNode = getRootNode();
   BinaryNodeInterface<T> parentNode = null;
   while (!found && (currentNode != null) )
   {
      T currentEntry = currentNode.getData();
      int comparison = entry.compareTo(currentEntry);

      if (comparison < 0)
      {
         parentNode = currentNode;
         currentNode = currentNode.getLeftChild();
      }
      else if (comparison > 0)
      {
         parentNode = currentNode;
         currentNode = currentNode.getRightChild();
      }
      else // comparison == 0
         found = true;
   } // end while

   if (found)
      result = new NodePair(currentNode, parentNode);
      // found entry is currentNode.getData()
   return result;
} // end findNode
```

11. Since the method `contains` invokes `getEntry`, the efficiency of these methods is the same. So if the tree's height is as small as possible, the efficiency is O(log *n*). If the tree's height is as large as possible, the efficiency is O(*n*).

12. O(1).

13.

```
public int getSize()
{
   return bst.getNumberOfNodes();
} // end getSize

public boolean isEmpty()
{
   return bst.isEmpty();
} // end isEmpty

public boolean contains(K key)
{
   Entry<K, V> findEntry = new Entry<K, V>(key, null);

   return bst.contains(findEntry);
} // end contains
```

```
public void clear()
{
   bst.clear();
} // end clear

public boolean contains(K key)
{
   return getValue(key) != null;
} // end contains
```

14.

```
public boolean contains(K key)
{
   return getValue(key) != null;
} // end contains
```

This page intentionally left blank

Chapter 26

A Heap Implementation

Contents

Prerequisites

Chapter	2	Bag Implementations That Use Arrays
Chapter	13	List Implementations That Use Arrays
Chapter	23	Trees

Objectives

After studying this chapter, you should be able to

- Use an array to represent a heap
- Add an entry to an array-based heap
- Remove the root of an array-based heap
- Create a heap from given entries
- Sort an array by using a heap sort

Recall from Chapter 23 that a heap is a complete binary tree whose nodes are ordered in a certain manner. When a binary tree is complete, you can use an array to represent it in an efficient and elegant way. The most common implementation of a heap uses an array, and that is the one we will describe in this chapter.

As you saw in Chapter 23, you can use a heap as an efficient implementation of the ADT priority queue. Later, this chapter will show you how to sort an array by using a heap.

Reprise: The ADT Heap

26.1 A heap is a complete binary tree whose nodes contain `Comparable` objects. In a maxheap, the object in each node is greater than or equal to the objects in the node's descendants. Segment 23.32 provided the following interface for the maxheap:

```
public interface MaxHeapInterface<T extends Comparable<? super T>>
{
   public void add(T newEntry);
   public T removeMax();
   public T getMax();
   public boolean isEmpty();
   public int getSize();
   public void clear();
} // end MaxHeapInterface
```

We will use this interface in our implementation of a maxheap.

> **Note:** You may also have heard the word "heap" used to refer to the collection of memory cells that are available for allocation to your program when the `new` operator executes. But that heap is not an instance of the ADT heap that we will discuss in this chapter. It would be considered, however, in a book about programming languages.

Using an Array to Represent a Heap

26.2 **Representing a complete binary tree.** We begin by using an array to represent a complete binary tree. A complete tree is full to its next-to-last level, and its leaves on the last level are filled in from left to right. Thus, until we get to the last leaf, a complete tree has no holes.

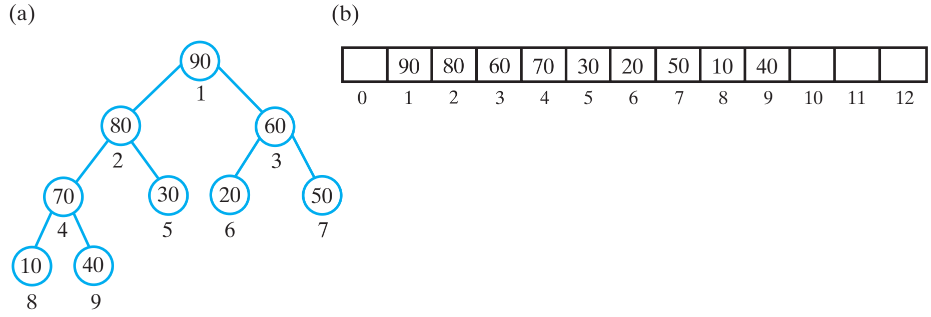
VideoNote
Implementing the ADT heap

Suppose that we number the nodes in a complete binary tree in the order in which a level-order traversal would visit them, beginning with 1. Figure 26-1a shows such a tree numbered in this way. Now suppose that we place the result of this tree's level-order traversal into consecutive array locations beginning at index 1, as Figure 26-1b shows. This representation of the data in the tree enables us to implement any needed tree operations. By beginning at index 1 instead of 0, we can simplify the implementation somewhat, as you will see.

26.3 Since the tree is complete, we can locate either the children or the parent of any node by performing a simple computation on the node's number. This number is the same as the node's corresponding array index. Thus, the children of the node i—if they exist—are stored in the array at indices $2i$ and $2i + 1$. The parent of this node is at array index $i/2$, unless of course the node is the root. In that case, $i/2$ is 0, since the root is at index 1. To detect the root, we can watch for either this index or a special value—called a **sentinel**—that we place at index 0.

> **Note:** When a binary tree is complete, using an array instead of linked nodes is desirable. You can use a level-order traversal to store the tree's data into consecutive locations of an array. This representation enables you to quickly locate the data in a node's parent or children. If you begin storing the tree at index 1 of the array—that is, if you skip the array's first element—the node at array index i
>
> - Has a parent at index $i/2$, unless the node is the root (i is 1)
> - Has any children at indices $2i$ and $2i + 1$

FIGURE 26-1 (a) A complete binary tree with its nodes numbered in level order; (b) its representation as an array



The complete binary tree in Figure 26-1 is actually a maxheap. We will now use the previous array representation of a complete tree in our implementation of a maxheap.

Question 1 If an array contains the entries of a heap in level order beginning at index 0, what array entries represent a node's parent, left child, and right child?

26.4 **Beginning the class `MaxHeap`.** As Listing 26-1 shows, our class begins with the following data fields: an array of `Comparable` heap entries, the index of the last entry in the array, and a constant for the default initial capacity of the heap. If `lastIndex` is less than 1, the heap is empty, since we begin the heap at index 1. Two simple constructors are similar to constructors we have seen before in array-based implementations. We allocate one extra array location, since we will not use the first one. The methods `getMax`, `isEmpty`, `getSize`, and `clear` have simple implementations that are shown in this listing. We consider the methods `add` and `removeMax` next.

LISTING 26-1 The class `MaxHeap`, partially completed

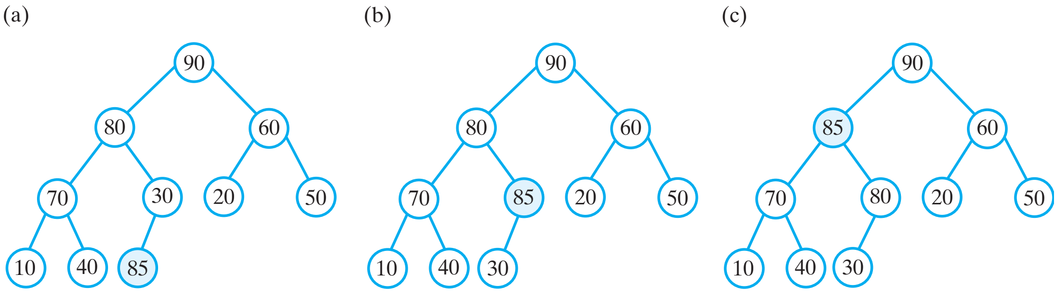
```java
import java.util.Arrays;
public class MaxHeap<T extends Comparable<? super T>>
       implements MaxHeapInterface<T>
{
   private T[] heap;      // array of heap entries
   private int lastIndex; // index of last entry
   private static final int DEFAULT_INITIAL_CAPACITY = 25;

   public MaxHeap()
   {
      this(DEFAULT_INITIAL_CAPACITY); // call next constructor
   } // end default constructor
   public MaxHeap(int initialCapacity)
   {
   // the cast is safe because the new array contains all null entries
      @SuppressWarnings("unchecked")
      T[] tempHeap = (T[]) new Comparable[initialCapacity + 1];
      heap = tempHeap;
      lastIndex = 0;
   } // end constructor

   public void add(T newEntry)
   {
       < See Segment 26.8. >
   } // end add

   public T removeMax()
   {
      < See Segment 26.12. >
   } // end removeMax

   public T getMax()
   {
      T root = null;
      if (!isEmpty())
         root = heap[1];
      return root;
   } // end getMax

   public boolean isEmpty()
   {
      return lastIndex < 1;
   } // end isEmpty

   public int getSize()
   {
      return lastIndex;
   } // end getSize

   public void clear()
   {
      for (; lastIndex > -1; lastIndex--)
          heap[lastIndex] = null;
      lastIndex = 0;
   } // end clear

   < Private methods >
   . . .
} // end MaxHeap
```

Adding an Entry

26.5 **The basic algorithm.** The algorithm to add an entry to a heap is not difficult. Recall that in a maxheap, the object in a node is greater than or equal to its descendant objects. Suppose that we want to add 85 to the maxheap in Figure 26-1. We first would place the new entry as the next leaf in the tree. Figure 26-2a shows that we add 85 as a left child of the 30. Notice that we actually would place 85 at index 10 of the array in Figure 26-1b.

Figure 26-2a is no longer a heap, since 85 is out of place. To transform the tree into a heap, we let 85 *float up* to its correct location. Since 85 is larger than its parent, 30, we swap it with the parent, as Figure 26-2b shows. The 85 is still larger than its new parent, 80, so we swap again (Figure 26-2c). Now 85 is less than its parent, so we have transformed the tree in Figure 26-2a into a maxheap.

FIGURE 26-2 The steps in adding 85 to the maxheap in Figure 26-1a



Question 2 What steps are necessary to add 100 to the heap in Figure 26-2c?

26.6 **Avoiding swaps.** Although the swaps mentioned in the previous segment make the algorithm easier to understand and to describe, they require more work than is actually necessary. Instead of placing the new entry in the next available position within the tree, as we did in Figure 26-2a, we need only reserve space for it. In an array-based implementation, we simply check that the array is not full. Figure 26-3a shows the new child as an empty circle.

We then compare the new entry—the 85 in this example—with the parent of the new child. Since 85 is larger than 30, we move the 30 to the new child, as Figure 26-3b shows. We treat the node that originally contained 30 as if it were empty. We now compare 85 with the parent, 80, of the empty node. Since 85 is larger than 80, we move the 80 to the empty node, as Figure 26-3c shows. Since 85 is not larger than the next parent, 90, we place the new entry into the empty node, as Figure 26-3d shows.

Note: To add a new entry to a heap, you begin at the next available position for a leaf. You follow a path from this leaf toward the root until you find the correct position for the new entry. As you do, you move entries from parent to child to ultimately make room for the new entry.

FIGURE 26-3 A revision of the steps shown in Figure 26-2, to avoid swaps

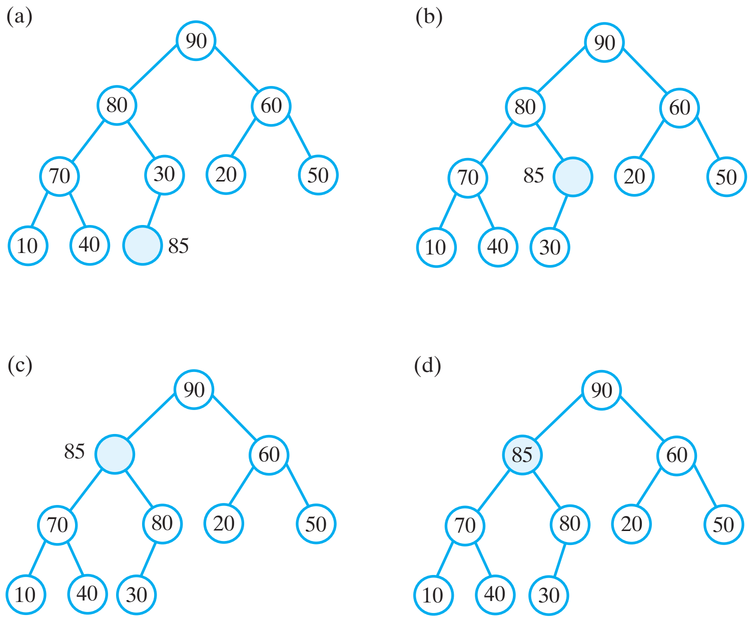
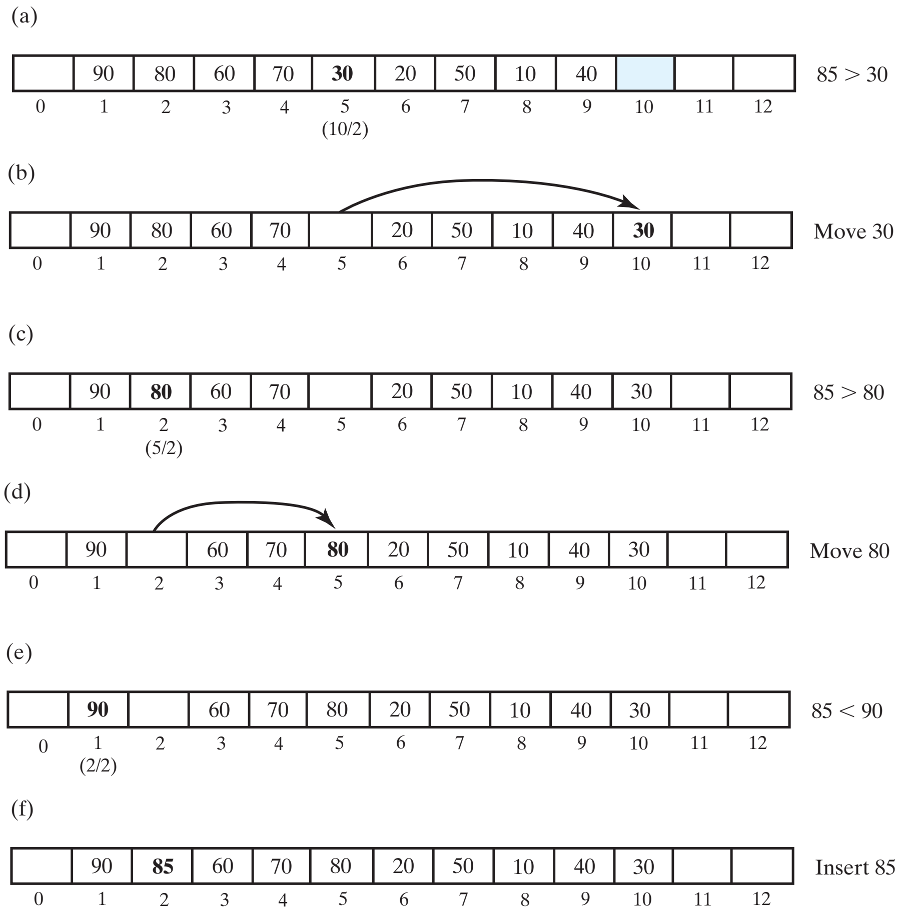


Figure 26-4 shows these same steps from the viewpoint of the array that represents the heap. In Part *a*, which is analogous to Figure 26-3a, we note that we have room for a new entry at index 10. The parent of this location is at location 10/2, or 5. We thus compare the new entry 85 to 30, the contents of the location at index 5. Since 85 > 30, we move 30 to the location at index 10 (Figures 26-4b and 26-3b.) The remaining steps proceed in a similar fashion. Note that Figure 26-4d corresponds to Figure 26-3c, and Figure 26-4f corresponds to Figure 26-3d.

26.7 **The refined algorithm.** The following algorithm summarizes the steps that add a new entry to a heap. Notice that the size of the array is expanded as necessary. To ignore the first location of the array, we simply ensure that `parentIndex` is greater than 0.

```
Algorithm add(newEntry)
if (the array heap is full)
   Double the size of the array

newIndex = index of next available array location
parentIndex = newIndex/2                  // index of parent of available location
while (parentIndex > 0 and newEntry > heap[parentIndex])
{
   heap[newIndex] = heap[parentIndex]  // move parent to available location

   // update indices
   newIndex = parentIndex
   parentIndex = newIndex/2
}

heap[newIndex] = newEntry                 // place new entry in correct location
```

FIGURE 26-4 An array representation of the steps in Figure 26-3



Question 3 Repeat Question 2 using the previous algorithm without swaps. Show the heap at each step as a tree and as an array.

26.8 **The method add.** We now implement the previous algorithm. The `add` method begins by incrementing the index `lastIndex` of the last array entry and then checks that the array is large enough to hold a new entry. If it is not, we double the size of the array, as we have done in previous chapters. (See Segment 2.34 of Chapter 2, for example.) The rest of the implementation closely follows the pseudocode.

```
public void add(T newEntry)
{
   lastIndex++;
   ensureCapacity();
```

```
      int newIndex = lastIndex;
      int parentIndex = newIndex / 2;
      while ( (parentIndex > 0) && newEntry.compareTo(heap[parentIndex]) > 0)
      {
         heap[newIndex] = heap[parentIndex];
         newIndex = parentIndex;
         parentIndex = newIndex / 2;
      } // end while

      heap[newIndex] = newEntry;
   } // end add
```

We can omit the test of `parentIndex` in the `while` statement if we place a sentinel value in the unused array location at index 0. We can use `newEntry` as this sentinel. You should answer Question 5 and convince yourself that this change will work.

In the worst case, this method follows a path from a leaf to the root. In Segment 23.9 of Chapter 23, we saw that the height of a complete tree having n nodes is $\log_2 (n + 1)$ rounded up. Thus, the `add` method is an O(log n) operation in the worst case.

? SELF-TEST

Question 4 Define the private method `ensureCapacity`.

Question 5 Revise the previous method `add` by placing `newEntry` as a sentinel value in the unused array location at index 0. You then can omit the test of `parentIndex` in the `while` statement.

Removing the Root

26.9 **The basic algorithm.** The `removeMax` method for a maxheap removes and returns the heap's largest object. This object is in the root of the maxheap. Let's remove the entry in the root of the heap in Figure 26-3d. Figure 26-5a shows this heap as if its root was empty.

We do not want to rip the root node out of the heap, as this will leave two disjoint subtrees. Instead we remove a leaf, namely the last one in the heap. To do so, we copy the leaf's data—30—to the root and then remove the leaf from the tree, as Figure 26-5b illustrates. Of course, in the array-based implementation, removing this leaf simply means adjusting `lastIndex`.

The 30 is out of place, so we no longer have a heap. We let the 30 *sink down* to its correct location. As long as 30 is less than its children, we swap it with its larger child. Thus, in Figure 26-5c, we have swapped 30 and 85. Continuing, we swap 30 and 80, as Figure 26-5d shows. In this case the 30 has settled at a leaf. In general, the out-of-place entry will settle at a node whose children are not greater than the entry.

? SELF-TEST

Question 6 What steps are necessary to remove the root from the heap in Figure 26-5d?

26.10 **Transforming a semiheap into a heap.** The tree in Figure 26-5b is called a **semiheap**. Except for the root, the objects in a semiheap are ordered as they are in a heap. In removing the root of the heap, we formed a semiheap and then transformed it back into a heap. As in the method `add`, we can save time by not swapping entries. Figure 26-6 shows the semiheap from Figure 26-5b and the steps that transform it into a heap without the swaps.

FIGURE 26-5 The steps to remove the entry in the root of the maxheap in Figure 26-3d

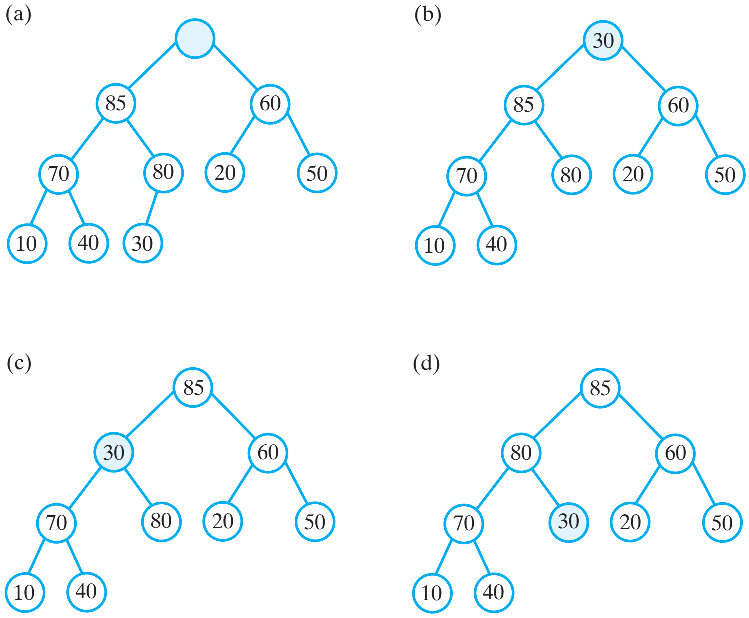
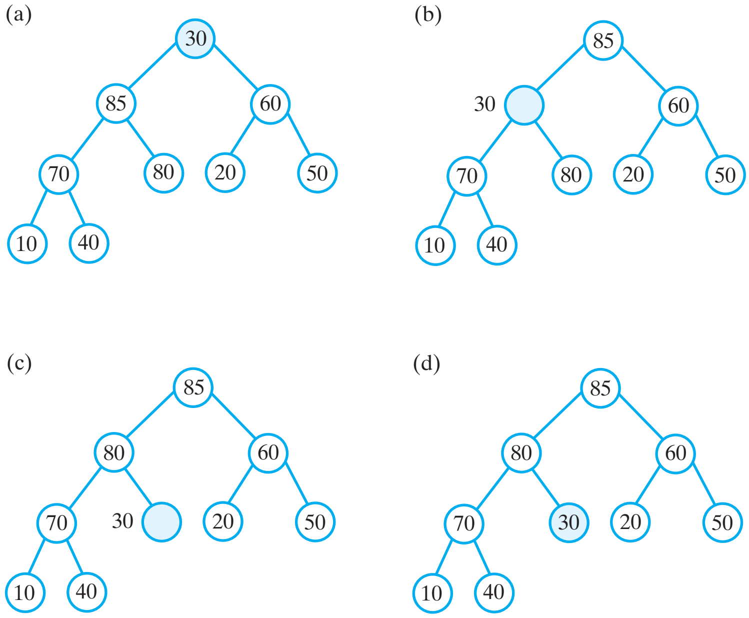


FIGURE 26-6 The steps that transform a semiheap into a heap without swaps



The following algorithm transforms a semiheap to a heap. To make the algorithm more general, we assume that the root of the semiheap is at a given index instead of at index 1.

```
Algorithm reheap(rootIndex)
// Transforms the semiheap rooted at rootIndex into a heap

done = false
orphan = heap[rootIndex]

while (!done and heap[rootIndex] has a child)
{
   largerChildIndex = index of the larger child of heap[rootIndex]
   if (orphan < heap[largerChildIndex])
   {
      heap[rootIndex] = heap[largerChildIndex]
      rootIndex = largerChildIndex
   }
   else
      done = true
}

heap[rootIndex] = orphan
```

As you will see, this algorithm has several uses.

Question 7 Show the contents of the array heap as you trace the steps of the algorithm reheap that correspond to those pictured in Figure 26-6.

26.11 **The method reheap.** The implementation of the reheap algorithm as a private method follows:

```
private void reheap(int rootIndex)
{
   boolean done = false;
   T orphan = heap[rootIndex];
   int leftChildIndex = 2 * rootIndex;

   while (!done && (leftChildIndex <= lastIndex) )
   {
      int largerChildIndex = leftChildIndex; // assume larger
      int rightChildIndex = leftChildIndex + 1;
      if ( (rightChildIndex <= lastIndex) &&
            heap[rightChildIndex].compareTo(heap[largerChildIndex]) > 0)
      {
         largerChildIndex = rightChildIndex;
      } // end if

      if (orphan.compareTo(heap[largerChildIndex]) < 0)
      {
         heap[rootIndex] = heap[largerChildIndex];
         rootIndex = largerChildIndex;
         leftChildIndex = 2 * rootIndex;
      }
```

```
         else
            done = true;
      } // end while

      heap[rootIndex] = orphan;
   } // end reheap
```

In the worst case, the method `reheap` follows a path from the root to a leaf. The number of nodes along this path is less than or equal to the height *h* of the heap. Thus, `reheap` is O(*h*). Recall that the height of a complete *n*-node tree is $\log_2 (n + 1)$ rounded up, so `reheap` is an O(log *n*) operation.

26.12 **The method `removeMax`.** The method `removeMax` replaces the heap's root with its last leaf to form a semiheap like the one in Figure 26-6a. The method then calls `reheap` to transform the semiheap back into a heap. Thus, `removeMax` has the following implementation:

```
public T removeMax()
{
   T root = null;

   if (!isEmpty())
   {
      root = heap[1];              // return value
      heap[1] = heap[lastIndex]; // form a semiheap
      lastIndex--;                 // decrease size
      reheap(1);                   // transform to a heap
   } // end if

   return root;
} // end removeMax
```

Since `reheap` is an O(log *n*) operation in the worst case, so is `removeMax`.

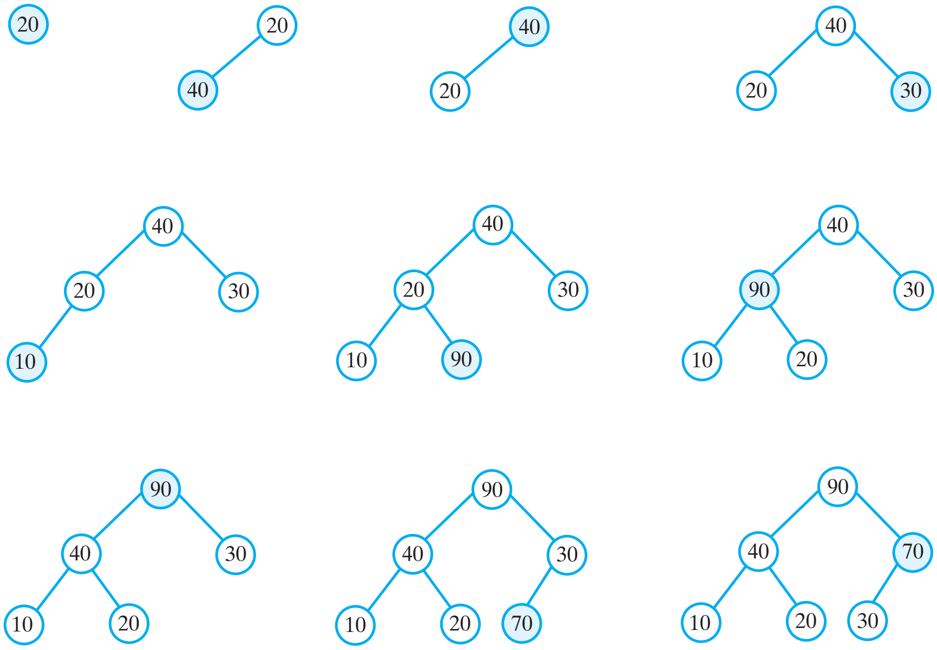
Note: To remove a heap's root, you first replace the root with the heap's last leaf. This step forms a semiheap, so you use the method `reheap` to transform the semiheap to a heap.

Creating a Heap

26.13 **Using `add`.** We could create a heap from a collection of objects by using the `add` method to add each object to an initially empty heap. Figure 26-7 shows the steps that this approach would take to add 20, 40, 30, 10, 90, and 70 to a heap. Since `add` is an O(log *n*) operation, creating the heap in this manner would be O(*n* log *n*).

Notice that we have a heap after each addition. This process does more than we really need. With less work, we can create one heap from a collection of objects without maintaining a heap at each intermediate step, as the next segment shows.

FIGURE 26-7 The steps in adding 20, 40, 30, 10, 90, and 70 to an initially empty heap



26.14 **Using `reheap`.** A more efficient way to create a heap uses the method `reheap`. We begin by placing the entries for the heap into an array beginning at index 1. Figure 26-8a provides an example of one such array. This array can represent the complete tree shown in Figure 26-8b. Does this tree contain any semiheaps that we can transform into heaps? The leaves are semiheaps, but they are also heaps. Thus, we can ignore the entries 70, 90, and 10. These entries are at the end of the array.

Moving toward the beginning of the array, we encounter 30, which is the root of a semiheap within the tree pictured in Figure 26-8b. If we apply `reheap` to this semiheap, we get the tree in Figure 26-8c. Continuing in this manner, we apply `reheap` to the semiheap rooted at 40 and then to the one rooted at 20. Parts *d*, *e*, and *f* of Figure 26-8 show the results of these steps. Figure 26-8f is the desired heap.

The following Java statements transform the array `heap`—whose entries are at the indices 1 through `lastIndex`—into a heap:

```
for (int rootIndex = lastIndex / 2; rootIndex > 0; rootIndex--)
   reheap(rootIndex);
```

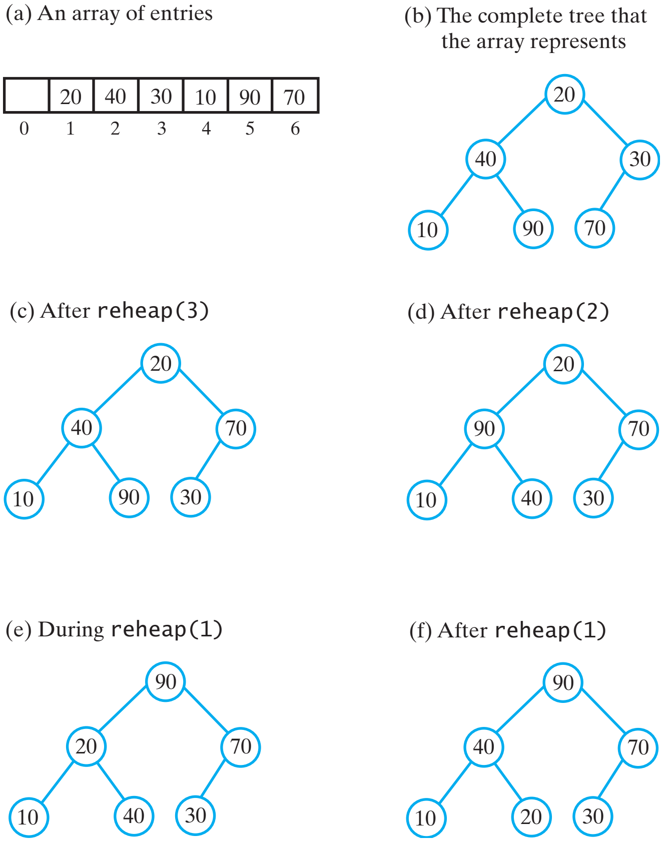
In applying `reheap`, we begin at the first nonleaf closest to the end of the array. This nonleaf is at index `lastIndex/2`, since it is the parent of the last leaf in the tree. We then work toward `heap[1]`.

? SELF-TEST

Question 8 If an array represents a heap, and `lastIndex` is the index of the heap's last leaf, show why the index of the first nonleaf closest to the end of the array is `lastIndex/2`.

26.15 Using `reheap` to transform an array of entries into a heap takes less work than if we used `add` to add the entries to the heap. In fact, building a heap in this manner is O(*n*), as we will now demonstrate.

FIGURE 26-8 The steps in creating a heap of the entries 20, 40, 30, 10, 90, and 70 by using `reheap`


(a) An array of entries

(b) The complete tree that the array represents

(c) After `reheap(3)`

(d) After `reheap(2)`

(e) During `reheap(1)`

(f) After `reheap(1)`

By the observation at the end of Segment 26.11, `reheap` is O(h_i), where h_i is the height of the subtree rooted at index *i*. In the worst case, the heap would be a full tree of height *h*. Each node in level $l < h$ is the root of a subtree whose height is $h - l + 1$. Moreover, level *l* contains 2^{l-1} nodes. Thus, the sum of the heights of the subtrees rooted at level *l* of a full heap is $(h - l + 1) \times 2^{l-1}$.

Since the loop in the previous segment ignores the nodes in the last level—level *h*—of the heap, its complexity is

$$O\left(\sum_{l=1}^{h-1} (h - l + 1) \cdot 2^{l-1}\right)$$

Exercise 6 at the end of this chapter asks you to show that this expression is equivalent to O(2^h), which is O(*n*).

Note: You can create a heap more efficiently by using the method `reheap` instead of the method `add`.

26.16 **Another constructor.** We can use the technique described in Segment 26.14 to implement another constructor for the class `MaxHeap`. Suppose that *n* entries for our heap are given in an array of exactly *n* locations. The following constructor takes this array, copies it into the data field `heap`, and uses `reheap` to create a heap. Although the entries in the given array begin at index 0, we place them into the array `heap` beginning at index 1.

```
public MaxHeap(T[] entries)
{
   // the cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempHeap = (T[]) new Comparable[entries.length + 1];
   heap = tempHeap;
   lastIndex = entries.length;

   // copy given array to data field
   for (int index = 0; index < entries.length; index++)
      heap[index + 1] = entries[index];

   // create heap
   for (int rootIndex = lastIndex / 2; rootIndex > 0; rootIndex--)
      reheap(rootIndex);
} // end constructor
```

Heap Sort

26.17 We can use a heap to sort an array. If we place the array items into a maxheap and then remove them one at a time, we will get the items in descending order. We saw in Segments 26.13 and 26.14 that using `reheap` instead of `add` is a more efficient way to create a heap from an array of items. In fact, we wrote a constructor in the previous segment that invoked `reheap` for this purpose. So if `a` is the array of items—strings, for example—we could use this constructor to create the heap, as follows:

VideoNote
The heap sort

```
MaxHeapInterface<String> myHeap = new MaxHeap<String>(a);
```

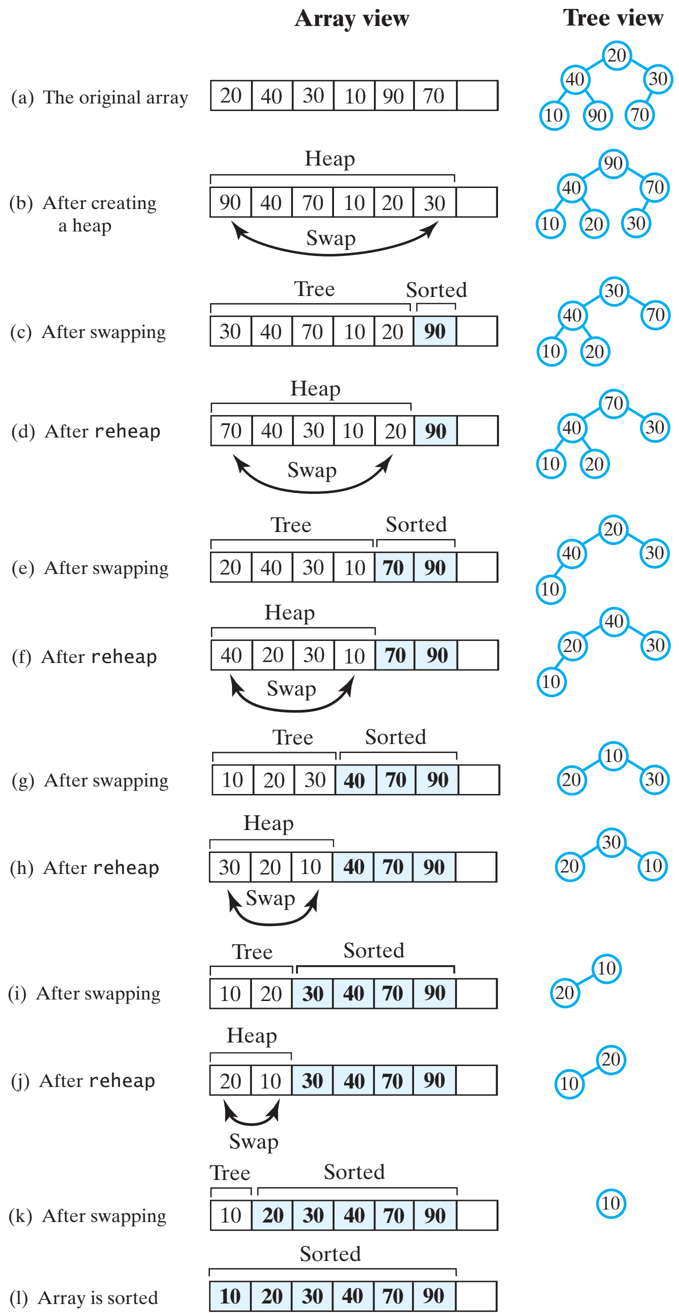
As we remove the items from `myHeap`, we could place them in order back into the array `a`. The problem with this approach is the additional memory required, since the heap uses an array besides the given array. However, by mimicking the heap's array-based implementation, we can make this approach more efficient without using the class `MaxHeap`. The resulting algorithm is called a **heap sort**.

26.18 To create an initial heap from the given array, we call `reheap` repeatedly, as we did in the constructor given in Segment 26.16. Parts *a* and *b* of Figure 26-9 show an array and the heap that results after this step. Since the array to be sorted begins at index 0, but in the constructor the heap begins at index 1, we must adjust `reheap`, as you will see.

The largest item in the array of Figure 26-9b is now first in the array, so we swap it with the last item in the array, as Figure 26-9c shows. The array is now partitioned into a tree region and a sorted region.

Following this swap, we call `reheap` on the tree portion—transforming it into a heap—and perform another swap, as Figures 26-9d and 26-9e illustrate. We repeat these operations until the tree region consists of one item (Figure 26-9k). The array is now sorted into ascending order. Notice that the array actually is sorted in Figure 26-9g, but the algorithm does not detect this fact.

FIGURE 26-9 A trace of heap sort



26.19 **Adjusting `reheap`.** We must revise the method `reheap` so that it is suitable for our sorting algorithm. The original method in Segment 26.11 uses the data fields `heap` and `lastIndex` of the class `MaxHeap`. Here, we make them parameters of the method. Thus, we change the method's header, as follows:

```
private static <T extends Comparable<? super T>>
        void reheap(T[] heap, int rootIndex, int lastIndex)
```

The portion of the array `heap` that represents the heap ranges from the index 0 to the index `lastIndex`. The semiheap is rooted at the index `rootIndex`.

Since the heap begins at index 0 instead of 1, as it did in Segment 26.11, the left child of the node at index *i* is at index 2*i* + 1 instead of 2*i*. Recall that Question 1 asked you to find this index. This change affects the two statements within `reheap` that determine `leftChildIndex`.

The revised `reheap` appears as follows:

```
private static <T extends Comparable<? super T>>
        void reheap(T[] heap, int rootIndex, int lastIndex)
{
   boolean done = false;
   T orphan = heap[rootIndex];
   int leftChildIndex = 2 * rootIndex + 1;

   while (!done && (leftChildIndex <= lastIndex))
   {
      int largerChildIndex = leftChildIndex;
      int rightChildIndex = leftChildIndex + 1;
      if ( (rightChildIndex <= lastIndex) &&
            heap[rightChildIndex].compareTo(heap[largerChildIndex]) > 0)
      {
         largerChildIndex = rightChildIndex;
      } // end if

      if (orphan.compareTo(heap[largerChildIndex]) < 0)
      {
         heap[rootIndex] = heap[largerChildIndex];
         rootIndex = largerChildIndex;
         leftChildIndex = 2 * rootIndex + 1;
      }
      else
         done = true;
   } // end while

   heap[rootIndex] = orphan;
} // end reheap
```

26.20 **The method `heapSort`.** The implementation of heap sort begins by calling `reheap` repeatedly, as we did in the constructor given in Segment 26.16, to create an initial heap from the given array. However, since the heap begins at index 0 instead of 1, we must adjust the loop somewhat:

```
for (int rootIndex = n / 2 - 1; rootIndex >= 0; rootIndex--)
   reheap(heap, rootIndex, n - 1);
```

This loop assumes *n* entries in the array `heap`, beginning at index 0. Exercise 3 at the end of this chapter asks you to verify the starting value of `rootIndex`.

The complete method appears as follows:

```
public static <T extends Comparable<? super T>>
       void heapSort(T[] array, int n)
{
   // create first heap
   for (int rootIndex = n / 2 - 1; rootIndex >= 0; rootIndex--)
      reheap(array, rootIndex, n - 1);
   swap(array, 0, n - 1);

   for (int lastIndex = n - 2; lastIndex > 0; lastIndex--)
   {
      reheap(array, 0, lastIndex);
      swap(array, 0, lastIndex);
   } // end for
} // end heapSort
```

Like merge sort and quick sort, heap sort is an O(n log n) algorithm. As implemented here, heap sort does not require a second array, but merge sort does. Recall from Chapter 9 that quick sort is O(n log n) most of the time, but is O(n^2) in its worst case. Since we usually can avoid quick sort's worst case by choosing appropriate pivots, it generally is the preferred sorting method.

Note: The time efficiency of heap sort

Although heap sort is O(n log n) in the average case, quick sort usually is the sorting method of choice.

Question 9 Trace the steps that the method `heapSort` takes when sorting the following array into ascending order: 9 6 2 4 8 7 5 3.

Chapter Summary

- Since a heap is a complete binary tree, it has an efficient array-based implementation.
- You add a new entry to a heap as the next leaf in a complete binary tree. You then make the entry float up to its proper location within the heap.
- You begin to remove the root entry of a heap by replacing it with the entry in the last leaf and then removing the leaf. The result is a semiheap. You transform the semiheap into a heap by making the new root entry sink down to its proper location within the heap.
- You could create a heap from a given array of entries by adding each entry to the heap. A more efficient approach considers the complete tree that the array represents and treats each nonleaf as a semiheap. You transform each such semiheap into a heap by using the same technique that you use when removing the root of the heap.
- A heap sort uses a heap to sort the entries in a given array.

Exercises

1. Trace the formation of a maxheap by the constructor given in Segment 26.16 for each of the following arrays:

a. 10 20 30 40 50
b. 10 20 30 40 50 60 70 80 90 100

2. Trace the addition of each of the following values to an initially empty maxheap:

10 20 30 40 50

Compare your trace with the results of Exercise 1a.

3. The method heapSort given in Segment 26.20 contains a loop that creates an initial heap from an array of *n* values. The loop variable rootIndex begins at n/2 - 1. Derive this starting value and show that the loop executes the same number of times as the corresponding loop in the constructor given in Segment 26.16.

4. Trace the steps of a heap sort on each of the following arrays:

   a. 10 20 30 40 50 60
   b. 60 50 40 30 20 10
   c. 20 50 40 10 60 30

5. Consider an array that represents a heap. Suppose that you replace the value at index *i* with a new value. It is likely that you will no longer have a heap. Write an algorithm that will give you a heap again.

6. Segment 26.15 showed that the complexity of creating a heap by using reheap is

$$O\left(\sum_{l=1}^{h-1}(h-l+1)\cdot 2^{l-1}\right)$$

Show that this expression is equivalent to $O(2^h)$, which is $O(n)$. *Hint*: First, change the summation variable from l to j, where $j = h - l + 1$. Then show by induction that

$$\sum_{j=2}^{h} j/2^j = 3/2 - \frac{h+2}{2^h}$$

7. Consider the loop in a heap sort that creates the initial heap from an array of *n* values (see Segment 26.20):

```
// create first heap
for (int rootIndex = n / 2 - 1; rootIndex >= 0; rootIndex--)
   reheap(array, rootIndex, n - 1);
```

Show that the actual number of calls to the method compareTo during the execution of this loop is no less than $n - 1$.

8. Consider again the loop mentioned in the previous exercise. Show that the actual number of calls to the method compareTo during the execution of this loop is no greater than $n \log_2 n$.

9. Heap sort is not the only way to sort an array using a heap. In this exercise you will explore a less efficient algorithm. After building an initial heap, as you would in the first step of a heap sort, the largest value will be in the first position of the array. If you leave this value in place and then build a new heap using the remaining values, you will get the next largest value in the entire array. By continuing in this manner, you can sort the array into descending order. If you use a minheap instead of a maxheap, you will sort the array into ascending order.

   a. Implement one of these sorts as the method newSortUsingAHeap.
   b. What is the Big Oh performance of this method?

Projects

1. Recall from Segment 23.31 of Chapter 23 that in a minheap, the object in each node is less than or equal to the objects in the node's descendants. While a maxheap has the method getMax, a minheap has the method getMin instead. Use an array to implement a minheap.

2. Compare the execution times of heap sort, merge sort, and quick sort on various arrays chosen at random. The "Projects" section of Chapter 4 described one way to time the execution of code.

3. Use a binary search tree in the implementation of MaxHeapInterface. Where in the tree will the largest entry occur? How efficient is this implementation?

4. Consider the problem of combining two heaps together into a single heap.
   - **a.** Write an efficient algorithm for combining two heaps, one with size n and the other with size 1. What is the Big Oh performance of your algorithm?
   - **b.** Write an efficient algorithm for combining two heaps of equal size n. What is the Big Oh performance of your algorithm?
   - **c.** Write an efficient algorithm for combining two arbitrary sized heaps into one heap. What is the Big Oh performance of your algorithm?
   - **d.** Implement the algorithm that you wrote in Part *c*.

5. You can study the average performance of the first step in a heap sort—building the initial heap—by taking the following steps:
   - Modify the method `reheap` so that it returns the number of calls made to `compareTo`.
   - Write a program that will iterate 1000 times. During each iteration, generate n random values and place them into an array. Count the number of comparisons needed by the code given in Exercise 7 to convert the array into a heap. Add the number of comparisons in each iteration into a total. After the loop has ended, compute the average number of comparisons needed to build the heap by dividing the number of comparisons by 1000.
   - In the previous step, let n = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 400, and 800. For each n, see whether the average number of calls to `compareTo` is greater than or equal to the lower bound $n - 1$ (see Exercise 7) and less than or equal to the upper bound $n \log_2 n$ (see Exercise 8).

6. Statisticians often are interested in the *median* value in a collection of data. In a collection, about the same number of values are greater than the median value as are less than the median value. When the data is sorted, the median value occurs at the midpoint of the collection. But when the data is not sorted, the median is not as easy to find.

   A problem more general than finding the median is to find the k^{th} smallest value in a collection of n values, where $0 < k < n$. To find the median, k would be $\lceil n/2 \rceil$—that is, the smallest integer greater than or equal to $n/2$. For example, the median value of 11 items is the 6^{th} smallest one.

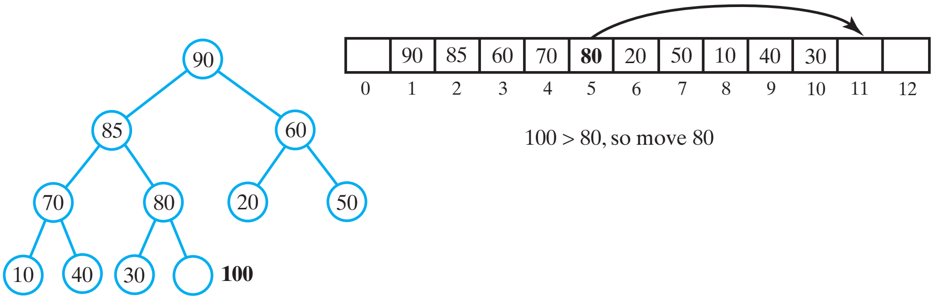
   Design an algorithm that uses a minheap to find the k^{th} smallest value in a collection of n values. Using the class of minheaps defined in Project 1, implement your algorithm as a method at the client level.

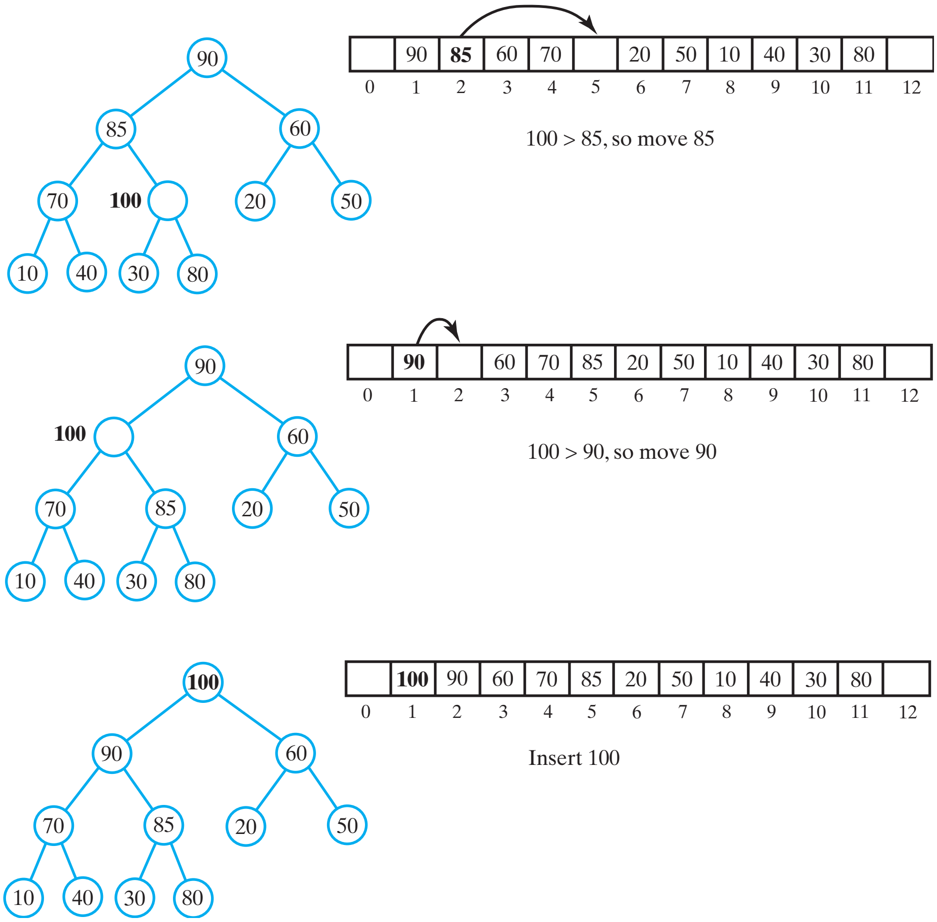
Answers to Self-Test Questions

1. The node at array index i
   - Has a parent at index $(i - 1)/2$, unless the node is the root (i is 0).
   - Has any children at indices $2i + 1$ and $2i + 2$.

2. Place 100 as a right child of 80. Then swap 100 with 80, swap 100 with 85, and finally swap 100 with 90.

3.





100 > 85, so move 85

100 > 90, so move 90

Insert 100

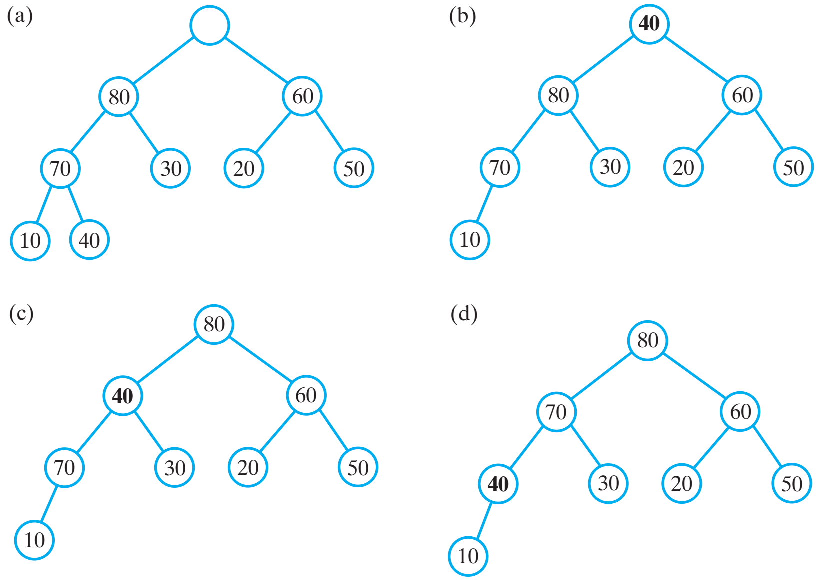
4.

```
private void ensureCapacity()
{
   if (lastIndex >= heap.length)
      heap = Arrays.copyOf(heap, 2 * heap.length);
} // end ensureCapacity
```

5.

```
public void add(T newEntry)
{
   lastIndex++;
   ensureCapacity();
   heap[0] = newEntry; // sentinel
   int newIndex = lastIndex;
   int parentIndex = newIndex / 2;

   while (newEntry.compareTo(heap[parentIndex]) > 0)
   {
      heap[newIndex] = heap[parentIndex];

      newIndex = parentIndex;
      parentIndex = newIndex / 2;
   } // end while

   heap[newIndex] = newEntry;
} // end add
```

6.



7.

(a)

85 is larger child of 30; 30 < 85, so move 85

	30	**85**	60	70	80	20	50	10	40	
0	1	2	3	4	5	6	7	8	9	10

(b)

80 is larger child of vacated node; 30 < 80, so move 80

	85		60	70	**80**	20	50	10	40	
0	1	2	3	4	5	6	7	8	9	10

(c)

Reached leaf

	85	80	60	70		20	50	10	40	
0	1	2	3	4	5	6	7	8	9	10

(d)

Insert 30

	85	80	60	70	**30**	20	50	10	40	
0	1	2	3	4	5	6	7	8	9	10

8. If a node at index *i* has children, they are at indices 2*i* and 2*i* + 1. The node at `lastIndex/2` then has a child at `lastIndex`. Since this child is the last leaf, any nodes beyond the one at `lastIndex/2` cannot have children and so must be leaves. Thus, the node at `lastIndex/2` must be the nonleaf closest to the end of the array.

Alternatively, examine some complete trees and notice that the desired nonleaf is the parent of the last child. This child is at index `lastIndex` of the array representation, so its parent has index `lastIndex/2`.

9.	9 6 2 4 8 7 5 3	Original array
	9 8 7 4 6 2 5 3	After repeated calls to `reheap`
	3 8 7 4 6 2 5 **9**	After swap
	8 6 7 4 3 2 5 **9**	After `reheap`
	5 6 7 4 3 2 **8 9**	After swap
	7 6 5 4 3 2 **8 9**	After `reheap`
	2 6 5 4 3 **7 8 9**	After swap
	6 4 5 2 3 **7 8 9**	After `reheap`
	3 4 5 2 **6 7 8 9**	After swap
	5 4 3 2 **6 7 8 9**	After `reheap`
	2 4 3 **5 6 7 8 9**	After swap
	4 2 3 **5 6 7 8 9**	After `reheap`
	3 2 **4 5 6 7 8 9**	After swap
	3 2 **4 5 6 7 8 9**	After `reheap`
	2 **3 4 5 6 7 8 9**	After swap
	2 3 4 5 6 7 8 9	Done

Chapter 27

Balanced Search Trees

Contents

Prerequisites

Chapter	23	Trees
Chapter	24	Tree Implementations
Chapter	25	A Binary Search Tree Implementation

Objectives

After studying this chapter, you should be able to

- Perform a rotation to restore the balance of an AVL tree after an addition
- Search for or add an entry to a 2-3 tree
- Search for or add an entry to a 2-4 tree
- Form a red-black tree from a given 2-4 tree
- Search for or add an entry to a red-black tree
- Describe the purpose of a B-tree

In Chapter 25, you saw that the operations on a binary search tree are O(log *n*) if the tree is balanced. Unfortunately, the add and remove operations do not ensure that a binary search tree remains balanced. This chapter will consider search trees that maintain their balance, and hence their efficiency.

Our goal is to introduce you to several types of balanced search trees and compare them. We will discuss the algorithms that add entries to a search tree while retaining its balance. We also will show you how to search the trees. We will not, however, cover the algorithms that remove entries, leaving this topic for a future course.

The entries in a tree are usually objects, but to make the pictures of trees clear and concise, we will show the entries as integers.

AVL Trees

27.1 Segment 23.28 in Chapter 23 showed that you can form several differently shaped binary search trees from the same collection of data. Some of these trees will be balanced and some will not. You could take an unbalanced binary search tree and rearrange its nodes to get a balanced binary search tree. Recall that every node in a balanced binary tree has subtrees whose heights differ by no more than 1.

VideoNote
AVL trees

This idea of rearranging nodes to balance a tree was first developed in 1962 by two mathematicians, Adel'son-Vel'skii and Landis. Named after them, the **AVL tree** is a binary search tree that rearranges its nodes whenever it becomes unbalanced. The balance of a binary search tree is upset only when you add or remove a node. Thus, during these operations, the AVL tree rearranges nodes as necessary to maintain its balance.

For example, Parts *a*, *b*, and *c* of Figure 27-1 show a binary search tree as we add 60, 50, and 20 to it. After the third addition, the tree is not balanced. An AVL tree would rearrange its nodes to restore balance, as shown in Figure 27-1d. This particular reorganization is called a **right rotation**, since you can imagine the nodes rotating about 50. If we now add 80 to the tree, it remains balanced, as Figure 27-2a shows. Adding 90 disrupts the balance (Figure 27-2b), but a **left rotation** restores it (Figure 27-2c). Here the rotation is about 80. Notice that after each rotation, the tree is still a binary search tree.

In discussing balance, we sometimes will mention a **balanced node**. A node is balanced if it is the root of a balanced tree, that is, if its two subtrees differ in height by no more than 1.

Single Rotations

27.2 **Right rotations.** Let's examine the previous rotations in more detail. Figure 27-3a shows a subtree of an AVL tree that is balanced. The heights of the subtrees T_1, T_2, and T_3 are the same. An addition that occurs in the left subtree T_1 of node *C* will add a leaf to T_1. Suppose that such an addition increases the height of T_1 by 1, as Figure 27-3b shows. The subtree rooted at node *N* is now unbalanced.

FIGURE 27-1 After inserting (a) 60; (b) 50; and (c) 20 into an initially empty binary search tree, the tree is not balanced; (d) a corresponding AVL tree rotates its nodes to restore balance

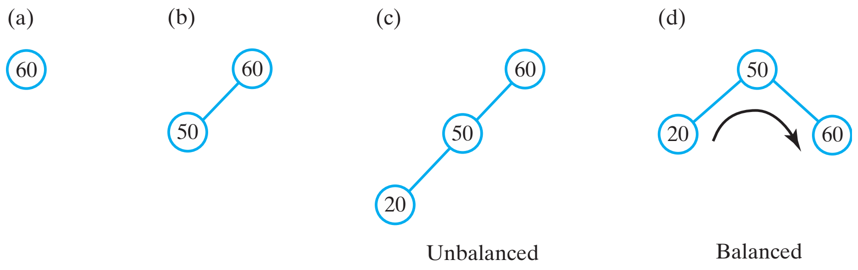
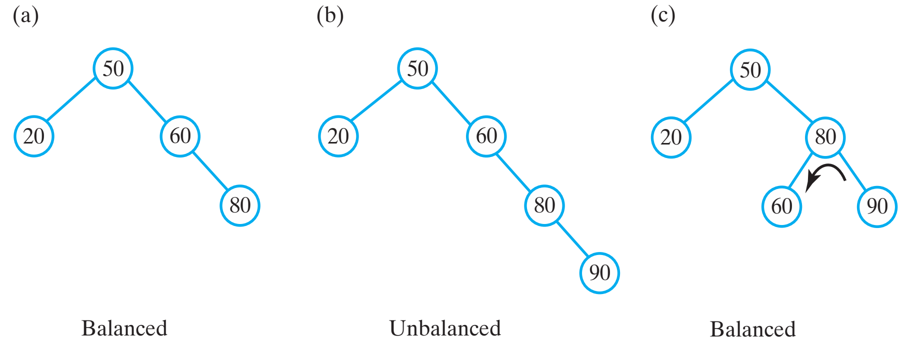


FIGURE 27-2 (a) Adding 80 to the tree in Figure 27-1d does not change the balance of the tree; (b) a subsequent addition of 90 makes the tree unbalanced ; (c) a left rotation restores its balance



N is the first node that is unbalanced along the path between the inserted leaf and *N*. A right rotation about node *C* restores the balance of the tree, as Figure 27-3c shows. After the rotation, *C* is above *N*, and the tree has the same height as it did before the addition of a node.

Since we had a binary search tree before the rotation (Figure 27-3b), the value in node *N* is greater than the value in node *C* and all the values in T_2. Moreover, all values in T_2 are greater than the value in node *C*. These relationships are maintained after the rotation (Figure 27-3c), since node *N* is a right child of node *C*, and T_2 is a left subtree of node *N*. Finally, the subtrees T_1 and T_3 have their original parents in the new tree. Thus, the resulting tree is still a binary search tree.

Figure 27-4 provides a specific instance of the right rotation depicted in Figure 27-3. Part *a* of the figure shows an imbalance at node *N* after 4 was added to the tree. A right rotation restores the tree's balance, as Part *b* illustrates. To simplify the figure, we have labeled only the roots of the subtrees T_1, T_2, and T_3. Here, node *N* was the root of the AVL tree, and node *C* became the root. However if node *N* had a parent before the rotation, we would make node *C* a child of that parent after the rotation.

FIGURE 27-3 Before and after an addition to an AVL subtree that requires a right rotation to maintain its balance

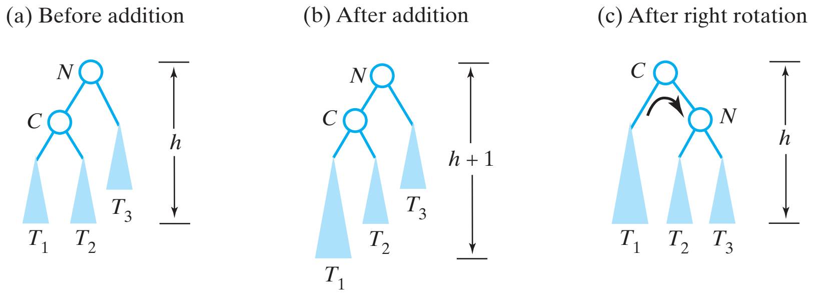
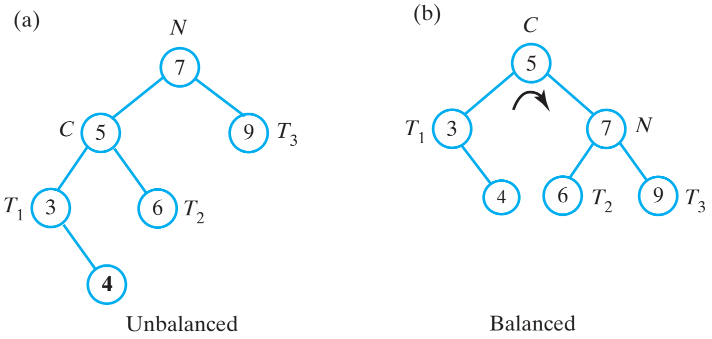


FIGURE 27-4 Before and after a right rotation restores balance to an AVL tree



The following algorithm performs the right rotation illustrated in Figures 27-3 and 27-4:

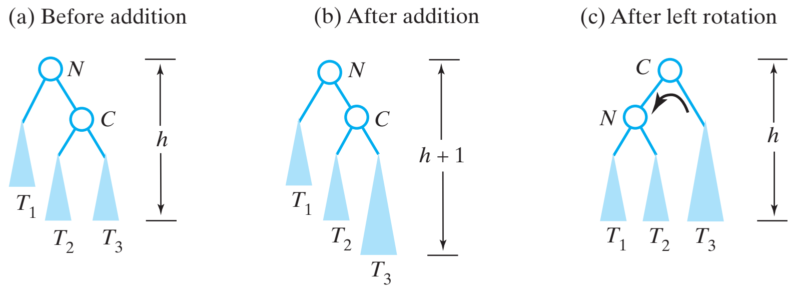
```
Algorithm rotateRight(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the left subtree of nodeN's left child.

nodeC = left child of nodeN
Set nodeN's left child to nodeC's right child
Set nodeC's right child to nodeN
return nodeC
```

Question 1 Using the notation of Figure 27-3, label nodes N and C, and subtrees T_1, T_2, and T_3, in Parts c and d of Figure 27-1.

27.3 **Left rotations.** Figure 27-5 shows a left rotation in a mirror image of Figure 27-3. The following algorithm performs this left rotation:

```
Algorithm rotateLeft(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the right subtree of nodeN's right child.

nodeC = right child of nodeN
Set nodeN's right child to nodeC's left child
Set nodeC's left child to nodeN
return nodeC
```

FIGURE 27-5 Before and after an addition to an AVL subtree that requires a left rotation to maintain its balance



Question 2 Why is the tree in Figure 27-5c a binary search tree?

Question 3 Using the notation of Figure 27-5, label nodes *N* and *C*, and subtrees T_1, T_2, and T_3, in Parts *b* and *c* of Figure 27-2.

Question 4 Just as Figure 27-4 gave an example of a right rotation, provide a specific example of the left rotation illustrated in Figure 27-5.

Note: An imbalance at node *N* of an AVL tree due to an addition to the tree can be corrected by a single rotation if

- The addition occurred in the left subtree of *N*'s left child *C* (right rotation), or
- The addition occurred in the right subtree of *N*'s right child *C* (left rotation)

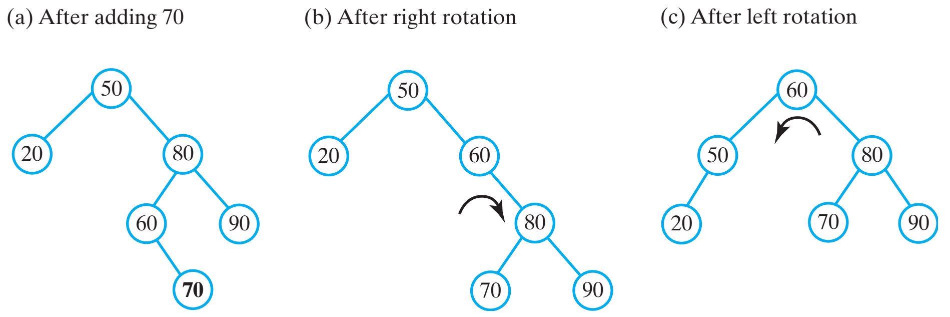
In both cases, we can imagine node *C* rotating above node *N*.

Double Rotations

27.4 **Right-left double rotations.** Now we add 70 to the AVL tree in Figure 27-2c. An imbalance occurs at the tree's root, as Figure 27-6a shows. A right rotation about the node containing 60 results in the tree in Figure 27-6b. The mechanics of this rotation are the same as the one in Figure 27-3, where the rotation is about node *C*. The subtree heights differ in these two figures, however.

Unfortunately, this rotation does not balance the tree. A subsequent left rotation about the node containing 60—corresponding to node *C* in Figure 27-5b—is necessary to restore the balance (Figure 27-6c). Together, these two rotations are called a **right-left double rotation**. First, 60 rotates above 80 and then it rotates above 50. Again, notice that after each rotation, the tree is still a binary search tree.

FIGURE 27-6 (a) Adding 70 to the tree in Figure 27-2c destroys its balance; to restore the balance, perform both (b) a right rotation and (c) a left rotation

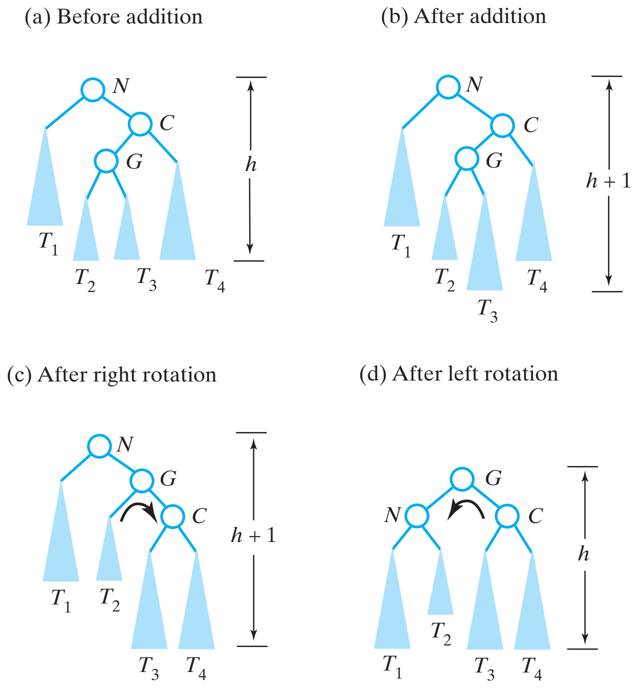


Let's look at the general case. Figure 27-7a shows a subtree of an AVL tree that is height balanced. Node *N* has a child *C* and a grandchild *G*. An addition that occurs in the right subtree T_3 of node *G* adds a leaf to T_3. When such an addition increases the height of T_3, as Figure 27-7b shows, the subtree rooted at *N* becomes unbalanced. Notice that nodes *N*, *C*, and *G* correspond to the nodes in Figure 27-6a that contain 50, 80, and 60, respectively.

Node *N* is the first node that is unbalanced along the path between the inserted leaf and *N*. After a right rotation about node *G*, the subtree rooted at *G* is unbalanced, as Figure 27-7c shows.

A left rotation about G restores the balance of the tree, as you can see in Figure 27-7d. Note that G first rotates above C and then above N.

FIGURE 27-7 Before and after an addition to an AVL subtree that requires both a right rotation and a left rotation to maintain its balance



The following algorithm performs the right-left double rotation illustrated in Figure 27-7:

```
Algorithm rotateRightLeft(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the left subtree of nodeN's right child.

nodeC = right child of nodeN
Set nodeN's right child to the node returned by rotateRight(nodeC)
return rotateLeft(nodeN)
```

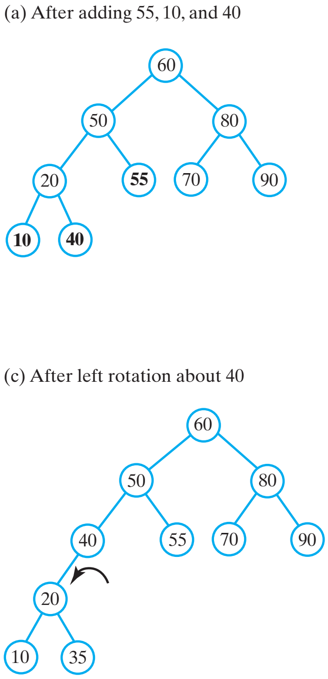
The right rotation transforms the tree in Figure 27-7b into the one in Figure 27-7c. The left rotation then transforms Figure 27-7c to Figure 27-7d.

? SELF-TEST

Question 5 Using the notation of Figure 27-7, label nodes N, C, and G, and subtrees T_1, T_2, T_3, and T_4 in Figure 27-6.

27.5 **Left-right double rotations.** Now we add 55, 10, and 40 to the tree in Figure 27-6c to get the tree in Figure 27-8a. As long as we add 55 first, these additions maintain the tree's balance without rotations. After we add 35, as Figure 27-8b shows, the tree is unbalanced at the node containing 50. To restore the balance, we perform a left rotation about the node containing 40—so 40 rotates above 20—to get the tree in Figure 27-8c. Then we perform a right rotation about the node containing 40—so 40 rotates above 50—to get the balanced tree in Figure 27-8d.

FIGURE 27-8 (a) The AVL tree in Figure 27-6c after additions that maintain its balance; (b) after an addition that destroys the balance; (c) after a left rotation; (d) after a right rotation

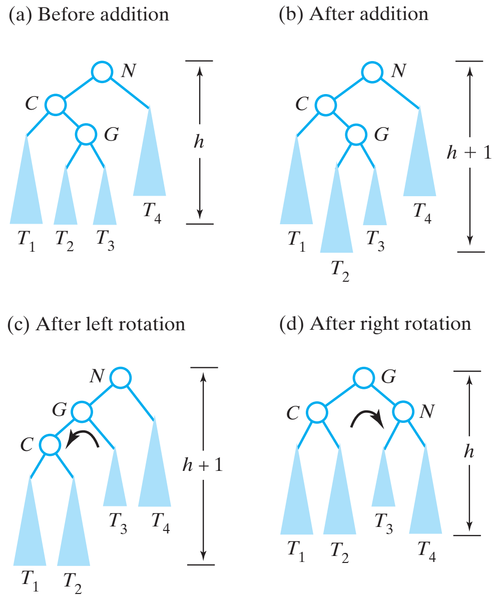

(a) After adding 55, 10, and 40

(b) After adding 35

(c) After left rotation about 40

(d) After right rotation about 40

FIGURE 27-9 Before and after an addition to an AVL subtree that requires both a left rotation and a right rotation to maintain its balance


(a) Before addition

(b) After addition

(c) After left rotation

(d) After right rotation

Figure 27-9 shows a left-right double rotation in general. It is a mirror image of the right-left double rotation pictured in Figure 27-7. Both left-right and right-left double rotations cause node *G* to rotate first above node *C* and then above node *N*.

The following algorithm performs the left-right double rotation illustrated in Figure 27-9:

```
Algorithm rotateLeftRight(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the right subtree of nodeN's left child.

nodeC = left child of nodeN
Set nodeN's left child to the node returned by rotateLeft(nodeC)
return rotateRight(nodeN)
```

Question 6 Using the notation of Figure 27-9, label nodes *N*, *C*, and *G*, and subtrees T_1, T_2, T_3, and T_4 in Figure 27-8.

Note: A double rotation is accomplished by performing two single rotations:

1. A rotation about node *N*'s grandchild *G* (its child's child)
2. A rotation about node *N*'s new child

We can imagine G rotating first above *N*'s original child *C* and then above *N*.

Note: An imbalance at node *N* of an AVL tree can be corrected by a double rotation if

- The addition occurred in the left subtree of *N*'s right child (right-left rotation), or
- The addition occurred in the right subtree of *N*'s left child (left-right rotation)

27.6 **Summary comments about rotation after an addition.** Following an addition to an AVL tree, a temporary imbalance might occur. Let *N* be an unbalanced node that is closest to the new leaf. Either a single or double rotation will restore the tree's balance. No other rotations are necessary. To see this, remember that before the addition, the tree was balanced; after all, it is an AVL tree. After an addition that causes a rotation, the tree has the same height as it did before the addition. Therefore, no node above *N* can be unbalanced now if it was balanced before the addition. Moreover, the four rotations cover the only four possibilities for the cause of the imbalance at node *N*:

- The addition occurred in the left subtree of *N*'s left child (right rotation)
- The addition occurred in the right subtree of *N*'s left child (left-right rotation)
- The addition occurred in the left subtree of *N*'s right child (right-left rotation)
- The addition occurred in the right subtree of *N*'s right child (left rotation)

Removing an entry from a binary search tree results in the removal of a node, but not necessarily the node that contained the entry. Thus, removing an entry from an AVL tree can lead to a temporary imbalance. We restore the tree's balance by using single or double rotations as described previously for addition. We leave the details for you to develop in Project 1.

Note: One single or double rotation during the addition of an entry will restore the balance of an AVL tree.

Question 7 What AVL tree results when you make the following additions to an initially empty AVL tree? 70, 80, 90, 20, 10, 50, 60, 40, 30

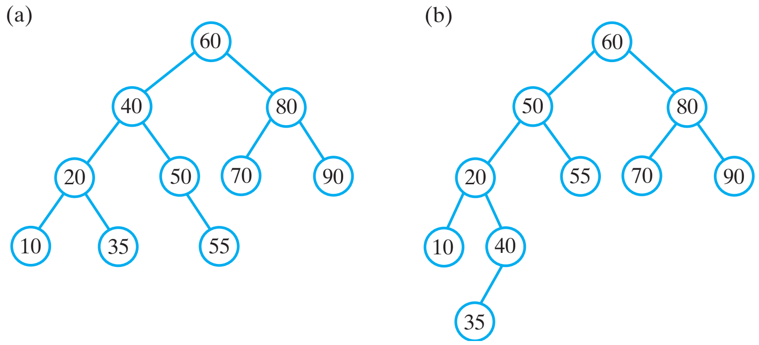
Question 8 What tree results when you make the same additions given in the previous question to an initially empty binary search tree? How does this tree compare to the AVL tree you created in the previous question?

Question 9 Why is the tree shown in Figure 27-7d a binary search tree?

Question 10 Why is the tree shown in Figure 27-9d a binary search tree?

27.7 **An AVL tree versus a binary search tree.** We created the AVL tree in Figure 27-8d by adding 60, 50, 20, 80, 90, 70, 55, 10, 40, and 35 to an initially empty AVL tree. Figure 27-10a shows that tree again. If we make the same additions to an initially empty binary search tree, we get the tree in Figure 27-10b. This tree is unbalanced and is taller than the AVL tree.

FIGURE 27-10 The result of adding 60, 50, 20, 80, 90, 70, 55, 10, 40, and 35 to an initially empty (a) AVL tree; (b) binary search tree



Implementation Details

27.8 **An outline of the class.** Listing 27-1 outlines our class of AVL trees. Since an AVL tree is also a binary search tree, we will derive the class `AVLTree` from the class `BinarySearchTree`, which we discussed in Chapter 25. The methods `add` and `remove` are like those in `BinarySearchTree` but require logic to detect and correct any imbalance that might occur. Thus, we need to override these methods. The other methods specified in `SearchTreeInterface` are inherited from `BinarySearchTree`.

LISTING 27-1 An outline of the class `AVLTree`

```
package TreePackage;
public class AVLTree<T extends Comparable<? super T>>
             extends BinarySearchTree<T>
             implements SearchTreeInterface<T>
{
   public AVLTree()
   {
      super();
   } // end default constructor

   public AVLTree(T rootEntry)
   {
      super(rootEntry);
   } // end constructor

   < Implementations of add and remove are here. A definition of add appears in Segment 27.12
     of this chapter. Other methods in SearchTreeInterface are inherited. >
   . . .
   < Implementations of private methods to rebalance the tree using rotations are here. >
   . . .
} // end AVLTree
```

27.9 **Rotations.** As we discussed earlier, an AVL tree uses rotations to maintain its balance following the addition or removal of a node. The methods that perform these rotations closely follow the pseudocode given in the previous segments.

For example, consider the algorithm for a single right rotation, as given in Segment 27.2:

```
Algorithm rotateRight(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the left subtree of nodeN's left child.

nodeC = left child of nodeN
Set nodeN's left child to nodeC's right child
Set nodeC's right child to nodeN
return nodeC
```

The following method implements this pseudocode as a private method of the class `AVLTree`:

```
// Corrects an imbalance at the node closest to a structural
// change in the left subtree of the node's left child.
// nodeN is a node, closest to the newly added leaf, at which
// an imbalance occurs and that has a left child.
private BinaryNodeInterface<T> rotateRight(BinaryNodeInterface<T> nodeN)
{
   BinaryNodeInterface<T> nodeC = nodeN.getLeftChild();
   nodeN.setLeftChild(nodeC.getRightChild());
   nodeC.setRightChild(nodeN);
   return nodeC;
} // end rotateRight
```

The method `rotateLeft` has a similar implementation and is left as an exercise.

Since a double rotation is equivalent to two single rotations, the methods that perform double rotations each call the methods that perform single rotations. For example, the algorithm for a right-left double rotation, as it appeared in Segment 27.4, is

```
Algorithm rotateRightLeft(nodeN)
// Corrects an imbalance at a given node nodeN due to an addition
// in the left subtree of nodeN's right child.

nodeC = right child of nodeN
Set nodeN's right child to the node returned by rotateRight(nodeC)
return rotateLeft(nodeN)
```

An implementation of this pseudocode follows:

```
// Corrects an imbalance at the node closest to a structural
// change in the left subtree of the node's right child.
// nodeN is a node, closest to the newly added leaf, at which
// an imbalance occurs and that has a right child.
private BinaryNodeInterface<T> rotateRightLeft(BinaryNodeInterface<T> nodeN)
{
   BinaryNodeInterface<T> nodeC = nodeN.getRightChild();
   nodeN.setRightChild(rotateRight(nodeC));
   return rotateLeft(nodeN);
} // end rotateRightLeft
```

The method `rotateLeftRight` has a similar implementation and is left as an exercise.

Question 11 Implement the algorithm given in Segment 27.3 for a single left rotation.

27.10 **Rebalancing.** As you saw previously, you can correct an imbalance at node *N* of an AVL tree caused by the addition of a node by performing only one of the following rotations, according to where in the tree the change to its structure occurred:

- Right rotation if the addition occurred in the left subtree of *N*'s left child
- Left-right rotation if the addition occurred in the right subtree of *N*'s left child
- Left rotation if the addition occurred in the right subtree of *N*'s right child
- Right-left rotation if the addition occurred in the left subtree of *N*'s right child

The following pseudocode uses these criteria and the rotation methods to rebalance the tree:

```
Algorithm rebalance(nodeN)
if (nodeN's left subtree is taller than its right subtree by more than 1)
{ // addition was in nodeN's left subtree
   if (the left child of nodeN has a left subtree that is taller than its right subtree)
      rotateRight(nodeN)       // addition was in left subtree of left child
   else
      rotateLeftRight(nodeN)  // addition was in right subtree of left child
}
else if (nodeN's right subtree is taller than its left subtree by more than 1)
{ // addition was in nodeN's right subtree
   if (the right child of nodeN has a right subtree that is taller than its left subtree)
      rotateLeft(nodeN)        // addition was in right subtree of right child
   else
      rotateRightLeft(nodeN)  // addition was in left subtree of right child
}
```

No rebalancing is needed if the heights of node *N*'s two subtrees either are the same or differ by 1.

27.11 **The method `rebalance`.** A method `getHeightDifference` that returns the difference in the heights of a node's left and right subtrees would help us to implement the previous algorithm. By giving a sign to the height difference it returns, `getHeightDifference` can indicate which subtree is taller. This method can be defined within either the class `AVLTree` or the class `BinaryNode`. The latter choice would be more efficient if each node maintained height information as one or more data fields instead of recomputing it each time the method is called. (See Project 4.)

A node is unbalanced if its two subtrees differ in height by more than 1, that is, if `getHeightDifference` returns a value either greater than 1 or less than -1. If this return value is greater than 1, the left subtree is taller; if it is less than -1, the right subtree is taller.

Using the method getHeightDifference, we can implement the previous pseudocode for rebalance within the class AVLTree as follows:

```
private BinaryNodeInterface<T> rebalance(BinaryNodeInterface<T> nodeN)
{
   int heightDifference = getHeightDifference(nodeN);

   if (heightDifference > 1)
   { // left subtree is taller by more than 1,
     // so addition was in left subtree
      if (getHeightDifference(nodeN.getLeftChild()) > 0)
         // addition was in left subtree of left child
         nodeN = rotateRight(nodeN);
      else
         // addition was in right subtree of left child
         nodeN = rotateLeftRight(nodeN);
   }
   else if (heightDifference < -1)
   { // right subtree is taller by more than 1,
     // so addition was in right subtree
      if (getHeightDifference(nodeN.getRightChild()) < 0)
         // addition was in right subtree of right child
         nodeN = rotateLeft(nodeN);
      else
         // addition was in left subtree of right child
         nodeN = rotateRightLeft(nodeN);
   } // end if
   // else nodeN is balanced

   return nodeN;
} // end rebalance
```

27.12 **The method add.** Adding to an AVL tree is just like adding to a binary search tree, but with a rebalancing step. For example, we can begin with the recursive implementations of the methods add and addEntry of BinarySearchTree (Segments 25.15 and 25.16 in Chapter 25) and insert calls to rebalance. The resulting methods in AVLTree are as follows:

```
public T add(T newEntry)
{
   T result = null;

   if (isEmpty())
      setRootNode(new BinaryNode<T>(newEntry));
   else
   {
      BinaryNodeInterface<T> rootNode = getRootNode();
      result = addEntry(rootNode, newEntry);
      setRootNode(rebalance(rootNode));
   } // end if

   return result;
} // end add

private T addEntry(BinaryNodeInterface<T> rootNode, T newEntry)
{
   assert rootNode != null;
   T result = null;
   int comparison = newEntry.compareTo(rootNode.getData());
   if (comparison == 0)
   {
      result = rootNode.getData();
      rootNode.setData(newEntry);
   }
   else if (comparison < 0)
   {
      if (rootNode.hasLeftChild())
      {
         BinaryNodeInterface<T> leftChild = rootNode.getLeftChild();
         result = addEntry(leftChild, newEntry);
         rootNode.setLeftChild(rebalance(leftChild));
      }
      else
         rootNode.setLeftChild(new BinaryNode<T>(newEntry));
   }
   else
   {
      assert comparison > 0;

      if (rootNode.hasRightChild())
      {
         BinaryNodeInterface<T> rightChild = rootNode.getRightChild();
         result = addEntry(rightChild, newEntry);
         rootNode.setRightChild(rebalance(rightChild));
      }
      else
         rootNode.setRightChild(new BinaryNode<T>(newEntry));
   } // end if

   return result;
} // end addEntry
```

Although `rebalance` is called several times during the course of executing these methods, a rebalancing of the tree occurs at most once. Most calls to `rebalance` simply check whether a rebalancing is needed.

As attractive as an AVL tree might seem, better search trees have been developed, as you will now see.

2-3 Trees

27.13 A **2-3 tree** is a general search tree whose interior nodes must have either two or three children. A **2-node** contains one data item *s* and has two children, like the nodes in a binary search tree. This data *s* is greater than any data in the node's left subtree and less than any data in the right subtree. That is, the data in the node's left subtree is less than *s*, and any data in the right subtree is greater than *s*, as Figure 27-11a shows.

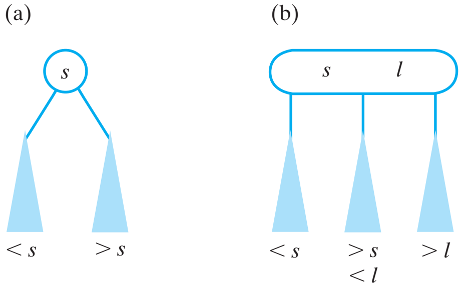
A **3-node** contains two data items, *s* and *l*, and has three children. Data that is less than the smaller data item *s* occurs in the node's left subtree. Data that is greater than the larger data item *l* occurs in the node's right subtree. Data that is between *s* and *l* occurs in the node's middle subtree. Figure 27-11b shows a typical 3-node.

VideoNote
2-3 trees

Because it can contain 3-nodes, a 2-3 tree tends to be shorter than a binary search tree. To make the 2-3 tree balanced, we require that all leaves occur on the same level. Thus, a 2-3 tree is completely balanced.

Note: A 2-3 tree is a general search tree whose interior nodes must have either two or three children and whose leaves occur on the same level. A 2-3 tree is completely balanced.

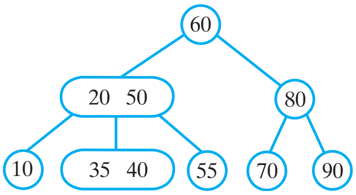
FIGURE 27-11 (a) A 2-node; (b) a 3-node



Searching a 2-3 Tree

27.14 If we had a 2-3 tree, such as the one in Figure 27-12, how would we search it? Notice that each 2-node adheres to the ordering of a binary search tree. The 3-node leaf <35 40> contains values that are between the values in its parent. Knowing this, we can search for the 40, for example, by first comparing 40 with the root value 60. We then move to 60's left subtree and compare 40 with the values in the root of this subtree. Since 40 lies between 20 and 50, it would occur in the middle subtree, if it appears at all. While searching the middle subtree, we compare 40 with 35 and then finally with 40.

FIGURE 27-12 A 2-3 tree



The search algorithm is an extension of the search algorithm for a binary search tree:

```
Algorithm search23Tree(23Tree, desiredObject)
// Searches a 2-3 tree for a given object.
// Returns true if the object is found.

if (23Tree is empty)
   return false
else if (desiredObject is in the root of 23Tree)
   return true
else if (the root of 23Tree contains two entries)
{
   if (desiredObject < smaller object in the root)
      return search23Tree(left subtree of 23Tree, desiredObject)
   else if (desiredObject > larger object in the root)
      return search23Tree(right subtree of 23Tree, desiredObject)
   else
      return search23Tree(middle subtree of 23Tree, desiredObject)
}
else if (desiredObject < object in the root)
   return search23Tree(left subtree of 23Tree, desiredObject)
else
   return search23Tree(right subtree of 23Tree, desiredObject)
```

Question 12 What comparisons are made while searching the 2-3 tree in Figure 27-12 for each of the following values?

a. 5 **b.** 55 **c.** 41 **d.** 30

Adding Entries to a 2-3 Tree

27.15 Using an example, we will describe how to add an entry to a 2-3 tree. So that we can compare our results with an AVL tree, we will make the same sequence of additions to an initially empty 2-3 tree that we made when forming the AVL tree in Figure 27-10a.

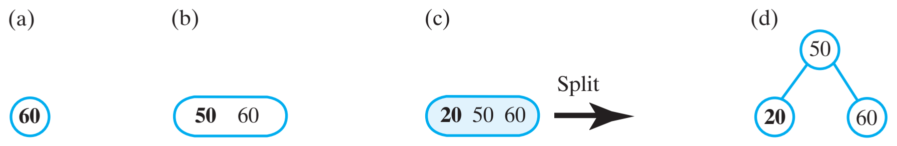
As we did when adding to a binary search tree, we add an entry to a 2-3 tree at a leaf. We locate this leaf by using the search algorithm that we described in the previous segment. Thus, once we make the addition, the search algorithm will be able to locate the new entry.

We will now add the following entries: 60, 50, 20, 80, 90, 70, 55, 10, 40, and 35.

27.16 **Adding 60, 50, and 20.** After we add 60, the 2-3 tree consists of a single 2-node. After we add 50, the tree is a single 3-node. Figures 27-13a and 27-13b show the tree after each of these additions.

Now we add 20. To facilitate our description of this addition, we show the 20 in Figure 27-13c within the only node in the tree. This is an imaginary placement, since a 3-node can contain only two data items. We would not actually place more data in this node. Since the node cannot accommodate the 20, we **split** it into three nodes, moving the middle value 50 up one level. In this case, we are splitting a leaf that is also the tree's root. Moving the 50 up requires that we create a new node that becomes the new root of the tree. This step increases the height of the tree by 1, as Figure 27-13d shows.

FIGURE 27-13 An initially empty 2-3 tree after adding (a) 60 and (b) 50; (c), (d) adding 20 causes the 3-node to split

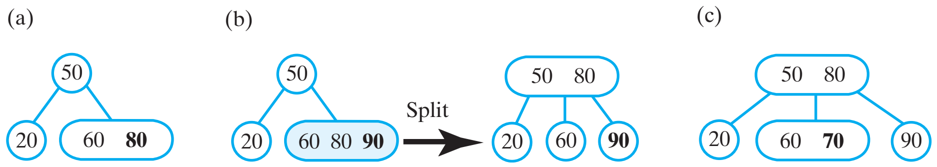


27.17 **Adding 80, 90, and 70.** To add 80, we note that the search algorithm would look for 80 in the tree's rightmost leaf. Since this leaf has room for another data entry, that is where we should add 80. Figure 27-14a shows the result of this addition.

The search algorithm would look for 90 in the leaf to which we just added 80. Although the leaf has no room for another entry, we imagine that we have added 90 there. We then move the middle value—the 80—up a level and split the leaf into two nodes for the 60 and 90, as Figure 27-14b shows. Since the root can accept the 80, the addition is complete.

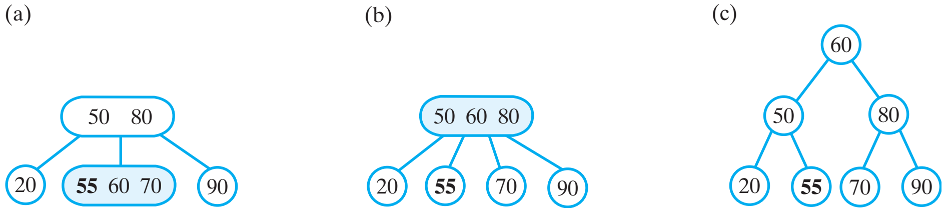
The entry 70 belongs in the root's middle subtree, and since this leaf can accept another entry, we add 70 there. Figure 27-14c shows the tree after this addition.

FIGURE 27-14 The 2-3 tree after adding (a) 80; (b) 90; (c) 70



27.18 **Adding 55.** When we add 55 to the tree in Figure 27-14c, the search algorithm terminates at the root's middle subtree—a leaf—as Figure 27-15a indicates. Since this leaf cannot accommodate another entry, we split the leaf and move 60 up a level to the root, as shown in Figure 27-15b. Moving the 60 causes the root to split, and 60 moves up another level to a new node that becomes the new root. Figure 27-15c shows the result of this addition.

FIGURE 27-15 Adding 55 to the 2-3 tree causes a leaf and then the root to split



27.19 **Adding 10, 40, and 35.** The leaf that contains 20 has room for 10 as an additional entry, as Figure 27-16a shows. An additional entry, 40, belongs in the same leaf. Since the leaf already contains two entries, we split it and move 20 up a level to the node that contains 50. Figures 27-16b and 27-16c show this result. Finally, Figure 27-17 shows the result of adding 35 to the tree. The leaf that contains 40 accommodates this new entry.

FIGURE 27-16 The 2-3 tree after adding (a) 10; (b), (c) 40

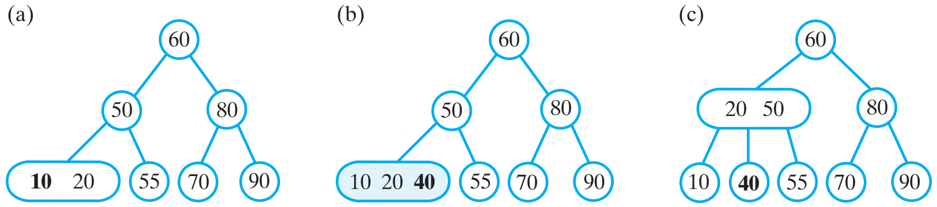
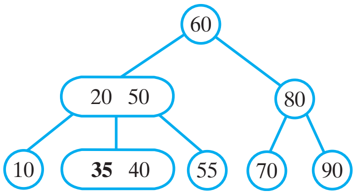


FIGURE 27-17 The 2-3 tree after adding 35



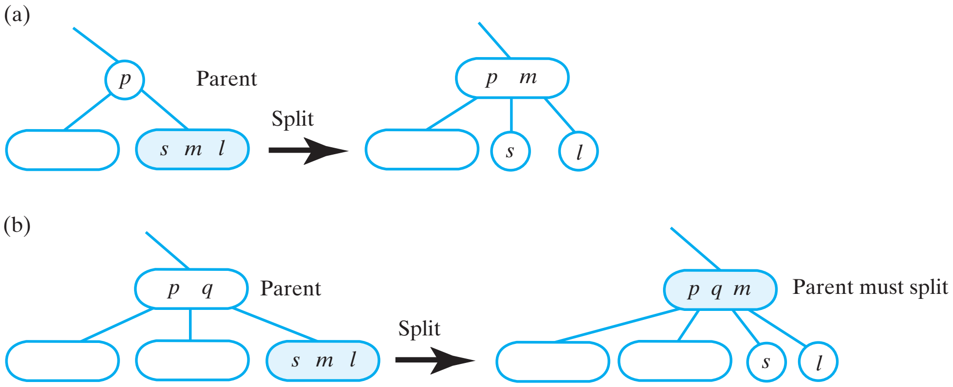
Compare the final 2-3 tree in Figure 27-17 with the AVL tree in Figure 27-10a. We used the same sequence of additions to form both trees. The 2-3 tree is completely balanced and shorter than the balanced AVL tree. Later, we will compare these trees with the 2-4 tree in the next section and draw some conclusions.

Splitting Nodes During Addition

27.20 **Splitting a leaf.** During the addition of a new entry to a 2-3 tree, the first node that splits is a leaf that already contains two entries. Figure 27-18a shows a leaf that would need to accommodate three entries. These entries are shown in ascending order as *s*, *m*, and *l*: *s* is the smallest entry in the node, *m* is the middle entry, and *l* is the largest. The node splits into two nodes that contain *s* and *l*, respectively, and the middle entry *m* moves up a level. If the parent of the leaf has room for *m*, no further action is necessary. This is the case in Figure 27-18a. But in Figure 27-18b, the parent already contains two entries, so we must split it as well. We consider that case next.

Although Figure 27-18 shows the leaf as a right child of its parent, other analogous configurations are possible.

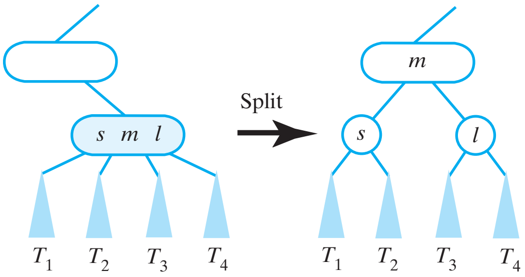
FIGURE 27-18 Splitting a leaf to accommodate a new entry when the leaf's parent contains (a) one entry; (b) two entries



27.21 **Splitting an internal node.** You just saw that splitting a leaf can cause the leaf's parent to have too many entries. This parent also has too many children, as illustrated in Figure 27-18b. Figure 27-19 shows such an internal node in general. This node must accommodate three entries *s*, *m*, and *l*, given in ascending order, and four children that are the roots of the subtrees T_1 through T_4. Thus, we split the node, move the middle entry *m* up to the node's parent, place *s* and *l* into their own nodes, and distribute the original node's subtrees between *s* and *l*. If the parent has room for *m*, no further splitting is necessary. If not, we split the parent as just described.

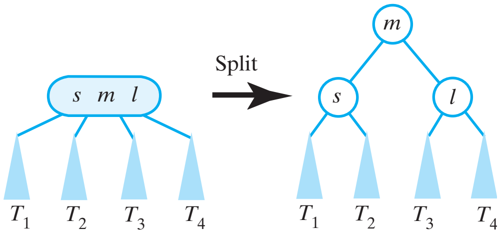
Other analogous configurations for an internal node are possible.

FIGURE 27-19 Splitting an internal node to accommodate a new entry



27.22 **Splitting the root.** Splitting a root proceeds just like the previous cases, except that when we move an entry up a level, we allocate a new node for the entry. This new node becomes the root of the tree, as Figure 27-20 illustrates. Notice that only this case increases the height of a 2-3 tree.

FIGURE 27-20 Splitting the root to accommodate a new entry



SELF-TEST

Question 13 What tree results when you add 30 to the 2-3 tree in Figure 27-17?

Question 14 What 2-3 tree results when you make the following additions to an initially empty 2-3 tree? 70, 80, 90, 20, 10, 50, 60, 40, 30

Question 15 How does the tree that you created in the previous question compare to the AVL tree you created in Question 7?

2-4 Trees

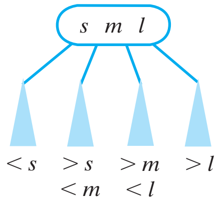
27.23 A **2-4 tree**, sometimes called a **2-3-4 tree**, is a general search tree whose interior nodes must have either two, three, or four children and whose leaves occur on the same level. In addition to 2-nodes and 3-nodes, as we described in the previous section, this tree also contains 4-nodes. A **4-node** contains three data items *s*, *m*, and *l* and has four children. Data that is less than the smallest data item *s*

VideoNote
2-4 and red-black trees

occurs in the node's left subtree. Data that is greater than the largest data item *l* occurs in the node's right subtree. Data that is between *s* and the middle data item *m* or between *m* and *l* occurs in the node's middle subtrees. Figure 27-21 illustrates a typical 4-node.

Searching a 2-4 tree is like searching a 2-3 tree, but with additional logic to handle the 4-nodes. This search forms the basis of an algorithm to add entries to a 2-4 tree.

FIGURE 27-21 A 4-node



Note: A 2-4 tree is a general search tree whose interior nodes must have two, three, or four children and whose leaves occur on the same level. A 2-4 tree is completely balanced.

Adding Entries to a 2-4 Tree

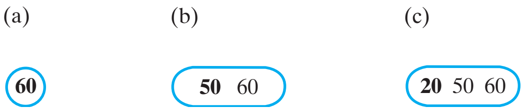
27.24 Recall how we add a new entry to a 2-3 tree. We make comparisons along a path that begins at the root and ends at a leaf. At this point, if the leaf is a 3-node, it already contains two data entries, and so we must split it. Since an entry would now move up a level, this split could require splits in nodes above the leaf. Thus, adding to a 2-3 tree can require us to retrace the path from the leaf back to the root.

In a 2-4 tree, we avoid this retrace by splitting each 4-node as soon as we first consider it during the search from root to leaf. After a split, the next node along the comparison path is the result of the split, and so is not a 4-node. If this node has a 4-node child that we consider next, it has room for the entry that moves up from this child. No other splits occur, as would happen in a 2-3 tree. You will see an example of this shortly.

As in the previous section, we will use an example to demonstrate how to add entries to a 2-4 tree. So that we can compare our results with previous trees, we will use the same sequence of additions—namely, 60, 50, 20, 80, 90, 70, 55, 10, 40, and 35—that we used earlier.

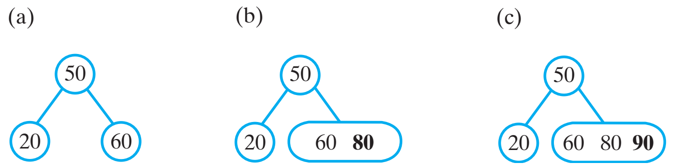
27.25 **Adding 60, 50, and 20.** Figure 27-22 shows the effect of adding 60, 50, and 20 to an initially empty 2-4 tree. The resulting tree consists of a single 4-node.

FIGURE 27-22 An initially empty 2-4 tree after adding (a) 60; (b) 50; (c) 20



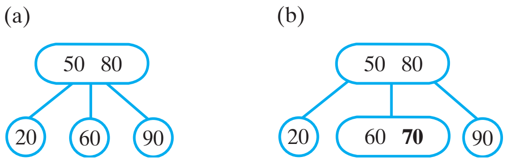
27.26 **Adding 80 and 90.** To add an entry to the 2-4 tree in Figure 27-22c, we find that the root is a 4-node. We split it by moving the middle entry, 50, up. Since we are at a root, we create a new node for the 50. That node becomes the new root of the tree, as shown in Figure 27-23a. We now can add 80 and 90 to the root's right leaf, as Figures 27-23b and 27-23c illustrate.

FIGURE 27-23 The 2-4 tree after (a) splitting the root; (b) adding 80; (c) adding 90



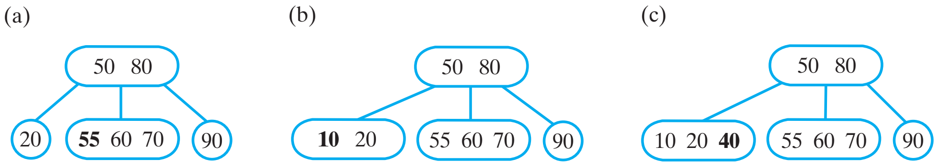
27.27 **Adding 70.** While searching the 2-4 tree in Figure 27-23c for a place to add 70, we encounter the 4-node that is the root's right child. We split this node into two nodes and move the middle entry 80 up to the root. The result of this split is shown in Figure 27-24a. We now have room to add 70 to the root's middle child, as Figure 27-24b shows.

FIGURE 27-24 The 2-4 tree after (a) splitting a 4-node; (b) adding 70



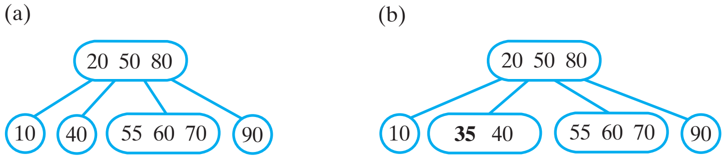
27.28 **Adding 55, 10, and 40.** The 2-4 tree in Figure 27-24b can accommodate the addition of 55, 10, and 40 without splitting nodes. Figure 27-25 shows the results of these additions.

FIGURE 27-25 The 2-4 tree after adding (a) 55; (b) 10; (c) 40



27.29 **Adding 35.** While adding 35 to the 2-4 tree in Figure 27-25c, our search encounters the root's left child, which is a 4-node. We split this node into two nodes and move the middle entry, 20, up to the root, as shown in Figure 27-26a. We now can add 35 to the root's middle left child, as Figure 27-26b shows. This is the final addition that we will make.

FIGURE 27-26 The 2-4 tree after (a) splitting the leftmost 4-node; (b) adding 35



Note: When adding a new entry to a 2-4 tree, you split any 4-node as soon as you encounter it during the search for the new entry's position in the tree. The addition is complete right after this search ends. Thus, adding to a 2-4 tree is more efficient than adding to a 2-3 tree.

SELF-TEST

Question 16 What comparisons are made while searching the 2-4 tree in Figure 27-26b for each of the following values?

a. 5 **b.** 56 **c.** 41 **d.** 30

Question 17 What tree results when you add 30 to the 2-4 tree in Figure 27-26b?

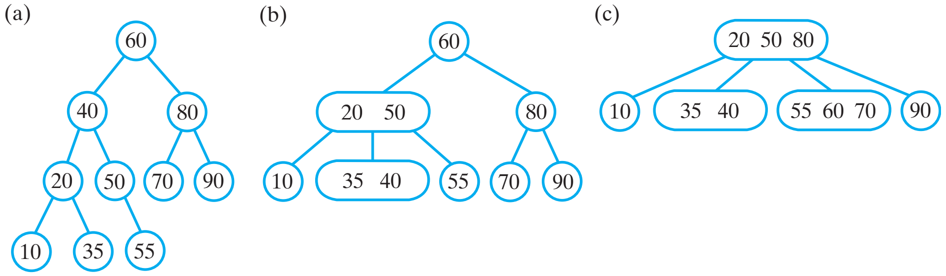
Question 18 What 2-4 tree results when you make the following additions to an initially empty 2-4 tree? 70, 80, 90, 20, 10, 50, 60, 40, 30

Question 19 How does the tree that you created in the previous question compare to the 2-3 tree you created in Question 14?

Comparing AVL, 2-3, and 2-4 Trees

27.30 Figure 27-27 compares the AVL tree in Figure 27-10a, the final 2-3 tree in Figure 27-17, and the 2-4 tree that we just constructed. The AVL tree is a balanced binary search tree of height 4. The other trees are completely balanced general search trees. The height of the 2-3 tree is 3; the height of the 2-4 tree is 2. In general, 2-4 trees are shorter than 2-3 trees, which are shorter than AVL trees.

FIGURE 27-27 Three balanced search trees obtained by adding 60, 50, 20, 80, 90, 70, 55, 10, 40, and 35: (a) AVL tree; (b) 2-3 tree; (c) 2-4 tree



You saw in Segment 25.42 of Chapter 25 that searching a balanced binary search tree, such as an AVL tree, is an O(log *n*) operation. Since 2-3 and 2-4 trees are no taller than a corresponding AVL tree, we usually can search them by examining fewer nodes. However, 3-nodes and 4-nodes contain more entries than 2-nodes, and so they require a longer search time. In general, searching an AVL, 2-3, or 2-4 tree is an O(log *n*) operation.

A 2-3 tree is appealing because maintaining its balance is easier than for an AVL tree. Maintaining the balance of a 2-4 tree is even easier. But defining search trees whose nodes contain more than three data items is usually counterproductive, because the number of comparisons per node increases. As you will see later in this chapter, such a search tree is attractive when it is maintained in external storage, such as a disk, instead of internal memory.

Red-Black Trees

27.31 You just saw that maintaining the balance of a 2-4 tree is easier than maintaining either an AVL tree or a 2-3 tree. While a 2-4 tree is a general tree, a **red-black tree** is a binary tree that is equivalent to a 2-4 tree. Adding an entry to a red-black tree is like adding an entry to a 2-4 tree, in that only one pass from root to leaf is necessary. But a red-black tree is a binary tree, so it uses simpler operations to maintain its balance than does a 2-4 tree. Additionally, the implementation of a red-black tree uses only 2-nodes, whereas a 2-4 tree requires 2-nodes, 3-nodes, and 4-nodes. This added requirement of a 2-4 tree makes it less desirable than a red-black tree.

Note: A red-black tree is a binary tree that is equivalent to a 2-4 tree. Conceptually, a red-black tree is more involved than a 2-4 tree, but its implementation uses only 2-nodes and so is more efficient.

27.32 When designing a node for the 2-4 tree, you need to decide how to represent the entries that are in the node. Since you must order these entries, you could use an ADT such as the sorted list for the entries. You might also use a binary search tree. For example, consider the 2-4 tree in Figure 27-27c. The entries in the root of this tree are 20, 50, and 80. We can represent these entries as a binary search tree whose root is 50 and whose subtrees are 20 and 80. Likewise, the entries in the 3-node leaf of this 2-4 tree are 35 and 40. We can represent these entries as one of two binary search trees: One has 35 as its root and 40 as its right subtree; the other has 40 as its root and 35 as its left subtree. Thus, we can convert all 3-nodes and 4-nodes to 2-nodes. The result is a binary search tree instead of a 2-4 tree.

Each time we convert a 3-node or a 4-node to a 2-node, we increase the height of the tree. We use color to highlight the new nodes that cause this increase in height. We use black for all the nodes in the original 2-4 tree. Since we do not change the 2-nodes, they remain black in the new tree.

Figure 27-28a shows how to represent a 4-node by using 2-nodes. The root of the resulting subtree remains black, but we color its children. The traditional color is red. Our figures use blue since that is our book's second color. Similarly, Figure 27-28b shows how to represent a 3-node by using one of two different subtrees, each having a black root and a red child.

With this notation, we can draw the 2-4 tree in Figure 27-27c as the balanced binary search tree in Figure 27-29. This binary search tree is called a red-black tree.

? SELF-TEST

Question 20 What comparisons are made while searching the 2-4 tree in Figure 27-27c and the equivalent red-black tree in Figure 27-29 for

a. 60 **b.** 55

FIGURE 27-28 Using 2-nodes to represent (a) a 4-node; (b) a 3-node

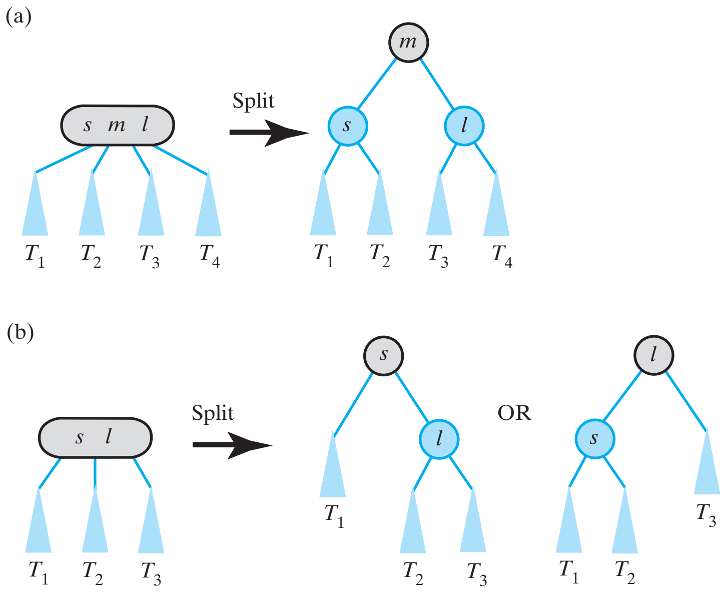
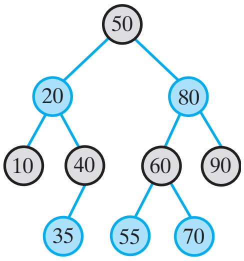


FIGURE 27-29 A red-black tree that is equivalent to the 2-4 tree in Figure 27-27c



Properties of a Red-Black Tree

27.33 The root of every red-black tree is black. If the original 2-4 tree had a 2-node as its root, the 2-node would be black. And if its root was either a 3-node or a 4-node, we would replace it with a subtree whose root is black, as shown in Figure 27-28.

Since we create red nodes only when we convert 3-nodes and 4-nodes to 2-nodes, every red node has a black parent, as you can see in Figure 27-29. It follows that a red node cannot have red children. If it did, a red child would have a red parent, and this contradicts our previous conclusion that every red node has a black parent.

When we formed a red-black tree equivalent to a 2-4 tree, 2-nodes stayed black and the representation of any other node contained one black node. Thus, every node in a 2-4 tree produced exactly one black node in the equivalent red-black tree. Since a 2-4 tree is completely balanced, all paths from its root to a leaf connect the same number of nodes. So every path from the root to a leaf in a red-black tree must contain the same number of black nodes.

Note: Properties of a red-black tree

1. The root is black.
2. Every red node has a black parent.
3. Any children of a red node are black; that is, a red node cannot have red children.
4. Every path from the root to a leaf contains the same number of black nodes.

SELF-TEST

Question 21 Show that the red-black tree in Figure 27-29 satisfies the four properties just given.

Question 22 What red-black tree is equivalent to the 2-4 tree in Figure 27-25c?

Question 23 Show that the red-black tree that answers Question 22 satisfies the four properties given previously.

Note: **Creating a red-black tree**
In practice, you do not convert 2-4 trees into red-black trees. You create a red-black tree by adding entries to an initially empty tree according to the steps described in the following section. These steps consider both the balance of the tree and the color of its nodes.

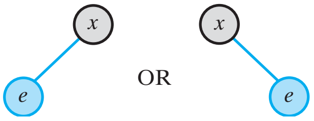
Adding Entries to a Red-Black Tree

27.34 **Adding a leaf.** What color should we assign to a new node that we add to the tree? An addition to a binary search tree always occurs at a leaf, so the same is true for a red-black tree. If we use black for a new leaf, we will increase the number of black nodes on the paths to that leaf. This increase violates the fourth property of a red-black tree. Thus, any new node must be red. However, do not assume that all the leaves in a red-black tree are red. Adding or removing entries can change the color of various nodes, including that of leaves added earlier.

Note: **The color of nodes added to a red-black tree**
Adding an entry to a red-black tree results in a new red leaf. The color of this leaf can change later when other entries are added or removed.

Consider some simple cases of adding to a red-black tree. A one-node red-black tree has one black node, its root. Figure 27-30 shows two possibilities when we add a new entry *e* to this tree. In each case, the new red node maintains the properties of a red-black tree, so it is legal.

FIGURE 27-30 The result of adding a new entry *e* to a one-node red-black tree



Now suppose that the red-black tree had two nodes before we added the new entry *e*. Figure 27-31a shows this original tree when it consists of a root *x* and right child *y*. Also pictured is the 2-4 tree that is equivalent to the original red-black tree. The rest of the figure shows the possible outcomes of the addition, depending on how *e* compares with *x* and *y*. In Part *b*, *e* is the left child of the root, and we are done. In Part *c*, a red node has a red child. These two consecutive red nodes are illegal in a red-black tree (properties 2 and 3). To understand what further action is necessary, consider the equivalent 2-4 tree. The original 2-node red-black tree is equivalent to the 2-4 tree that contains the one node <*x y*> (Figure 27-31a). If we add an entry *e* that is larger than *y*, the 2-4 tree becomes the single node <*x y e*> (Figure 27-31c). Notice the red-black tree that is equivalent to this 3-node. This tree is the one we need as the result of adding *e*. We can get it from the first red-black tree shown in Part *c* by first performing a single left rotation about the node containing *y*. You have seen this rotation before in Figures 27-5b and 27-5c when we talked about AVL trees. After the rotation, we need to reverse the colors of the nodes containing *x* and *y*—that is, the original parent and grandparent of the new node. We call this step a **color flip**.

FIGURE 27-31 The possible results of adding a new entry *e* to a two-node red-black tree

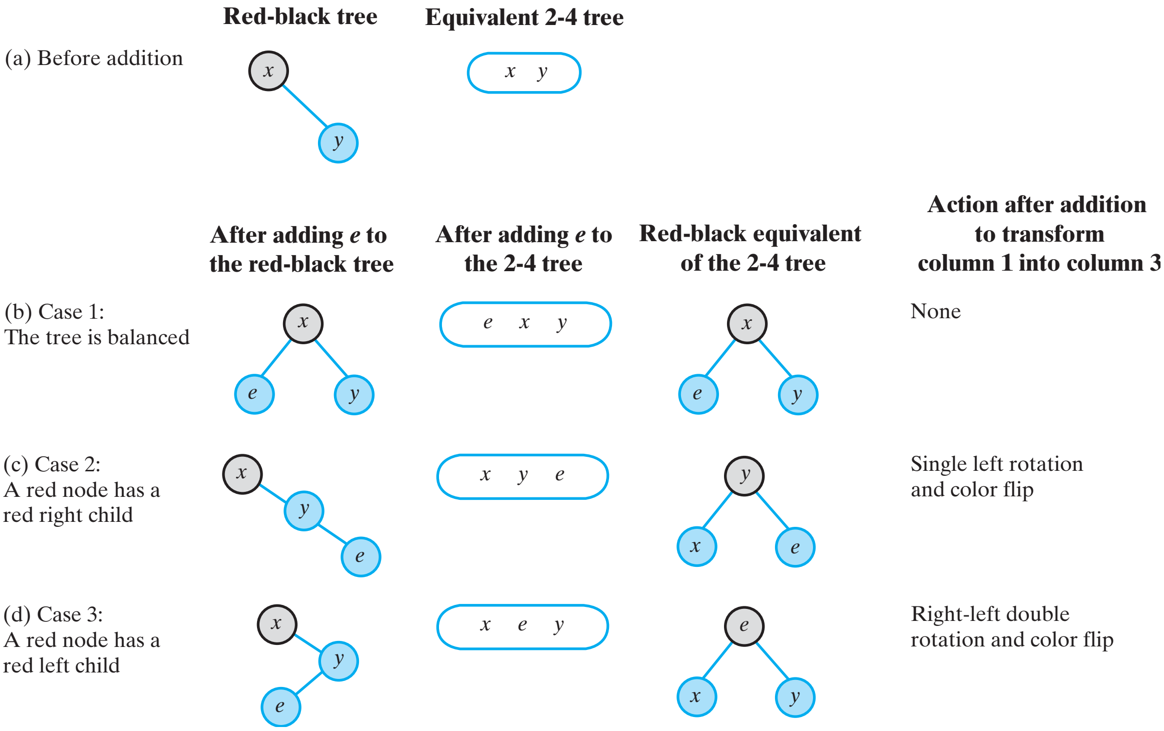
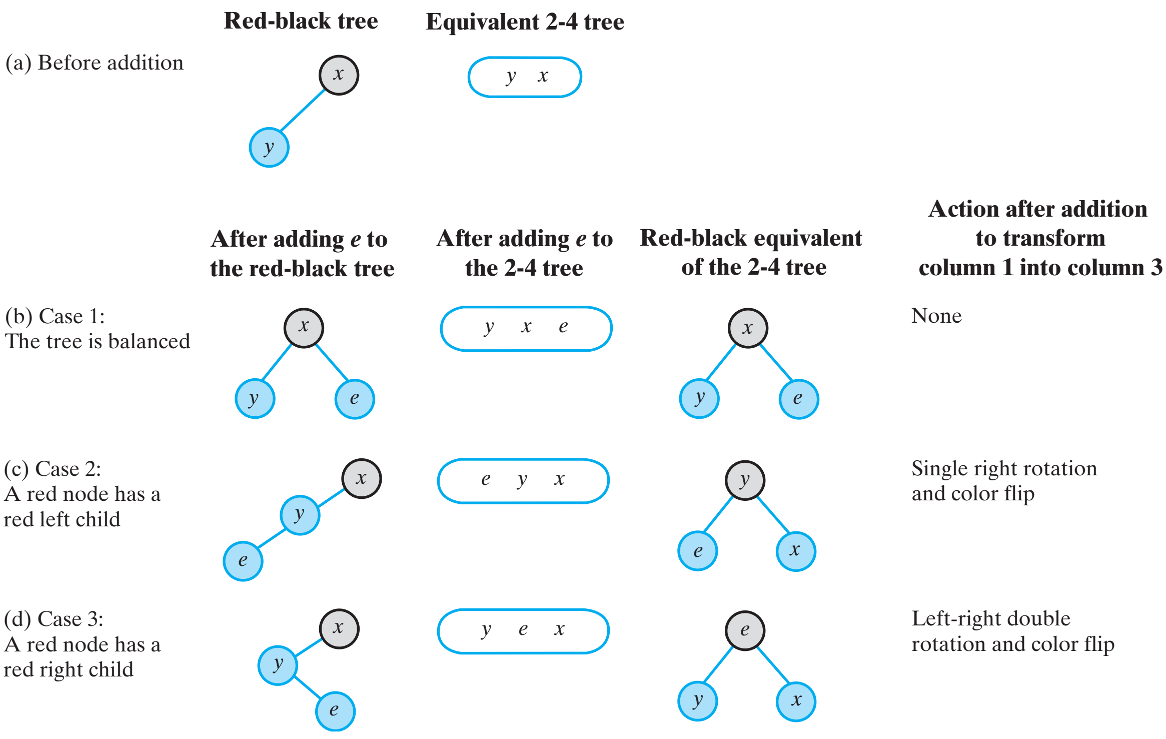

Red-black tree
Equivalent 2-4 tree
(a) Before addition
x
y
x y
After adding e to the red-black tree
After adding e to the 2-4 tree
Red-black equivalent of the 2-4 tree
Action after addition to transform column 1 into column 3
(b) Case 1: The tree is balanced
x
e
y
e x y
x
e
y
None
(c) Case 2: A red node has a red right child
x
y
e
x y e
y
x
e
Single left rotation and color flip
(d) Case 3: A red node has a red left child
x
y
e
x e y
e
x
y
Right-left double rotation and color flip

Figure 27-31d shows the last possible result of adding *e* to the two-node red-black tree. Here, a right-left double rotation followed by a color flip of the new node and its original grandparent are necessary to avoid two consecutive red nodes. Figures 27-7b, 27-7c, and 27-7d show the rotation in general in the context of an AVL tree.

Figure 27-32 shows mirror images of the cases in Figure 27-31.

FIGURE 27-32 The possible results of adding a new entry *e* to a two-node red-black tree: mirror images of Figure 27-31


Red-black tree
Equivalent 2-4 tree
(a) Before addition
x
y
y x
After adding e to the red-black tree
After adding e to the 2-4 tree
Red-black equivalent of the 2-4 tree
Action after addition to transform column 1 into column 3
(b) Case 1: The tree is balanced
x
y
e
y x e
None
(c) Case 2: A red node has a red left child
e y x
Single right rotation and color flip
(d) Case 3: A red node has a red right child
y e x
Left-right double rotation and color flip

27.35 **Splitting a 4-node whose parent is black.** During an addition to a 2-4 tree, we split any 4-nodes that we encounter as we move along the path from the root to the eventual insertion point. We must perform an equivalent action during an addition to a red-black tree. Figure 27-28a shows that when a black node has two red children, we have encountered the red-black representation of a 4-node. We will call this configuration a red-black 4-node, or simply a 4-node.

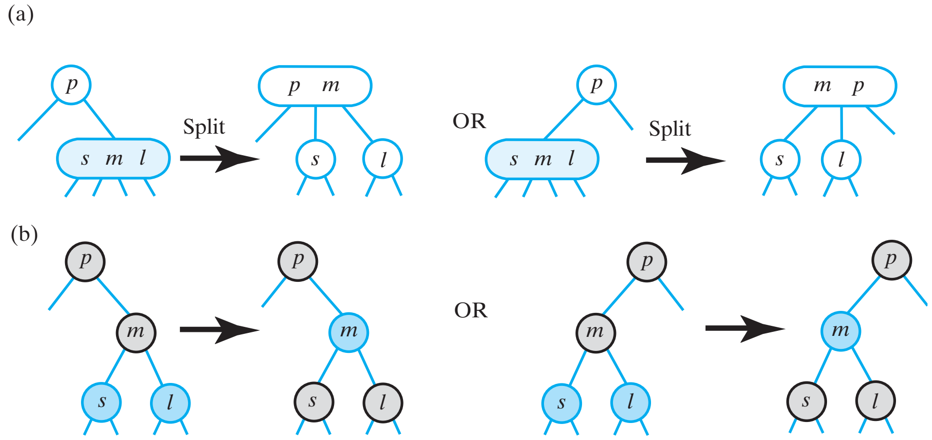
Note: A red-black 4-node

A red-black 4-node consists of a black node and two red children.

Figure 27-33a recalls how to split a 4-node when its parent in the 2-4 tree is a 2-node. The middle entry *m* moves up to the node's parent, and the other entries *s* and *l* are given their own nodes as replacement children of the parent. Figure 27-33b shows the corresponding red-black trees. Notice that the three nodes in the subtree rooted at *m* reverse colors. Thus, we split the red-black representation of a 4-node by performing a color flip.

A color flip is all that is necessary when a red-black 4-node has a black parent. As you can see from Figure 27-33, a black parent corresponds to a 2-node in the 2-4 tree. If a 4-node in a 2-4 tree has a 3-node as its parent, the red-black 4-node will have a red parent. We examine this situation in the next segment.

FIGURE 27-33 Splitting a 4-node whose parent is a 2-node in (a) a 2-4 tree; (b) a red-black tree



27.36 **Splitting a 4-node whose parent is red: Case 1.** Figure 27-34a shows the splitting of a 4-node that has a 3-node parent within a 2-4 tree. Here, the 4-node is the right child of its parent. Figure 27-34b shows the red-black representations of the two trees in Part *a*. How can we transform the first red-black tree into the second? Figure 27-35 shows the necessary steps. In Part *a*, we detect a 4-node at *m*, since this black node has two red children. A color flip results in two adjacent red nodes, as shown in Figure 27-35b. Earlier, in Figure 27-31c, we saw this configuration of a black node and two consecutive right descendants that are red. As we did then, we perform a left rotation about *p*, as Figure 27-35c shows, and then we reverse the colors of the nodes containing *p* and *g*. This color flip, together with the rotation, resolves the illegal red nodes. The result in Figure 27-35d is the desired red-black tree that we saw in Figure 27-34b.

Since a 3-node has two different red-black representations, we can replace Figure 27-35a with a different red-black tree. We leave it to you to show that the final result will be the same, but with less work. (See Exercise 13.)

27.37 **Splitting a 4-node whose parent is red: Case 2.** The 4-node in Figure 27-34a is a right child of its parent. If it were a left child, the red-black representation would be as in Figure 27-36a. The rest of this figure shows that both color flips and a right rotation are necessary to split the 4-node.

As before, we can replace Figure 27-36a with a different red-black tree and get the same final result. Again we leave the details to you as an exercise. (See Exercise 14.)

FIGURE 27-34 Splitting a 4-node that has a 3-node parent within (a) a 2-4 tree; (b) a red-black tree

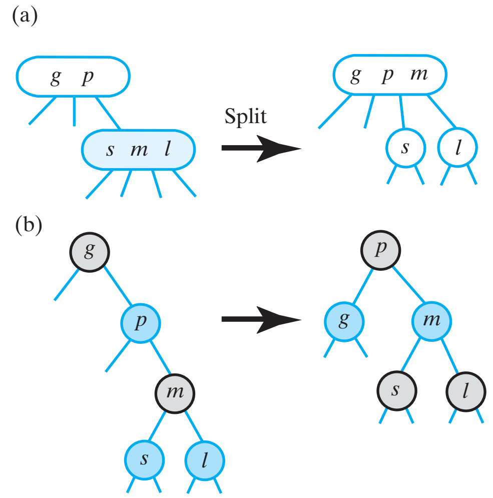


FIGURE 27-35 Splitting a 4-node that has a red parent within a red-black tree: Case 1

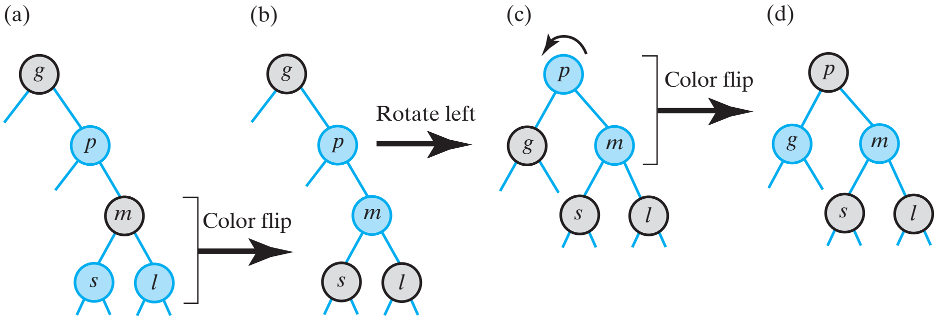
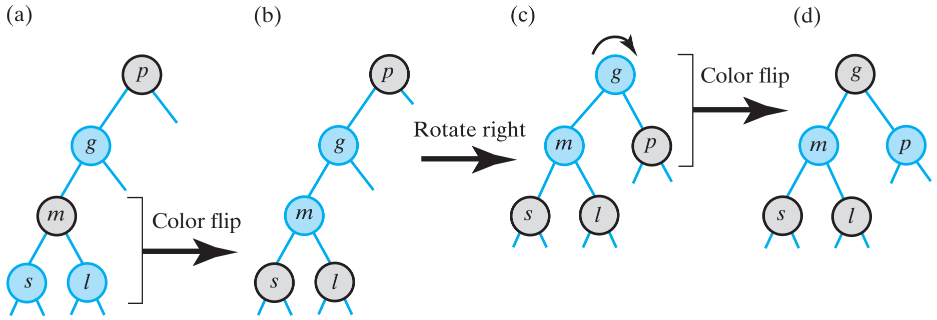


FIGURE 27-36 Splitting a 4-node that has a red parent within a red-black tree: Case 2



27.38 **Splitting a 4-node whose parent is red: Cases 3 and 4.** Now consider the case in which the 4-node is the middle child of its 3-node parent. This time, we look at both red-black representations that the 3-node parent produces. Figure 27-37a shows one possible red-black tree. After the color flip in Part *b*, we resolve the consecutive red nodes as we did in Figure 27-31d. A right-left double rotation followed by a color flip produces the desired results, as you can see in the rest of Figure 27-37.

Figure 27-38a shows the second possible red-black tree. After the color flip in Part *b*, we resolve the consecutive red nodes as we did in Figure 27-32d. A left-right double rotation followed by a color flip is necessary, as the rest of Figure 27-38 shows. Notice that the tree in Figure 27-38e is the same as the one in Figure 27-37e.

FIGURE 27-37 Splitting a 4-node that has a red parent within a red-black tree: Case 3

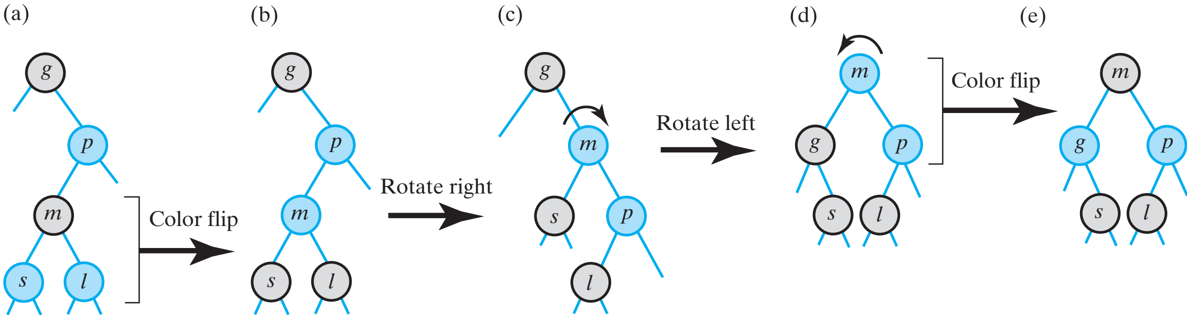
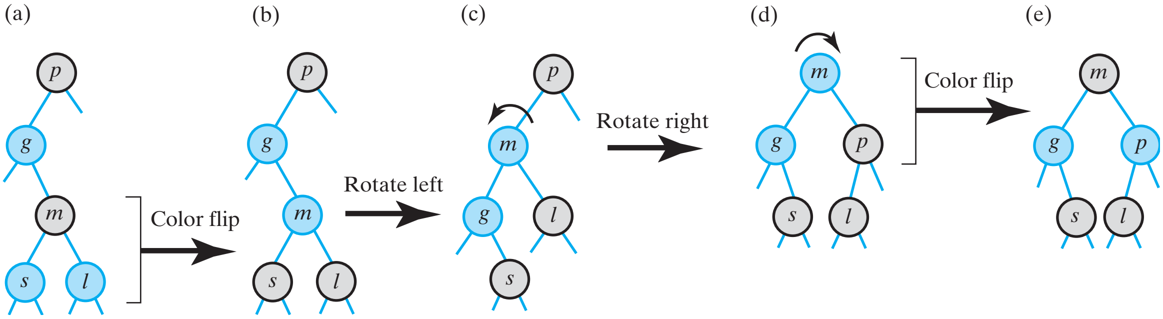


FIGURE 27-38 Splitting a 4-node that has a red parent within a red-black tree: Case 4



Note: Splitting a red-black 4-node

When splitting a red-black 4-node, the color of its parent determines the necessary operations. If the parent is black, a color flip is sufficient. But if the parent is red, a color flip, a rotation, and another color flip are necessary.

Java Class Library: The Class `TreeMap`

27.39 The package `java.util` contains the class `TreeMap<K, V>`. This class uses a red-black tree to implement the methods in the interface `SortedMap<K, V>` in the same package. `SortedMap` extends the interface `Map<K, V>`, which we described in Segment 19.22 of Chapter 19. Recall that the interface `Map` is similar to our interface for the ADT dictionary. `SortedMap` specifies a sorted dictionary in which the search keys are maintained in ascending order. Because `TreeMap` uses a red-black tree, methods such as `get`, `put`, `remove`, and `containsKey` are each an O(log n) operation.

B-Trees

27.40 A **multiway search tree of order *m***—or sometimes an ***m*-way search tree**—is a general tree whose nodes have up to m children each. A node that has $k - 1$ data items and k children is called a ***k*-node**. An order m multiway search tree can contain k-nodes for values of k ranging from 2 to m.

A binary search tree is a multiway search tree of order 2. You know that not all binary search trees are balanced; likewise, not all multiway search trees are balanced. However, 2-3 trees and 2-4 trees are balanced multiway search trees of orders 3 and 4, respectively. We maintained the balance of a 2-3 tree, for example, by insisting that every interior node have two or three children and that all leaves occur on the same level.

A **B-tree of order *m*** is a balanced multiway search tree of order m that has the following additional properties to maintain its balance:

- The root has either no children or between 2 and m children.
- Other interior nodes (nonleaves) have between $\lceil m/2 \rceil$ and m children each.
- All leaves are on the same level.

2-3 and 2-4 trees satisfy these constraints, and so are examples of B-trees.

27.41 The search trees that you have seen so far maintain their data within the main memory of a computer. At some point, we probably will save this data in external memory, such as a disk. As long as we can read the data back into internal memory, we can use any of the previous search trees. But what happens when your database becomes too large to be retained entirely within internal memory? Typically, you use a B-tree.

Accessing data in external memory is much slower than accessing data in main memory. When reading external data, the major cost is locating it on the storage device. Data on a disk, for example, is organized sequentially into **blocks**, whose size depends on the physical characteristics of the disk. When you read data from a disk, an entire block is read. Locating the block takes much more time than reading the data. If each block contains the data for at least one node, you can reduce the access time by placing numerous data items in each node. Although many comparisons per node could be necessary, their cost is much less than the cost of accessing external data.

Since increasing the number of data items per node decreases the tree's height, you decrease the number of nodes that you must search and hence the number of disk accesses. A high-order B-tree fits these requirements. You would choose the order m so that $m - 1$ data items fit into a block on the disk.

Note: Although a high-order B-tree is usually counterproductive for an internal database because the number of comparisons per node increases, it is attractive when it is maintained in external storage such as a disk.

Chapter Summary

- An AVL tree is a balanced binary search tree that rearranges its nodes whenever it becomes unbalanced. If adding a node causes an imbalance, one single or double rotation restores the tree's balance.
- A 2-node is a node that has two children and one data item. A 3-node has three children and two data items.
- A 2-3 tree is a balanced search tree that contains 2-nodes and 3-nodes. When an addition to the tree would cause a leaf to have three data items, the leaf splits into two 2-nodes. These nodes contain the smallest and largest of the three data items and become the children of the former leaf's parent. The middle data item moves up to this parent node, possibly causing the node to split.
- A disadvantage of the 2-3 tree is that the addition algorithm follows a path from the root to a leaf and then returns along that path as nodes split.
- A 4-node has four children and three data items.
- A 2-4 (or 2-3-4) tree is a balanced search tree that contains 2-nodes, 3-nodes, and 4-nodes. During an addition to the tree, each 4-node is split as it is considered during the search from root to leaf. Thus, returning along the path to the root is unnecessary.
- A red-black tree is a binary search tree that is logically equivalent to a 2-4 tree. Conceptually, a red-black tree is more involved than a 2-4 tree, but its implementation is more efficient because it uses only 2-nodes.
- Additions to a red-black tree maintain the tree's balance and status as a red-black tree by using color flips as well as rotations like those used for an AVL tree.
- A *k*-node is a node that has *k* children and *k* -1 data items.
- A multiway search tree of order *m* is a general tree that contains *k*-nodes for values of *k* ranging from 2 to *m*. A B-tree of order *m* is a balanced multiway search tree of order *m*. To maintain its balance, a B-tree requires every interior node to have a certain number of children and has all its leaves on the same level.
- A 2-3 tree is a B-tree of order 3; a 2-4 tree is a B-tree of order 4.
- A B-tree is useful when data is maintained in external storage, such as a disk.

Exercises

1. Implement the algorithm for a left-right double rotation, as given in Segment 27.5.
2. Add 62 and 65 to the AVL tree in Figure 27-27a.
3. Add 62 and 65 to the 2-3 tree in Figure 27-27b.
4. Add 62 and 65 to the 2-4 tree in Figure 27-27c.
5. Add 62 and 65 to the red-black tree in Figure 27-29.
6. Each of the trees in Figures 27-27 and 27-29 contains the same values. Exercises 2 through 5 asked you to add 62 and 65 to each of them. Describe the effect that these additions had on each tree.
7. What red-black tree is equivalent to the 2-4 tree in Figure 27-25b?

8. What tree results when you add the values 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 to each of the following initially empty trees?

a. An AVL tree **b.** A 2-3 tree **c.** A 2-4 tree **d.** A red-black tree

9. Add the values given in Exercise 8 to an initially empty binary search tree. Compare the resulting tree with the trees you created in Exercise 8. Which tree could you search most efficiently?

10. Draw the shortest possible tree that contains 20 values for each of the following kinds of trees:

a. An AVL tree **b.** A 2-3 tree **c.** A 2-4 tree **d.** A red-black tree

11. Draw the tallest possible tree that contains 20 values for each of the following kinds of trees:

a. An AVL tree **b.** A 2-3 tree **c.** A 2-4 tree **d.** A red-black tree

12. Using pseudocode, describe an inorder traversal of

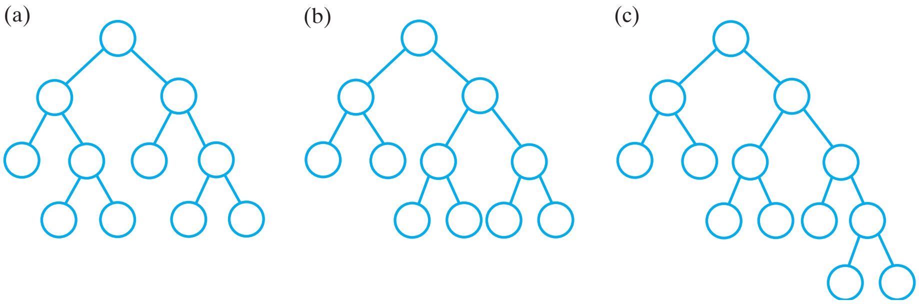
a. A 2-3 tree **b.** A 2-4 tree

13. Figure 27-34a shows a 4-node within a 2-4 tree that is the right child of a 3-node parent containing data items *g* and *p*. When converting these nodes to red-black notation, make *p* be the parent of *g*. Revise Figure 27-35 to show that a color flip is all that is necessary to get the desired red-black tree.

14. Repeat Exercise 13, but this time assume that the 4-node is a left child.

15. Color the nodes in each tree in Figure 27-39 so that it is a red-black tree.

FIGURE 27-39 Three binary trees for Exercise 15



16. Which of the trees that you studied in this chapter could be used to implement a priority queue? Recall that we discussed the priority queue in Chapter 10.

17. How efficiently could a red-black tree or an AVL tree implement the `add` and `remove` methods of a priority queue?

18. Consider a B-tree of order 1000 whose height is 3. What is the smallest number of values that this tree can contain? What is the largest number of values? Generalize your results to a B-tree of order 1000 and height *h*.

Projects

1. You remove an entry from an AVL tree in the same way that you remove an entry from a binary search tree. However, after you remove the appropriate node from the tree, an imbalance can occur that you must correct by performing single or double rotations. Develop an algorithm that removes a node from an AVL tree.

2. Implement a class of AVL trees.

3. Consider the implementation of the method `rebalance` for an AVL tree, as given in Segment 27.11. The performance of this method depends on the cost of a rotation and the cost of the method `getHeightDifference`.

   a. Assume that the tree has height *h* and that `nodeN` is at height *k*. What is the cost, using Big O notation, of each of the following tasks?

   - A rotation
   - Executing the method `getHeightDifference`
   - Executing the method `rebalance`

   b. Suppose that a node is added at the bottom of the tree by the method `add`. How many times will `rebalance` be called? Give a Big Oh expression for the cost of adding a node.

4. Design and implement a class of nodes that you can use in the implementation of an AVL tree. You can derive this class from `BinaryNode`. Each node, as the root of a subtree, should contain the height of this subtree. Implement a class of AVL trees using your new class of nodes.

5. Design a class of nodes that you can use in the implementation of a 2-4 tree. Is one class enough, or will you need several?

6. Implement a class of 2-4 trees in which only additions and retrievals are permitted.

7. Implement a class of red-black trees that permit only additions and retrievals.

8. Implement a class of sets that stores its entries in an instance of the class of red-black trees defined in the previous project. You can omit the methods `remove` and `clear` from your class. Recall that Chapter 1 defined a set as a bag that does not permit duplicate entries.

9. Design and carry out an experiment to compare the heights of ordinary binary search trees with the heights of either AVL trees or red-black trees. You first will need to complete either Project 2 or Project 7.

10. Design a class `BTreeNode` of nodes for a B-tree of order *m*. Consider operations that allow you to

    - Get a particular value from the node
    - Insert a value into the node while maintaining the links to subtrees (remember that the B-tree is a search tree)
    - Get a count of the number of values in the node
    - Replace a value in the node, if the search tree is maintained
    - Replace a subtree
    - Split a node into two nodes, each containing half the values
    - Give an algorithm for adding a new value to a B-tree whose implementation uses your definition of `BTreeNode`.

11. Implement a priority queue by using one of the balanced search trees in this chapter.

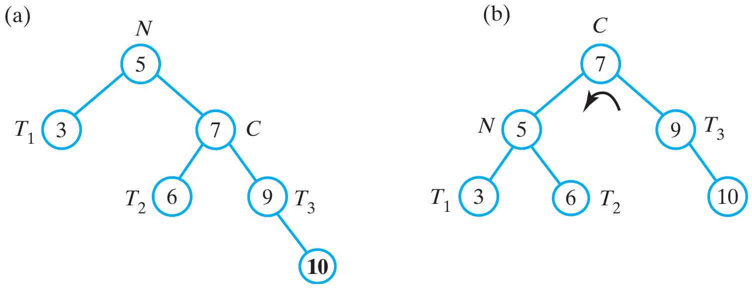
Answers to Self-Test Questions

1. Node *N* contains 60, and node *C* contains 50. T_1 is the one-node subtree containing 20. T_2 and T_3 are empty.

2. Since we had a binary search tree before the rotation, the value in node *N* is less than the value in node *C* and all the values in T_2. Moreover, all values in T_2 are less than the value in node *C*. These relationships are maintained after the rotation, since node *N* is a le‘ft child of node *C*, and T_2 is a right subtree of node *N*. Also, the subtrees T_1 and T_3 have their original parents in the new tree. Thus, the resulting tree is a binary search tree.

3. Node *N* contains 60, and node *C* contains 80. T_3 is the one-node subtree containing 90. T_1 and T_2 are empty.

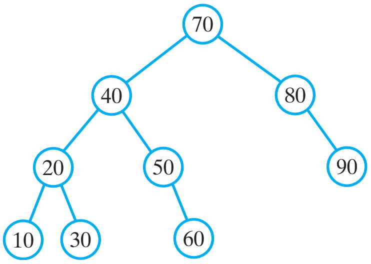
4. Part *a* of the following figure shows an imbalance at node *N* after 10 was added to the tree. A left rotation restores the tree's balance, as Part *b* illustrates.



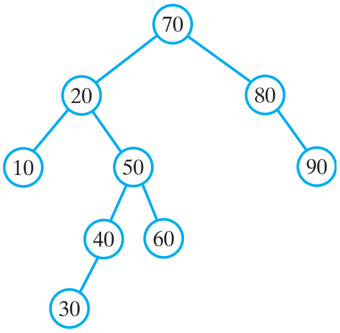
5. Node *N* contains 50, node *C* contains 80, and node G contains 60. T_1, T_3, and T_4 are one-node subtrees: T_1 contains 20, T_3 contains 70, and T_4 contains 90. T_2 is empty. In Parts *a* and *b*, T_2 is the left subtree of 60. In Part *c*, it is the right subtree of 50.

6. Node *N* contains 50, node *C* contains 20, and node G contains 40. T_1, T_2, and T_4 are one-node subtrees: T_1 contains 10, T_2 contains 35, and T_4 contains 55. T_3 is empty. In Parts *a*, *b* and *c*, T_3 is the right subtree of 40. In Part *d*, it is the left subtree of 50.

7.



8. The following binary search tree is taller than the previous AVL tree, and it is not balanced:



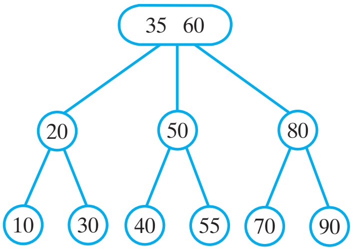
9. Since we had a binary search tree before the rotation, the value in node *N* is less than the values in nodes *C* and *G* and all the values in T_2 and T_3. The value in node *C* is greater than the value in node *G*, and each of these two values is greater than all values in T_2. Moreover, all values in T_3 are less than the value in node *C* but greater than the value in node *G*. These relationships are maintained after the rotation: Node *G* has node *N* as its left child and node *C* as its right child; T_2 is a right subtree of node *N*, and T_3 is a left subtree of node *C*. The subtrees T_1 and T_4 have their original parents in the new tree. Thus, the resulting tree is a binary search tree.

10. Since we had a binary search tree before the rotation, the value in node *N* is greater than the values in nodes *C* and *G* and all the values in T_2 and T_3. The value in node *C* is less than the value in node *G*, and each of these two values is less than all values in T_3. Moreover, all values in T_2 are less than the value in node *G* but greater than the value in node *C*. These relationships are maintained after the rotation: Node *G* has node *C* as its left child and node *N* as its right child; T_2 is a right subtree of node *C*, and T_3 is a left subtree of node *N*. The subtrees T_1 and T_4 have their original parents in the new tree. Thus, the resulting tree is a binary search tree.

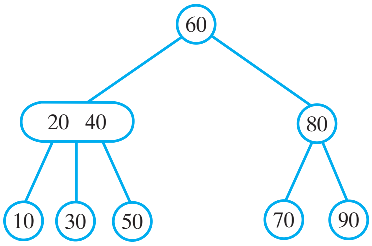
11.
```
private BinaryNodeInterface<T> rotateLeft(BinaryNodeInterface<T> nodeN)
{
   BinaryNodeInterface<T> nodeC = nodeN.getRightChild();
   nodeN.setRightChild(nodeC.getLeftChild());
   nodeC.setLeftChild(nodeN);
   return nodeC;
} // end rotateLeft
```

12. 60, 20, 10.
60, 20, 50, 55.
60, 20, 50, 35, 40.
60, 20, 50, 35.

13.



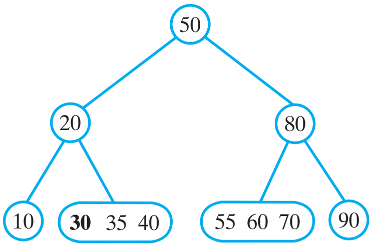
14.



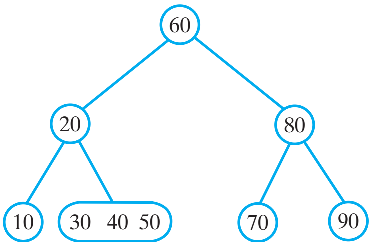
15. The 2-3 tree is shorter than the AVL tree and is balanced.

16. **a.** 20, 10.
b. 20, 50, 80, 55, 60.
c. 20, 50, 35, 40.
d. 20, 50, 35.

17.



18.

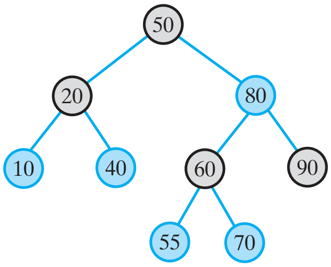


19. Both trees are completely balanced, and they have the same height. The 2-4 tree has fewer nodes.

20. **a.** 2-4: 20, 50, 80, 55, 60; Red-black: 50, 80, 60.
b. 2-4: 20, 50, 80, 55; Red-black: 50, 80, 60, 55.

21. The first three properties follow immediately by observing the tree. Every path from the root to a leaf contains two black nodes.

22.



If you tried to form a red-black tree whose root is 80, you would find that the tree was not balanced and so could not be a red-black tree. (You normally do not convert 2-4 trees into red-black trees, but doing so as an exercise should give you some insight into the rationale of red-black trees.)

23. The first three properties follow immediately by observing the tree. Every path from the root to a leaf contains two black nodes.

Chapter 28

Graphs

Contents

Prerequisites

Chapter	5	Stacks
Chapter	10	Queues, Deques, and Priority Queues
Chapter	23	Trees

Objectives

After studying this chapter, you should be able to

- Describe the characteristics of a graph, including its vertices, edges, and paths
- Give examples of graphs, including those that are undirected, directed, unweighted, and weighted
- Give examples of vertices that are adjacent and that are not adjacent for both directed and undirected graphs
- Give examples of paths, simple paths, cycles, and simple cycles
- Give examples of connected graphs, disconnected graphs, and complete graphs
- Perform a depth-first traversal and a breadth-first traversal on a given graph
- List a topological order for the vertices of a directed graph without cycles

- Detect whether a path exists between two given vertices of a graph
- Find the path with the fewest edges that joins one vertex to another
- Find the path with the lowest cost that joins one vertex to another in a weighted graph
- Describe the operations for the ADT graph

The news media often use line graphs, pie charts, and bar graphs to help us visualize certain statistics. But these common graphs are *not* examples of the kind of graph that we will study in this chapter. The graphs that computer scientists and mathematicians use include the trees that you saw in Chapter 23. In fact, a tree is a special kind of graph. These graphs represent the relationships among data elements. This chapter will present the terminology we use when discussing graphs, the operations on them, and some typical applications.

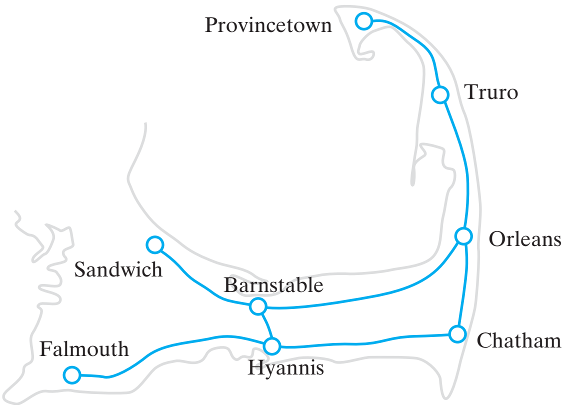
Some Examples and Terminology

Although the graphs you have drawn in the past likely are not the kind of graph we will discuss here, the examples in this section will be familiar. But you probably have never called them graphs!

Road Maps

28.1 Figure 28-1 contains a portion of a road map for Cape Cod, Massachusetts. Small circles represent the towns, and the lines that join them represent the roads. A road map is a graph. In a graph, the circles are called **vertices**, or **nodes**, and the lines are called **edges**. A **graph**, then, is a collection of distinct vertices and distinct edges. A **subgraph** is a portion of a graph that is itself a graph, just as the road map in Figure 28-1 actually is a part of a larger map.

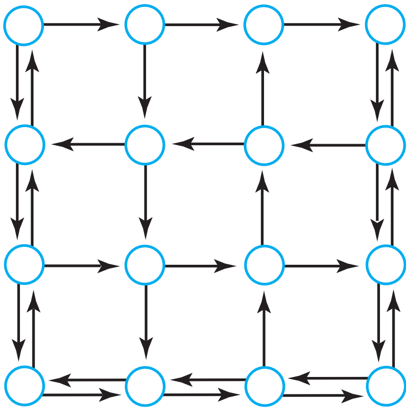
FIGURE 28-1 A portion of a road map



VideoNote
Graph concepts and terminology

Since you can travel in both directions along the roads in Figure 28-1, the corresponding graph and its edges are said to be **undirected**. But cities often have one-way streets. The graph in Figure 28-2 has a vertex for each intersection in a city's street map. The edges each have a direction and are called **directed edges**. A graph with directed edges is called a **directed graph**, or **digraph**. You can transform an undirected graph into a directed graph by replacing each undirected edge with two directed edges that have opposite directions.

FIGURE 28-2 A directed graph representing a portion of a city's street map



28.2 **Paths.** A **path** between two vertices in a graph is a sequence of edges. A path in a directed graph must consider the direction of the edges, and is called a **directed path**. The **length** of a path is the number of edges that it comprises. If the path does not pass through any vertex more than once, it is a **simple path**. Figure 28-1 contains a simple path from Provincetown to Orleans of length 2.

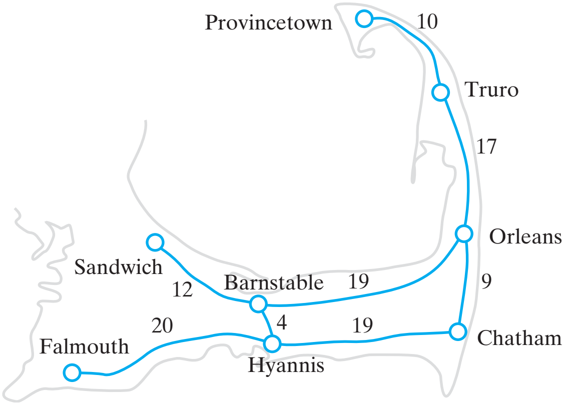
A **cycle** is a path that begins and ends at the same vertex. A **simple cycle** passes through other vertices only once each. In Figure 28-1, the cycle Chatham-Hyannis-Barnstable-Orleans-Chatham is a simple cycle. A graph that has no cycles is **acyclic**.

You use a road or street map to see how to get from point A to point B. The path you choose between these points will usually be a simple path. In doing so, you avoid retracing your steps or going around in circles. People who take a ride to view the autumn leaves, however, would follow a cycle that begins and ends at home.

28.3 **Weights.** You might be happy just to get from one place to another, but you often have a choice of several paths. You could choose the shortest, the fastest, or the cheapest path, for example. To do so, you use a **weighted graph**, which has values on its edges. These values are called either **weights** or **costs**. For example, Figure 28-3 shows the road map from Figure 28-1 as a weighted graph. In this version, each weight represents the distance in miles between two towns. Other types of weights you might use could represent the driving time or the cost of traveling by taxi.

A path in a weighted graph also has a weight, or cost, that is the sum of its edge weights. For example, the weight of the path from Provincetown to Orleans in Figure 28-3 is 27.

FIGURE 28-3 A weighted graph



Question 1 Consider the graph in Figure 28-3.

a. What is the length of the path that begins in Provincetown, passes through Truro and Orleans, and ends in Chatham?

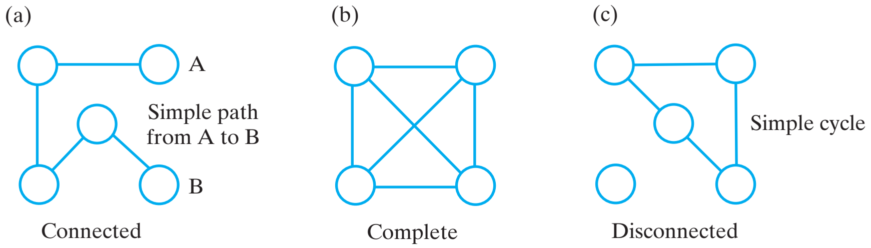
b. What is the weight of the path just described?

c. Consider all paths from Truro to Sandwich that do not have cycles. Which path has the shortest length?

d. Of the paths you considered in Part *c*, which one has the smallest weight?

28.4 **Connected graphs.** The towns on a road map are connected by roads in a way that enables you to go from any town to any other town. That is, you can get from here to there. A graph that has a path between every pair of distinct vertices is **connected**. A **complete graph** goes even further; it has an edge between every pair of distinct vertices. Figure 28-4 provides examples of undirected graphs that are connected, complete, or **disconnected**—that is, not connected. Notice the simple path in Part *a* and the simple cycle in Part *c*.

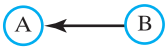
FIGURE 28-4 Undirected graphs



28.5 **Adjacent vertices.** Two vertices are **adjacent** in an undirected graph if they are joined by an edge. In Figure 28-3, Orleans and Chatham are adjacent, but Orleans and Sandwich are not. Adjacent vertices are called **neighbors**. In a directed graph, vertex *i* is adjacent to vertex *j* if a directed edge begins at *j* and ends at *i*. In Figure 28-5, vertex *A* is adjacent to vertex *B*, but vertex *B* is not adjacent to vertex *A*. That is, vertex *A* is vertex *B*'s neighbor, but the converse is not true.

When convenient, we will place vertex labels within the circles that represent the vertices, as in Figure 28-5. But sometimes, the vertex labels will appear next to the circles, as in Figure 28-3.

FIGURE 28-5 Vertex *A* is adjacent to vertex *B*, but *B* is not adjacent to *A*



28.6 **The number of edges.** If a directed graph has *n* vertices, how many edges can it have? If the graph is complete, each vertex is a neighbor of all the other vertices. Thus, each vertex ends $n - 1$ directed edges. Consequently, the graph has $n (n - 1)$ edges. A complete undirected graph has half that number of edges. For example, the graph in Figure 28-4b has 4 vertices and $4 \times 3 / 2$, or 6, edges. To make the graph directed and complete, we would replace each edge with two directed edges, which results in a graph having 12 edges.

Note: If a graph has *n* vertices, it can have at most

- $n (n - 1)$ edges if the graph is directed
- $n (n - 1) / 2$ edges if the graph is undirected

A graph is **sparse** if it has relatively few edges. It is **dense** if it has many edges. While these terms have no precise definition, we will say that a sparse graph has $O(n)$ edges, and a dense graph has $O(n^2)$ edges. The graph in Figure 28-1 has eight vertices and eight edges. It is sparse.

Note: Typical graphs are sparse.

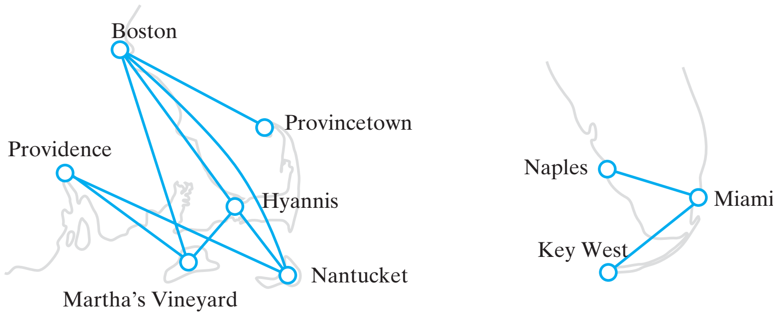
Airline Routes

28.7 A graph that represents the routes that an airline flies is similar to one that represents a road map. They are different, however, because not every city has an airport, and not every airline flies to or from every airport. For example, the graph in Figure 28-6 shows the flights for a small airline on the East Coast of the United States. The graph is undirected and consists of two subgraphs that are each connected. The entire graph, however, is disconnected.

Notice that you can fly from Boston to Provincetown, but not from Boston to Key West. Algorithms exist that see whether a flight between given cities is possible.

Note: Figure 28-6 contains one graph that consists of two distinct subgraphs. Although each subgraph is connected, the entire graph is disconnected.

FIGURE 28-6 Airline routes

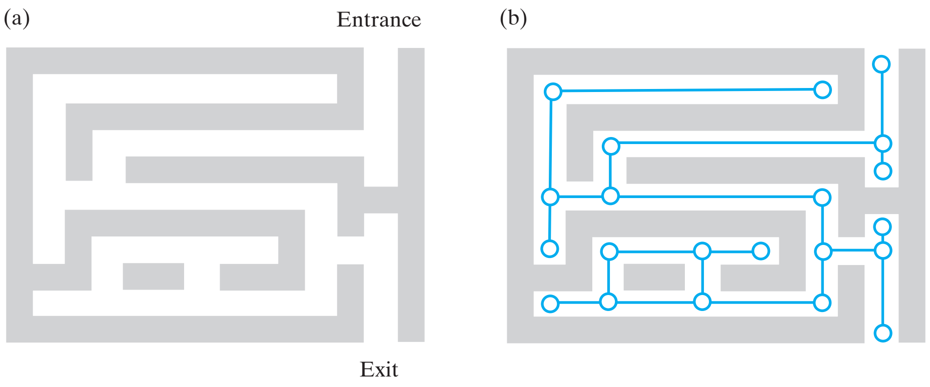


Mazes

28.8 Mazes have been constructed in Victorian English gardens and modern-day cornfields. A typical maze, like the one in Figure 28-7a, has a path from its entrance to its exit. Other paths begin at the entrance, but some lead to dead ends, rather than to the exit. Can you find your way through the maze?

We can represent this maze as a graph by placing a vertex at the entrance and exit, at each turn in the path, and at each dead end, as Figure 28-7b shows. This graph, like the road map in Figure 28-1, is connected. For such graphs, we can find a path between any two vertices, as you will see later in this chapter.

FIGURE 28-7 (a) A maze; (b) its representation as a graph



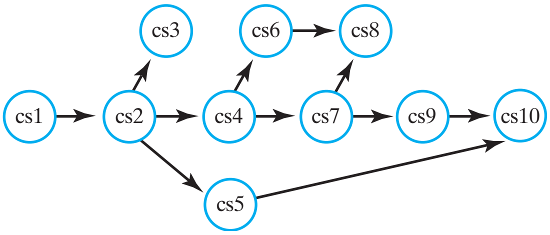
Course Prerequisites

28.9 As a college student, you must take a sequence of courses in your major. Each course has certain prerequisite courses that you must complete first. In what order can you take the required courses and satisfy the prerequisites?

To answer this question, we first create a directed graph to represent the courses and their prerequisites. Figure 28-8 is an example of such a graph. Each vertex represents a course, and each directed edge begins at a course that is a prerequisite to another. Notice, for example, that you must complete cs1, cs2, cs4, cs7, cs9, *and* cs5 before you can take cs10.

This graph has no cycles. In a directed graph without cycles, we can arrange the vertices so that vertex *a* precedes vertex *b* whenever a directed edge exists from *a* to *b*. The order of the vertices in this arrangement is called a **topological order**. Later in this chapter, you will see how to find this order and, therefore, the order in which you should complete your course requirements.

FIGURE 28-8 The prerequisite structure for a selection of courses as a directed graph without cycles



Trees

28.10 The ADT tree is a kind of graph that uses parent-child relationships to organize its nodes in a hierarchical fashion. One particular node, the root, is the ancestor of all other nodes in the tree. But not all graphs have a hierarchical organization, and so not all graphs are trees.

Note: All trees are graphs, but not all graphs are trees. A tree is a connected graph without cycles.

SELF-TEST

Question 2 What physical systems in a typical house could you represent as a graph?

Question 3 Is the graph in Figure 28-1 connected? Is it complete?

Question 4 Is the graph in Figure 28-8 a tree? Explain.

Question 5 For the graph in Figure 28-8,

a. Is cs1 adjacent to cs2? **c.** Is cs1 adjacent to cs4?
b. Is cs2 adjacent to cs1? **d.** Is cs4 adjacent to cs1?

Traversals

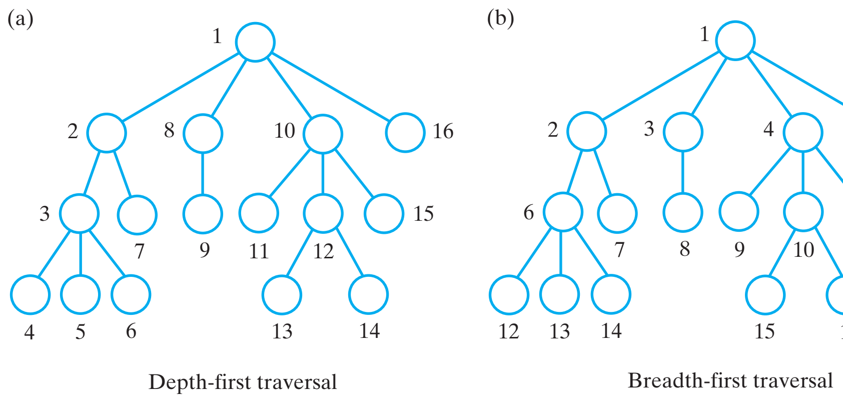
28.11 As you learned in earlier chapters, you usually search a tree for a node that contains a particular value. Graph applications, however, focus on the connections between vertices, rather than the contents of vertices. These applications often are based on a traversal of the graph's vertices.

VideoNote
Graph operations

In Chapter 23, we examined several orders in which we could visit the nodes of a tree. The preorder, inorder, and postorder traversals are examples of a **depth-first traversal**. This kind of traversal follows a path that descends the levels of a tree as deeply as possible until it reaches a leaf, as Figure 28-9a shows. More generally, a depth-first traversal of a graph follows a path that goes as deeply into the graph as possible before following other paths. After visiting a vertex, this traversal visits the vertex's neighbor, the neighbor's neighbor, and so on.

The level-order traversal of a tree is an example of a **breadth-first traversal**. It follows a path that explores an entire level before moving to the next level, as Figure 28-9b shows. In a graph, a breadth-first traversal visits all neighbors of a node before visiting the neighbors' neighbors.

FIGURE 28-9 The visitation order of two traversals: (a) depth first; (b) breadth first



Note: Visiting a node in either a tree or a graph is an action that we perform during a traversal. In a tree, "visit a node" means to "process the node's data." In a graph, "visit a node" means simply to "mark the node as visited."

A traversal of a tree visits all of the tree's nodes beginning with the root. However, a graph traversal begins at any vertex—called the **origin vertex**—and visits only the vertices that it can reach. Only when a graph is connected can such a traversal visit all the vertices.

Breadth-First Traversal

28.12 Given an origin vertex, a breadth-first traversal visits the origin and the origin's neighbors. It then considers each of these neighbors and visits their neighbors. The traversal uses a queue to hold the visited vertices. When we remove a vertex from this queue, we visit and enqueue the vertex's unvisited neighbors. The traversal order is then the order in which vertices are added to the queue. We can retain this traversal order in a second queue.

The following algorithm performs a breadth-first traversal of a nonempty graph beginning at a given vertex.

```
Algorithm getBreadthFirstTraversal(originVertex)
traversalOrder = a new queue for the resulting traversal order
vertexQueue = a new queue to hold vertices as they are visited
```

```
Mark originVertex as visited
traversalOrder.enqueue(originVertex)
vertexQueue.enqueue(originVertex)

while (!vertexQueue.isEmpty())
{
   frontVertex = vertexQueue.dequeue()
   while (frontVertex has a neighbor)
   {
      nextNeighbor = next neighbor of frontVertex
      if (nextNeighbor is not visited)
      {
         Mark nextNeighbor as visited
         traversalOrder.enqueue(nextNeighbor)
         vertexQueue.enqueue(nextNeighbor)
      }
   }
}
return traversalOrder
```

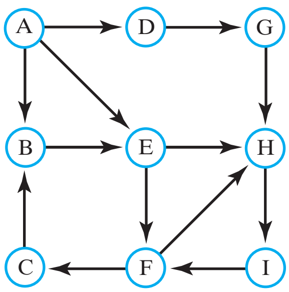
Figure 28-10 traces this algorithm for a directed graph.

Note: Breadth-first traversal

A breadth-first traversal visits a vertex and then each of the vertex's neighbors before advancing. The order in which these neighbors are visited is not specified and can depend on the graph's implementation.

Question 6 In what order does a breadth-first traversal visit the vertices in the graph shown in Figure 28-10 when you begin at vertex *E* and visit neighbors in alphabetic order?

FIGURE 28-10 A trace of a breadth-first traversal beginning at vertex *A* of a directed graph



`frontVertex`	`nextNeighbor`	Visited vertex	`vertexQueue` (front to back)	`traversalOrder` (front to back)
		A	A	A
A			*empty*	
	B	B	B	AB
	D	D	BD	ABD
	E	E	BDE	ABDE
B			DE	
D			E	
	G	G	EG	ABDEG
E			G	
	F	F	GF	ABDEGF
	H	H	GFH	ABDEGFH
G			FH	
F			H	
	C	C	HC	ABDEGFHC
H			C	
	I	I	CI	ABDEGFHCI
C			I	
I			*empty*	

Depth-First Traversal

28.13 Given an origin vertex, a depth-first traversal visits the origin, then a neighbor of the origin, and a neighbor of the neighbor. It continues in this fashion until it finds no unvisited neighbor. Backing up by one vertex, it considers another neighbor. This traversal has a recursive feel, since traversing from the origin leads to a traversal from the origin's neighbor. It should not surprise you, then, that we use a stack in the iterative description of this traversal.

We begin by pushing the origin vertex into the stack. When the vertex at the top of the stack has an unvisited neighbor, we visit and push that neighbor onto the stack. If no such neighbor exists, we pop the stack. The traversal order is the order in which vertices are added to the stack. We can maintain this traversal order in a queue.

The following algorithm performs a depth-first traversal of a nonempty graph, beginning at a given vertex:

```
Algorithm getDepthFirstTraversal(originVertex)
traversalOrder = a new queue for the resulting traversal order
vertexStack = a new stack to hold vertices as they are visited

Mark originVertex as visited
traversalOrder.enqueue(originVertex)
vertexStack.push(originVertex)

while (!vertexStack.isEmpty())
{
   topVertex = vertexStack.peek()
   if (topVertex has an unvisited neighbor)
   {
      nextNeighbor = next unvisited neighbor of topVertex
      Mark nextNeighbor as visited
      traversalOrder.enqueue(nextNeighbor)
      vertexStack.push(nextNeighbor)
   }
   else // all neighbors are visited
      vertexStack.pop()
}
return traversalOrder
```

Figure 28-11 traces this algorithm for the same directed graph as in Figure 28-10.

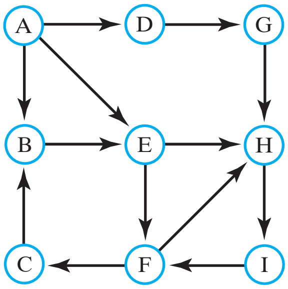
Note: Depth-first traversal

A depth-first traversal visits a vertex, then a neighbor of the vertex, a neighbor of the neighbor, and so on, advancing as far as possible from the original vertex. It then backs up by one vertex and considers another neighbor. The order in which these neighbors are visited is not specified and can depend on the graph's implementation.

SELF-TEST

Question 7 In what order does a depth-first traversal visit the vertices in the graph shown in Figure 28-11 when you begin at vertex *E* and visit neighbors in alphabetic order?

FIGURE 28-11 A trace of a depth-first traversal beginning at vertex *A* of a directed graph



`topVertex`	`nextNeighbor`	Visited vertex	`vertexStack` (top to bottom)	`traversalOrder` (front to back)
		A	A	A
A			A	
	B	B	BA	AB
B			BA	
	E	E	EBA	ABE
E			EBA	
	F	F	FEBA	ABEF
F			FEBA	
	C	C	CFEBA	ABEFC
C			FEBA	
F			FEBA	
	H	H	HFEBA	ABEFCH
H			HFEBA	
	I	I	IHFEBA	ABEFCHI
I			HFEBA	
H			FEBA	
F			EBA	
E			BA	
B			A	
A			A	
	D	D	DA	ABEFCHID
D			DA	
	G		GDA	ABEFCHIDG
G			DA	
D			A	
A			*empty*	ABEFCHIDG

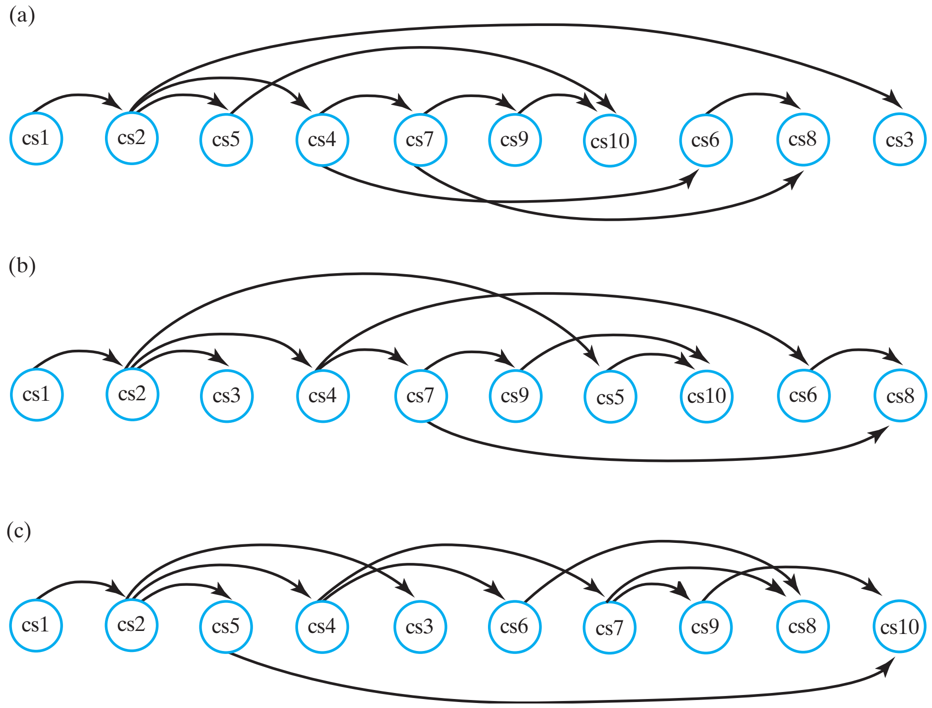
Topological Order

28.14 Figure 28-8 shows a graph that represents the prerequisite structure of a group of computer science courses. This graph is a directed graph without cycles. Recall that you can place the vertices of such a graph in a topological order.

> **Note:** In a topological order of the vertices in a directed graph without cycles, vertex *a* precedes vertex *b* whenever a directed edge exists from *a* to *b*.

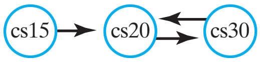
The vertices in a graph can have several different topological orders. For example, one such order for the graph in Figure 28-8 is cs1, cs2, cs5, cs4, cs7, cs9, cs10, cs6, cs8, cs3. That is, if you complete the courses in this order, you will satisfy all prerequisites. Suppose that you can move the vertices in the graph so that they align in this order, stretching the edges as needed. The result will be like the graph in Figure 28-12a. Each edge points toward a node that comes after the edge's origin node. You will be able to find at least one such arrangement for every directed graph, if the graph has no cycles. Figure 28-12 shows two other topological orders for the graph in Figure 28-8 as well. As is true for this example, any one topological order is usually sufficient to solve a given problem.

FIGURE 28-12 Three topological orders for the graph in Figure 28-8



A topological order is not possible for a graph that has a cycle. If vertices *a* and *b* are on the cycle, a path exists from *a* to *b* and from *b* to *a*. One of these paths will contradict any order that we choose for *a* and *b*. For example, the graph in Figure 28-13 contains a cycle. You need to complete cs15 and cs20 before taking cs30. But you need to complete cs30 before taking cs20. This circular logic is caused by the cycle and creates an impossible situation.

FIGURE 28-13 An impossible prerequisite structure for three courses, as a directed graph with a cycle



Question 8 What is another topological order of the vertices in the graph in Figure 28-8?

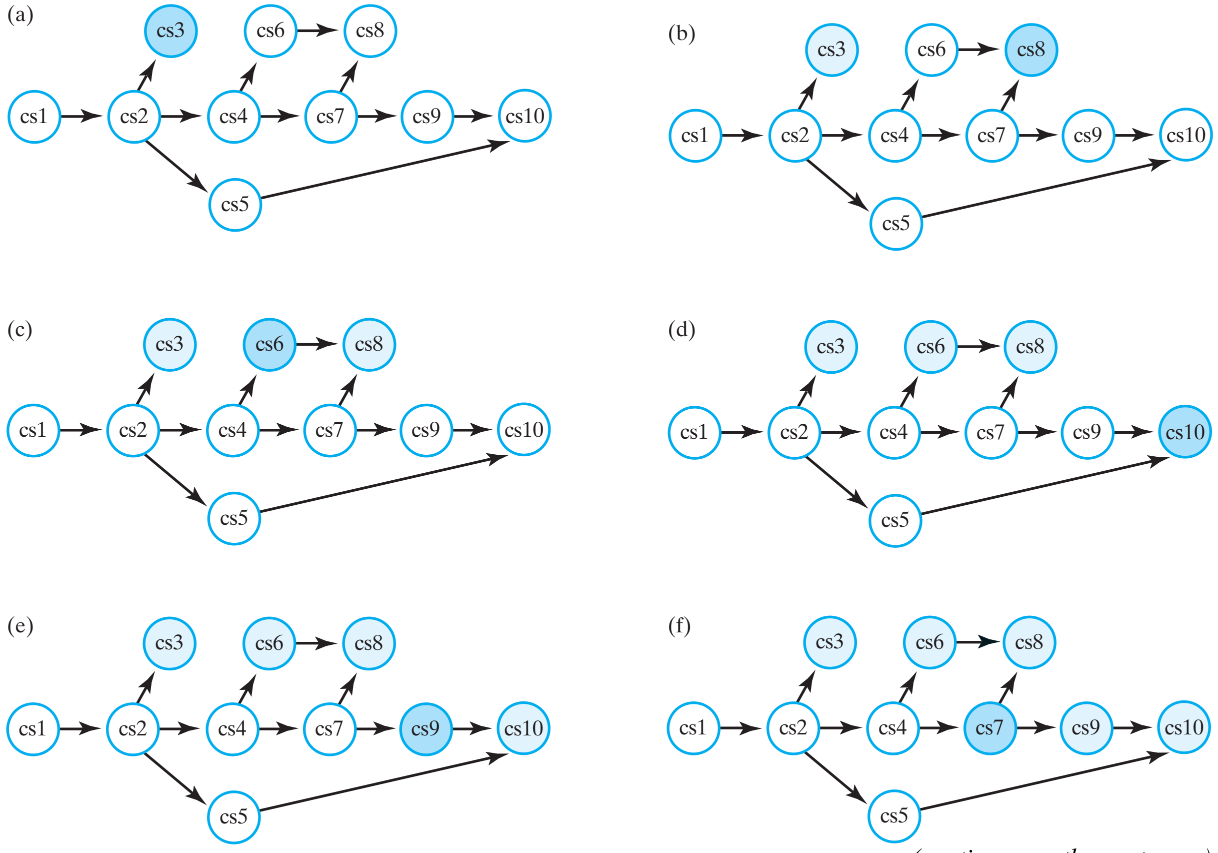
28.15 The process that discovers a topological order for the vertices in a graph is called a **topological sort**. Several algorithms for this process are possible. We can begin a topological sort by locating a vertex that has no successor, that is, no adjacent vertex. Finding this vertex is possible because the graph has no cycles. We mark the vertex as visited and push it onto a stack. We continue by finding another vertex *u* that is unvisited and whose neighbors, if any, are visited. We mark *u* as visited and push it onto the stack. We proceed in this way until we have visited all the vertices. At that time, the stack contains the vertices in topological order, beginning at the top of the stack.

The following algorithm describes this topological sort:

```
Algorithm getTopologicalOrder()
vertexStack = a new stack to hold vertices as they are visited
numberOfVertices = number of vertices in the graph
for (counter = 1 to numberOfVertices)
{
   nextVertex = an unvisited vertex whose neighbors, if any, are all visited
   Mark nextVertex as visited
   vertexStack.push(nextVertex)
}
return vertexStack
```

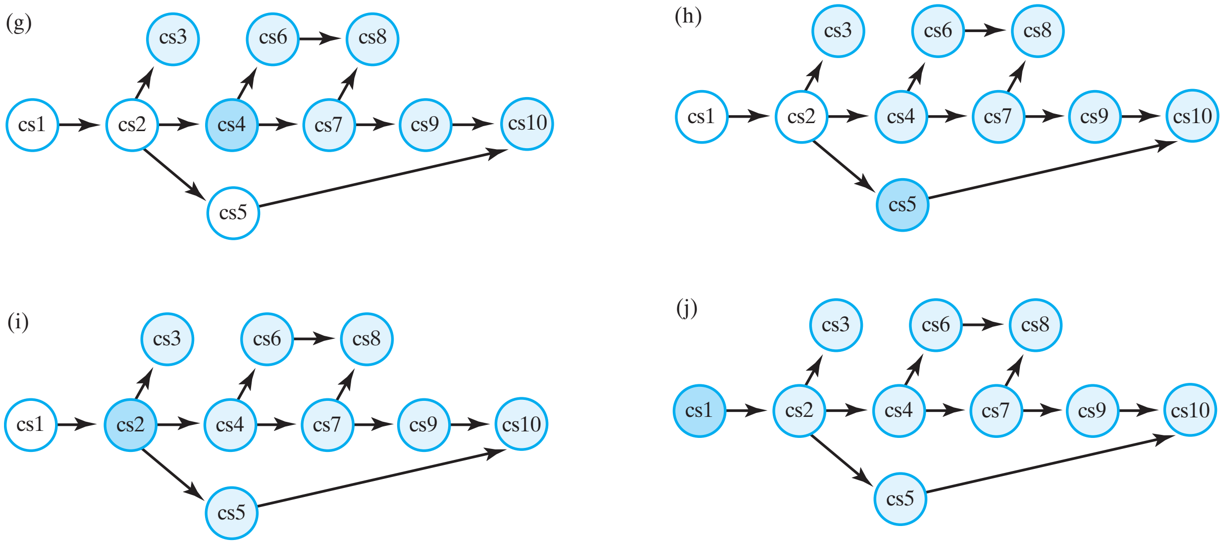
Figure 28-14 traces this algorithm for the graph in Figure 28-8. At each iteration of the algorithm's loop, `nextVertex` becomes shaded in the figure as it is visited. The topological order is the opposite of the order in which this shading occurs. In this example, the topological order is the one pictured in Figure 28-12a.

FIGURE 28-14 Finding a topological order for the graph in Figure 28-8



(continues on the next page)

(Figure 28-14 continued)



Paths

Learning whether a particular airline flies between two given cities is important to the average traveler. We can obtain this information by using a graph—such as the one in Figure 28-6—to represent the airline's routes and testing whether a path exists from vertex *a* to vertex *b*. If a path exists, we can also find out what it is. If not any path will do, we can find the one that is shortest or cheapest.

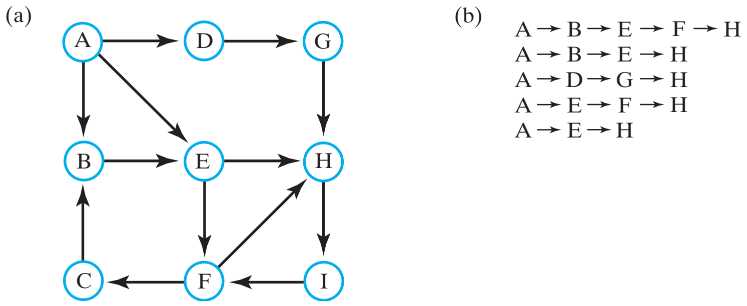
Finding a Path

28.16 For the moment we are content to find any path, not necessarily the best one. A depth-first traversal—discussed in Segment 28.13—stays on a path through the graph as it visits as many vertices as possible. We can easily modify this traversal to locate a path between two vertices. We begin at the origin vertex. Each time we visit another vertex, we see whether that vertex is the desired destination. If so, we are done and the resulting stack contains the path. Otherwise, we continue the traversal until either we are successful or the traversal ends. We leave the development of this algorithm as an exercise.

The Shortest Path in an Unweighted Graph

28.17 **Example.** A graph can have several different paths between the same two vertices. In an unweighted graph, we can find the path with the shortest length, that is, the path that has the fewest edges. For example, consider the unweighted graph in Figure 28-15a. Suppose that we want to know the shortest path from vertex *A* to vertex *H*. By inspecting the graph, we can see that several simple paths—shown in Part *b* of the figure—are possible between these two vertices. The path from *A* to *E* to *H* has length 2 and is the shortest.

FIGURE 28-15 (a) An unweighted graph and (b) the possible paths from vertex *A* to vertex *H*



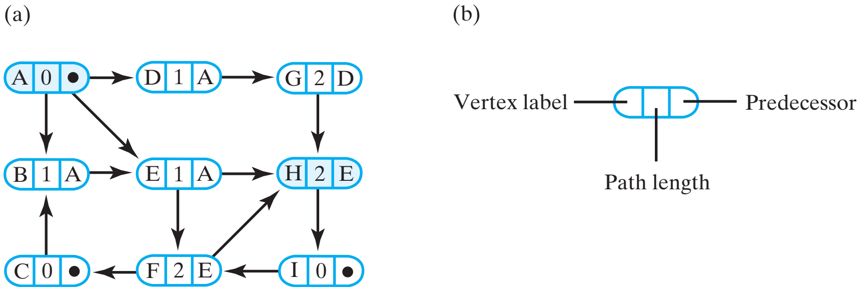
A → B → E → F → H
A → B → E → H
A → D → G → H
A → E → F → H
A → E → H

28.18 **Developing the algorithm.** The algorithm to find the shortest path between two given vertices in an unweighted graph is based on a breadth-first traversal. Recall that this traversal visits the origin vertex, then the origin's neighbors, the neighbors of each of these neighbors, and so on. Each vertex is placed into a queue as it is visited.

To find the shortest path, we enhance the breadth-first traversal as follows. When we visit a vertex *v* and mark it as visited, we note the vertex *p* that we just left to reach *v*. That is, *p* precedes *v* in the graph. We also note the length of the path that the traversal followed to reach *v*. This length is 1 more than the length of the path to vertex *p*. We place both the length of the path to *v* and a reference to *p* into vertex *v*. At the end of the traversal, we will use this data in the vertices to construct the shortest path. Let's jump ahead to that part of the algorithm.

Figure 28-16a shows the state of the graph in Figure 28-15a after the algorithm has traversed from vertex *A* to vertex *H*. Each vertex contains its label, the length of the path to it, and the vertex that precedes it on this path, as shown in Figure 28-16b. Although a vertex also contains other data fields, we have ignored them in this figure.

FIGURE 28-16 (a) The graph in Figure 28-15a after the shortest-path algorithm has traversed from vertex *A* to vertex *H*; (b) the data in a vertex



Now, by examining the destination vertex—*H*—we find that the length of the shortest path from *A* to *H* is 2. We also find that *H*'s predecessor along this shortest path is vertex *E*. From vertex *E* we see that its predecessor is vertex *A*. Thus, the desired shortest path from vertex *A* to vertex *H* is *A* → *E* → *H*. Our algorithm has discovered what we of course knew to be true by inspecting this simple graph.

Note: In an unweighted graph, the shortest path between two given vertices has the shortest length—that is, it has the fewest edges. The algorithm to find this path is based on a breadth-first traversal. If several paths have the same shortest length, the algorithm will find only one of them.

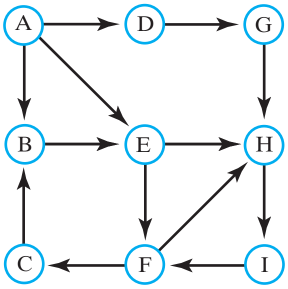
28.19 **The algorithm.** The following algorithm finds the shortest path in an unweighted graph between the vertices `originVertex` and `endVertex`. Like the breath-first traversal in Segment 28.12, the algorithm uses a queue to hold the vertices as they are visited. It then uses the given, initially empty stack `path` to construct the shortest path.

```
Algorithm getShortestPath(originVertex, endVertex, path)
done = false
vertexQueue = a new queue to hold vertices as they are visited
Mark originVertex as visited
vertexQueue.enqueue(originVertex)

while (!done && !vertexQueue.isEmpty())
{
   frontVertex = vertexQueue.dequeue()

   while (!done && frontVertex has a neighbor)
   {
      nextNeighbor = next neighbor of frontVertex
      if (nextNeighbor is not visited)
      {
         Mark nextNeighbor as visited
         Set the length of the path to nextNeighbor to 1 + length of path to frontVertex
         Set the predecessor of nextNeighbor to frontVertex
         vertexQueue.enqueue(nextNeighbor)
      }

      if (nextNeighbor equals endVertex)
         done = true
   }
}

// traversal ends - construct shortest path
pathLength = length of path to endVertex
path.push(endVertex)

vertex = endVertex
while (vertex has a predecessor)
{
   vertex = predecessor of vertex
   path.push(vertex)
}
return pathLength
```

When the algorithm ends, the stack `path` contains the vertices along the shortest path, with the origin at the top of the stack. The value returned is the length of this shortest path.

28.20 **Tracing the algorithm.** Figure 28-17 traces the steps that the algorithm takes to produce the path information shown in Figure 28-16a for the unweighted graph in Figure 28-15a. After adding the origin—vertex *A*—to the queue, we visit the origin's three neighbors—*B*, *D*, and *E*—and enqueue them.

FIGURE 28-17 A trace of the traversal in the algorithm to find the shortest path from vertex *A* to vertex *H* in an unweighted graph



`frontVertex`	`nextNeighbor`	Visited vertex	`vertexQueue` (front to back)
		A	A 0 ●
A 0 ●			*empty*
	B	B	B 1 A
	D	D	B 1 A D 1 A
	E	E	B 1 A D 1 A E 1 A
B 1 A			D 1 A E 1 A
D 1 A			E 1 A
	G	G	E 1 A G 2 D
E 1 A			G 2 D
	F	F	G 2 D F 2 E
	H	H	G 2 D F 2 E H 2 E

The length of each path from *A* to these neighbors is 1. Since vertex *A* has no more neighbors, we remove vertex *B* from the queue. This vertex has vertex *E* as a neighbor, but *E* has been visited already. This implies that we can get to *E* from *A* without first going through *B*. That is, *B* is not on any shortest path that begins at *A* and goes through *E*. Indeed, the path *A* → *B* → *E* is longer than the path *A* → *E*. We do not know whether our final path involves *E*, but if it does, it will not pass through *B*.

The algorithm now removes vertex *D* from the queue. Its neighbor *G* is unvisited, so we set *G*'s path-length field to 2 and its predecessor to *D*. We then enqueue *G*. The algorithm continues in this manner and eventually encounters the destination vertex, *H*. After *H* is updated, the outer loop ends. We then construct the path by working back from *H*, as we did earlier in Segment 28.18.

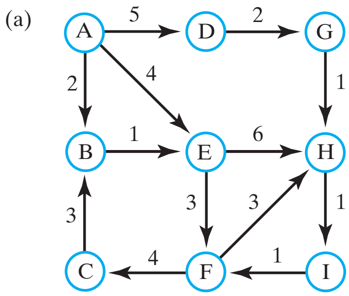
Question 9 Continue the trace begun in Figure 28-17 to find the shortest path from vertex *A* to vertex *C*.

The Shortest Path in a Weighted Graph

28.21 **Example.** In a weighted graph, the shortest path is not necessarily the one with the fewest edges. Rather, it is the one with the smallest edge-weight sum. Figure 28-18a shows a weighted graph obtained by adding weights to the graph in Figure 28-15a. The possible paths from vertex *A* to vertex *H* are the same as you saw in Figure 28-15b. This time, however, we list each path with its weight—that is, the sum of the weights of its edges—in Figure 28-18b.

We can see that the smallest path weight is 8, so the shortest path is *A* → *D* → *G* → *H*. When the weights are distances, the term "shortest" is appropriate. When the weights represent costs, we might think of this path as the "cheapest" path.

FIGURE 28-18 (a) A weighted graph and (b) the possible paths from vertex *A* to vertex *H*, with their weights



(b)

Path	Weight
A → B → E → F → H	9
A → B → E → H	9
A → D → G → H	8
A → E → F → H	10
A → E → H	10

28.22 **Developing the algorithm.** The algorithm to find the shortest, or cheapest, path between two given vertices in a weighted graph is based on a breadth-first traversal. It is similar to the algorithm we developed for an unweighted graph. In that algorithm, we noted the number of edges in the path that led to the vertex under consideration. Here, we compute the sum of the edge weights in the path leading to a vertex. In addition, we must record the cheapest of the possible paths. Whereas before we used a queue to order vertices, this algorithm uses a priority queue.

Note: In a weighted graph, the shortest path between two given vertices has the smallest edge-weight sum. The algorithm to find this path is based on a breadth-first traversal. Several paths in a weighted graph might share the same minimum edge-weight sum. Our algorithm will find only one of these paths.

Each entry in the priority queue is an object that contains a vertex, the cost of the path to that vertex from the origin vertex, and the previous vertex on that path. The priority value is the cost of the path, with the smallest value having the highest priority. Thus, the cheapest path is at the front of the priority queue, and it is thus the first one removed. Note that several entries in the priority queue might contain the same vertex but different costs.

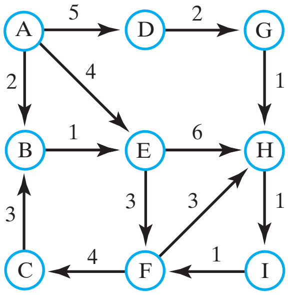
At the conclusion of the algorithm, the vertices in the graph contain predecessors and costs that enable us to construct the cheapest path, much as we constructed the path with the fewest edges from the graph in Figure 28-16a.

28.23 **Tracing the algorithm.** Figure 28-19 traces the traversal portion of the algorithm for the weighted graph in Figure 28-18a when vertex *A* is the origin. Initially, an object containing *A*, zero, and `null` is placed in the priority queue. We begin a loop by removing the front entry from the priority queue. We use the contents of this entry to change the state of the indicated vertex—*A* in this case—in the graph. Thus, we store a path length of zero and a `null` predecessor into *A*. We also mark *A* as visited.

Vertex *A* has three unvisited neighbors, *B*, *D*, and *E*. The costs of the paths from *A* to each of these neighbors is 2, 5, and 4, respectively. These costs, along with *A* as the previous vertex, are used to create objects that are placed into the priority queue. The priority queue orders these objects so that the cheapest path is first.

We remove the front entry from the priority queue. The entry contains vertex *B*, so we visit *B*. We also store within vertex *B* the path cost 2 and its predecessor *A*. Now *B* has vertex *E* as its sole unvisited neighbor. The cost of the path $A \rightarrow B \rightarrow E$ is the cost of the path $A \rightarrow B$ plus the weight of

FIGURE 28-19 A trace of the traversal in the algorithm to find the cheapest path from vertex *A* to vertex *H* in a weighted graph



`frontVertex`	Visited vertex	`nextNeighbor`	Priority queue (front to back)
			A 0 ●
A 0 ●	A		*empty*
		B	B 2 A
		D	B 2 A D 5 A
		E	B 2 A E 4 A D 5 A
B 2 A	B		E 4 A D 5 A
		E	E 3 B E 4 A D 5 A
E 3 B	E		E 4 A D 5 A
		F	E 4 A D 5 A F 6 E
		H	E 4 A D 5 A F 6 E H 9 E
			D 5 A F 6 E H 9 E
D 5 A	D		F 6 E H 9 E
		G	F 6 E G 7 D H 9 E
F 6 E	F		G 7 D H 9 E
		H	G 7 D H 9 E H 9 F
		C	G 7 D H 9 E H 9 F C 10 F
G 7 D	G		H 9 E H 9 F C 10 F
		H	H 8 G H 9 E H 9 F C 10 F
H 8 G	H		H 9 E H 9 F C 10 F

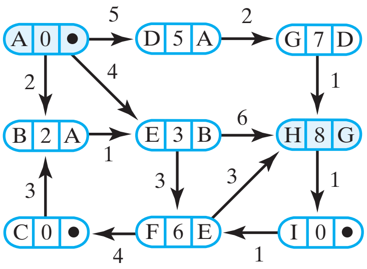
the edge from *B* to *E*. This total cost is 3. We encapsulate *E*, the cost 3, and the predecessor *B* into an object that we add to the priority queue. Notice that two objects in the priority queue involve vertex *E*, but the most recent one has the cheapest path.

We again remove the front entry from the priority queue. The entry contains vertex *E*, so we visit it and store into *E* the path cost 3 and *E*'s predecessor *B*. Vertex *E* has two unvisited neighbors, *F* and *H*. The cost of each path to a neighbor is the cost of the path to *E* plus the weight of the edge to the neighbor. Two new objects are added to the priority queue.

The next object removed from the priority queue contains the vertex *E*, but since *E* has been visited, we ignore it. We then remove the next object from the priority queue. The algorithm continues until the destination vertex *H* is visited.

Figure 28-20 shows the state of the graph at the conclusion of the trace given in Figure 28-19. By looking at the destination vertex *H*, we can see that the weight of the cheapest path from *A* to *H* is 8. Tracing back from *H* to *A,* we see that this path is *A* → *D* → *G* → *H*, as we noted in Segment 28.21.

FIGURE 28-20 The graph in Figure 28-18a after finding the cheapest path from vertex *A* to vertex *H*



28.24 **The algorithm.** The pseudocode for the algorithm we just described follows. Objects in the priority queue are instances of a private class `EntryPQ`. Following the traversal, the algorithm pushes the vertices that occur along the cheapest path from `originVertex` to `endVertex` into a given, initially empty stack `path`.

```
Algorithm getCheapestPath(originVertex, endVertex, path)
done = false
priorityQueue = a new priority queue

priorityQueue.add(new EntryPQ(originVertex, 0, null))

while (!done && !priorityQueue.isEmpty())
{
   frontEntry = priorityQueue.remove()
   frontVertex = vertex in frontEntry

   if (frontVertex is not visited)
   {
      Mark frontVertex as visited
      Set the cost of the path to frontVertex to the cost recorded in frontEntry
      Set the predecessor of frontVertex to the predecessor recorded in frontEntry

      if (frontVertex equals endVertex)
         done = true
      else
      {
         while (frontVertex has a neighbor)
         {
            nextNeighbor = next neighbor of frontVertex
            weightOfEdgeToNeighbor = weight of edge to nextNeighbor

            if (nextNeighbor is not visited)
            {
               nextCost = weightOfEdgeToNeighbor + cost of path to frontVertex
               priorityQueue.add(new EntryPQ(nextNeighbor, nextCost,
                                                  frontVertex))
            }
         }
      }
   }
}

// traversal ends; construct cheapest path
```

```
pathCost = cost of path to endVertex
path.push(endVertex)

vertex = endVertex
while (vertex has a predecessor)
{
   vertex = predecessor of vertex
   path.push(vertex)
}
return pathCost
```

The origin of the cheapest path will be at the top of the stack `path`. At the bottom of the stack is the destination vertex. The cost of the path is returned by the algorithm.

This algorithm is based on Dijkstra's algorithm, which finds the shortest paths from an origin to all other vertices.

SELF-TEST

Question 10 Continue the trace begun in Figure 28-19 to find the shortest (cheapest) path from vertex *A* to vertex *C*.

Question 11 Why do we place instances of `EntryPQ` into the priority queue, instead of placing vertices?

Java Interfaces for the ADT Graph

28.25 The ADT graph is a bit different from other ADTs in that, once you create it, you do not add, remove, or retrieve components. Instead, you use a graph to answer questions based on the relationships among its vertices.

We will divide the graph operations into two Java interfaces. You use the operations in the first interface to create the graph and to obtain basic information such as the number of vertices. The second interface specifies operations such as the traversals and path searches that we discussed earlier in this chapter. For convenience, we define a third interface, `GraphInterface`, that combines the first two interfaces.

To make these interfaces as general as possible, we have them specify graphs that are either directed or undirected, and weighted or unweighted. The first interface appears in Listing 28-1. The generic type `T` represents the data type of the objects that label the graph's vertices.

LISTING 28-1 An interface of basic graph operations

```
package GraphPackage;
/** An interface of methods providing basic operations for directed
    and undirected graphs that are either weighted or unweighted. */
public interface BasicGraphInterface<T>
{
  /** Adds a given vertex to the graph.
      @param vertexLabel  an object that labels the new vertex and is
                          distinct from the labels of current vertices
      @return true if the vertex is added, or false if not */
  public boolean addVertex(T vertexLabel);
  /** Adds a weighted edge between two given distinct vertices that
      are currently in the graph. The desired edge must not already
      be in the graph. In a directed graph, the edge points toward
      the second vertex given.
      @param begin  an object that labels the origin vertex of the edge
      @param end    an object, distinct from begin, that labels the end
                    vertex of the edge
      @param edgeWeight  the real value of the edge's weight
      @return true if the edge is added, or false if not */
  public boolean addEdge(T begin, T end, double edgeWeight);

  /** Adds an unweighted edge between two given distinct vertices
      that are currently in the graph. The desired edge must not
      already be in the graph. In a directed graph, the edge points
      toward the second vertex given.
      @param begin  an object that labels the origin vertex of the edge
      @param end    an object, distinct from begin, that labels the end
                    vertex of the edge
      @return true if the edge is added, or false if not */
  public boolean addEdge(T begin, T end);

  /** Sees whether an edge exists between two given vertices.
      @param begin  an object that labels the origin vertex of the edge
      @param end    an object that labels the end vertex of the edge
      @return true if an edge exists */
  public boolean hasEdge(T begin, T end);

  /** Sees whether the graph is empty.
      @return true if the graph is empty */
  public boolean isEmpty();

  /** Gets the number of vertices in the graph.
      @return the number of vertices in the graph */
  public int getNumberOfVertices();

  /** Gets the number of edges in the graph.
      @return the number of edges in the graph */
  public int getNumberOfEdges();

  /** Removes all vertices and edges from the graph. */
  public void clear();
} // end BasicGraphInterface
```

28.26 **Example.** The following statements create the graph shown in Figure 28-21, which is a portion of the graph in Figure 28-6:

```
BasicGraphInterface<String> airMap = new UndirectedGraph<String>();
airMap.addVertex("Boston");
airMap.addVertex("Provincetown");
airMap.addVertex("Nantucket");
airMap.addEdge("Boston", "Provincetown");
airMap.addEdge("Boston", "Nantucket");
```

At this point,

```
airMap.getNumberOfVertices()
```

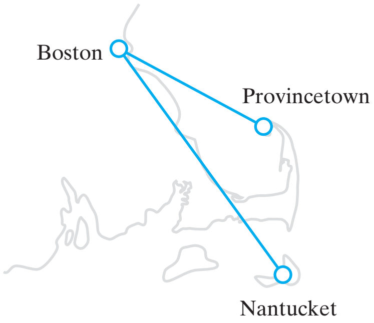
returns 3, and

```
airMap.getNumberOfEdges()
```

returns 2.

Question 12 What revisions to the previous Java statements are necessary to make `airMap` a weighted graph?

FIGURE 28-21 A portion of the flight map in Figure 28-6



28.27 The algorithms discussed earlier in this chapter use graph operations that are not specified in the previous interface. Although we could add these operations to the interface so the client could implement various algorithms, such as the topological sort, we choose not to do so. Instead, methods that implement the graph algorithms will be a part of the graph class. The interface in Listing 28-2 specifies these methods. Again, the data type of the objects that label the graph's vertices are represented by the generic type `T`.

LISTING 28-2 An interface of operations on an existing graph

```
package GraphPackage;
import ADTPackage.*; // classes that implement various ADTs
/** An interface of methods that process an existing graph. */
public interface GraphAlgorithmsInterface<T>
```

```
{
  /** Performs a breadth-first traversal of a graph.
      @param origin  an object that labels the origin vertex of the
                     traversal
      @return a queue of labels of the vertices in the traversal, with
              the label of the origin vertex at the queue's front */
  public QueueInterface<T> getBreadthFirstTraversal(T origin);

  /** Performs a depth-first traversal of a graph.
      @param origin  an object that labels the origin vertex of the
                     traversal
      @return a queue of labels of the vertices in the traversal, with
              the label of the origin vertex at the queue's front */
  public QueueInterface<T> getDepthFirstTraversal(T origin);

  /** Performs a topological sort of the vertices in a graph
      without cycles.
      @return a stack of vertex labels in topological order, beginning
              with the stack's top */
  public StackInterface<T> getTopologicalOrder();

  /** Finds the path between two given vertices that has the
      shortest length.
      @param begin  an object that labels the path's origin vertex
      @param end    an object that labels the path's destination vertex
      @param path   a stack of labels that is empty initially;
                    at the completion of the method, this stack contains
                    the labels of the vertices along the shortest path;
                    the label of the origin vertex is at the top, and
                    the label of the destination vertex is at the bottom
      @return the length of the shortest path */
  public int getShortestPath(T begin, T end, StackInterface<T> path);

  /** Finds the least-cost path between two given vertices.
      @param begin  an object that labels the path's origin vertex
      @param end    an object that labels the path's destination vertex
      @param path   a stack of labels that is empty initially;
                    at the completion of the method, this stack contains
                    the labels of the vertices along the cheapest path;
                    the label of the origin vertex is at the top, and
                    the label of the destination vertex is at the bottom
      @return the cost of the cheapest path */
  public double getCheapestPath(T begin, T end, StackInterface<T> path);
} // end GraphAlgorithmsInterface
```

The interface in Listing 28-3 combines `BasicGraphInterface` and `GraphAlgorithmsInterface`.

LISTING 28-3 An interface for the ADT graph

```
package GraphPackage;
public interface GraphInterface<T> extends BasicGraphInterface<T>,
                                           GraphAlgorithmsInterface<T>
{
} // end GraphInterface
```

28.28 **Example.** Imagine that we want to find the shortest route between the towns of Truro and Falmouth. By “shortest route” we mean the route with the least number of miles, not the path with the fewest edges. We first could create the graph in Figure 28-3, using statements much like those in Segment 28.26. We then could use the method `getCheapestPath` to answer our question. The following statements indicate how to perform these steps and to display the names of the cities along the shortest route:

```
GraphInterface<String> roadMap = new UndirectedGraph<String>();
roadMap.addVertex("Provincetown");
roadMap.addVertex("Truro");
. . .
roadMap.addVertex("Falmouth");

roadMap.addEdge("Provincetown", "Truro", 10);
. . .
roadMap.addEdge("Hyannis", "Falmouth", 20);

StackInterface<String> bestRoute = new LinkedStack<String>();
double distance = roadMap.getCheapestPath("Truro", "Falmouth", bestRoute);
System.out.println("The shortest route from Truro to Falmouth is " +
                   distance + " miles long and " +
                   "passes through the following towns:");
while (!bestRoute.isEmpty())
   System.out.println(bestRoute.pop());
```

Note: The operations of the ADT graph enable you to create a graph and answer questions about the relationships among its vertices.

Question 13 The previous example finds the shortest route between two towns. Why did we invoke the method `getCheapestPath` instead of `getShortestPath`?

Chapter Summary

- A graph is a collection of distinct vertices and distinct edges. Each edge joins two vertices. A subgraph is a portion of a graph that is itself a graph.
- A tree is a special graph that has a hierarchical order and a root that is the ancestor of all other nodes—that is, vertices—in the tree.

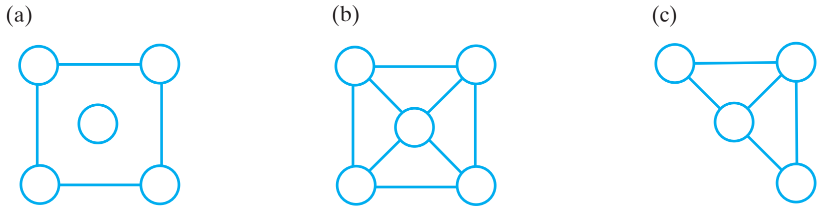
- Each edge in a directed graph has a direction from one vertex to another. The edges in an undirected graph are bidirectional.
- A path from one vertex to another is a sequence of edges. The length of the path is the number of these edges. A simple path passes through each of its vertices once. A cycle is a path that begins and ends at the same vertex. A simple cycle passes through its other vertices once.
- The edges in a weighted graph have values called weights or costs. A path in a weighted graph has a weight, or cost, that is the sum of its edge weights.
- A graph that has a path between every pair of distinct vertices is connected. A complete graph has an edge between every pair of distinct vertices.
- Two vertices in an undirected graph are adjacent if they are joined by an edge. In a directed graph, vertex *i* is adjacent to vertex *j* if a directed edge begins at *j* and ends at *i*. Adjacent vertices are called neighbors.
- You can traverse the vertices in a graph by using either a depth-first traversal or a breadth-first traversal. A depth-first traversal follows a path that goes as deeply into the graph as possible before following other paths. A breadth-first traversal visits all neighbors of a vertex before visiting the neighbors' neighbors.
- A directed graph without cycles imposes an order on its vertices called a topological order. This order is not unique. You use a topological sort to discover these orders.
- You can use a depth-first traversal of a graph to see whether a path exists between two given vertices.
- You can modify the breadth-first traversal of a graph to find the path between two given vertices that has the fewest edges.
- You can modify the breadth-first traversal of a weighted graph to find the path between two given vertices that has the lowest cost.

EXERCISES

1. Suppose that five vertices are arranged at the corners of an imaginary pentagon. Draw a connected graph that contains these vertices.

2. Describe each graph in Figure 28-22, using the terms introduced in Segments 28.1 through 28.4.

FIGURE 28-22 Graphs for Exercise 2



3. Consider a graph that represents acquaintances among people. Each vertex represents a person. Each edge represents an acquaintance between two people.

a. Is this graph directed or undirected?
b. Consider all vertices adjacent to a given vertex *x*. What does this set of vertices represent?
c. What does a path in this graph represent?
d. In what circumstance might one want to know the shortest path between two vertices in this graph?
e. Is the graph associated with all the people alive on January 1, 1995, connected? Justify your answer.

4. In what order does a breadth-first traversal visit the vertices in the graph in Figure 28-10 when you begin at

   a. Vertex *G*
   b. Vertex *F*

5. Repeat the previous exercise, but perform a depth-first traversal instead.

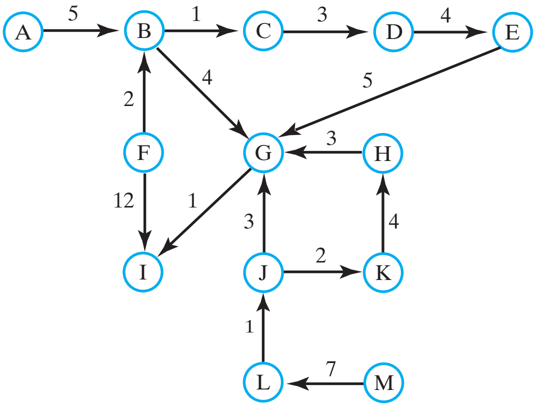
6. Consider the directed graph that appears in Figure 28-10, and remove the edge between vertices *E* and *F*, and the edge between vertices *F* and *H*.

   a. In what order will a breadth-first traversal visit the vertices when you begin at vertex *A*?
   b. Repeat Part *a*, but perform a depth-first traversal instead.

7. Draw a directed graph that depicts the prerequisite structure of the courses required for your major. Find a topological order for these courses.

8. Construct the topological ordering for the weighted, directed, acyclic graph in Figure 28-23.

FIGURE 28-23 A graph for Exercises 8 and 22



9. A computer network such as the Internet or a local area network can be represented as a graph. Each computer is a vertex in the graph. An edge between two vertices represents a direct connection between two computers. Explain when and why you would be interested in each of the following tasks:

   a. Finding a path in this graph
   b. Finding multiple paths from one particular vertex to another
   c. Finding the shortest path from one particular vertex to another
   d. Seeing whether the graph is connected

10. Write an algorithm that finds a path from vertex *a* to vertex *b* in a directed graph by using a slightly modified depth-first traversal. Segment 28.16 outlines an approach to this problem.

11. A tree is a connected graph without cycles.

    a. What is the smallest number of edges that could be removed from the graph in Figure 28-1 to make it a tree?
    b. Give one example of such a set of edges.

12. Figure 28-7b shows a graph that represents a maze. Label the vertices of this graph, with the uppermost vertex labeled *S* (the entrance to the maze) and the lowest vertex labeled *T* (the exit from the maze).

    a. Is this graph a tree?
    b. What is the shortest path from *S* to *T*?
    c. What is the longest simple path in this graph?

13. Revise the unweighted, directed graph in Figure 28-15a by adding a directed edge from *D* to *H*. The resulting graph has two paths from *A* to *H* that are shortest among all paths between these two vertices. Which of these two paths will the algorithm `getShortestPath` in Segment 28.19 find?

14. Repeat the previous exercise, but remove the directed edge from *E* to *H* instead of adding a directed edge from *D* to *H*.

15. Revise the weighted, directed graph in Figure 28-18a by adding a directed edge from *D* to *H*. Let the weight of this new edge be 3. The resulting graph has two paths from *A* to *H* that are cheapest among all paths between these two vertices. Which of these two paths will the algorithm `getCheapestPath` in Segment 28.24 find?

16. Find a map of the routes of a major U. S. airline. Such maps are usually printed at the back of in-flight magazines. You could also search the Internet for one. The map is a graph like the one in Figure 28-6. Consider the following pairs of cities:

    Providence (RI) and San Diego (CA)
    Albany (NY) and Phoenix (AZ)
    Boston (MA) and Baltimore (MD)
    Dallas (TX) and Detroit (MI)
    Charlotte (NC) and Chicago (IL)
    Portland (ME) and Portland (OR)

    a. Which pairs of cities in this list have edges (nonstop flights) between them?
    b. Which pairs are not connected by any path?
    c. For each of the remaining pairs, find the path with the fewest edges.

17. Find the trail map of a cross-country ski area. Represent the trail map as an undirected graph, where each intersection of trails is a vertex, and each section of trail between intersections is an edge. Consider a cross-country skier who wishes to take the longest tour possible, but does not want to ski on any trail more than once. What is the longest path that starts and ends at the ski lodge and does not traverse any section of trail more than once? (Intersections may be passed through more than once, and some sections of trail may be left unskied.)

18. Find the trail map of a downhill ski area. Represent the trail map as a graph, where each intersection of trails is a vertex, and each section of trail between intersections is an edge.

    a. Is the graph directed or undirected?
    b. Does the graph have cycles?
    c. Find the longest path possible that begins at the top of the mountain and ends at the ski lodge.

19. Write statements appropriate for the client of the class `UndirectedGraph` that create the graph in Figure 28-3. Assume that `UndirectedGraph` implements `GraphInterface`.

20. Write statements appropriate for the client of the class `DirectedGraph` that create the graph in Figure 28-8. Assume that `DirectedGraph` implements `GraphInterface`. Then write statements to find and display a topological order for this graph.

21. A graph is said to be **biconnected** if two paths that do not share edges or vertices exist between every pair of vertices.

    a. Which graphs in Figure 28-1 and 28-4 are biconnected?
    b. What are some applications that would use a biconnected graph?

22. A **critical path** in a weighted, directed, acyclic graph is the path with the greatest weight. Let's assume that all edge weights are positive. Give each vertex a value equal to the weight of a path to that vertex. Initially, each vertex's value is zero.

    We can find the critical path by considering the vertices one at a time in topological order. For each vertex, consider all the edges that leave the vertex. For each of these edges, add the weight of the edge and the value of the edge's source vertex. Compare the sum with the value of the edge's destination vertex. Make the larger of these values the value of the destination vertex. After all vertices have been visited, the largest value stored in a vertex will be the weight of the critical path.

    Find the critical path for the graph in Figure 28-23.

Projects

1. In a search tree, it is easy to search for any value. For other trees in which the children of a node are not ordered in any particular way, you can use a breadth-first traversal, as described for graphs, to find a path from the root to some other node (vertex) *v*. Implement such a method for a general tree.

2. Write Java code that creates the graph given in Figure 28-1. Find the shortest path from Sandwich to Falmouth. Do the same for the weighted graph in Figure 28-3. (See Exercise 19.)

3. Write Java code that creates the graph in Figure 28-10. Perform a breadth-first traversal of the graph, beginning at the node labeled *A*.

4. In the game of Nim, an arbitrary number of chips are divided into an arbitrary number of piles. Each player can remove as many chips as desired from any single pile. The last player to remove a chip wins.

   Consider a limited version of this game, in which three piles contain 3, 5, and 8 chips, respectively. You can represent this game as a directed graph. Each vertex in this graph is a possible configuration of the piles (chips in each pile). The initial configuration, for example, is (3, 5, 8). Each edge in the graph represents a legal move in the game.

   a. Write Java statements that will construct this directed graph.
   b. Discuss how a computer program might use this graph to play Nim.

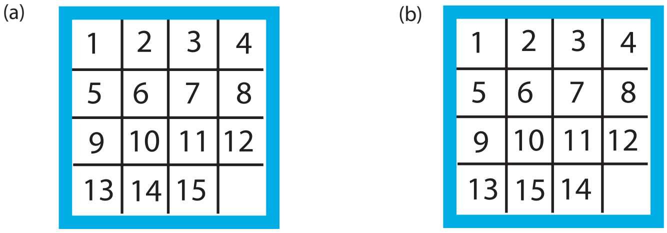
5. The **diameter** of an unweighted graph is the maximum of all the shortest distances between pairs of vertices in the graph.

   a. Give an algorithm for computing the diameter of a graph.
   b. What is the Big Oh performance of your algorithm in terms of the number of vertices and edges in the graph?
   c. Implement your algorithm.
   d. Discuss possible ways that you can improve the performance of the algorithm.

6. Exercise 22 described how to find the critical path in a weighted, directed, acyclic graph. Write a method that will find the critical path. You may assume the existence of a method that tests whether a graph is acyclic.

7. The *n*-puzzle is a one-person game that involves a square or rectangular frame that can contain exactly *n* + 1 square tiles. The game begins with *n* tiles numbered from 1 to *n* positioned randomly in the frame. With one empty space in the frame, the objective is to slide the tiles—one at a time and either horizontally or vertically—until they appear in numerical order, as shown in Figure 28-24a. This solved configuration is for a 15-puzzle using a 4-by-4 frame.

FIGURE 28-24 (a) A solved 15-puzzle; (b) an unsolvable 15-puzzle



(a)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

(b)

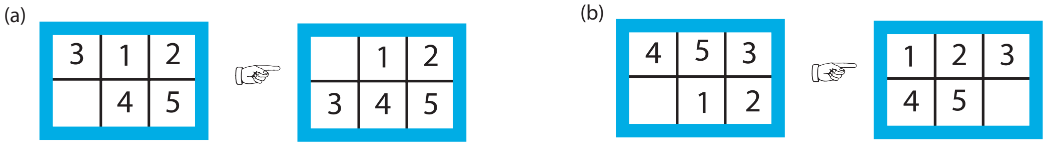
1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Not all initial configurations of an *n*-puzzle can be solved. For example, if the initial configuration of a 15-puzzle were as pictured in Figure 28-24b, with only the 14 and 15 tiles interchanged, no solution would be possible. A solvable 15-puzzle can take up to 80 moves to reach the solution; an 8-puzzle using a 3 × 3 frame will take at most 31 moves to solve, if it has a solution. To reduce our effort even further, we will consider 5-puzzles in 2 × 3 frames. Figure 28-25 shows two such puzzles with their solutions. Note that the empty space in a solution can be either before the 1 or after the 5.

Figure 23-22 in Chapter 23 showed a game tree for tic-tac-toe. A game tree, which is a kind of graph, contains the possible moves for a particular game. Because you cannot change a move made in tic-tac-toe, the game tree is a directed graph. Such is not the case for the *n*-puzzle, as you can change your mind about any move. Thus, an undirected graph can represent all of the possible moves.

Write Java code that creates an undirected graph of the possible board configurations for the 5-puzzle. Using a shortest-path search, find a solution to any given initial configuration.

FIGURE 28-25 The initial and final configurations of two 5-puzzles



Answers to Self-Test Questions

1. **a.** 3.
   b. 36.
   c. Truro-Orleans-Barnstable-Sandwich, with a length of 3.
   d. Truro-Orleans-Barnstable-Sandwich, with a weight of 48.

2. Electrical (or telephone or TV) wires, plumbing, hallways or other connections between rooms.

3. The graph is connected but not complete.

4. No; cs8 and cs10 each would have 2 parents.

5. **a.** No; **b.** Yes; **c.** No; **d.** No.

6. *E*, *F*, *H*, *C*, *I*, *B*.

7. *E*, *F*, *C*, *B*, *H*, *I*.

8. cs1, cs2, cs4, cs7, cs9, cs5, cs10, cs6, cs8, cs3; or cs1, cs2, cs3, cs4, cs5, cs6, cs7, cs8, cs9, cs10.

9. Remove vertex *G* from the queue. *G*'s neighbor *H* has been visited already. Now remove *F* from the queue. *F*'s neighbor *C* is unvisited. Set *C*'s predecessor to *F* and its path-length field to 3 (1 + the length recorded in *F*). Add *C* to the queue. Since *C* is the destination, construct the path by working backward from *C*, as we did in Segment 28.18. The shortest path is $A \rightarrow E \rightarrow F \rightarrow C$, with a length of 3.

10. Vertex *H* has one unvisited neighbor, *I*. The cost of the path to *I* is the cost of the path to *H* plus the weight of the edge from *H* to *I*. This total cost is 9. Encapsulate vertex *I*, the cost 9, and the predecessor *H* into an object and add it to the priority queue.

    Now remove the front entry from the priority queue. The entry contains *H*, and since *H* is visited already, ignore the entry. Remove the next entry. This entry also contains *H*, which is visited, so ignore it and remove the next entry. This entry contains *I*, which is unvisited. Visit *I*. *I*'s neighbor *F* is visited, so remove the next entry from the priority queue. This entry contains *C*, so visit *C*.

    Since *C* is the destination, construct the path by working backward from *C*, as we did in Segment 28.18. The shortest (cheapest) path is $A \rightarrow B \rightarrow E \rightarrow F \rightarrow C$, with a weight (cost) of 10.

11. Two or more entries in the priority queue can record data about the same vertex. For example, consider the trace in Figure 28-19. After we visit vertex *B*, the first two entries in the priority queue record data about vertex *E*. The first entry involves the path from *B* to *E*, while the second entry involves the path from *A* to *E*. While vertex *E* can record similar data for one path, it cannot do so for multiple paths.

12. In the two calls to `addEdge`, you would add an edge weight as a third argument.

13. The method `getCheapestPath` finds the path with the smallest weight sum, which gives us the shortest route when measured in miles; `getShortestPath` finds the path that contains the fewest number of edges.

Chapter 29

Graph Implementations

Contents

Prerequisites

Chapter	5	Stacks
Chapter	10	Queues, Deques, and Priority Queues
Chapter	12	Lists
Chapter	15	Iterators
Chapter	19	Dictionaries
Chapter	23	Trees
Chapter	28	Graphs

Objectives

After studying this chapter, you should be able to
- Describe an adjacency matrix
- Describe an adjacency list
- Specify and implement the classes that represent the vertices and edges of a graph
- Implement the ADT graph by using adjacency lists

Like the ADTs you have seen previously, graphs have several implementations. Each implementation must represent the vertices in the graph and the edges between the vertices. In general, you use either a list or a dictionary to hold the vertices, and an array or a list to represent the edges. Each representation of the edges has its own advantages, but the list representation is most typical.

An Overview of Two Implementations

Two common implementations of the ADT graph use either an array or a list to represent the graph's edges. The array is typically a two-dimensional array called an **adjacency matrix**. The list is called an **adjacency list**. Each of these constructs represents the connections—that is, the edges—among the vertices in the graph.

The Adjacency Matrix

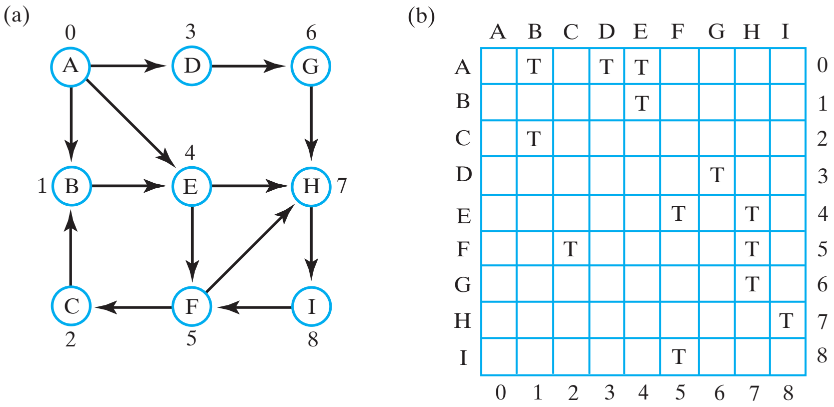
29.1 The adjacency matrix for a graph of n vertices has n rows and n columns. Each row and each column corresponds to a vertex in the graph. You number the vertices from 0 through $n - 1$ to match the row indices and the column indices. If a_{ij} is the element in row i and column j of the matrix, a_{ij} indicates whether an edge exists between vertex i and vertex j. For an unweighted graph, you can use boolean values in the matrix. For a weighted graph, you can use edge weights when edges exist and a representation of infinity otherwise.

VideoNote
The adjacency matrix

Figure 29-1 provides an example of an unweighted, directed graph and its adjacency matrix. Let's consider vertex A of the graph, which we have numbered as vertex 0. Since directed edges exist from vertex A to each of the vertices B, D, and E, the matrix elements a_{01}, a_{03}, and a_{04} are true. We have used a "T" in the figure to represent true. The other entries in the first row are false (blank in the figure).

Although a directed edge exists from vertex A to vertex B, the converse is not true. Therefore, a_{10} is false, even though a_{01} is true. The adjacency matrix for an undirected graph, however, is **symmetric**; that is, a_{ij} and a_{ji} have the same value. When an undirected graph has an edge from vertex i to vertex j, it also has an edge from vertex j to vertex i.

FIGURE 29-1 (a) An unweighted, directed graph and (b) its adjacency matrix



	A	B	C	D	E	F	G	H	I	
A		T		T	T					0
B					T					1
C		T								2
D							T			3
E						T		T		4
F			T					T		5
G								T		6
H									T	7
I						T				8
	0	1	2	3	4	5	6	7	8	

29.2 From an adjacency matrix, you quickly can see whether an edge exists between any two given vertices. This operation is O(1). But if you want to know all the neighbors of a particular vertex, you need to scan an entire row of the matrix, an O(n) task. Additionally, the matrix occupies a considerable, fixed amount of space that depends on the number of vertices but not on the number of edges. In fact, an adjacency matrix represents every possible edge in a graph, regardless of whether the edges actually exist. However, most graphs have relatively few of the many edges possible—that is, they are sparse. For such graphs, an adjacency list uses less space, as you will now see.

Note: An adjacency matrix uses a fixed amount of space that depends on the number of vertices, but not the number of edges, in a graph. The adjacency matrix for a sparse graph wastes space, because the graph has relatively few edges.

Note: Seeing whether an edge exists between any two given vertices of a graph can be done quickly when you use an adjacency matrix. But you need to scan an entire row of the matrix if you want to know all the neighbors of a particular vertex.

SELF-TEST

Question 1 Consider the graph in Figure 28-4b of the previous chapter. Number the vertices from 0 through 3, starting at the vertex in the upper left corner and moving in a clockwise direction. What adjacency matrix represents this graph?

The Adjacency List

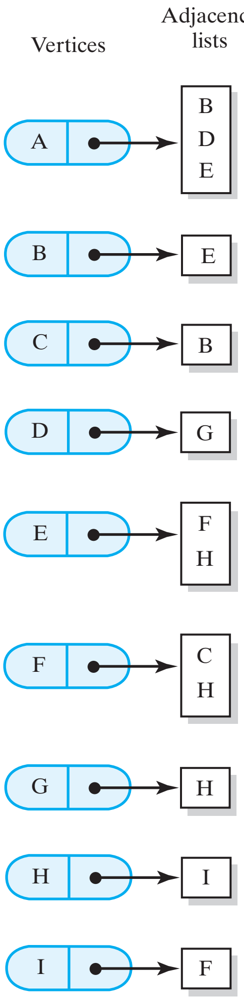
29.3 An adjacency list for a given vertex represents only those edges that originate from the vertex. In Figure 29-2, each vertex of the graph in Figure 29-1a references a list of adjacent vertices. Space is not reserved for edges that do not exist. Thus, the adjacency lists, taken together, use less memory than the corresponding adjacency matrix in Figure 29-1b. For this reason, implementations of sparse graphs use adjacency lists. The implementation that we present in this chapter will do so also, since typical graphs are sparse.

Although the adjacency lists in our diagram contain vertices, they will actually contain edges in our implementation. Each of these edges, however, will contain the illustrated vertex as its terminal vertex.

Note: An adjacency list for a given vertex represents only those edges that originate from the vertex. For a sparse graph, an adjacency list uses less memory than an adjacency matrix. For a dense graph, an adjacency matrix can be the better choice.

Note: Using adjacency lists, you can find all the neighbors of a particular vertex by traversing a list. If you want to know whether an edge exists between any two given vertices, you need to search a list. If the graph contains n vertices, each of these operations is O(n) at worst, but is faster on average.

FIGURE 29-2 Adjacency lists for the directed graph in Figure 29-1a



Question 2 What adjacency lists represent the graph described in Question 1?

Vertices and Edges

29.4 While designing a class that implements the ADT graph, we encounter two other types of objects, the vertex and the edge. These objects are interrelated: A vertex has edges that leave it, and an edge is defined by the vertices at its ends.

A vertex in a graph is somewhat like a node in a tree. Both vertices and nodes are implementation details that we hide from the client. In the implementation of a binary tree, we used a package-friendly class `BinaryNode`. (See Segment 24.3 of Chapter 24.) Here, a graph will have package access to the class `Vertex`. Previously, we simplified the implementation of a binary tree by giving

BinaryNode more than simple accessor and mutator operations. The same is true now for the implementation of the ADT graph. In fact, the specifications of the ADT graph that you saw in the previous chapter (Segment 28.25) omit the operations necessary to implement various graph algorithms. We assign these operations to the vertices.

The structure of a vertex is more like the structure of a node in a general tree than the structure of a node in a binary tree. Both the general node in Figure 24-8 and a vertex reference a list that you use to address other nodes or vertices.

Specifying the Class Vertex

29.5 **Identifying vertices.** First, we need a way to identify the vertices in a graph. One simple way is to use either integers or strings. A more general approach—the one we used in the previous chapter—labels each vertex with an object. This label will be a data field of the class Vertex. One operation of Vertex, then, is to retrieve a vertex's label. We'll use the constructor to set the label, omitting a mutator method for this field.

29.6 **Visiting vertices.** The algorithms that we discussed in the previous chapter required us to mark certain vertices when they were visited. We therefore give operations to Vertex that mark a vertex as visited, test whether a vertex has been visited, and remove the mark.

29.7 **The adjacency list.** As we mentioned earlier in this chapter, a vertex's adjacency list indicates its neighbors. Rather than placing this list within the class of graphs, it is more convenient to make it a part of the class Vertex. Soon we will define a simple class Edge whose instances we will place in these adjacency lists. Thus, a particular vertex's adjacency list contains the edges that leave the vertex. Each edge indicates its weight, if any, and references the vertex that ends the edge. Vertex then needs methods to add edges to the adjacency list. These methods essentially connect a vertex to its neighbors.

In addition, we must provide access to the adjacency list for a given vertex. Thus, we define an iterator that returns a vertex's neighbors, as well as an iterator that returns the weights of the edges to those neighbors. For convenience, we also include a method to test whether a vertex has at least one neighbor.

As you will see, the adjacency list is the only place where we need instances of Edge. Thus, Edge is an implementation detail that we can hide within Vertex as an inner class.

29.8 **Path operations.** While finding a path through a graph, we must be able to locate the vertex that comes before a given vertex on the path—in other words, the vertex's predecessor. Thus, we need set, get, and test operations for a vertex's predecessor. Certain algorithms find the path with the shortest length or the path that has the smallest weight, or cost. A vertex can record either the length or the weight of the path from the origin to itself. Thus, we have operations that set and get this recorded value.

29.9 **The Java interface.** The interface in Listing 29-1 specifies the vertex operations that we have just introduced. The generic type T represents the data type of the object that labels a vertex.

LISTING 29-1 An interface for the vertices in a graph

```
package GraphPackage;
import java.util.Iterator;
public interface VertexInterface<T>
```

```
{
   /** Gets the vertex's label.
       @return the object that labels the vertex */
   public T getLabel();

   /** Marks the vertex as visited. */
   public void visit();

   /** Removes the vertex's visited mark. */
   public void unvisit();

   /** Sees whether the vertex is marked as visited.
       @return true if the vertex is visited */
   public boolean isVisited();

   /** Connects this vertex and a given vertex with a weighted edge.
       The two vertices cannot be the same, and must not already
       have this edge between them. In a directed graph, the edge
       points toward the given vertex.
       @param endVertex   a vertex in the graph that ends the edge
       @param edgeWeight  a real-valued edge weight, if any
       @return true if the edge is added, or false if not */
   public boolean connect(VertexInterface<T> endVertex,
                          double edgeWeight);

   /** Connects this vertex and a given vertex with an unweighted
       edge. The two vertices cannot be the same, and must not
       already have this edge between them. In a directed graph,
       the edge points toward the given vertex.
       @param endVertex   a vertex in the graph that ends the edge
       @return true if the edge is added, or false if not */
   public boolean connect(VertexInterface<T> endVertex);

   /** Creates an iterator of this vertex's neighbors by following
       all edges that begin at this vertex.
       @return an iterator of the neighboring vertices of this vertex */
  public Iterator<VertexInterface<T>> getNeighborIterator();

   /** Creates an iterator of the weights of the edges to this
       vertex's neighbors.
       @return an iterator of edge weights for edges to neighbors of this
               vertex */
   public Iterator<Double> getWeightIterator();

   /** Sees whether this vertex has at least one neighbor.
       @return true if the vertex has a neighbor */
   public boolean hasNeighbor();
   /** Gets an unvisited neighbor, if any, of this vertex.
       @return either a vertex that is an unvisited neighbor or null
               if no such neighbor exists */
   public VertexInterface<T> getUnvisitedNeighbor();

   /** Records the previous vertex on a path to this vertex.
       @param predecessor  the vertex previous to this one along a path
   */
   public void setPredecessor(VertexInterface<T> predecessor);

   /** Gets the recorded predecessor of this vertex.
       @return either this vertex's predecessor or null if no predecessor
               was recorded */
   public VertexInterface<T> getPredecessor();

   /** Sees whether a predecessor was recorded.
       @return true if a predecessor was recorded for this vertex */
   public boolean hasPredecessor();

   /** Records the cost of a path to this vertex.
       @param newCost  the cost of the path */
   public void setCost(double newCost);

   /** Gets the recorded cost of the path to this vertex.
       @return the cost of the path */
   public double getCost();
} // end VertexInterface
```

The Inner Class Edge

29.10 As we mentioned, we will place instances of the class Edge in a vertex's adjacency list to indicate the edges that originate at the vertex. Thus, each edge must record both the vertex that ends it and the edge's weight, if any. Recording an edge weight is the only reason we need a class of edges. For unweighted graphs, we could simply place vertices in the adjacency list. Using edge objects, however, allows us to use one class of vertices for both weighted and unweighted graphs.

Since Vertex is the only class that uses Edge, we make Edge an inner class of Vertex. Listing 29-2 shows an implementation of Edge. We provide a data field for an edge's weight, if any. For unweighted graphs, we will set this field to zero rather than creating a class of unweighted edges.

LISTING 29-2 The protected class Edge, as an inner class of Vertex

```
protected class Edge
{
   private VertexInterface<T> vertex; // end vertex
   private double weight;
   protected Edge(VertexInterface<T> endVertex, double edgeWeight)
   {
      vertex = endVertex;
      weight = edgeWeight;
   } // end constructor

   protected VertexInterface<T> getEndVertex()
   {
      return vertex;
   } // end getEndVertex

   protected double getWeight()
   {
      return weight;
   } // end getWeight
} // end Edge
```

Note: An instance of the inner class `Edge` contains both the vertex that ends it and the edge's weight, if any. Although not necessary for unweighted graphs, `Edge` allows us to use one class of vertices for both weighted and unweighted graphs.

Implementing the Class `Vertex`

29.11 **An outline of the class.** To hide `Vertex` from the clients of the graph, we place it within the package `GraphPackage` that we introduced in the previous chapter. Listing 29-3 outlines the class and shows its data fields and constructor. An ADT list that has an iterator serves as the adjacency list `edgeList`. We have chosen the linked implementation `LinkedListWithIterator` discussed in Segment 15.19 of Chapter 15.

LISTING 29-3 An outline of the class `Vertex`

```
package GraphPackage;
import java.util.Iterator;
import java.util.NoSuchElementException;
import ADTPackage.*; // classes that implement various ADTs
class Vertex<T> implements VertexInterface<T>
{
   private T label;
   private ListWithIteratorInterface<Edge> edgeList; // edges to
                                                    // neighbors
   private boolean visited;                         // true if visited
   private VertexInterface<T> previousVertex; // on path to this vertex
   private double cost;                       // of path to this vertex

   public Vertex(T vertexLabel)
   {
      label = vertexLabel;
      edgeList = new LinkedListWithIterator<Edge>();
      visited = false;
      previousVertex = null;
      cost = 0;
   } // end constructor

   < Implementations of the vertex operations go here. >
   . . .

   protected class Edge
   {
      < See Listing 29-2. >
   } // end Edge
} // end Vertex
```

Note: The data fields of the class `Vertex` facilitate the implementation of the algorithms presented in the previous chapter. For example, the fields `previousVertex` and `cost` are useful in a breadth-first search for the cheapest path from one vertex to another.

29.12 **The two `connect` methods.** Each method `connect` places an edge into a vertex's adjacency list. We first implement the method for weighted graphs and then use it to implement the method for unweighted graphs. Preventing the addition of an edge that either exists in the graph already or connects a vertex with itself consumes most of the method's effort. Once those details are complete, `connect` simply calls the ADT list's `add` method to add the edge.

```
public boolean connect(VertexInterface<T> endVertex, double edgeWeight)
{
   boolean result = false;

   if (!this.equals(endVertex))
   {  // vertices are distinct
      Iterator<VertexInterface<T>> neighbors = this.getNeighborIterator();
      boolean duplicateEdge = false;

      while (!duplicateEdge && neighbors.hasNext())
      {
         VertexInterface<T> nextNeighbor = neighbors.next();
         if (endVertex.equals(nextNeighbor))
            duplicateEdge = true;
      } // end while

      if (!duplicateEdge)
      {
         edgeList.add(new Edge(endVertex, edgeWeight));
         result = true;
      } // end if
   } // end if

   return result;
} // end connect

public boolean connect(VertexInterface<T> endVertex)
{
   return connect(endVertex, 0);
} // end connect
```

Although adding to a list can be done in O(1) time, scanning the list to prevent duplicate edges takes more time. Since each vertex in a graph of *n* vertices can be the origin of at most *n* – 1 edges, `connect` is an O(*n*) operation. For a sparse graph, however, the number of edges originating at any vertex is much less than *n*. In this case, `connect` is significantly faster than O(*n*).

29.13 **The iterators.** The method `getNeighborIterator` returns an iterator to a vertex's adjacent vertices, that is, its neighbors. We define a private inner class—`neighborIterator`—within `Vertex` that implements Java's interface `Iterator`. Thus, `getNeighborIterator` has the following implementation:

```
public Iterator<VertexInterface<T>> getNeighborIterator()
{
   return new neighborIterator();
} // end getNeighborIterator
```

The class `neighborIterator` appears in Listing 29-4. Its constructor establishes an instance of the iterator defined in `LinkedListWithIterator`. The method `next` uses this iterator to traverse the edges in the vertex's adjacency list. Then, using `Edge`'s method `getEndVertex`, `next` accesses the neighboring vertex and returns it.

LISTING 29-4 The private class `neighborIterator`, as an inner class of `Vertex`

```
private class neighborIterator implements Iterator<VertexInterface<T>>
{
   private Iterator<Edge> edges;

   private neighborIterator()
   {
      edges = edgeList.getIterator();
   } // end default constructor

   public boolean hasNext()
   {
      return edges.hasNext();
   } // end hasNext

   public VertexInterface<T> next()
   {
      VertexInterface<T> nextNeighbor = null;

      if (edges.hasNext())
      {
         Edge edgeToNextNeighbor = edges.next();
         nextNeighbor = edgeToNextNeighbor.getEndVertex();
      }
      else
         throw new NoSuchElementException();

      return nextNeighbor;
   } // end next
   public void remove()
   {
      throw new UnsupportedOperationException();
   } // end remove
} // end neighborIterator
```

In a similar manner, the method `getWeightIterator` returns an instance of a private inner class `weightIterator`. This class is similar to the class `neighborIterator`.

29.14 **The methods `hasNeighbor` and `getUnvisitedNeighbor`.** The method `hasNeighbor` uses the method `isEmpty` of `LinkedListWithIterator` to test whether `edgeList` is empty:

```
public boolean hasNeighbor()
{
   return !edgeList.isEmpty();
} // end hasNeighbor
```

Using the iterator returned by `getNeighborIterator`, the method `getUnvisitedNeighbor` returns an adjacent vertex that is unvisited. This task is necessary in a topological sort.

```
public VertexInterface<T> getUnvisitedNeighbor()
{
   VertexInterface<T> result = null;

   Iterator<VertexInterface<T>> neighbors = getNeighborIterator();
   while ( (neighbors.hasNext() && (result == null) )
   {
      VertexInterface<T> nextNeighbor = neighbors.next();
      if (!nextNeighbor.isVisited())
         result = nextNeighbor;
   } // end while

   return result;
} // end getUnvisitedNeighbor
```

29.15 **The remaining methods.** `Vertex` should override the method `equals`. Two vertices are equal if their labels are equal.

```
public boolean equals(Object other)
{
   boolean result;

   if ((other == null) || (getClass() != other.getClass()))
      result = false;
   else
   {
      Vertex<T> otherVertex = (Vertex<T>)other;
      result = label.equals(otherVertex.label);
   } // end if

   return result;
} // end equals
```

The remaining methods of `Vertex` have uncomplicated implementations and are left as exercises.

SELF-TEST

Question 3 Given the interface `VertexInterface` and the class `Vertex`, write Java statements that create the vertices and edges for the following directed, weighted graph. This graph contains three vertices—*A*, *B*, and *C*—and four edges, as follows: $A \to B$, $B \to C$, $C \to A$, $A \to C$. These edges have the weights 2, 3, 4, and 5, respectively.

An Implementation of the ADT Graph

We now consider how to use `Vertex` in an implementation of a directed graph that can be either weighted or unweighted.

Basic Operations

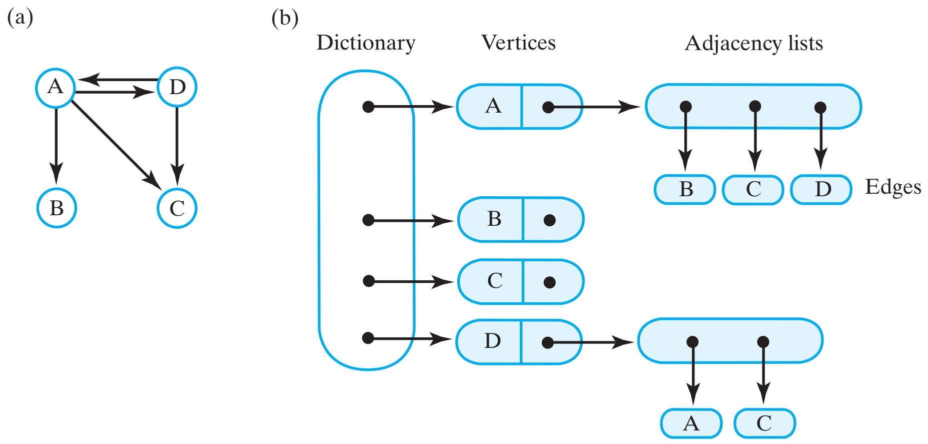
29.16 **Beginning the class.** Whether our implementation uses an adjacency list—as it will here—or an adjacency matrix, it must have a container for the graph's vertices. If we use integers to identify the vertices, a list would be a natural choice for this container, since each integer could correspond to a position within the list. If we use an object such as a string to identify them, a dictionary is a better choice. That is what we will do here.

> **Note:** Regardless of the kind of graph or how you implement it, you need a container such as a dictionary for the graph's vertices.

VideoNote
Implementing graph operations

Figure 29-3 illustrates a dictionary of vertices for a small directed graph. Each of the vertices *A* and *D* has an adjacency list of the edges that originate at that vertex. The letters within these edges represent references to corresponding vertices within the dictionary. Since the ADT dictionary consists of key-value pairs, we can use the vertex labels as the search keys and the vertices themselves as the corresponding values. This organization allows us to quickly locate a particular vertex, given its label.

FIGURE 29-3 (a) A directed graph and (b) its implementation using adjacency lists



Our class begins as shown in Listing 29-5. Recall that the generic type `T` represents the data type of the objects that label the graph's vertices. The class's first data field is a dictionary of vertices. A count of vertices is not necessary, since the dictionary will count the vertices for us.

Since each vertex maintains its own adjacency list, the edges in the graph are not easily counted. Thus, we should maintain an edge count as a data field within the graph class.

LISTING 29-5 An outline of the class `DirectedGraph`

```
package GraphPackage;
import java.util.Iterator;
import ADTPackage.*; // classes that implement various ADTs
```

```
public class DirectedGraph<T> implements GraphInterface<T>
{
   private DictionaryInterface<T, VertexInterface<T>> vertices;
   private int edgeCount;

   public DirectedGraph()
   {
      vertices = new LinkedDictionary<T, VertexInterface<T>>();
      edgeCount = 0;
   } // end default constructor

   < Implementations of the graph operations go here. >
   . . .

} // end DirectedGraph
```

29.17 **Adding vertices.** The method `addVertex` uses `Vertex's` constructor to create a new vertex. It then adds the vertex to the dictionary by invoking the dictionary's method `add`:

```
public boolean addVertex(T vertexLabel)
{
   VertexInterface<T> isDuplicate =
                  vertices.add(vertexLabel, new Vertex(vertexLabel));
   return isDuplicate == null; // was add to dictionary successful?
} // end addVertex
```

Notice that `vertexLabel` is the search key for the dictionary entry, and the new vertex is the associated value. Recall from the interface in Segment 19.4 of Chapter 19 that `add` returns `null` if the addition to the dictionary is successful. We use this fact to determine the return value for `addVertex`.

29.18 **Adding edges.** Methods such as `addEdge` that identify an existing vertex by its label must locate the vertex within the dictionary `vertices`. To do this, they invoke the dictionary method `getValue`, using the vertex label as the search key. Having located the two vertices that delineate the edge to be added, `addEdge` adds an edge to the adjacency list of the origin vertex. It does this by invoking `Vertex`'s `connect` method. If the edge is added successfully, `edgeCount` is incremented. Thus, our graph's `addEdge` methods have the following definitions, one for weighted graphs and one for unweighted graphs:

```
public boolean addEdge(T begin, T end, double edgeWeight)
{
   boolean result = false;

   VertexInterface<T> beginVertex = vertices.getValue(begin);
   VertexInterface<T> endVertex = vertices.getValue(end);

   if ( (beginVertex != null) && (endVertex != null) )
      result = beginVertex.connect(endVertex, edgeWeight);

   if (result)
      edgeCount++;

   return result;
} // end addEdge

public boolean addEdge(T begin, T end)
{
   return addEdge(begin, end, 0);
} // end addEdge
```

29.19 **Testing for an edge.** The method hasEdge begins like addEdge, by locating the two vertices that define the desired edge. With the origin vertex in hand, hasEdge invokes Vertex's method getNeighborIterator and searches the origin's adjacency list for the desired edge. In the following implementation, you can see why defining the equals method in Vertex is important.

```
public boolean hasEdge(T begin, T end)
{
   boolean found = false;

   VertexInterface<T> beginVertex = vertices.getValue(begin);
   VertexInterface<T> endVertex = vertices.getValue(end);

   if ( (beginVertex != null) && (endVertex != null) )
   {
      Iterator<VertexInterface<T>> neighbors =
                                   beginVertex.getNeighborIterator();
      while (!found && neighbors.hasNext())
      {
         VertexInterface<T> nextNeighbor = neighbors.next();
         if (endVertex.equals(nextNeighbor))
            found = true;
      } // end while
   } // end if

   return found;
} // end hasEdge
```

29.20 **Miscellaneous methods.** The methods isEmpty, clear, getNumberOfVertices, and getNumberOfEdges have the following simple implementations:

```
public boolean isEmpty()
{
   return vertices.isEmpty();
} // end isEmpty

public void clear()
{
   vertices.clear();
   edgeCount = 0;
} // end clear

public int getNumberOfVertices()
{
   return vertices.getSize();
} // end getNumberOfVertices

public int getNumberOfEdges()
{
   return edgeCount;
} // end getNumberOfEdges
```

29.21 **Resetting vertices.** You saw in Segment 29.11 that the class Vertex has the data fields visited, previousVertex, and cost. These fields are necessary for the implementation of the graph algorithms that we introduced in the previous chapter. Once you have searched a graph for a shortest path, for example, many of the vertices will have been visited and marked accordingly. Before you could perform a topological sort on the same graph, you would have to reset the field visited for each vertex in the graph.

The following method resetVertices sets the fields visited, previousVertex, and cost to their initial values. To do so, the method uses one of the iterators declared in the interface DictionaryInterface. The method is not public, as we will call it only from methods declared in GraphAlgorithmsInterface.

```
protected void resetVertices()
{
   Iterator<VertexInterface<T>> vertexIterator = vertices.getValueIterator();
   while (vertexIterator.hasNext())
   {
      VertexInterface<T> nextVertex = VertexIterator.next();
      nextVertex.unvisit();
      nextVertex.setCost(0);
      nextVertex.setPredecessor(null);
   } // end while
} // end resetVertices
```

Question 4 Create an instance of the class DirectedGraph for the graph described in Question 3.

29.22 **Efficiency.** Adding a vertex to a graph is an O(*n*) operation, since the vertex is added to a linked dictionary. Adding an edge involves retrieving two vertices from the dictionary and then calling Vertex's method connect. Thus, the method addEdge is also O(*n*). Likewise, hasEdge is O(*n*), as it first retrieves two vertices from the dictionary. It then iterates through the edges that leave the first vertex to see whether one of them ends at the second vertex. As you can see, the performance of these three graph operations depends on the number of vertices in the graph. The remaining methods in BasicGraphInterface are each O(1). Figure 29-4 summarizes these observations.

FIGURE 29-4 The performance of basic operations of the ADT graph when implemented by using adjacency lists

addVertex	O(*n*)
addEdge	O(*n*)
hasEdge	O(*n*)
isEmpty	O(1)
getNumberOfVertices	O(1)
getNumberOfEdges	O(1)
clear	O(1)

Graph Algorithms

29.23 **Breadth-first traversal.** Segment 28.12 of the previous chapter presented an algorithm for a breadth-first traversal of a nonempty graph, beginning at a given origin vertex. Recall that the traversal first visits the origin and the origin's neighbors. It then visits each neighbor of the origin's neighbors. The traversal uses a queue to hold the vertices as they are visited. The traversal order is the order in which vertices are added to this queue. But since the algorithm must remove vertices from this queue, we maintain the traversal order in a second queue. Since this second queue is returned to the client, we enqueue vertex labels instead of vertices. Remember that the class Vertex is unavailable to the client.

The following implementation of the method `getBreadthFirstTraversal` closely follows the pseudocode given in the previous chapter. The parameter `origin` is an object that labels the origin vertex of the traversal.

```java
public QueueInterface<T> getBreadthFirstTraversal(T origin)
{
   resetVertices();
   QueueInterface<T> traversalOrder = new LinkedQueue<T>();
   QueueInterface<VertexInterface<T>> vertexQueue =
                                      new LinkedQueue<VertexInterface<T>>();

   VertexInterface<T> originVertex = vertices.getValue(origin);
   originVertex.visit();
   traversalOrder.enqueue(origin);    // enqueue vertex label
   vertexQueue.enqueue(originVertex); // enqueue vertex

   while (!vertexQueue.isEmpty())
   {
      VertexInterface<T> frontVertex = vertexQueue.dequeue();

      Iterator<VertexInterface<T>> neighbors =
                                   frontVertex.getNeighborIterator();
      while (neighbors.hasNext())
      {
         VertexInterface<T> nextNeighbor = neighbors.next();
         if (!nextNeighbor.isVisited())
         {
            nextNeighbor.visit();
            traversalOrder.enqueue(nextNeighbor.getLabel());
            vertexQueue.enqueue(nextNeighbor);
         } // end if
      } // end while
   } // end while

   return traversalOrder;
} // end getBreadthFirstTraversal
```

The implementation of a similar method to perform a depth-first traversal is left as an exercise.

Question 5 Write Java statements that display the vertices in a breadth-first traversal of the graph that you created in Question 4, beginning with vertex *A*.

29.24 **Shortest path.** The shortest path of all paths from one vertex to another in an unweighted graph is the path that has the fewest edges. The algorithm that finds this path—as you saw in Segment 28.19 of the previous chapter—is based on a breadth-first traversal. When we visit a vertex *v*, we mark it as visited, note the vertex *p* that precedes *v* in the graph, and note the length of the path that the traversal followed to reach *v*. We place both this path length and a reference to *p* into the vertex *v*. When the traversal reaches the desired destination, we can construct the shortest path from this data in the vertices.

The implementation of the method `getShortestPath` closely follows the pseudocode given in the previous chapter. The parameters `begin` and `end` are objects that label the origin and destination vertices of the path. The third parameter `path` is an initially empty stack. At the conclusion of the method, this stack contains the labels of the vertices along the shortest path. The method returns the length of this path.

```
public int getShortestPath(T begin, T end, StackInterface<T> path)
{
   resetVertices();
   boolean done = false;
   QueueInterface<VertexInterface<T>> vertexQueue =
                                      new LinkedQueue<VertexInterface<T>>();

   VertexInterface<T> originVertex = vertices.getValue(begin);
   VertexInterface<T> endVertex = vertices.getValue(end);

   originVertex.visit();
   // Assertion: resetVertices() has executed setCost(0)
   // and setPredecessor(null) for originVertex

   vertexQueue.enqueue(originVertex);

   while (!done && !vertexQueue.isEmpty())
   {
      VertexInterface<T> frontVertex = vertexQueue.dequeue();

      Iterator<VertexInterface<T>> neighbors =
                                   frontVertex.getNeighborIterator();
      while (!done && neighbors.hasNext())
      {
         VertexInterface<T> nextNeighbor = neighbors.next();

         if (!nextNeighbor.isVisited())
         {
            nextNeighbor.visit();
            nextNeighbor.setCost(1 + frontVertex.getCost());
            nextNeighbor.setPredecessor(frontVertex);
            vertexQueue.enqueue(nextNeighbor);
         } // end if

         if (nextNeighbor.equals(endVertex))
            done = true;
      } // end while
   } // end while

   // traversal ends; construct shortest path
   int pathLength = (int)endVertex.getCost();
   path.push(endVertex.getLabel());

   VertexInterface<T> vertex = endVertex;
   while (vertex.hasPredecessor())
   {
      vertex = vertex.getPredecessor();
      path.push(vertex.getLabel());
   } // end while

   return pathLength;
} // end getShortestPath
```

The implementation of the method `getCheapestPath` for a weighted graph is left as an exercise.

? SELF-TEST

Question 6 Write Java statements that display the vertices in the shortest path from vertex *A* to vertex *C* for the graph that you created in Question 4. Also, display the length of this path.

Chapter Summary

- An adjacency list for a given vertex contains references to the vertex's neighbors.
- Using an adjacency list, you can quickly find all the neighbors of a particular vertex. But if you want to know whether an edge exists between any two given vertices, you need to search a list.
- An adjacency matrix is a two-dimensional array that represents the edges in a graph. If you number the vertices from 0 through $n - 1$, the entry in row i and column j of the matrix indicates whether an edge exists between vertex i and vertex j. For an unweighted graph, you can use boolean values in the matrix. For a weighted graph, you can use edge weights when edges exist and a representation of infinity otherwise.
- Using an adjacency matrix, you can quickly discover whether an edge exists between any two given vertices. But if you want to know all the neighbors of a particular vertex, you need to scan an entire row of the matrix.
- Each adjacency list represents only those edges that originate from a vertex, but an adjacency matrix reserves space for every possible edge in a graph. Thus, when a graph is sparse, adjacency lists use less memory than a corresponding adjacency matrix. For this reason, typical graph implementations use adjacency lists.
- One way to implement an adjacency list is to make it a data field of a class `Vertex`. So that you can represent weighted graphs as well as unweighted graphs, you place instances of a class `Edge` in the adjacency list. `Edge`'s data fields include the terminal vertex of an edge and the edge weight. `Vertex` is accessed from within a package instead of publicly. `Edge` is an inner class of `Vertex`. Thus, both `Vertex` and `Edge` are hidden from the graph's client.
- To facilitate the implementation of various graph algorithms, an instance of the class `Vertex` can indicate whether it has been visited. It also can record data about a path to it, such as the previous vertex and the path's cost.

Exercises

1. What adjacency matrix represents the graph in Figure 28-15a of the previous chapter?
2. What adjacency matrix represents the graph in Figure 28-18a of the previous chapter?
3. What adjacency lists represent the graph in Figure 28-15a of the previous chapter?
4. What adjacency lists represent the graph in Figure 28-18a of the previous chapter?
5. When is an adjacency matrix just as space-efficient as adjacency lists?
6. Suppose that you want only to test whether an edge exists between two particular vertices. Does an adjacency matrix or an adjacency list provide a more efficient way of doing this?
7. Suppose that you want only to find all vertices that are adjacent to some particular vertex. Does an adjacency matrix or an adjacency list provide a more efficient way of doing this?
8. Complete the implementation of the class `Vertex` that was begun in Segment 29.11.
9. What is the Big Oh of the methods `getBreadthFirstTraversal` and `getShortestPath`, as given in Segments 29.23 and 29.24?

10. Implement the method `getDepthFirstTraversal`. Segment 28.13 of the previous chapter presents the pseudocode for this method. What is its Big Oh?

11. Implement the method `getCheapestPath` for a weighted graph. The pseudocode for this method appears in Segment 28.24 of the previous chapter. What is its Big Oh?

12. Draw a class diagram that shows the relationships among the classes `DirectedGraph`, `Vertex`, and any other supporting classes such as `LinkedDictionary`.

13. The **out degree** of a vertex is the number of edges that originate at the vertex. The **in degree** of a vertex is the number of edges that terminate at the vertex. Modify the class `DirectedGraph` so that it can compute the in degree and out degree of any of its vertices.

14. Suppose that you have a weighted, directed graph in which the out degree and in degree of every vertex is at most 4. (See the previous exercise.) If the graph has *n* vertices, you could represent it by using an array that has *n* rows and 4 columns. Each of the *n* rows is associated with a different vertex in the graph. The entries in a row associated with vertex *v* are the vertices at the ends of the edges that begin at *v*. Since the out degree of a vertex can be less than 4, some entries in a row might be `null`.

    What is the Big Oh of each of the following operations?

    a. Testing whether two given vertices are adjacent
    b. Finding all vertices adjacent to a given vertex

15. A graph is said to be **bipartite** if the vertices can be divided into two groups such that every edge goes from a vertex in one group to a vertex in the other group. Figure 28-1 of the previous chapter contains a bipartite graph. We could put Sandwich, Hyannis, Orleans, and Provincetown in group *A*, and Barnstable, Falmouth, Chatham, and Truro in group *B*. Every edge goes from a vertex of group *A* to a vertex of group *B*.

    a. Which of the graphs in Figure 28-4, 28-6, and 28-7b are bipartite?
    b. How might the implementation of a bipartite graph differ from that of a regular graph to take advantage of its bipartite nature?

16. Consider a directed graph that has *n* nodes and *e* edges, where $0 \le e \le n^2$.

    a. What is the time complexity, using Big Oh notation, for each of the following operations when an adjacency matrix is used to represent the graph?

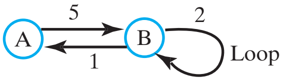
    - Testing whether two vertices are joined by an edge
    - Finding the successors of a given vertex
    - Finding the predecessors of a given vertex

    b. Repeat part *a*, but assume that the graph uses an adjacency list in its implementation instead of an adjacency matrix.

17. A **loop** is an edge that starts and ends at the same vertex. Figure 29-5 shows an example of a loop in a directed, weighted graph.

    a. Give an example of a problem where allowing loops would be useful.
    b. Can the adjacency matrix and adjacency list representations of a graph support loops?

FIGURE 29-5 A graph for Exercise 17

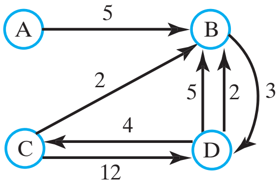


18. A **multiple edge** occurs when two vertices are joined by two or more edges in the same direction. Figure 29-6 shows a directed, weighted graph that has a multiple edge from *D* to *B*.

    a. Give an example of a problem in which allowing multiple edges would be useful.
    b. Can an adjacency matrix represent an unweighted graph that has multiple edges?
    c. Can an adjacency matrix represent a weighted graph that has multiple edges?
    d. Can adjacency lists represent an unweighted graph that has multiple edges?
    e. Can adjacency lists represent a weighted graph that has multiple edges?

19.

FIGURE 29-6 A graph for Exercise 18



Projects

1. Complete the implementation of the class `DirectedGraph` that was begun in Segment 29.16 of this chapter.

2. Implement a class of undirected graphs by extending the class `DirectedGraph`. What methods should you override? What methods, if any, in `DirectedGraph` do not apply to an undirected graph? If such methods exist, what should you do in your new class? Note that the method `getNumberOfEdges` is the only accessor method to a data field of `DirectedGraph`.

3. Revise the class `DirectedGraph` by defining protected mutator methods for the data fields `vertices` and `edgeCount`. Also, define a protected accessor method for `vertices`. Then repeat Project 2, using your revised `DirectedGraph`. Compare the performance of the `addEdge` methods in this implementation of an undirected graph versus the implementation possible under the assumptions of Project 2.

4. Implement the classes `Vertex` and `DirectedGraph` by using an adjacency matrix.

5. Assuming an implementation of a class of undirected graphs, implement a method that detects whether an undirected graph is acyclic. You can look for cycles during either a breadth-first traversal or a depth-first traversal by discovering an edge to a vertex that is not the predecessor and has already been visited. To simplify the problem initially, you can assume that the graph is connected. Then remove this assumption.

6. Implement a method that detects whether a graph is connected.

7. Create the classes `LimitedVertex` and `LimitedDirectedGraph` that use the representation described in Exercise 14.

8. A **graph coloring** assigns a color to every vertex in a graph, with the restriction that two vertices of the same color cannot be adjacent. A graph is said to be ***k*-colorable** if it can be colored in *k* or fewer colors.

    - Give an algorithm that will return true if a graph is 2-colorable and false otherwise.
    - Exercise 15 defined a bipartite graph. Show that a graph is bipartite if and only if it is 2-colorable. Then, using this fact, implement a method that detects whether a graph is bipartite.

9. Repeat Project 15 in Chapter 13 to create a simple social network. Use a graph to track the friend relationships among members of the network. Add a feature to enable people to see a list of their friends' friends.

Answers to Self-Test Questions

1.

	0	1	2	3
0		T	T	T
1	T		T	T
2	T	T		T
3	T	T	T	

2. Vertex 0 references the list 1, 2, 3.
Vertex 1 references the list 0, 2, 3.
Vertex 2 references the list 0, 1, 3.
Vertex 3 references the list 0, 1, 2.

3.

```
VertexInterface<String> vertexA = new Vertex<String>("A");
VertexInterface<String> vertexB = new Vertex<String>("B");
VertexInterface<String> vertexC = new Vertex<String>("C");
vertexA.addEdge(vertexB, 2.0);
vertexB.addEdge(vertexC, 3.0);
vertexC.addEdge(vertexA, 4.0);
vertexA.addEdge(vertexC, 5.0);
```

4.

```
DirectedGraph<String> myGraph = new DirectedGraph<String>();
myGraph.addVertex("A");
myGraph.addVertex("B");
myGraph.addVertex("C");
myGraph.addEdge("A", "B", 2.0);
myGraph.addEdge("B", "C", 3.0);
myGraph.addEdge("C", "A", 4.0);
myGraph.addEdge("A", "C", 5.0);
```

5.

```
QueueInterface<String> bfs = myGraph.getBreadthFirstTraversal("A");
while (!bfs.isEmpty())
   System.out.print(bfs.dequeue() + " ");
System.out.println();
```

6.

```
StackInterface<String> path = new LinkedStack<String>();
int pathLength = myGraph.getShortestPath("A", "C", path);
System.out.println("The shortest path from A to C has length " + pathLength +
                   " and passes through the following vertices:");
while (!path.isEmpty())
   System.out.print(path.pop() + " ");
System.out.println();
```

This page intentionally left blank

Mutable, Immutable, and Cloneable Objects

Chapter 30

Contents

Prerequisites

Appendix C	Creating Classes from Other Classes
Appendix E	Exception Handling
Chapter 13	List Implementations That Use Arrays
Chapter 14	A List Implementation That Links Data
Chapter 15	Iterators
Chapter 16	Sorted Lists
Chapter 17	Inheritance and Lists

Objectives

After studying this chapter, you should be able to

- Distinguish among mutable and immutable objects
- Define a class of immutable objects
- Define companion classes, one of immutable objects and the other of mutable objects
- Define a method `clone` for a given class
- Clone an array or chain of objects
- Implement a collection such as a sorted list that clones the objects added to it

When a class has public mutator, or set, methods, a client can use these methods to alter objects of that class. Although this ability seems reasonable, it is unreasonable if another class organizes those objects in a particular way. For example, a sorted list of names maintains the names in alphabetical order. If a client can alter a name in the list, it can destroy the order of the list.

This chapter looks at two strategies that prevent this problem. The first one simply requires a client to place into a collection only objects that cannot be altered. The second strategy requires the collection to make a copy, or clone, of any object that a client adds to it. With this technique, the client has no reference to the copy and so cannot change it. In describing this approach, we discuss how to write methods that make clones of objects.

Mutable and Immutable Objects

VideoNote
Mutable and immutable objects

30.1 Many of the classes you have studied so far have private data fields and public methods that either look at or change these fields. As you know, such methods are called accessor methods and mutator methods—or, alternatively, get and set methods. An object that belongs to a class that has public mutator methods is said to be mutable—as we mentioned in Chapter 10—because the client can use the set methods to change the values of the object's data fields. For example, you saw the class `Name` of two-part names in Segment B.16 of Appendix B. It has the following two data fields:

```
private String first; // first name
private String last;  // last name
```

To change these fields, the class has the mutator methods `setFirst` and `setLast`. To look at the fields, it has the accessor methods `getFirst` and `getLast`.

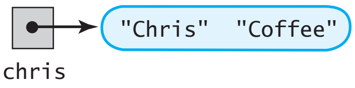
Note: A mutable object belongs to a class that has mutator (set) methods for its data fields.

30.2 Let's use this class to create an object for *Chris Coffee* by writing the following Java statement:

```
Name chris = new Name("Chris", "Coffee");
```

Figure 30-1 illustrates this object and the reference variable `chris`.

FIGURE 30-1 An object and its reference variable `chris`



Now suppose we create a list and then add `chris` to the list by writing

```
ListInterface<Name> nameList = new LList<Name>();
nameList.add(1, chris);
```

Since `chris` is a mutable object, we can change its data fields by writing, for example,

```
chris.setLast("Smith");
```

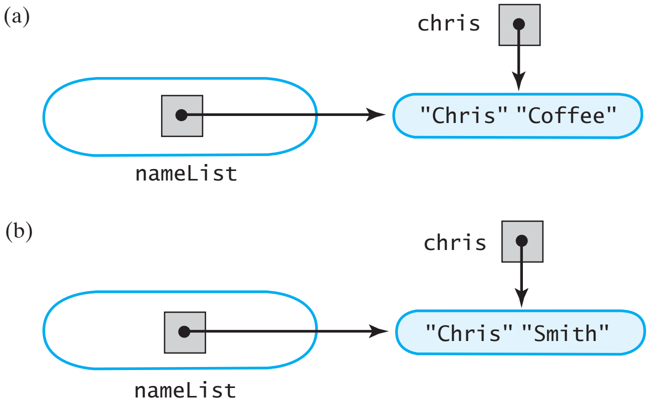
After this change, the object `chris` represents the name *Chris Smith*. Nothing is surprising here. What might be surprising, however, is that the list has changed! That's right: If we retrieve the first item in the list by writing, for instance,

```
System.out.println(nameList.getEntry(1));
```

we will get *Chris Smith* instead of *Chris Coffee*.

30.3 How can it be that the list, which we created before changing the name, contains the changed name? Remember that in Java, the list contains references to the actual objects that the client places in it. So the list has a reference to its first item, but so does the client, since it has the variable `chris`, as Figure 30-2a shows.

FIGURE 30-2 An object in the list `nameList` (a) initially; (b) after the reference variable `chris` is used to change it



When we altered the object by executing

```
chris.setLast("Smith");
```

we changed the one and only copy of the object, as Figure 30-2b shows. Since the list still references that object, `nameList.getEntry(1)` returns a reference to the object. This aspect of Java can be a convenient way for the client to alter the objects it has placed in a list.

> **Note:** When a client creates a mutable object and adds it to an ADT list, only one copy of the object ordinarily exists. Thus, if the client alters the object, the list changes. Ideally, the client will use the `replace` operation to revise an entry in the list, but we cannot force the client to do so.

30.4 The ability to alter mutable objects in a collection can permit a client to destroy the collection's integrity. For example, suppose that we create a sorted list of names instead of a list of names. If we write

```
Name jesse = new Name("Jesse", "Java");
Name rob = new Name("Rob", "Bean");
SortedListInterface<Name> alphaList = new SortedList<Name>();
alphaList.add(jesse);
alphaList.add(rob);
```

we would get the sorted list

Rob Bean
Jesse Java

assuming that `Name` implements the interface `Comparable`. (See Question 9 of Appendix D.) Now if we write

```
rob.setLast("Smith");
```

the list changes to

Rob Smith
Jesse Java

This sorted list is no longer alphabetical. One solution to this problem is to require the client to use immutable objects, as the next segment describes.

30.5 As Chapter 10 indicated, an immutable object is one whose data fields cannot be altered by a client. The class to which an immutable object belongs has no public mutator (set) methods, so once you create the object, you cannot change its data fields. If you need to change them, you will have to discard the object and create a new one with the revised fields. Such a class is said to be **read only**. A client that places immutable objects into a sorted list cannot alter those objects and thus cannot destroy the sorted order of the list.

Note: An immutable object belongs to a read-only class. Such a class prevents a client from changing the values of its data fields.

Note: When a collection—such as a sorted list—organizes objects in a certain way, a client should not destroy this organization by altering the objects directly. Yet if the client retains references to the objects, that is exactly what a client could do. You can prevent this problem by adding only immutable objects to the collection.

30.6 **Mutable or immutable?** Most classes have set methods, so their instances are mutable. Being able to change an object's data is convenient and efficient, particularly when an object's state must change often during the course of a program's execution. For example, a bank must regularly update the object that represents your checking account. If that object were immutable, it would be discarded and a new object representing the updated data would be created each time there was a change. But revising an object takes less time than replacing it.

On the other hand, sharing a mutable object can be dangerous. Suppose that you have two references, `a` and `b`, to the same object. If you use `a` to modify the object, you might get confused when you use `b` to reference it. But sharing immutable objects is safe, since no matter how you reference them, they remain unchanged.

Programming Tip: Use an immutable object if it will be shared or added to a collection that can be corrupted by changes to the object. Use a mutable object if its data will change frequently.

Creating a Read-Only Class

30.7 To convert the previous class `Name` into a read-only class, we can change the access modifiers of the methods `setFirst`, `setLast`, and `setName` from `public` to `private` to prevent a client from invoking them. (We could instead omit these methods altogether, modifying the other methods that invoke them.) We also omit `Name`'s method `giveLastNameTo`.

Let's call the resulting class `ImmutableName`. If we place instances of `ImmutableName` in a list or a sorted list, we will not be able to change these objects by using any references that we might have retained to them. Of course, we can use the `replace` operation of the ADT list to replace a particular item in a list, but no such operation exists for a sorted list. To change an entry in a sorted list, we would remove the entry and add a new one. In this way, the sorted list maintains its sorted order.

Suppose that `ImmutableName` had protected mutator methods such as `setFirst` and `setLast`. A programmer could use inheritance to alter the behavior of the class. Imagine deriving a class of mutable objects from `ImmutableName`. You then could add instances of this new class to a sorted list of `ImmutableName` objects. Since these entries are mutable, you could change them and destroy the order of the sorted list. To prevent this from happening, we can make the class final, as described in Segment C.19 of Appendix C.

30.8 Listing 30-1 defines `ImmutableName` as a final class. Since the class has no set methods, we did not define a default constructor. You could define one, even though its use is unnecessary.

LISTING 30-1 The read-only class `ImmutableName`

```java
public final class ImmutableName
{
   private String first; // first name
   private String last;  // last name

   public ImmutableName(String firstName, String lastName)
   {
      first = firstName;
      last = lastName;
   } // end constructor

   public String getFirst()
   {
      return first;
   } // end getFirst

   public String getLast()
   {
      return last;
   } // end getLast

   public String getName()
   {
      return toString();
   } // end getName

   public String toString()
   {
      return first + " " + last;
   } // end toString
} // end ImmutableName
```

30.9 We have one more concern. Imagine a class that has a `Name` object as a data field. The class has accessor methods, including one that returns the `Name` field, but has no mutator methods. However, a client of this class can access the `Name` field and then use `Name`'s set methods to alter the field's value. In other words, the class is not read only. To make it read only, we can define the field as final. Note that the fields of `ImmutableName` are strings, which are immutable, so we need not make them final.

Note: Design guidelines for a read-only class

- The class should be final.
- Data fields should be private.
- The class should not have public set methods.
- Data fields that are mutable objects should be final.

SELF-TEST

Question 1 Define a constructor for `ImmutableName` that has a `Name` object as a parameter.

Companion Classes

30.10 Although immutable objects are desirable for certain applications, mutable objects have their place. Sometimes we will want to represent an object in both immutable and mutable forms. In such cases, a pair of **companion classes** can be convenient. The classes `ImmutableName` and `Name` are examples of two such companion classes. The objects in both classes represent names, but one type of object cannot be altered, while the other can be.

To make the classes even more convenient, you could include constructors and/or methods that convert an object from one type to the other. For example, we might add the following constructor and method to the class `ImmutableName`:

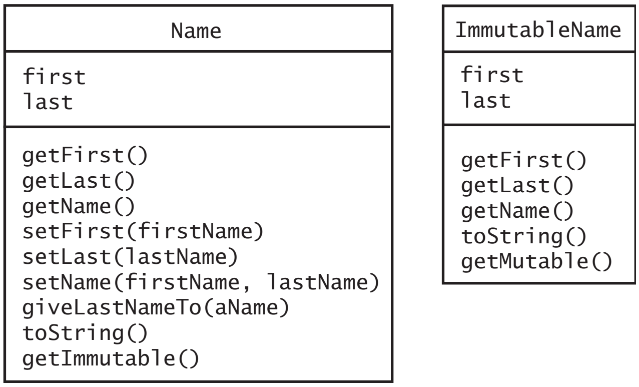
```
// add to the class ImmutableName:
public ImmutableName(Name aName)
{
  first = aName.getFirst();
  last  = aName.getLast();
} // end constructor

public Name getMutable()
{
  return new Name(first, last);
} // end getMutable
```

Similarly, we could add the following constructor and method to the class `Name`:

```
// add to the class Name
public Name(ImmutableName aName)
{
  first = aName.getFirst();
  last  = aName.getLast();
} // end constructor

public ImmutableName getImmutable()
{
  return new ImmutableName(first, last);
} // end getMutable
```

Figure 30-3 illustrates the two classes `Name` and `ImmutableName`.

FIGURE 30-3 The classes `Name` and `ImmutableName`



30.11 **Example.** Let's see how we can use the previous additions to our companion classes. If we have a `Name` object such as `flexibleName` in the statement

```
Name flexibleName = new Name("Maria", "Mocha");
```

and we no longer need the capability to change it, we can use `ImmutableName`'s constructor, as follows:

```
ImmutableName fixedName = new ImmutableName(flexibleName);
```

The new object `fixedName` has the same data fields as `flexibleName`, but it is immutable. Alternatively, we could have invoked `Name`'s method `getImmutable`, as follows:

```
ImmutableName fixedName = flexibleName.getImmutable();
```

Similarly, if we have another instance of `ImmutableName`, such as

```
ImmutableName persistent = new ImmutableName("Jesse", "Java");
```

and we find that we need to alter it, we can define a new mutable object as either

```
Name transient = new Name(persistent);
```

or

```
Name transient = persistent.getMutable();
```

The new object `transient` has the same data fields as `persistent`, but it also has set methods to change them.

Question 2 Write Java statements that take the following steps:

- Create an object of the class `Name`.
- Convert the object to an immutable object without changing its data fields.
- Add the immutable object to the sorted list `nameList`.

Question 3 Write Java statements that take the following steps:

- Create an object of the class `Immutable Name`.
- Convert the object to a mutable object without changing its data fields.
- Change the last name of the new object.
- Convert the revised mutable object to an immutable object.

Note: Java's class String is a read-only class. That is, instances of String are immutable. Once you create a string, you cannot change it. Frequently, however, string applications require that you either remove a portion of a string or join two strings together. For such applications, Java provides the class StringBuilder of mutable strings. StringBuilder provides several methods that modify a string by adding, removing, or replacing substrings. Appendix A describes some of the methods that belong to these two classes.

String and StringBuilder are a pair of companion classes. String has a constructor that takes an instance of StringBuilder as an argument and produces an immutable string with the same value. StringBuilder has an analogous constructor that creates mutable strings from immutable ones. StringBuilder also has the methods substring and toString that return instances of String.

Cloneable Objects

VideoNote
Cloneable objects

30.12 In Segment 30.4, we created a sorted list of mutable objects. Unfortunately, the client of this list can modify the objects so that they are no longer sorted. As you saw then, one solution is to always place immutable objects in a sorted list.

A more involved solution has the sorted list copy the client's objects. The sorted list then can control what the client can and cannot do to the copies. This section examines how to make a copy of an object.

30.13 In Java, a **clone** is a copy of an object. Typically, we clone only mutable objects. Since sharing an immutable object is safe, cloning it is usually unnecessary.

The class Object contains a protected method clone that returns a copy of an object. The method has the following header:

```
protected Object clone() throws CloneNotSupportedException
```

Since clone is protected, and since Object is the superclass of all other classes, the implementation of any method can contain the invocation

```
super.clone()
```

But clients of a class cannot invoke clone unless the class overrides it and declares it public. Making copies of objects can be expensive, so it might be something you do not want a class to do. By making clone a protected method, the designers of Java force you to think twice about cloning.

Programming Tip: Not all classes should have a public clone method. In fact, most classes, including read-only classes, do not have one.

30.14 If you want your class to contain a public method clone, the class needs to state this fact by implementing the Java interface Cloneable, which is in the package java.lang of the Java Class Library. Such a class would begin as follows:

```
public class MyClass implements Cloneable
{ . . .
```

The interface Cloneable is simply

```
public interface Cloneable
{
}
```

As you can see, the interface is empty. It declares no methods and serves only as a way for a class to indicate that it implements clone. If you forget to write implements Cloneable in your class definition, instances of your class that invoke clone will cause the exception CloneNotSupportedException. This result can be confusing at first, particularly if you did implement clone.

Programming Tip: If your program produces the exception CloneNotSupportedException even though you implemented a method clone in your class, you probably forgot to write implements Cloneable in your class definition.

Note: The Cloneable interface

The empty Cloneable interface is not a typical interface. A class implements it to indicate that it will provide a public clone method. Since the designers of Java wanted to provide a default implementation of the method clone, they included it in the class Object and not in the interface Cloneable. But because the designers did not want every class to automatically have a public clone method, they made clone a protected method.

Note: Cloning

Cloning is not an operation that every class should be able to do. If you want your class to have this ability, you must

- Declare that your class implements the Cloneable interface
- Override the protected method clone that your class inherits from the class Object with a public version

30.15 **Example: Cloning a Name object.** Let's add a method clone to the class Name of Segment B.16 in Appendix B and Segment 30.10. Before we begin, we should add implements Cloneable to the first line of the class definition, as follows:

```
public class Name implements Cloneable
```

The public method clone within Name must invoke the method clone of its superclass by executing super.clone(). Because Name's superclass is Object, super.clone() invokes Object's protected method clone. Object's version of clone can throw an exception, so we must enclose each call to it in a try block and write a catch block to handle the exception. The method's final action should return the cloned object.

Thus, Name's method clone could appear as follows:

```
public Object clone()
{
   Name theCopy = null;

   try
   {
      theCopy = (Name)super.clone(); // Object can throw an exception
   }
   catch (CloneNotSupportedException e)
   {
      System.err.println("Name cannot clone: " + e.toString());
   }

   return theCopy;
} // end clone
```

Since `super.clone()` returns an instance of `Object`, we must cast this instance to `Name`. After all, we are creating a `Name` object as the clone. The `return` statement will implicitly cast `theCopy` to `Object`, as required.

The exception that `Object`'s method `clone` can throw is `CloneNotSupportedException`. Since we are writing a `clone` method for our class `Name`, this exception will never occur. Even so, we still must use `try` and `catch` blocks when invoking `Object`'s `clone` method. Instead of the `println` statement in the `catch` block, we could write the simpler statement

```
throw new Error(e.toString());
```

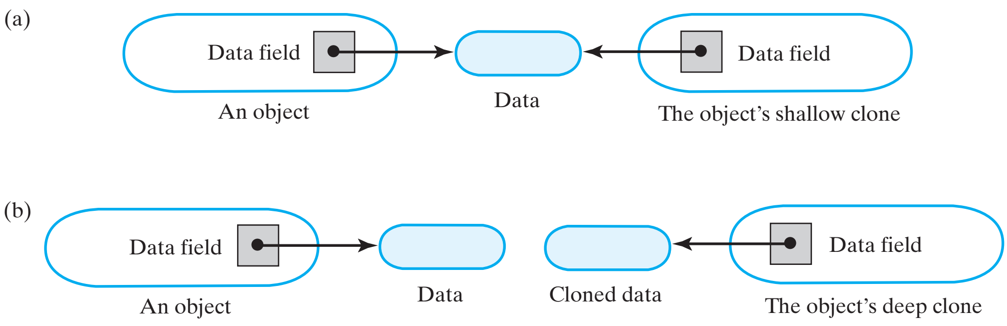
Programming Tip: Every public `clone` method must invoke the method `clone` of the base class by executing `super.clone`. Ultimately, `Object`'s protected `clone` method will be invoked. That invocation must appear in a `try` block, even though a `CloneNotSupportedException` will never occur.

30.16 **Two ways to copy.** What does this method `clone` actually do? You want it to make copies of the data fields associated with the invoking object. When a data field is an object, you can copy it in one of two ways:

- You can copy the reference to the object and share the object with the clone, as illustrated in Figure 30-4a. This copy is called a **shallow copy**; the clone is a **shallow clone**.
- You can copy the object itself, as illustrated in Figure 30-4b. This copy is called a **deep copy**; the clone is a **deep clone**.

Note: `Object`'s `clone` method returns a shallow clone.

FIGURE 30-4 (a) A shallow clone; (b) a deep clone

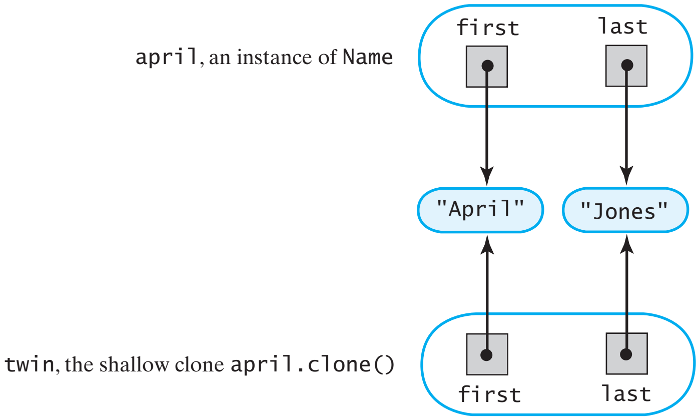


30.17 **`Name`'s clone is shallow.** The class `Name` has the data fields `first` and `last`, which are instances of `String`. Each field contains a reference to a string. It is these references that are copied when `clone` invokes `super.clone()`. For example, Figure 30-5 illustrates the objects that the following statements create:

```
Name april = new Name("April", "Jones");
Name twin = (Name)april.clone();
```

The clone `twin` is a shallow clone because the strings that are the first and last names are not copied.

FIGURE 30-5 An instance of Name and its shallow clone

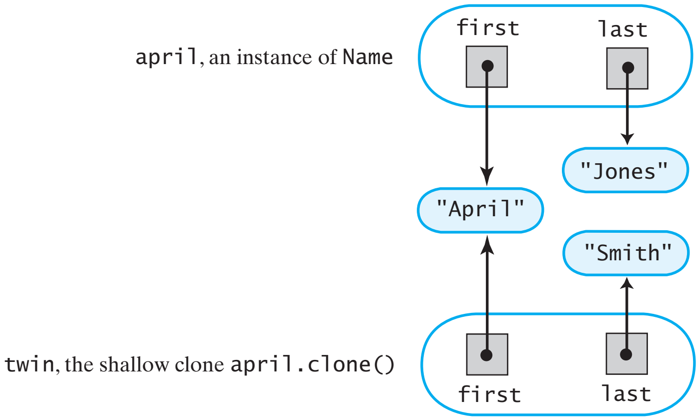

april, an instance of Name
first
last
"April"
"Jones"
twin, the shallow clone april.clone()
first
last

A shallow clone is good enough for the class Name. Recall that instances of String are immutable. Having an instance of Name and its clone share the same strings is not a problem because no one can change the strings. This is good news since, like many classes that Java provides, String has no method clone. Thus, if we change the clone's last name by writing

```
twin.setLast("Smith");
```

twin's last name will be *Smith*, but april's will still be *Jones*, as Figure 30-6 shows. That is, setLast changes twin's data field last so that it references another string *Smith*. It does not change april's last, so it still references *Jones*.

FIGURE 30-6 The clone twin, after the statement twin.setLast("Smith") changes one of its data fields


april, an instance of Name
first
last
"Jones"
"April"
"Smith"
twin, the shallow clone april.clone()
first
last

Programming Tip: Shallow copies of data fields that reference immutable objects are typically sufficient for a clone. Sharing an immutable object is usually safe.

30.18 **Example: Creating a deep clone of a single field.** Sometimes a shallow clone is unsuitable. If a class has mutable objects as data fields, you must clone the objects and not simply copy their references. For example, let's add a method `clone` to the class `Student` that we encountered in Segment C.2 of Appendix C. The class has the following form after we add the required `implements` clause:

```
public class Student implements Cloneable
{
   private Name   fullName;
   private String id;

   < Constructors and the methods setStudent, setName, setId, getName, getId, and
     toString >
   . . .
} // end Student
```

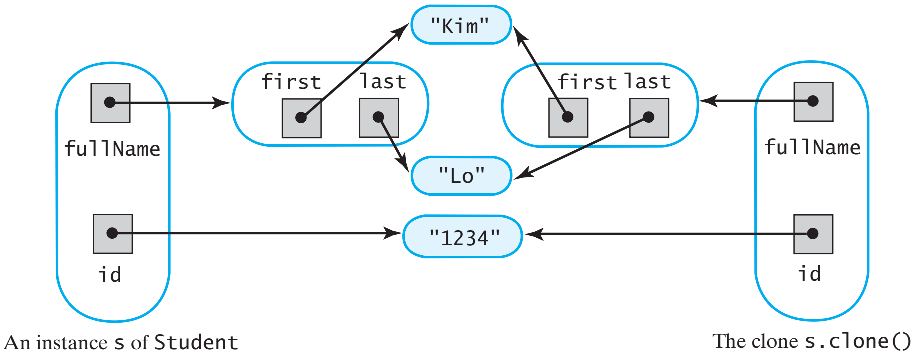
Since the class `Name` has set methods, the data field `fullName` is a mutable object. Therefore, we should be sure to clone `fullName` within the definition of `Student`'s `clone` method. We can do that because we added a `clone` method to `Name` in Segment 30.15. Since `String` is read only, `id` is immutable, and so cloning it is unnecessary. Thus, we can define a `clone` method for the class `Student` as follows:

```
public Object clone()
{
   Student theCopy = null;

   try
   {
      theCopy = (Student)super.clone(); // Object can throw an exception
   }
   catch (CloneNotSupportedException e)
   {
      throw new Error(e.toString());
   }

   theCopy.fullName = (Name)fullName.clone();
   return theCopy;
} // end clone
```

After invoking `super.clone()`, we clone the mutable data field `fullName` by calling `Name`'s public `clone` method. This latter invocation need not be within a `try` block. Only `Object`'s `clone` method contains a `throws` clause.

Figure 30-7 illustrates an instance of `Student` and the clone that this method returns. As you can see, the `Name` object that represents the student's full name is copied, but the strings that represent the first and last names, as well as the ID number, are not.

FIGURE 30-7 An instance of `Student` and its clone, including a deep copy of `fullName`

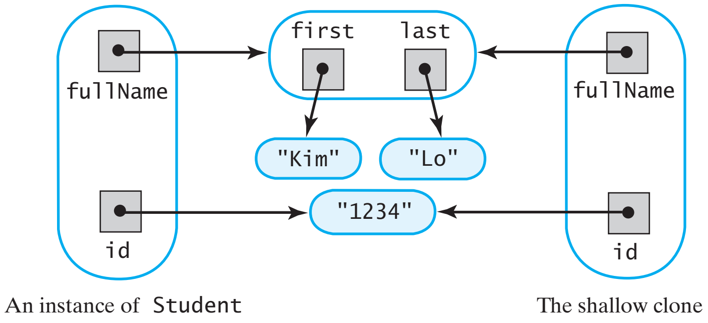


Had we failed to clone the data field `fullName`—that is, had we omitted the statement

```
theCopy.fullName = (Name)fullName.clone();
```

the student's full name would be shared by the original instance and its clone. Figure 30-8 illustrates this situation.

FIGURE 30-8 An instance of `Student` and its clone, including a shallow copy of `fullName`



Question 4 Suppose that `x` is an instance of `Student` and `y` is its clone; that is,

```
Student y = (Student)x.clone();
```

a. If you change `x`'s last name by executing

```
Name xName = x.getName();
xName.setLast("Smith");
```

does `y`'s last name change? Explain.

b. If you fail to clone `fullName` within `Student`'s `clone` method, will changing `x`'s last name change `y`'s last name as well? Explain.

Note: Within each public `clone` method, you typically perform the following tasks:

- Invoke the `clone` method of the superclass by writing `super.clone()`.
- Enclose this call to `clone` in a `try` block, and write a `catch` block to handle the possible exception `CloneNotSupportedException`. You can skip this step if `super.clone()` invokes a public `clone` method.
- Clone the mutable data fields of the object that `super.clone()` returned, when possible.
- Return the clone.

30.19 **Example: Cloning a `CollegeStudent` object.** Now let's add a `clone` method to a subclass of `Student`. Segment C.8 of Appendix C defines such a subclass, namely the class `CollegeStudent`. After we add an `implements` clause to its definition, the class has the following form:

```
public class CollegeStudent extends Student implements Cloneable
{
   private int    year;   // year of graduation
   private String degree; // degree sought
```

```
      < Constructors and the methods setStudent, setYear, getYear, setDegree, getDegree,
        and toString >
      . . .
   } // end CollegeStudent
```

The data fields of a CollegeStudent object are a primitive value and an immutable object, and so they do not need to be cloned. Thus, the definition of clone that we add to CollegeStudent is as follows:

```
   public Object clone()
   {
      CollegeStudent theCopy = (CollegeStudent)super.clone();
      return theCopy;
   } // end clone
```

The method must call Student's clone method, and it does so with the invocation super.clone(). Note that since Student's clone method does not throw an exception, no try block is necessary when we call it. Had CollegeStudent defined fields that needed to be cloned, we would clone them right before the return statement.

Cloning an Array

30.20 The class AList that you saw in Chapter 13 uses an array to implement the ADT list. Suppose that we want to add a clone method to this class.

While making a copy of the list, clone needs to copy the array and all the objects in it. Thus, the objects in the list must have a clone method as well. Recall that AList defines a generic type T for the objects it contains. Beginning AList as

```
   public class AList<T extends Cloneable> . . . // incorrect
```

will not work correctly, since the interface Cloneable is empty.

Instead, we define a new interface that declares a public method clone to override Object's protected method:

```
   public interface Copyable extends Cloneable
   {
      public Object clone();
   } // end Copyable
```

We then can begin AList with any one of the following statements:

- `public class AList<T extends Copyable> implements ListInterface<T>, Cloneable`
- `public class AList<T extends Copyable> implements ListInterface<T>, Copyable`
- `public class AList<T extends Copyable> implements CloneableListInterface<T>`

where CloneableListInterface is defined as follows:

```
   public interface CloneableListInterface<T>
                    extends ListInterface<T>, Copyable // or Cloneable
   {
   } // end CloneableListInterface
```

The notation

```
   AList<T extends Copyable>
```

requires the objects in the list to belong to a class that implements our interface Copyable.

Note that CloneableListInterface extends two interfaces, ListInterface and either Copyable or Cloneable. As noted in Segment D.25 of Appendix D, an interface can extend more than one interface, even though a class can extend only one other class.

Programming Tip: When bounding generic types, use an interface that declares a public method `clone` instead of using `Cloneable`. The new interface must extend `Cloneable`, however.

30.21 Using `Copyable` as a bound for `T` requires us to change the implementation of `AList`'s constructor. Recall from Listing 13-1 in Chapter 13 that one field of `AList` is an array of list entries:

```
private T[] list;
```

Writing

```
list = (T[]) new Object[initialCapacity];
```

in the constructor causes a `ClassCastException`. Instead, we write

```
list = (T[]) new Copyable[initialCapacity];
```

30.22 Now we can implement `clone`. We will invoke `super.clone()` within a `try` block but perform the rest of the tasks after the `catch` block. Thus, we have the following outline for `AList`'s method `clone`:

```
public Object clone()
{
   AList<T> theCopy = null;

   try
   {
      theCopy = (AList<T>)super.clone(); // not enough by itself
   }
   catch (CloneNotSupportedException e)
   {
      throw new Error(e.toString());
   }

   < For a deep copy, we need to do more here, as you will see. >
   . . .
   return theCopy;
} // end clone
```

The method first invokes `super.clone` and casts the returned object to `AList<T>`. To perform a deep copy, we need to clone the data fields that are or could be mutable objects. Thus, we need to clone the array `list`.

30.23 Arrays in Java have a public `clone` method; in other words, they implement `Cloneable`. So we can add the following statement to the list's `clone` method:

```
theCopy.list = (T[])list.clone();
```

No `try` and `catch` blocks are necessary here.

An array's `clone` method creates a shallow copy of each object in the array. For our deep copy, we need to clone each array entry. Since we insisted that the list's entries have a public `clone` method, we can write a loop whose body contains the following statement:

```
theCopy.list[index] = (T)list[index].clone();
```

We can control the loop by using `AList`'s data field `numberOfEntries`, which records the number of entries in the list.

Thus, we have the following definition of `clone` for the class `AList`:

```
public Object clone()
{
   AList<T> theCopy = null;
   try
   {
      theCopy = (AList<T>)super.clone();
   }
   catch (CloneNotSupportedException e)
   {
      throw new Error(e.toString());
   }

   theCopy.list = (T[])list.clone();

   for (int index = 0; index < numberOfEntries; index++)
      theCopy.list[index] = (T)list[index].clone();

   return theCopy;
} // end clone
```

Note: To make a deep clone of an array a of cloneable objects, you invoke `a.clone()` and then clone each object in the array. For example, if `myArray` is an array of `Thing` objects, and `Thing` implements `Cloneable`, you would write

```
Thing[] clonedArray = (Thing[])myArray.clone();

for (int index = 0; index < myArray.length; index++)
   clonedArray[index] = (Thing)myArray[index].clone();
```

Cloning a Chain

30.24 Now suppose that we want to add a `clone` method to a linked implementation of the ADT list, such as the class `LList` of Chapter 14 or the class `LinkedChainBase` of Chapter 17. (The `clone` methods for these classes are virtually identical.) Given the interface `Copyable`, we can begin the class in one of the ways given in Segment 30.20. Ultimately, the class and the objects in the list must implement the interface `Cloneable`. Thus, `LList`, for example, could begin as follows:

```
public class LList<T extends Copyable> implements CloneableListInterface<T>
{
   private Node firstNode; // reference to first node
   private int  numberOfEntries;
   . . .
```

The first part of the `clone` method would be like the code that you saw in Segment 30.22, except that we would replace `AList` with `LList`. If we invoked only `super.clone()`, our method would produce a shallow copy of the list, as Figure 30-9 illustrates. In other words, both the original list and its clone would reference the same chain of nodes, and these nodes would reference one set of data.

As before, `clone` needs to do more to perform a deep copy. It needs to clone the chain of nodes as well as the data that the nodes reference. Figure 30-10 shows a list with its deep clone.

30.25 **Cloning a node.** To clone the nodes in the chain, we need to add a method `clone` to the inner class `Node`. First, we add `implements Cloneable` to the declaration of the class `Node`. Note that `Node` is private in `LList` but protected in `LinkedChainBase`. `Node`'s `clone` method begins like other `clone` methods, but then it goes on to clone the data portion of the node. We do not bother cloning the

FIGURE 30-9 A list and its shallow clone: linked implementation

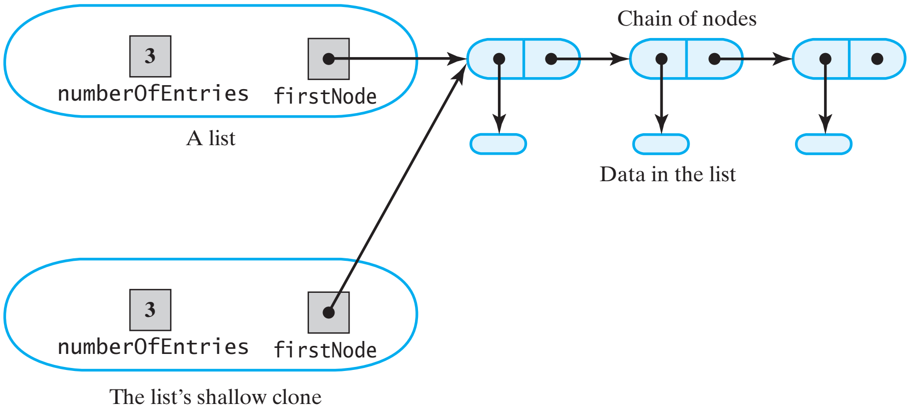
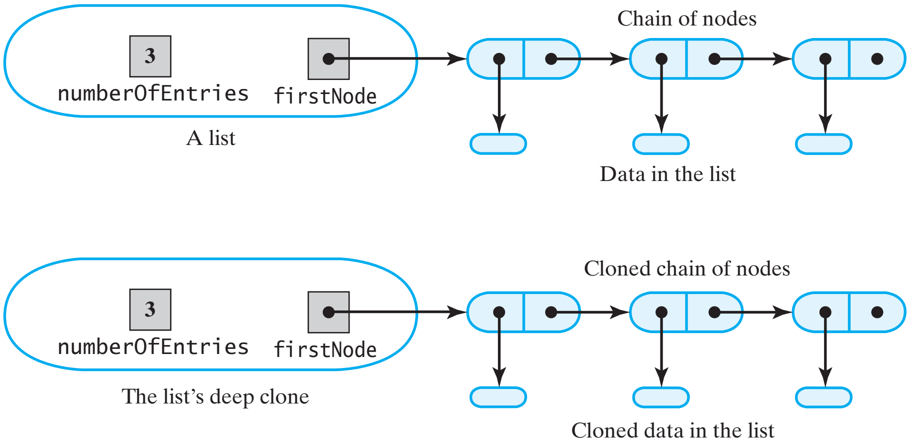


FIGURE 30-10 A list and its deep clone: linked implementation



link, since the list's `clone` method will set it. With these changes, the revised class `Node` appears in `LList` as follows (changes are highlighted):

```
public class Node implements Cloneable
{
   private T    data;
   private Node next;

   < Constructors >
   . . .
   < Accessor and mutator methods getData, setData, getNextNode, and setNextNode >
   . . .
   protected Object clone()
   {
      Node theCopy = null;
      try
      {
         theCopy = (Node)super.clone();
      }
      catch (CloneNotSupportedException e)
      {
         throw new Error(e.toString());
      }

      theCopy.data = (T)data.clone();
      theCopy.next = null; // don't clone link; it’s set later

      return theCopy;
   } // end clone
} // end Node
```

Remember that `data` invokes a public method `clone` that does not throw an exception, and so `data.clone()` can appear outside of a `try` block.

30.26 **Cloning the chain.** `LList`'s `clone` method invokes `super.clone()` in a statement such as

```
LList<T> theCopy = (LList<T>)super.clone();
```

The method then must clone the chain of nodes that stores the list's data. To do so, the method needs to traverse the chain, clone each node, and link the cloned nodes appropriately. We begin by cloning the first node so that we can set the data field `firstNode` correctly:

```
// make a copy of the first node
theCopy.firstNode = (Node)firstNode.clone();
```

Next, we traverse the rest of the chain. A reference `newRef` references the last node that we have added to the new chain, while the reference `oldRef` keeps track of where we are in the traversal of the original chain. The statement

```
newRef.setNextNode((Node)oldRef.clone()); // attach cloned node
```

clones the current node in the original chain, along with its data, and then links the clone to the end of the new chain. Recall that `Node`'s `clone` method also clones the data that a node references.

The following statements incorporate the previous ideas and clone the rest of the chain:

```
Node newRef = theCopy.firstNode;       // last node in new chain
Node oldRef = firstNode.getNextNode(); // next node in old chain

for (int count = 2; count <= numberOfEntries; count++)
{
   newRef.setNextNode((Node)oldRef.clone()); // attach cloned node
   newRef = newRef.getNextNode();            // update references
   oldRef = oldRef.getNextNode();
} // end for
```

30.27 The code in the previous segment assumes a nonempty chain of nodes. The complete `clone` method that follows checks for an empty chain.

```
public Object clone()
{
   LList<T> theCopy = null;
```

```
      try
      {
         theCopy = (LList<T>) super.clone();
      }
      catch (CloneNotSupportedException e)
      {
         throw new Error(e.toString());
      }

      // copy underlying chain of nodes

      if (firstNode == null) // if chain is empty
      {
         theCopy.firstNode = null;
      }
      else
      {
         // make a copy of the first node
         theCopy.firstNode = (Node)firstNode.clone();

         // make a copy of the rest of chain
         Node newRef = theCopy.firstNode;
         Node oldRef = firstNode.getNextNode();

         for (int count = 2; count <= numberOfEntries; count++)
         {
            // clone node and its data; link clone to new chain
            newRef.setNextNode((Node)oldRef.clone());
            newRef = newRef.getNextNode();
            oldRef = oldRef.getNextNode();
         } // end for
      } // end if

      return theCopy;
   } // end clone
```

Note: To make a deep clone of a chain of linked nodes that reference cloneable objects, you must clone the nodes as well as the objects.

SELF-TEST

Question 5 The `for` statement in Segment 30.27 is controlled by the number of nodes in the chain. Revise this statement and its associated body so that it is controlled by `oldRef`.

A Sorted List of Clones

30.28 Segment 30.4 talked about the danger of placing mutable objects in a collection such as a sorted list. If the client retains a reference to any of the objects, it could alter those objects and destroy the integrity of the collection. In the case of a sorted list, the client could destroy the sorted order of the objects.

Segment 30.5 offered one solution to this problem, namely to place only immutable objects in the collection. This section offers another solution that enables you to place mutable objects in the collection.

Suppose that a client adds an object to a collection. Imagine that the collection clones the object before adding it to its data. The client then would be able to access or change the collection's data only by using ADT operations. It would not have a reference to the clone that it could use to alter the clone. Of course, this scenario requires that the added object be `Cloneable`. Let's examine the details of such an implementation of the ADT sorted list.

30.29 Segment 16.1 of Chapter 16 noted that objects in a sorted list must be Comparable—that is, they must have a compareTo method. In this case, we also want the objects to be Cloneable. Segment 30.20 defined an interface Copyable that declares a public method clone. Using that interface, let's create another one:

```
public interface ComparableAndCopyable<T> extends Comparable<T>, Copyable
{
} // end ComparableAndCopyable
```

This interface will enable us to bound the generic type of the objects we place into the sorted list, as the next segment will show.

A class that implements ComparableAndCopyable must define the methods compareTo and clone. For example, the class Name mentioned in Segment 30.15 could begin as follows:

```
public class Name implements ComparableAndCopyable<Name>
```

Name's method clone is given in Segment 30.15, and you wrote compareTo when you answered Question 8 in Appendix D.

30.30 Since we want the sorted list to contain only objects that are both Comparable and Copyable, we can begin the definition of a class SortedList as follows:

```
public class SortedList<T extends ComparableAndCopyable<? super T>>
```

We introduced this notation in Segments 8.1 and 8.2 of Chapter 8. The class that T represents must implement the interface ComparableAndCopyable. The notation ? super T, which means any superclass of T, affects the interface Comparable—as you can see by looking at ComparableAndCopyable—and hence the method compareTo.

We can revise our interface for a sorted list by beginning it as follows:

```
public interface SortedListInterface
                 <T extends ComparableAndCopyable<? super T>>
```

and then use it in the definition of SortedList:

```
public class SortedList<T extends ComparableAndCopyable<? super T>>
             implements SortedListInterface<T>
```

30.31 With these logistics out of the way, we propose the following changes to the implementation of the ADT sorted list. You can apply these changes to the implementations discussed in Chapters 16 and 17:

- In add, place a clone of the desired entry into the sorted list instead of the entry itself. That is, place newEntry.clone() into the list instead of newEntry. Thus, the body of the method could begin with

  ```
  Node newNode = new Node((T)newEntry.clone());
  ```

  Since clone returns an Object, the cast to the generic type T is necessary.

- In getEntry, return a clone of the desired entry instead of the entry itself. For example, you could return (T)result.clone() instead of result.

Let's examine these changes more closely. Suppose that a client has a reference, newEntry, to an object, and it adds the object to a collection. The collection clones the object and adds the clone instead of the original object, as Figure 30-11 illustrates. The client has no reference to the collection's data. If the client modifies the object that newEntry references, the collection is not changed.

What if getEntry did not return a clone of the desired entry but instead returned a reference to the desired entry in the collection? As Figure 30-12 illustrates, the client would be able to change the entry within the collection. So even though the collection contains a clone of the client's original object, getEntry would give the client access to the clone. Thus, it is necessary for getEntry to return a clone of the desired entry. This is a clone of the clone of the client's original object, as Figure 30-13 shows.

FIGURE 30-11 A collection and its client after the clone of an object is added to the collection

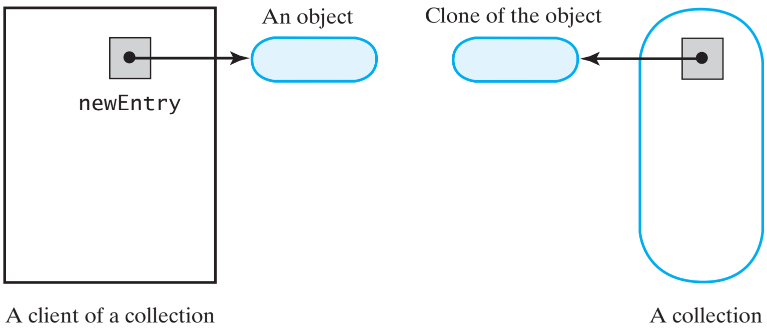


FIGURE 30-12 The effect of getEntry if it did not return a clone

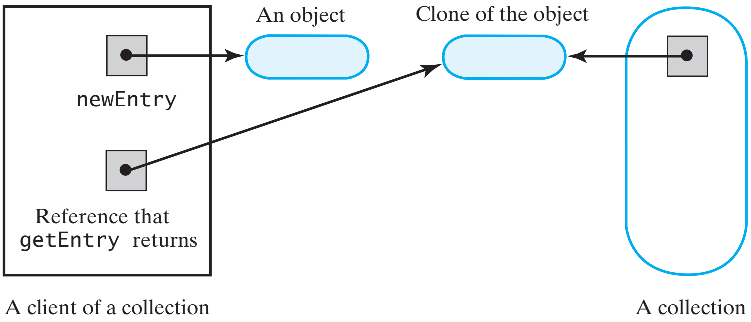
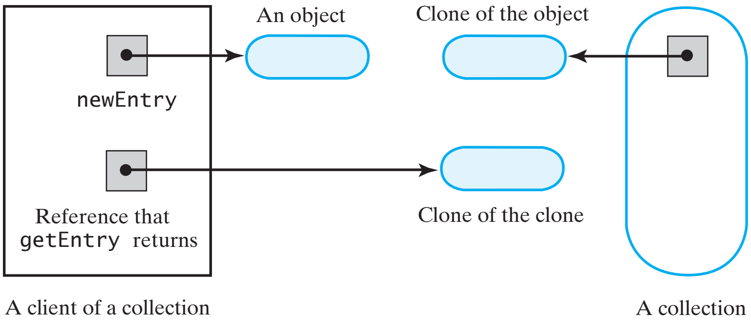


FIGURE 30-13 The effect of getEntry when it returns a clone



Note: A collection can clone the objects that a client adds to it, but you will have duplicates of each entry in the collection. For complex objects, the time and memory needed to make each copy can be substantial.

30.32 Realize that some devious programming could defeat the purpose of our sorted list of clones. You could write a `clone` method that retains a reference to the clone. Then, knowing when a sorted list calls `clone`, you could get and retain the reference to the clone. Restricting the contents of a collection to immutable objects is a safer way to maintain its integrity.

Chapter Summary

- An object that belongs to a class having public mutator (set) methods is mutable because the client can use the methods to change the values of the object's data fields. If a client cannot change the values of an object's data fields, the object's class is read only, and the object is immutable.
- A read-only class is final, has private data fields that, if mutable, are final, and has no public set methods.
- Companion classes represent the same data in both immutable and mutable forms.
- The class `Object` includes a protected method `clone` that makes an identical copy of an object. A class can override `clone` and declare it public, thus making it available to a client of the class. Such a class must implement the Java interface `Cloneable`. If a class does not override `clone`, it has no `clone` method, since `clone` is protected in `Object`.
- A `clone` method should invoke `super.clone()` to ensure that all aspects of an object are copied.
- Every array has a `clone` method that copies the array but not the objects in it. A separate step is necessary to clone these objects.
- To create a deep clone of a chain of linked nodes, you must clone the nodes, including their data objects.

Programming Tips

- Use an immutable object if it will be shared or added to a collection that can be corrupted by changes to the object. Use a mutable object if its data will change frequently.
- Not all classes should have a public `clone` method. In fact, most classes, including read-only classes, do not have one.
- If your program produces the exception `CloneNotSupportedException` even though you implemented a method clone in your class, you probably forgot to write `implements Cloneable` in your class definition.
- Every public `clone` method must invoke the method `clone` of the base class by executing `super.clone`. Ultimately, `Object`'s protected `clone` method will be invoked. That invocation must appear in a `try` block, even though a `CloneNotSupportedException` will never occur.
- Shallow copies of data fields that reference immutable objects are typically sufficient for a clone. Sharing an immutable object is usually safe.
- When bounding generic types, use an interface that declares a public method `clone` instead of using `Cloneable`. The new interface must extend `Cloneable`, however.

Exercises

1. Is the class `Student` that is mentioned in Segment 30.18 mutable or immutable? Describe what you would do to create a companion class for `Student`.

2. **a.** Using the method `getEntry` of the ADT list, describe a client-level method that modifies the entry at a given position within a list.
   b. What must you know about the entries in the list for your method to work?
   c. Does the method `replace` of the ADT list have any advantage over your method?

3. Implement the method `compareTo` for the class `ImmutableName`.

4. Consider the following class definition:

```
public class Friend
{
   private Name   friendsName;
   private String friendsAddress;

   public Friend(Name aName, String anAddress)
   {
      friendsName = aName;
      friendsAddress = anAddress;
   } // end constructor

   public Name getName()
   {
      return friendsName;
   } // end getName

   public String getAddress()
   {
      return friendsAddress;
   } // end getAddress
} // end Friend
```

   Is this class read only? Why?

5. Given the class `Name`, define a companion class `ImmutableName` by using composition. That is, `ImmutableName` should contain an instance of `Name` as a data field. What are the advantages and disadvantages of using composition to define companion classes?

6. Given the class `ImmutableName`, define a companion class `Name` by using inheritance. What are the advantages and disadvantages of using inheritance to define companion classes?

7. Define an interface, `ImmutableNameInterface`, that declares the methods in the class `ImmutableName`. Define another interface, `MutableNameInterface`, that extends `ImmutableNameInterface` and declares the methods in the class `Name`. Then modify the classes `ImmutableName` and `Name` so that they implement `ImmutableNameInterface` and `MutableNameInterface`, respectively.

8. A variable of type `ImmutableNameInterface` can reference objects of what type?

9. A variable of type `MutableNameInterface` can reference objects of what type?

10. A variable of type `Name` can reference objects of what type?

11. A variable of type `ImmutableName` can reference objects of what type?

12. The class NickName given in Segment C.3 of Appendix C has a data field that is an instance of the class Name and two methods, setNickName and getNickName. Implement a method clone for NickName, assuming that Name implements the interface Cloneable.

13. Consider a class that implements ListInterface but does not make clones of objects added to the list. Suggest two ways that we can make a list of clones.

14. Consider a class that implements ListInterface and makes clones of objects added to the list. Suggest two ways that we can make a list of the original objects.

15. Consider the following class definition:

```
public class FamilyMember implements Cloneable
{
   private Name myName;
   private FamilyMember mySpouse;

   public FamilyMember(Name aName)
   {
      myName = aName;
   } // end constructor

   public Name getName()
   {
      return myName;
   } // end getName

   public FamilyMember getSpouse()
   {
      return mySpouse;
   } // end getSpouse

   public void setSpouse(FamilyMember spouse)
   {
      mySpouse = spouse;
   } // end setSpouse

   public Object clone()
   {
      FamilyMember theCopy = null;
      try
      {
         theCopy = (FamilyMember)super.clone();
      }
      catch(CloneNotSupportedException e)
      {
         System.err.println("FamilyMember cannot clone:" + e.toString());
      }
      return theCopy;
   } // end clone
} // end FamilyMember
```

Draw a diagram showing the state of the objects after the following code executes.

```
FamilyMember jack = new FamilyMember(new Name("Jack", "Buck"));
FamilyMember jane = new FamilyMember(new Name("Jane", "Doe"));
jack.setSpouse(jane);
jane.setSpouse(jack);
FamilyMember copy = (FamilyMember)jane.clone();
```

16. Replace the method copy, as discussed in Segment 24.6 of Chapter 24, with a method clone. That is, make BinaryNode implement the interface Cloneable. Cloning a node should also clone the node's data object.

Projects

1. Define a pair of companion classes modeled after the class `Student`, as given in Segment C.2 of Appendix C.
2. Project 5 in Chapter 7 asked you to write a class `KnapsackItem` of items that would be placed in a list. Design and implement a pair of companion classes for such knapsack items.
3. Project 6 in Chapter 7 asked you to write a class `Activity` of activities that would be placed in a list. Design and implement a pair of companion classes for such activities.
4. Chapter 13 describes an implementation of the ADT list that uses an instance of the class `Vector` to represent the entries in the list. Write a `clone` method for this implementation and demonstrate that it works. Note that `Vector` implements the interface `Cloneable`.
5. Revise the linked implementation of the ADT sorted list, as given in Chapter 16, according to the suggestions about cloning given in Segment 30.31. That is, the method `add` should place a clone of the desired entry, instead of the entry itself, into the sorted list. Additionally, the method `getEntry` should return a clone of the desired entry, instead of returning the entry itself.
6. Suppose that you wanted to implement a deep copy for the class `FamilyMember` defined in Exercise 15 in this chapter.

   a. What difficulty will arise?
   b. Implement a `clone` method for `FamilyMember` that makes a deep copy.
7. Define three classes—`A`, `B`, and `C`—such that `C` is a subclass of `B`, and `B` is a subclass of `A` . Each class should define a `clone` method and have at least one data field that is a mutable, cloneable object.

Answers to Self-Test Questions

1.

```
public ImmutableName(Name aName)
{
   first = aName.getFirst();
   last  = aName.getLast();
} // end constructor
```

2.

```
// Create an object of the class Name
Name derek = new Name("Derek", "Greene");
// Convert the object to an immutable object; don't change its data fields
ImmutableName derekI = derek.getImmutable();
// Add the immutable object to the sorted list nameList
SortedListInterface<ImmutableName> nameList = new SortedList<ImmutableName>();
nameList.add(derekI);
```

3.

```
// Create an object of the class ImmutableName
ImmutableName lila = new ImmutableName("Lila", "Bleu");
// Convert the object to a mutable object; don't change its data fields
Name changer = lila.getMutable();
// Change the last name of the new object
changer.setLast("Greene");
// Convert the revised mutable object to an immutable object
ImmutableName unchanger = changer.getImmutable();
```

4. **a.** No. The clone `y` has a name object that is distinct from `x`'s name object, because a deep copy was made. (See Figure 30-7.)
   b. Yes. Both objects share one name object. (See Figure 30-8.)

5.

```
Node newRef = theCopy.firstNode;
for (Node oldRef = firstNode.getNextNode(); oldRef != null; oldRef = oldRef.getNextNode())
{
   newRef.setNextNode((Node)oldRef.clone());
   newRef = newRef.getNextNode();
} // end for
```

Index

Appendix

A

Java Essentials

Contents

Prerequisite

Knowledge of a programming language

This book assumes that you know how to write programs in Java. If you know some other programming language, this appendix will help you to learn Java by reviewing the essential elements of the language. Appendices B, C, and D supplement the coverage of Java that is presented here by discussing methods, classes, and inheritance. Appendices E, F, and G cover exceptions, file I/O, and comments.

If you already know Java, note that this book uses applications, not applets. If you know only about applets, you should read at least the first few pages of this appendix.

Introduction

Applications and Applets

A.1 There are two kinds of Java programs, applications and applets. An **application** is simply a program that runs on your computer like any other program. It is a **stand-alone** program. In contrast, an **applet** is a program that cannot run without the support of a browser or a viewer. Typically, an applet is sent to another location on the Internet and is run there. The term “applet” is meant to suggest a little application.

Applets and applications are almost identical. Once you know how to design and write one, it is easy to learn to write the other. This book uses applications rather than applets.

Objects and Classes

A.2 An **object** is a program construct that contains data and can perform certain actions. When a Java program is run, the objects interact with one another to accomplish a particular task. The actions performed by objects are defined by **methods** in the program. When you ask an object to perform an action, you **invoke**, or **call**, a method. Java has two kinds of methods. A **valued method** uses a `return` statement to return a result, but a **void method** does not.

All objects of the same kind are said to be in the same class. So a **class** is a category or kind or type of object. All objects in the same class have the same types of data and the same methods.

You will see some objects and methods in the next section and again when we discuss the classes `Math`, `Scanner`, and `String` later in this appendix. Appendix B reviews classes, objects, and methods in more detail. If you are not familiar with these concepts, you should read at least Segment B.1 in Appendix B now and the rest of Appendix B and Appendix C later.

A First Java Application Program

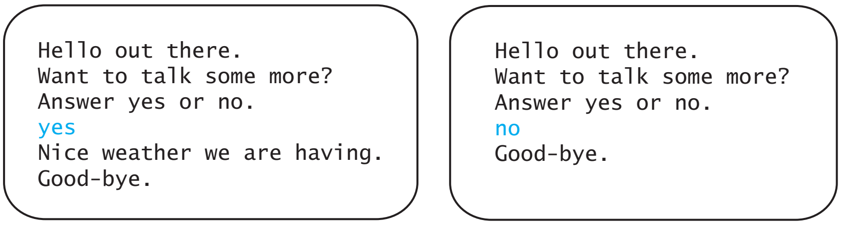
A.3 To give you a feel for the Java language, let's take a brief, informal look at the following sample Java application program:

```
import java.util.Scanner;
public class FirstProgram
{
   public static void main(String[] args)
   {
      Scanner keyboard = new Scanner(System.in);
      System.out.println("Hello out there.");
      System.out.println("Want to talk some more?");
      System.out.println("Answer yes or no.");

      String answer = keyboard.next();
      if (answer.equals("yes"))
         System.out.println("Nice weather we are having.");
      System.out.println("Good-bye.");
   } // end main
} // end FirstProgram
```

The program is stored in the file `FirstProgram.java`.

Figure A-1 shows two screen displays that might be produced when a user runs and interacts with this program. The text typed by the user is shown in blue.

FIGURE A-1 Two possible results when running the sample program



```
Hello out there.
Want to talk some more?
Answer yes or no.
yes
Nice weather we are having.
Good-bye.
```

```
Hello out there.
Want to talk some more?
Answer yes or no.
no
Good-bye.
```

A.4 This program uses the class `Scanner`, which is a part of the **Java Class Library**. This library contains many standard classes that you can use in your Java programs. The classes in the library are organized into groups called **packages**. The import statement indicates that this program uses the class Scanner from the package java.util.

The program contains the definition of a class that begins with the lines

```
public class FirstProgram
{
```

and ends with

```
} // end FirstProgram
```

Within the class definition is a method called main that begins with the statements

```
public static void main(String[] args)
{
```

and ends with

```
} // end main
```

Statements within this pair of braces are the **body** of the method main. Generally, a class contains several methods, each defining a specific task and each having a name of our choosing. An application program, however, must contain a method that is called main.

A.5 The line

```
Scanner keyboard = new Scanner(System.in);
```

gets us ready to read data from the keyboard using Java's class Scanner. It defines the Scanner object keyboard and associates it with the object System.in. This latter object represents the computer's keyboard. System.in is in the class System, which is in the package java.lang of the Java Class Library.

Programming Tip: No import statement is necessary when you use a class from the package java.lang of the Java Class Library.

A.6 The next three lines display text for the user of the program:

```
System.out.println("Hello out there.");
System.out.println("Want to talk some more?");
System.out.println("Answer yes or no.");
```

Each of these lines causes the quoted characters, or **string**, given within the parentheses to be displayed on the screen. System.out is an object within the class System that can send output to the screen via its method println. You invoke an object's method by writing the object name followed by a period, or **dot**, followed by the method name and some parentheses that may or may not have something inside them. The text inside the parentheses is called an **argument** and provides the information the method needs to carry out its action. In each of these three lines, the method println writes the value of its argument—here, the characters inside the quotes—to the screen.

The method println is an example of a void method. It performs an action, but does not return a value.

A.7 The next line of the program reads the characters that are typed at the keyboard and stores them in the variable answer as a string:

```
String answer = keyboard.next();
```

The Scanner object keyboard invokes its method next to read the word that the user types at the keyboard. The user presses the Enter key (also called the Return key) after typing the word. Although the method next has no arguments, the parentheses are required. You will learn more about the class Scanner later in this appendix.

The method `next` is an example of a valued method. It returns the value read. Its **invocation**, `keyboard.next()`, represents this value.

A.8 The next two lines of the program make a decision to do or not do something based on the value of the variable `answer`. The first line asks whether the string stored in `answer` is the string *yes*. If it is, a message is displayed on the screen. Otherwise, the message is not displayed.

Notice that the first sample dialogue in Figure A-1 displays the string *Nice weather we are having,* and the second one does not. That is because, in the first run of the program, the string *yes* is stored in the variable `answer`, and in the second run of the program, the string *no* is stored in `answer`.

A.9 Of course, precise rules govern how you write each part of a Java program. For example, a final semicolon ends each Java statement. These rules form the **grammar** for the Java language, just as there are rules for the grammar of the English language. The grammar rules for a programming language (or any language) are called the **syntax** of the language. We now look at the elements of Java in more detail.

Java Basics

In this section, we examine how to use Java to perform arithmetic computations.

Identifiers

A.10 You use **identifiers** to name certain parts of a program. An identifier in Java consists entirely of letters, digits, the underscore character _, and the dollar sign $. An identifier cannot start with a digit and must not contain a space or any other special character such as a period or an asterisk. There is no official limit to the length of a name, though in practice, there is always a limit. Although Java allows identifiers to contain a dollar sign, it is reserved for special purposes, and so you should not use $ in a Java identifier.

Java is **case sensitive**. This means that it treats uppercase letters and lowercase letters as different characters. For example, `mystuff`, `myStuff`, and `MyStuff` are three different identifiers. Having these identifiers in the same program could be confusing to human readers, and therefore doing so is a poor programming practice. But the Java compiler would be happy with them.

Java uses a character set, called **Unicode**, that includes characters from languages other than English. Java allows you to use these extra characters in identifiers, but you are not likely to find them on most keyboards. Segment A.68 discusses Unicode further.

A.11 Although it is not required by the Java language, the common practice, and the one followed in this book, is to start the names of classes with uppercase letters and to start the names of objects, methods, and variables (which you are about to see) with lowercase letters. These names are usually spelled using only letters and digits. We separate multiword names by using uppercase letters, since we cannot use spaces. For example, the following are all legal identifiers that follow this well-established convention:

```
inputStream   YourClass   CarWash   hotCar   theTimeOfDay
```

Some people use an underscore to separate the words in an identifier, but typically we will not.

The following are all illegal identifiers in Java, and the compiler will complain if you use any of them:

```
.MyClass   goTeam-   7eleven
```

The first two contain an illegal character, either a dot or a dash. The last name is illegal because it starts with a digit.

Reserved Words

A.12 Some words, such as the word `if`, have a special predefined meaning in the Java language. You cannot use these words, called **reserved words** or **keywords**, for anything other than their intended meaning. A full list of reserved words for Java is given on the inside cover of this book. Within a programming environment, the names of reserved words are often highlighted in some way. In this book, they will appear in boldface.

Some other words, such as `String`, name classes that are supplied with Java. They have a predefined meaning but are not reserved words. This means that you can change their meaning, but doing so could easily confuse you or somebody else reading your program.

Variables

A.13 A **variable** in a program represents a memory location that stores data such as numbers and letters. The number, letter, or other data item in a variable is called its **value**. This value can be changed, so that at one time the variable contains, say, 6, and at another time after the program has run for a while, the variable contains a different value, such as 4.

You use an identifier to name a variable. Besides following the rules and conventions for identifiers, you should choose variable names that suggest their use or the kind of data they will hold. For example, if a variable is used to count something, you might name the variable `count`. If the variable is used to hold the speed of an automobile, you might call the variable `speed`. You should almost never use single-letter variable names like `x` and `y`.

A.14 A variable's **data type**—or simply **type**—determines what kind of value the variable can hold. If the type is `int`, the variable can hold integers. If the type is `double`, the variable can hold numbers with a decimal point and a fractional part after the decimal point. If the type is `char`, the variable can hold any one character from the computer keyboard.

Java has two kinds of types, reference types and primitive types. A **reference type**—also called a **class type**—is a type that represents a class, that is, a type for objects of a class. For example, `String` is a class type. A variable of a reference type is called a **reference variable**. A **primitive type** is a simpler type. Values of a primitive type are not complex items but simple, indecomposable values, such as a single number or a single letter. The types `int`, `double`, and `char` are primitive types. The names of primitive types begin with a lowercase letter. By convention, the names of class types begin with an uppercase letter. Also by convention, variable names of either class types or primitive types begin with a lowercase letter.

> **Note:** Naming conventions help you to distinguish among identifiers when reading a program.

A.15 A **variable declaration** indicates the type of data the variable will hold. Different types of data are stored in the computer's memory in different ways. Variable declarations are necessary so that the value of a variable can be correctly stored in or retrieved from the computer's memory. Even though the mechanisms for storing values in the variables of class types differ from the mechanisms used for primitive types, you declare variables for class types and primitive types in the same way.

You declare a variable by writing a type name followed by a list of variable names separated by commas and ending with a semicolon. All the variables named in the list will have the type given at the start of the declaration. For example:

```
int numberOfBaskets, eggsPerBasket, totalEggs;
String myName;
```

The first line declares that the three variables numberOfBaskets, eggsPerBasket, and totalEggs will contain values of type int. The second line declares that myName will store a String object.

You must declare a variable in a Java program before you use it. Normally, you declare a variable either just before it is used or at the start of a method definition.

Primitive Types

A.16 A whole number without a decimal point, such as 0, 1, or –2, is called an **integer**. A number with a decimal point, such as 3.14159, –8.63, or 5.0, is called a **floating-point number**. Notice that 5.0 is a floating-point number, not an integer. If a number has a fractional part, even if the fractional part is zero, it is a floating-point number.

All the Java primitive types appear inside the cover of this book. Notice that there are four types for integers—namely byte, short, int, and long. The only difference among the various integer types is the range of integers they can store and the amount of computer memory they use. If you cannot decide which integer type to use, use the type int. It has a large enough range for most purposes and does not use as much memory as the type long.

Java has two types for floating-point numbers, float and double. If you cannot decide between the types float and double, use double. It allows a wider range of values and is used as a default type for floating-point numbers.

You use the primitive type char for single characters, such as letters, digits, or punctuation. For example, the following declares the variable symbol to be of type char, stores the character for uppercase *A* in symbol, and then displays that value—the *A*—on the screen:

```
char symbol;
symbol = 'A';
System.out.println(symbol);
```

Notice that we enclose the character *A* in single quotes. Again note that uppercase letters and lowercase letters are different characters. For example, 'a' and 'A' represent two different characters.

Finally, the primitive type boolean has two values, true and false. You can use a variable of type boolean to store the answer to a true/false question such as “Is myTime less than yourTime?”

Constants

A.17 A variable can have its value changed; its value *varies*. A literal number like 2 cannot change. It is always 2. It is never 3. Values like 2 or 4.8 are called **constants**, or **literals**, because their values do not change.

You write constants of integer types with an optional plus sign or minus sign, but without commas or decimal points. Floating-point constants have an optional plus sign or a minus sign and no commas. You can write a floating-point constant in one of two ways. One way looks like the everyday way of writing numbers. For example, 9.8, -3.14, and 5.0 are floating-point constants, because they contain a decimal point. The second way is to include a multiplier that is a power of 10. You use the letter e to represent both the multiplication sign and the 10. For example, you would write 8.65×10^8 in Java as 8.65e8 (or in the less convenient form 865000000.0). The two forms, 8.65e8 and 865000000.0, are equivalent in a Java program. Similarly, the number 4.83×10^{-4}, which is equal to 0.000483, can be written as 4.83e-4 in Java.

The e stands for “exponent,” since it is followed by a number that is thought of as an exponent of 10. The number before the e can be a number with or without a decimal point. The number after the e cannot contain a decimal point.

Other types of literal expressions are also called constants. You write constants of type char by placing the character in single quotes. For example, 'Y' is a constant of type char. A string constant is a sequence of characters enclosed in double quotes, as in "Java".

Assignment Statements

A.18 You can use an **assignment statement** to give a value to a variable. For example, if `answer` is a variable of type `int` and we want to give it the value 42, we could use the following assignment statement:

```
answer = 42;
```

An assignment statement always consists of a single variable on the left-hand side of an equal sign and an expression on the right-hand side followed by a semicolon. The expression can be another variable, a constant, or a more complicated expression made up by combining **operators**, such as + and *, with variables and constants. The value of the expression is assigned to the variable on the left of the equal sign.

For example, the following are all examples of assignment statements:

```
amount = 3.99;
firstInitial = 'B';
score = numberOfCards + handicap;
```

Here we assume that `amount` is a variable of type `double`, `firstInitial` is of type `char`, and the rest of the variables are of type `int`. If the variable `numberOfCards` has the value 7 and `handicap` has the value 2, the value of the variable `score` is 9.

The equal sign, =, which is called the **assignment operator**, does not mean equality. You can think of the assignment operator as saying, "Make the value of the variable equal to what follows." For example, in the statement

```
eggsPerBasket = eggsPerBasket - 2;
```

the variable `eggsPerBasket` occurs on both sides of the assignment operator. This statement subtracts 2 from the present value of `eggsPerBasket` and assigns the new value to `eggsPerBasket`. In effect, the statement decreases the value of `eggsPerBasket` by 2.

A.19 A variable that has been declared but that has not yet been given a value by the program is **uninitialized**. Such a varible might literally have no value, or it might have some default value. For example, an integer varible might have a default value of zero, and a reference variable might have a default value of **`null`**, which is a predefined constant in Java. However, your program will be clearer if you explicitly give the variable a value, even if you are simply reassigning it the default value. (The exact details on default values have been known to change and should not be counted on.)

One easy way to ensure that you do not have an uninitialized variable is to initialize it within the declaration. Simply combine the declaration and an assignment statement, as in the following examples:

```
int count = 0;
double taxRate = 0.075;
char grade = 'A';
int balance = 1000, newBalance;
```

Note that a single declaration, such as the last statement, can initialize some variables and not others.

Sometimes the compiler may complain that you have failed to initialize a variable. In most cases, this is indeed true. Occasionally, the compiler is mistaken about this. However, the compiler will not compile your program until you convince it that the variable in question is initialized. To make the compiler happy, initialize the variable when it is declared, even if the variable will be given a different value before you use it for anything. In such cases, you cannot argue with the compiler.

Note: A reference variable that contains `null` does not reference any object.

Assignment Compatibilities

A.20 You cannot put a square peg in a round hole, and similarly you cannot put a value of one type in a variable of another type. You cannot put an `int` value like 42 in a variable of type `char`. You cannot put a `double` value like 3.5 in a variable of type `int`. You cannot even put the `double` value 3.0 in a variable of type `int`. You cannot store a value of one type in a variable of another type unless the value is somehow converted to match the type of the variable.

When dealing with numbers, however, this conversion will sometimes—but not always—be performed for you automatically. For example, you can always assign a value of an integer type to a variable of a floating-point type, such as when you write either

```
double interestRate = 7;
```

or

```
int wholeRate = 7;
double interestRate = wholeRate;
```

More generally, you can assign a value of any type on the following list to a variable of any type that appears further down on the list:

```
byte → short → int → long → float → double
```

For example, you can assign a value of type `long` to a variable whose type is either `long`, `float`, or `double`. Notice that you can assign a value of any integer type to a variable of any floating-point type. This is not an arbitrary ordering of the types. As you move down the list from left to right, the types become more complex, or **wider**, either because they allow larger values or because they allow decimal points in the numbers. Thus, you can assign a value of one type to a variable of either the same type or a wider type.

In addition, you can assign a value of type `char` to a variable of type `int` or to any of the numeric types that follow `int` in the previous list of types. However, we do not advise doing so, because the result could be confusing.[1]

If you want to assign a value of type `double` to a variable of type `int`, you must change the type of the value explicitly by using a type cast, as we explain next.

Type Casting

A.21 A **type cast** is the changing of the type of a value to some other type, such as changing the type of 2.0 from `double` to `int`. The previous segment described when a change in type is done for you automatically. In all other cases, if you want to assign a value of one type to a variable of another type, you must perform a type cast. For example, you cannot simply assign a value of type `double` to a variable of type `int`, even if the value of type `double` happens to have all zeros after the decimal point and so is conceptually a whole number. Thus, the second of the following statements is illegal:

```
double distance = 9.0;
int points = distance; // ILLEGAL
```

[1] Readers who have used certain other languages, such as C or C++, may be surprised to learn that we cannot assign a value of type `char` to a variable of type `byte`. This is because Java uses the Unicode character set rather than the ASCII character set, and so Java reserves two bytes of memory for each value of type `char`, but naturally reserves only one byte of memory for values of type `byte`. This is one of the few cases where we might notice that Java uses the Unicode character set. Indeed, if we convert from an `int` to a `char` or vice versa, we can expect to get the usual correspondence of ASCII numbers and characters.

To cast the type of distance to int, you enclose int within parentheses and place it in front of distance. For example, we would replace the preceding illegal assignment with

```
int points = (int)distance; // casting from double to int
```

Note that when you type-cast from any floating-point type to any integer type, the value is not rounded. The part after the decimal point is simply discarded, or **truncated**. For example, if the variable distance contains 25.86, (int)distance has an int value of 25. A type cast does not really change the value of a variable; distance is still 25.86, but points is 25.

Recall that when you assign an integer value to a variable of a floating-point type, the type cast is done automatically for you.

Note: When casting, some programmers place a space before the variable, as in (int) sum. We prefer to treat casting much like a minus sign. Just as we write minus five as –5, we cast sum to an integer by writing (int)sum.

Arithmetic Operators and Expressions

A.22 In Java, you perform arithmetic by using the **arithmetic operators** +, -, *, /, and %. You combine variables and constants with these operators and parentheses to form an **arithmetic expression**. The variables and constants in an expression are called **operands**. Spaces around the operators, operands, and parentheses within an expression are ignored.

A **unary operator** is one that has only one operand, like the operator - in the assignment statement

```
bankBalance = -cost;
```

A **binary operator** has two operands, like the operators + and * in

```
total = cost + (tax * discount);
```

Note that the operators - and + can be used as both unary and binary operators.

The meaning of an expression is basically what you expect it to be, but there are some subtleties about the type of the result and occasionally even about the value of the result. The type of the value produced when an expression is evaluated depends on the types of the values being combined. Consider an expression with only two operands, such as

```
amount - adjustment
```

If both amount and adjustment are of type int, the result of the subtraction has type int. If either amount or adjustment, or both, is of type double, the result is of type double. If we replace the operator - with any of the operators +, *, /, or %, the type of the result is determined in the same way. However, the operator % is typically used with integers, as you will see soon.

Larger expressions using more than two operands are viewed as a series of steps, each of which involves only two operands. For example, to evaluate the expression

```
balance + (balance * rate)
```

we evaluate balance * rate and obtain a number, and then we add that number to balance. Thus, if balance is int and rate is double, balance * rate is double and so is the entire expression.

Note: If all the items in an arithmetic expression have the same type, the result has that type. If at least one of the items has a floating-point type, the result has a floating-point type.

Knowing whether the value produced has an integer type or a floating-point type is typically all that you need. However, if you need to know the exact type of the value produced by an arithmetic expression, you can use the following rule:

Note: The data type of an arithmetic expression's value matches the most complex, or widest, data type among the operands in the expression. In other words, the data type matches the type that appears rightmost in the following list:

```
byte → short → int → long → float → double
```

For example, if `sum` is `float` and `more` is `int`, `sum + more` is `float`.

A.23 The division operator `/` deserves special attention, because the type of its operands can affect the value produced in a dramatic way. When you combine two numbers with the division operator `/` and at least one of the numbers has a floating-point type, the result has a floating-point type. For example, `9.0 / 2` has one operand of type `double`, namely `9.0`. Hence, the result is the type `double` number 4.5. However, when both operands have an integer type, the result can be surprising. For example `9 / 2` has two operands of type `int`, and so it yields the result 4 of type `int`, not 4.5. The fraction after the decimal point is simply lost. When you divide two integers, the result is truncated, *not* rounded. The part after the decimal point is discarded no matter how large it is. So, `11 / 3` is 3, not 3.6666 . . . . If nothing but a zero is after the decimal point, that decimal point and zero are still lost. Even this seemingly trivial difference can be of some significance. For example, `8.0 / 2` has the value 4.0 of type `double`, which technically is only an approximate quantity. However, `8 / 2` has the `int` value 4, which is an exact quantity. The approximate nature of 4.0 can affect the accuracy of any further calculation that is performed with this result.

Often, the `%` operator has operands only of integer types. You use it to recover the equivalent of the fraction after the decimal point. When you divide one integer by another, you get a result (which some call a quotient) and a remainder. For example, 14 divided by 4 yields 3 with a remainder of 2 (or with 2 left over). The `%` operation gives the remainder—that is, the amount left over after doing the division. So `14 / 4` is 3 and `14 % 4` is 2, because 14 divided by 4 is 3 with 2 left over. The `%` operator is called the **remainder operator**.

The `%` operator has more applications than you might at first suspect. It allows your program to count by 2s, 3s, or any other number. For example, if you want to do something to every other integer, you need to know whether the integer is even or odd. An integer `n` is even if `n % 2` is zero, and it is odd if `n % 2` is not zero. Similarly, if you want your program to do something to every third integer, you test whether the integer `n` is divisible by 3. It will be if `n % 3` is zero.

Programming Tip: To make your arithmetic expressions more readable, place a space on both sides of each binary operator.

Parentheses and Precedence Rules

A.24 You can use parentheses to group portions of an arithmetic expression in the same way that you use parentheses in algebra and arithmetic. With the aid of parentheses, you can indicate which operations are performed first, second, and so forth. For example, consider the following two expressions that differ only in the positioning of their parentheses:

```
(cost + tax) * discount
cost + (tax * discount)
```

To evaluate the first expression, the computer first adds `cost` and `tax` and then multiplies the result by `discount`. To evaluate the second expression, it multiplies `tax` by `discount` and then adds the result to `cost`. If you use some numbers for the values of the variables and carry out the two evaluations, you will see that they produce different results.

If you omit parentheses, as in the assignment statement

```
total = cost + tax * discount;
```

multiplication occurs before addition. Thus, the previous statement is equivalent to

```
total = cost + (tax * discount);
```

More generally, when the order of operations is not determined by parentheses, the operations occur in an order determined by the following **precedence rules**:

Note: Precedence of arithmetic operators
Arithmetic operators in an expression execute in the following order:
The unary operators `+`, `-`
The binary operators `*`, `/`, `%`
The binary operators `+`, `-`

Operators that are listed higher on the list are said to have **higher precedence**. Operators of higher precedence execute before operators of lower precedence, unless parentheses override this order. Operators at the same level have the same precedence. When two operators have equal precedence, the operations are performed using this convention:

Note: Binary operators of equal precedence in an expression are performed in left-to-right order.

Increment and Decrement Operators

A.25 The increment and decrement operators increase or decrease the value of a variable by 1. The **increment operator** is written as two plus signs, `++`. For example, the following Java statement will increase the value of the variable `count` by 1:

```
count++;
```

If the variable `count` has the value 5 before this statement is executed, it will have the value 6 after the statement is executed. Thus, this statement is equivalent to

```
count = count + 1;
```

You can use the increment operator with variables of any numeric type, but it is used most often with variables of an integer type such as `int`.

The **decrement operator** is similar, except that it subtracts 1 rather than adds 1 to the value of the variable. The decrement operator is written as two minus signs, `--`. For example, the following will decrease the value of the variable `count` by 1:

```
count--;
```

If the variable `count` has the value 5 before this statement is executed, it will have the value 4 after the statement is executed. This statement is equivalent to

```
count = count - 1;
```

A.26 You can use the increment and decrement operators within expressions, but when you do, the increment operator or the decrement operator changes the value of the variable it is applied to and returns a value. Although we do not recommend using the increment and decrement operators in expressions, you should be familiar with them used in this way, because you might see this use in other people's code.

In expressions, you can place the ++ or -- either before or after a variable, but your choice affects the result. For example, consider the code

```
int n = 3;
int m = 4;
int result = n * (++m);
```

After this code executes, the value of n is unchanged at 3, the value of m is 5, and the value of result is 15. Thus, ++m changes the value of m and returns that changed value to the multiply operator.

In the previous example, we placed the increment operator before the variable m. If we place it after the variable m, something slightly different happens. Consider the code

```
int n = 3;
int m = 4;
int result = n * (m++);
```

Now, after the code executes, the value of n is 3 and the value of m is 5, just as in the previous example. However, the value of result is 12, not 15. What happened?

The two expressions n * (++m) and n * (m++) both increase the value of m by 1, but the first expression increases the value of m *before* it does the multiplication, whereas the second expression increases the value of m *after* it does the multiplication. Both ++m and m++ have the same effect on the final value of m, but when we use them as part of an arithmetic expression, they give a different value to the expression.

Similarly, both --m and m-- have the same effect on the final value of m, but when we use them as part of an arithmetic expression, they give a different value to the expression. If the -- is *before* the m, the value of m is decreased *before* its value is used in the expression. If the -- is *after* the m, the value of m is decreased *after* its value is used in the expression.

The increment and decrement operators can be applied only to variables. They cannot be applied to constants or to entire, more complicated arithmetic expressions.

Programming Tip: To avoid errors and confusing code, use the operator ++ or -- only in a statement that involves one variable and no other operators.

Special Assignment Operators

A.27 You can combine the simple assignment operator (=) with an arithmetic operator, such as +, to produce a kind of special-purpose assignment operator. For example, the following will increase the value of the variable amount by 5:

```
amount += 5;
```

This is really just a shorthand for

```
amount = amount + 5;
```

You can do the same thing with any of the other arithmetic operators -, *, /, and %. For example, the statement

```
amount *= 25;
```

is equivalent to

```
amount = amount * 25;
```

Named Constants

A.28 You probably recognize the number 3.14159 as the approximate value of *pi,* the number that is used in many circle calculations and that is often written as π. However, when you see 3.14159, you might not be sure that it is π and not some other number; somebody other than you might have no idea of where the number 3.14159 came from. To avoid such confusion, you should always give a name to constants, such as 3.14159, and use the name instead of writing out the number. For example, we might give the number 3.14159 the name `PI`. Then the assignment statement

```
area = 3.14159 * radius * radius;
```

could be written more clearly as

```
area = PI * radius * radius;
```

How do you give a number, or other constant, a name like `PI`? You could use a variable named `PI` and initialize it to the desired value 3.14159. But then you might inadvertently change the value of this variable. However, Java provides a mechanism that allows you to define and initialize a variable and moreover fix the variable's value so it cannot be changed. The syntax is

```
public static final type  name = constant;
```

For example, the statement

```
public static final double PI = 3.14159;
```

gives the name `PI` to the constant 3.14159. The part

```
double PI = 3.14159;
```

simply declares `PI` as a variable and initializes it to 3.14159. The word `public` says that there are no restrictions on where we can use the name `PI`. The word `static` defines one copy of `PI` that every object of the class can access instead of having its own copy of `PI`. The word `final` means that the value 3.14159 is the *final* value assigned to `PI` or, to phrase it another way, it means that the program cannot change the value of `PI`. Appendix B provides more details about `static` and `final`.

It is a good practice to place named constants near the beginning of a class and outside of any method definitions. That way, your named constants are handy in case you need to modify them. You might, for example, want to change the number of digits you provide for a constant.

Note: The class `Math` in the Java Class Library defines a static named constant `PI` just as we did in this segment, but with more decimal places. The following segment describes this class and shows how to access `PI`. You should use `Math`'s `PI` instead of defining your own.

Programming Tip: Programmers typically use all uppercase letters when naming constants to distinguish constants from ordinary variables. They use an underscore as a separator in multiword names. For example, `FEET_PER_MILE` follows this convention.

The Class `Math`

A.29 The class `Math` in the package `java.lang` of the Java Class Library provides a number of standard mathematical methods. These methods are static methods. (Segment B.28 of Appendix B discusses static methods in more detail.) When you invoke a static method, you write the class name—`Math`, in this case—a dot, the name of the method, and a pair of parentheses. Using the name of a class to invoke a method is not typical. Ordinarily, you use the name of an object to invoke a method.

Most `Math` methods require that you specify items within the pair of parentheses. As we noted earlier in this appendix, these items are called arguments to the method. Thus, a typical invocation of a method in this class has the form `Math.`*method_name*`(`*arguments*`)`.

You can invoke the method in an assignment statement, such as

variable `= Math.`*method_name*`(`*arguments*`);`

or embed it within an arithmetic expression. That is, you can use `Math.`*method_name*`(`*arguments*`)` anyplace that you can use a variable of a primitive data type. Figure A-2 describes some of the available methods in this class.

The class `Math` also has two predefined named constants. `E` is the base of the natural logarithm system—often written *e* in mathematical formulas—and is approximately 2.72. `PI` is used in calculations involving circular geometric figures—often written π in mathematical formulas—and is approximately 3.14159. Because these constants are defined in the class `Math`, you use them by writing `Math.E` and `Math.PI`.

Simple Input and Output Using the Keyboard and Screen

The input and output of data is usually referred to as **I/O**. A Java program can perform I/O in many different ways. In this section, we present some ways to handle simple text input that we type at the keyboard and simple text output displayed on the screen.

Screen Output

A.30 Statements like

```
System.out.println("Enter a whole number from 1 to 99.");
```

and

```
System.out.println(quarters + " quarters");
```

send output to the display screen. As we mentioned near the beginning of this appendix, `System.out` is an object within the class `System`, which is a class in the Java Class Library. This object has `println` as one of its methods. So the preceding output statements are calls to the method `println` of the object `System.out`. You simply follow `System.out.println` with a pair of parentheses that contain what you want to display. You end the statement with a semicolon.

Within the parentheses can be strings of text in double quotes, like `"Enter a whole number from 1 to 99."`, variables like `quarters`, numbers like `5` or `7.3`, and almost any other object or value. To display more than one thing, simply place a plus sign between them. For example,

```
System.out.println("Lucky number = " + 13 +
                   "Secret number = " + number);
```

FIGURE A-2 Some methods in the class `Math`

In each of the following methods, the argument and the return value are double:

`Math.cbrt(x)`	Returns the cube root of x.
`Math.ceil(x)`	Returns the nearest whole number that is $\geq x$.
`Math.cos(x)`	Returns the trigonometric cosine of the angle x in radians.
`Math.exp(x)`	Returns e^x.
`Math.floor(x)`	Returns the nearest whole number that is $\leq x$.
`Math.hypot(x, y)`	Returns the square root of the sum $x^2 + y^2$.
`Math.log(x)`	Returns the natural (base e) logarithm of x.
`Math.log10(x)`	Returns the base 10 logarithm of x.
`Math.pow(x, y)`	Returns x^y.
`Math.random( )`	Returns a random number that is ≥ 0 but < 1.
`Math.sin(x)`	Returns the trigonometric sine of the angle x in radians.
`Math.sqrt(x)`	Returns the square root of x, assuming that $x \geq 0$.
`Math.tan(x)`	Returns the trigonometric tangent of the angle x in radians.
`Math.toDegrees(x)`	Returns an angle in degrees equivalent to the angle x in radians.
`Math.toRadians(x)`	Returns an angle in radians equivalent to the angle x in degrees.

In each of the following methods, the argument and the return value have the same type–
either `int,long,float,` or `double:`

`Math.abs(x)`	Returns the absolute value of x.
`Math.max(x, y)`	Returns the larger of x and y.
`Math.min(x, y)`	Returns the smaller of x and y.

`Math.round(x)`	Returns the nearest whole number to x. If x is `float`, returns an `int`; if x is `double`, returns a `long`.

If the value of `number` is 7, the output will be

```
Lucky number = 13Secret number = 7
```

Notice also that no spaces are added. If we want a space between the 13 and the word "Secret" in the preceding output—and we probably do—we should add a space to the beginning of the string `"Secret number = "` so that it becomes `" Secret number = "`.

Notice that you use double quotes, not single quotes, and that the opening and closing quotes are the same symbol. Finally, notice that you can place the statement on two lines if it is too long.

However, you should break the line before or after a + sign, not in the middle of a quoted string or a variable name. You also should indent the second line to make the entire statement easier to read.

Later, in the section about the class String, you will see that the + operator joins, or **concatenates**, two strings. In the preceding System.out.println statement, Java converts the number 13 to the string "13". Likewise, it converts the integer 7 in the variable number to the string "7". Then the + operator joins the strings and the System.out.println statement displays the result. You do need to be a bit careful, however. If you write a + between two numeric values or variables, they will be added rather than concatenated.

You can also use the println method to display the value of a String variable, as illustrated by the following:

```
String greeting = "Hello Programmers!";
System.out.println(greeting);
```

This will cause the following to be written on the screen.

```
Hello Programmers!
```

A.31 Every invocation of println ends a line of output. If you want the output from two or more output statements to appear on a single line, use print instead of println. For example,

```
System.out.print("One, two,");
System.out.print(" buckle my shoe.");
System.out.println(" Three, four,");
System.out.println("shut the door.");
```

will produce the following output:

```
One, two, buckle my shoe. Three, four,
shut the door.
```

Notice that a new line is not started until you use println, rather than print. Also notice that the new line starts *after* displaying the items specified in the println statement. This is the only difference between print and println.

Keyboard Input Using the Class Scanner

A.32 A Java program can read data from either the keyboard or another source such as a disk, and place it into memory. The Java Class Library provides the class Scanner for this purpose. Here, we will use the methods in Scanner to read data typed at the keyboard and place it into variables that we specify.

As we noted earlier, the class Scanner is in the package java.util. To use Scanner in your program, you must **import** it from this package by writing the following import statement before the rest of your program:

```
import java.util.Scanner;
```

Before you can use any of the methods in Scanner, you must create a Scanner object by writing a statement such as

```
Scanner keyboard = new Scanner(System.in);
```

The variable keyboard—which could be any variable of your choosing—is assigned a Scanner object that is associated with the input device that System.in represents. This device by convention is the keyboard. The variable keyboard has a class type.

A.33 `Scanner` provides several methods that read input data. You can use any of these methods by writing a statement that has the following form, where `keyboard` is the `Scanner` object that we defined previously:

```
variable = keyboard.method_name();
```

The named method reads a value from the keyboard and returns it. That is, the expression `keyboard.`*method_name*`()` represents the value that was read. The previous statement then assigns this value to the indicated variable.

You can read integers and real numbers by using the following expressions:

`keyboard.nextInt()`— Returns the next integer encountered in the input data.

`keyboard.nextDouble()`— Returns the next real number encountered in the input data.

Each of these expressions ignores any **white space** that might precede or follow the number typed at the keyboard. Whitespace characters are the characters that appear as spaces when printed on paper or displayed on the screen. The blank-space character is likely the only white space character that will concern us at first, but the start of a new line and the tab symbol are also white space characters.

A.34 **Example.** To read an integer from the keyboard, you can write a statement such as

```
size = keyboard.nextInt();
```

where `size` has been declared previously as an `int` variable. The user of your program would type an integer and press the Enter, or Return, key. The value is read by the method `nextInt`, returned, and assigned to the variable `size`.

Typically, you should display a message, or **prompt**, for the user to enter data. For example, your program might contain the following statements:

```
System.out.println("What is your age?");
int age = keyboard.nextInt();
```

Whatever the user types appears in the same window as the prompt. Here, the prompt would appear on one line and the user would type his or her age on the next.

A.35 **Example.** To read a real number from the keyboard, you can write statements such as

```
System.out.print("Enter the area of your room in square feet: ");
double area = keyboard.nextDouble();
```

After the user types a real number at the keyboard and presses the Enter key, the value of the number is assigned to the variable `area`. Since we have used `print` instead of `println`, both the prompt and the input data appear on the same line on the display.

A.36 **Example.** You can read more than one value per line of input. For example,

```
System.out.println("Please enter your height in feet and inches:");
int feet = keyboard.nextInt();
int inches = keyboard.nextInt();
```

The user could type either

```
6  2
```

on one line or

```
6
2
```

on two lines. In either case, `feet` is 6 and `inches` is 2.

Note: Streams

The characters that a user types at the keyboard are directed into the memory assigned to your program by an object known as an **input stream**. The name of the input stream associated with the keyboard is `System.in`. Likewise, an **output stream** is an object that directs data from your program to an output device. `System.out` is such an object, directing characters to a display.

A.37 **More input methods.** The class `Scanner` includes the following method to read a string:

`nextLine()`— Returns the string that appears next in the input data.

For example, if `keyboard` is defined as shown earlier, the statement

```
String message = keyboard.nextLine();
```

reads the entire string that the user types before pressing the Enter key—including any spaces—and assigns it to the variable `message`.

The method `next` in the class `Scanner` reads the next group of contiguous characters that are not white space and returns it as a string. For example, you can use this method to read the next word that appears in the input data, as follows:

```
String word = keyboard.next();
```

We used `next` in Segment A.3 to read the user's yes or no response.

The `Scanner` methods we've looked at—`nextInt`, `nextDouble`, `nextLine`, and `next`—often are invoked in simple assignment statements, although that is not necessary. Since each method returns a value, you can call it within an arithmetic expression, for example.

The `if-else` Statement

A.38 In programs, as in everyday life, things can sometimes go in one of two different ways. If you have money in your checking account, some banks will pay you a little interest. On the other hand, if you have overdrawn your checking account, you will be charged a penalty. This might be reflected in the bank's accounting program by the following Java statement, known as an **`if-else` statement**:

```
if (balance >= 0)
   balance = balance + (INTEREST_RATE * balance) / 12;
else
   balance = balance - OVERDRAWN_PENALTY;
```

The two-symbol operator >= means "greater than or equal to" in Java. We use two symbols because the one-character symbol ≥ is not on the keyboard.

The meaning of an `if-else` statement is really just the meaning it would have if read as an English sentence. When your program executes an `if-else` statement, it first checks the expression in parentheses after the `if`. This expression must evaluate to either true or false. If it is true, the statement after the `if` is executed. If the expression is false, the statement after the `else` is executed. In the preceding example, if `balance` is positive or zero, the following action occurs:

```
balance = balance + (INTEREST_RATE * balance) / 12;
```

(We divide by 12 because the interest is for only 1 of 12 months.) On the other hand, if the value of `balance` is negative, the following is executed instead:

```
balance = balance - OVERDRAWN_PENALTY;
```

The indentation in the `if-else` statement is conventional as an aid in reading the statement; it does not affect the statement's meaning.

A.39 If you want to include more than one statement in either of the two portions of the `if-else` statement, you simply enclose the statements in braces, as in the following example:

```
if (balance >= 0)
{
   System.out.println("Good for you. You earned interest.");
   balance = balance + (INTEREST_RATE * balance) / 12;
}
else
{
   System.out.println("You will be charged a penalty.");
   balance = balance - OVERDRAWN_PENALTY;
} // end if
```

When you enclose several statements within braces, you get one larger statement called a **compound statement**. Compound statements are seldom used by themselves but often are used as substatements of a larger statement such as an `if-else` statement.

Programming Tip: Some programmers always use compound statements within other statements such as `if-else`, even when only a single statement appears between the braces. Doing so makes it easier to add another statement to the compound statement, but more importantly, it avoids the error that would occur if you forgot to add the braces. We encourage you to follow this convention, even though we do not always do so in this book to save space.

A.40 You can omit the `else` part. If you do, nothing happens when the tested expression is false. For example, if your bank does not charge an overdraft penalty, the statement would be the following, instead of the previous one:

```
if (balance >= 0)
{
   System.out.println("Good for you. You earned interest.");
   balance = balance + (INTEREST_RATE * balance) / 12;
} // end if
```

If `balance` is negative, the statement after the closing brace executes next.

Boolean Expressions

A.41 A **boolean expression** is an expression that is either true or false. The expression

```
balance >= 0
```

that we used in the previous `if-else` statement is an example of a simple boolean expression. Such expressions compare two things, like numbers, variables, or other expressions. Figure A-3 shows the various Java **comparison operators** you can use to compare two expressions.

FIGURE A-3 Java comparison operators

Math Notation	Name	Java Operator	Java Examples
≥	Greater than or equal to	`>=`	`points >= 60`
≤	Less than or equal to	`<=`	`expenses <= income`
>	Greater than	`>`	`expenses > income`
<	Less than	`<`	`pressure < max`

=	Equal to	`==`	`balance == 0`
			`answer == 'y'`

≠	Not equal to	`!=`	`income != tax`
			`answer != 'y'`

A.42 **Logical operators.** Often, when you write an if-else statement, you will want to use a boolean expression that is more complicated than a simple comparison. You can form more-complicated boolean expressions from simpler ones by joining expressions with either the Java version of "and," which is &&, or the Java version of "or," which is ||. For example, consider the following:

```
if ((pressure > min) && (pressure < max))
   System.out.println("Pressure is OK.");
else
   System.out.println("Warning: Pressure is out of range.");
```

If the value of pressure is greater than min, *and* the value of pressure is less than max, the output will be

```
Pressure is OK.
```

Otherwise, the output is

```
Warning: Pressure is out of range.
```

Note that you *cannot* use a string of inequalities in Java, like the following:

```
min < pressure < max
```
← Illegal!

Instead, you must express each inequality separately and connect them with &&, as follows:

```
(pressure > min) && (pressure < max)
```

The parentheses in the previous expression are not necessary, but we typically include them. The parentheses that surround the entire expression in an if-else statement are required, however.

The binary operators && and || together with the unary operator ! are **logical operators**. We look at each of them next.

A.43 **The operator &&.** When you form a larger boolean expression by connecting two smaller expressions with &&, the entire larger expression is true provided that both of the smaller expressions are true. Thus, if at least one of pressure > min and pressure < max is false, the larger expression is false. Moreover, if the first part of the larger expression is false, the second part is ignored, since the larger expression must be false regardless of the value of the second part. For example, if pressure is less than min, we know that

```
(pressure > min) && (pressure < max)
```

is false without looking at pressure < max.

A.44 **The operator `||`.** You also can use `||` to form a larger boolean expression from smaller ones in the same way that you use `&&`, but with different results. The meaning is essentially the same as the English word "or." For example, consider

```
if ((salary > expenses) || (salary + savings > expenses))
   System.out.println("Solvent");
else
   System.out.println("Bankrupt");
```

If the value of `salary` is greater than the value of `expenses` *or* the value of `salary + savings` is greater than the value of `expenses`—or both—the output will be

```
Solvent
```

Otherwise, the output will be

```
Bankrupt
```

The entire larger expression is true if either of the smaller expressions is true. Moreover, if the first part of the larger expression is true, the second part is ignored, since the larger expression must be true regardless of the value of the second part. For example, if `salary` is greater than `expenses`, we know that

```
(salary > expenses) || (salary + savings > expenses)
```

is true without looking at `salary + savings > expenses`.

You use parentheses in expressions containing the `||` operator in the same way that you use them with `&&`.

Note: Short-circuit evaluation

When two boolean expressions are joined by either `&&` or `||`, the second expression is not evaluated if the value of the first expression implies the value of the entire expression. Such is the case if the first expression is false when the operator is `&&` or true when the operator is `||`. This behavior is known as the **short-circuit evaluation** of a boolean expression.

Besides saving execution time, short-circuit evaluation can prevent execution errors. For example, the following statement prevents a division by zero:

```
if ((count != 0) && (sum / count > minimum))
```

If `count` is zero, the expression `count != 0` is false. Thus, the expression `sum / count > minimum` is not evaluated, thereby avoiding the erroneous division.

A.45 **The operator `!`.** You can negate a boolean expression by preceding it with the operator `!`. For example, the expression

```
!(number >= min)
```

has the same meaning as the expression

```
number < min
```

In this case, you can and should avoid using `!`.

Sometimes, however, the use of `!` makes perfect sense. For example, if you have two strings that should be the same for normal processing to continue, you would compare them and issue a warning if they are not equal. Later, in the section about the class `String`, you will see that you use the `equals` method to compare two strings. For example,

```
stringOne.equals(stringTwo)
```

is true if the strings `stringOne` and `stringTwo` are equal. But if we want to know whether these strings are not equal, we could write

```
if (!stringOne.equals(stringTwo))
  System.out.println("Warning: The strings are not the same.");
```

The precedence of the boolean operators in relation to each other and to the arithmetic operators follows:

Note: Precedence of a selection of Java operators
Operators in the same expression execute in the following order:
The unary operators +, -, !
The binary arithmetic operators *, /, %
The binary arithmetic operators +, -
The comparison operators <, >, <=, >=
The comparison operators ==, !=
The logical operator &&
The logical operator ||

Nested Statements

A.46 Notice that an if-else statement contains smaller statements within it. These smaller statements can be any sort of Java statements. In particular, you can use one if-else statement within another if-else statement to get **nested** if-else statements, as illustrated by the following:

```
if (balance >= 0)
   if (INTEREST_RATE >= 0)
      balance = balance + (INTEREST_RATE * balance) / 12;
   else
      System.out.println("Cannot have a negative interest.");
else
   balance = balance - OVERDRAWN_PENALTY;
```

If the value of balance is greater than or equal to zero, the entire following if-else statement is executed:

```
if (INTEREST_RATE >= 0)
   balance = balance + (INTEREST_RATE * balance) / 12;
else
   System.out.println("Cannot have a negative interest.");
```

When writing nested if-else statements, you may sometimes become confused about which if goes with which else. To eliminate this confusion, you can add braces as follows:

```
if (balance >= 0)
{
   if (INTEREST_RATE >= 0)
      balance = balance + (INTEREST_RATE * balance) / 12;
   else
      System.out.println("Cannot have a negative interest.");
}
else
   balance = balance - OVERDRAWN_PENALTY;
```

Here, the braces are an aid to clarity but are not, strictly speaking, needed. In other cases, they are needed. While you should use indentation to indicate your intentions, remember that it is ignored by the compiler.

A.47 If you omit an else, things get a bit trickier. The following two if-else statements differ only in that one has a pair of braces, but they do not have the same meaning:

```
// First Version
if (balance >= 0)
{
   if (INTEREST_RATE >= 0)
      balance = balance + (INTEREST_RATE * balance) / 12;
}
else
   balance = balance - OVERDRAWN_PENALTY;

// Second Version
if (balance >= 0)
   if (INTEREST_RATE >= 0)
      balance = balance + (INTEREST_RATE * balance) / 12;
else
   balance = balance - OVERDRAWN_PENALTY;
```

In the second version, without braces, the `else` is paired with the second `if`, not the first one, as the indentation leads us to believe. Thus, the meaning is

```
// Equivalent to Second Version
if (balance >= 0)
{
   if (INTEREST_RATE >= 0)
      balance = balance + (INTEREST_RATE * balance) / 12;
   else
      balance = balance - OVERDRAWN_PENALTY;
}
```

To clarify the difference a bit more, consider what happens when `balance` is less than zero. The first version causes the following action:

```
balance = balance - OVERDRAWN_PENALTY;
```

However, the second version takes no action.

Note: In an `if-else` statement, each `else` is paired with the nearest previous unmatched `if`.

Programming Tip: Indentation within an `if-else` statement does not affect the action of the statement. For clarity, you should use indentation that matches the logic of the statement.

Multiway `if-else` Statements

A.48 Since an `if-else` statement has two outcomes, and each of these two outcomes can have an `if-else` statement with two outcomes, you can use nested `if-else` statements to produce any number of possible effects. Convention provides a standard way of doing this. Let's start with an example.

Suppose `balance` is a variable that holds your checking account balance and you want to know whether your balance is positive, negative (overdrawn), or zero. To avoid any questions about accuracy, let's assume that `balance` is of type `int`—that is, `balance` is the number of dollars in your account, with the cents ignored. To find out if your balance is positive, negative, or zero, you could use the following nested `if-else` statement:

```
if (balance > 0)
   System.out.println("Positive balance");
else if (balance < 0)
```

```
      System.out.println("Negative balance");
   else if (balance == 0)
      System.out.println("Zero balance");
```

This is really an ordinary nested `if-else` statement, but it is indented differently than before. The indentation reflects the logic more clearly and is preferred. Although this is not a separate kind of `if-else` statement, we call this nested construction a **multiway `if-else` statement**.

When a multiway `if-else` statement is executed, the computer tests the boolean expressions one after the other, starting from the top. When it finds the first true boolean expression, it executes the statement after the expression. The rest of the `if-else` statement is ignored. For example, if `balance` is greater than zero, the preceding statements will display

```
   Positive balance
```

Exactly one of the three possible messages will be displayed, depending on the value of the variable `balance`.

A.49 The previous example has three possibilities, but you can have any number of possibilities by adding more `else-if` parts. In this example, the possibilities are **mutually exclusive**. That is, only one of the three possibilities can actually occur for any given value of `balance`. However, you can use any boolean expressions, even if they are not mutually exclusive. If more than one boolean expression is true, only the action associated with the first true boolean expression is executed. A multiway `if-else` statement never performs more than one action.

If none of the boolean expressions is true, nothing happens. However, it is a good practice to add an `else` clause—without any `if`—at the end, to be executed in case none of the boolean expressions is true. In fact, we can rewrite our previous example in this way. We know that if `balance` is neither positive nor negative, it must be zero. So we do not need the test

```
   if (balance == 0)
```

Thus, we can and should write the previous `if-else` statement as

```
   if (balance > 0)
      System.out.println("Positive balance");
   else if (balance < 0)
      System.out.println("Negative balance");
   else
      System.out.println("Zero balance");
```

The Conditional Operator *(Optional)*

A.50 To allow compatibility with older programming styles, Java includes an operator that is a notational variant on certain forms of the `if-else` statement. A **conditional operator expression** consists of a boolean expression followed by a question mark and two expressions separated by a colon. For example, the expression on the right side of the assignment operator in the following statement is a conditional operator expression:

```
   max = (n1 > n2) ? n1 : n2;
```

The ? and : together form a **ternary operator** that has three operands and is known as the **conditional operator**. If the boolean expression is true, the value of the first of the two expressions is returned; otherwise, the value of the second of the two expression is returned. Thus, the logic of this example is equivalent to

```
   if (n1 > n2)
      max = n1;
   else
      max = n2;
```

This book will not use conditional operator expressions, as they are less clear than equivalent if-else statements. If you decide to use them in your program, realize that not everyone will know their meaning.

The switch Statement

A.51 Multiway if-else statements can become unwieldy when you must choose from among many possible courses of action. If the choice is based on the value of an integer or character expression, the **switch statement** can make your code easier to read.

The switch statement begins with the word switch followed by an expression in parentheses. This expression is called the **controlling expression**. Its value must be of type int, char, byte, short or, as of Java 7, String. The switch statement in the following example determines the price of a ticket according to the location of the seat in a theater. An integer code that indicates the seat location is the controlling expression:

```
int seatLocationCode;
< Code here assigns a value to seatLocationCode >
. . .
double price = -0.01;
switch (seatLocationCode)
{
   case 1:
      System.out.println("Balcony.");
      price = 15.00;
      break;
   case 2:
      System.out.println("Mezzanine.");
      price = 30.00;
      break;
   case 3:
      System.out.println("Orchestra.");
      price = 40.00;
      break;
   default:
      System.out.println("Unknown ticket code.");
      break;
} // end switch
```

The switch statement contains a list of cases, each consisting of the reserved word case, a constant, a colon, and a list of statements that are the actions for the case. The constant after the word case is called a **case label**. When the switch statement executes, the controlling expression—in this example, seatLocationCode—is evaluated. The list of alternative cases is searched until a case label that matches the current value of the controlling expression is found. Then the action associated with that label is executed. You are not allowed to have duplicate case labels, as that would be ambiguous.

If no match is found, the case labeled default is executed. The default case is optional. If there is no default case, and no match is found to any of the cases, no action takes place. Although the default case is optional, we encourage you to always use it. If you think your cases cover all the possibilities without a default case, you can insert an error message or an assertion as the default case. You never know when you might have missed some obscure case.

Notice that the action for each case in the previous example ends with a **break statement**. If you omit the break statement, the action just continues with the statements in the next case until it reaches either a break statement or the end of the switch statement. Sometimes this feature is desirable, but sometimes omitting the break statement causes unwanted results.

Note: The controlling expression in a switch statement provides an entry point to a case within the statement. Execution continues from this point until it reaches either a break statement or the end of the switch.

A.52 At times, you will want to take the same action in more than one case. You can list cases one after the other so that they all apply to the same action. In the following example, we have changed the seat location code to a character instead of an integer. A code of *B* or *b*, for example, indicates a balcony seat:

```
char seatLocationCode;
< Code here assigns a value to seatLocationCode >
. . .
double price = -0.01;
switch (seatLocationCode)
{
   case 'B':
   case 'b':
      System.out.println("Balcony.");
      price = 15.00;
      break;
   case 'M': case 'm':
      System.out.println("Mezzanine.");
      price = 30.00;
      break;
   case 'O': case 'o':
      System.out.println("Orchestra.");
      price = 40.00;
      break;
   default:
      System.out.println("Unknown ticket code.");
      break;
} // end switch
```

The first case, *B*, has no break statement; in fact, the case has no action statements at all. Execution continues with the case for *b*, as desired. Note that we have written the cases in two ways to show two common programming styles.

The controlling expression in a switch statement need not be a single variable. It can be a more complicated expression, but it must evaluate to a single value. The expression cannot indicate a range of values. That is, the expression cannot be a boolean expression like the ones you use in an if-else statement. Thus, if you want to take one action when the controlling expression has values from 1 to 10 and a second action for values from 11 to 20, you would need a case label for each value. In situations like this, a switch statement would be harder to write than an if-else statement.

Programming Tip: Omitting a break statement

If you test a program that contains a switch statement and it executes two cases when you expect it to execute only one case, you probably have forgotten to include a break statement where one is needed.

A.53 Our last example of a switch statement revises the previous ones to use strings as the case labels:

```
String seatLocationCode;
```

```
< Code here assigns a value to seatLocationCode >
. . .
double price = -0.01;
if (seatLocationCode != null)
{
   switch (seatLocationCode)
   {
      case "balcony":
         System.out.println("Balcony.");
         price = 15.00;
         break;
      case "mezzanine":
         System.out.println("Mezzanine.");
         price = 30.00;
         break;
      case "orchestra":
         System.out.println("Orchestra.");
         price = 40.00;
         break;
      default:
         System.out.println("Unknown ticket code.");
         break;
   } // end switch
} // end if
```

Because strings are objects, you must be careful that the value of the controlling expression is not `null`, as we have done here. You should also take care that the possible strings assigned to `seatLocationCode` match the case labels exactly. In this example, the labels use lowercase letters. To ensure that the value of the controlling expression uses lowercase letters, we can replace the beginning of the `switch` statement with

```
switch (seatLocationCode.toLowercase())
```

Segment A.76 later in this appendix will describe the `String` method `toLowercase` further.

Enumerations

A.54 To compute a student's quality-point average—also known as a grade-point average—a program could assign the number of quality points for a given letter grade to the `double` variable `qualityPoints`. You could use a `char` variable `grade` for the letter grade, but then it could contain any character, not just the letters *A*, *B*, *C*, *D*, and *F*. Instead, to restrict the contents of `grade` to the values you specify, you could declare it as an **enumerated data type**, or **enumeration**. An enumeration itemizes the values that a variable can have.

For example, the following statement defines `LetterGrade` as an enumeration:

```
enum LetterGrade {A, B, C, D, F}
```

`LetterGrade` behaves as a class type, so we can declare `grade` to have this type, as follows:

```
LetterGrade grade;
```

The items listed between the braces in the definition of `LetterGrade` are objects that `grade` can reference. For example, you can write

```
grade = LetterGrade.A;
```

to assign `A` to `grade`. Assigning a value other than `A`, `B`, `C`, `D`, or `F` to `grade` will cause a syntax error. These values behave as static constants. You qualify each of them with the name of the enumeration just as you qualify the constant `PI` with the name of its class `Math`.

Note: An enumeration is actually a class. Therefore, you cannot define an enumeration within a method. Instead, define enumerations outside of any method definitions, preferably near the beginning of your class. Also, note that no semicolon follows an enumeration's definition. Writing one, however, will not cause a syntax error; the semicolon will simply be ignored.

A.55 **Example.** You can use a `switch` statement with a variable whose data type is an enumeration. For example, if we define `LetterGrade` and `grade` as in the previous segment, the following `switch` statement assigns the correct number of quality points to the `double` variable `qualityPoints`:

```
switch (grade)
{
   case A:
      qualityPoints = 4.0;
      break;
   case B:
      qualityPoints = 3.0;
      break;
   case C:
      qualityPoints = 2.0;
      break;
   case D:
      qualityPoints = 1.0;
      break;
   case F:
      qualityPoints = 0.0;
      break;
   default:
      qualityPoints = -9.0;
} // end switch
```

Since the data type of the expression in the `switch` statement is an enumeration, the case labels are assumed to belong to that enumeration without qualification. In fact, writing `case LetterGrade.A`, for example, is a syntax error. However, if you need to reference one of the enumerated values elsewhere within the `switch` statement, you must qualify it.

Since we know that `grade` cannot have values other than those in the enumeration, a default case is unnecessary. However, if you choose to omit the default case, you must assign a value to `qualityPoints` prior to the `switch` statement to avoid a syntax error. Without this initialization, the compiler would think it possible for `qualityPoints` to remain uninitialized after the `switch` statement.

A.56 When the compiler encounters an enumeration, it creates a class that has several methods. Among them is the method `ordinal`, which you can use to access the **ordinal value** of an object within an enumeration. These values begin at zero. Thus, in `LetterGrade`, the ordinal values of *A* and *F* are 0 and 4, respectively. For example, if you have the following assignment:

```
LetterGrade yourGrade = LetterGrade.A;
```

the expression

```
yourGrade.ordinal()
```

returns 0. Likewise, the expression

```
LetterGrade.B.ordinal()
```

returns 1.

The method `equals` tests whether `yourGrade` is equal to a given object within the enumeration. For example, you might write

```
if (yourGrade.equals(LetterGrade.A))
   System.out.println("Congratulations, your grade is A!");
```

Finally, the static method `valueOf` takes a string and returns a matching object in a given enumeration. For example, the expression

```
LetterGrade.valueOf("A")
```

returns `LetterGrade.A`. The string passed to `valueOf` must match the name of the constant exactly.

Appendix B discusses enumerations further, beginning at Segment B.30.

Scope

A.57 The **scope** of a variable (or a named constant) is the portion of a program in which the variable is available. That is, a variable does not exist outside of its scope. A variable's scope begins at its declaration and ends at the closing brace of the pair of braces that enclose the variable's declaration.

For example, consider the following statements that involve two variables, `counter` and `greeting`:

```
{
   // counter and greeting are not available here
   . . .
   int counter = 1;
   // counter is available here
   . . .
   {
      String greeting = "Hello!";
      // both greeting and counter are available here
      . . .
   } // end scope of greeting
   . . .
   // only counter is available here
   . . .
} // end scope of counter
```

The variable `counter` is available anywhere after its declaration. The variable `greeting` is available only within the inner pair of braces.

The concept of scope applies to every pair of braces within a Java program, regardless of whether they delineate the definition of a class or a method, appear within an `if-else` statement or `switch` statement, or appear within the loops described in the next section.

Loops

A.58 Programs often need to repeat some action. For example, a grading program would contain some code that assigns a letter grade to a student based on the student's scores on assignments and exams. To assign grades to the entire class, the program would repeat this action for each student in the class. A portion of a program that repeats a statement or group of statements is called a **loop**. The statement (or group of statements) to be repeated in a loop is called the **body** of the loop. Each repetition of the loop body is called an **iteration** of the loop.

When you design a loop, you need to decide what action the body of the loop should take and when the loop should stop repeating this action. Once you have made these choices, you can pick one of three Java statements to implement the loop: the `while` statement, the `for` statement, or the `do-while` statement.

The while Statement

A.59 One way to construct a loop in Java is with a **while statement**, which is also known as a **while loop**. A while statement repeats its action again and again until a controlling boolean expression becomes false. That is, the loop is repeated *while* the controlling boolean expression is true. The general form of a while statement is

```
while (expression)
   statement;
```

The while loop starts with the reserved word while followed by a boolean expression in parentheses. The loop body is a statement, typically a compound statement enclosed in braces {}. The loop body is repeated while the boolean expression is true. The loop body normally contains some action that can change the value of the boolean expression from true to false and so end the loop.

For example, the following while statement displays the integers from 1 to a given integer number:

```
int number;
. . . // assign a value to number here
int count = 1;
while (count <= number)
{
   System.out.println(count);
   count++;
} // end while
```

Let's suppose that number is 2. The variable count begins at 1. Since the boolean expression count <= number is true at this point, the body of the loop executes. Thus, 1 is displayed and then count becomes 2. The expression count <= number is still true, so the loop's body executes a second time, displaying 2 and incrementing count to 3. Now count <= number is false, so the while loop ends. Execution continues with the statement, if any, that follows the loop.

Notice that if number is zero or negative in the previous example, nothing is displayed. The body of the loop would not execute at all, since count, which is 1, would be greater than number.

! **Programming Tip: A while loop can perform zero iterations**

The body of a while loop can execute zero times. When a while loop executes, its first action is to check the value of the boolean expression. If the boolean expression is false, the loop body is not executed even one time. Perhaps the loop adds up the sum of all your expenses for the day. If you did not go shopping on a given day, you do not want the loop body to execute at all.

A.60 **Infinite loops.** A common program bug is a loop that does not end but simply repeats its loop body again and again. A loop that iterates its body repeatedly without ever ending is called an **infinite loop**. Normally, a statement in the body of the loop will change some variables so that the controlling boolean expression becomes false. If this variable does not change in the right way, you can get an infinite loop.

For instance, let's consider a slight variation to the previous example of a while loop. If we forget to increment count, the boolean expression will never change and the loop will be infinite:

```
int count = 1;
while (count <= number)
{
   System.out.println(count);
} // end while
```

Some infinite loops will not really run forever but will instead end your program abnormally when a system resource is exhausted. However, some infinite loops will run forever if left alone. To end a program that is in an infinite loop, you should learn how to force a program to stop running. The way to do this depends on your particular operating system. For example, in a Unix operating system, you would press the key combination Control-C.

Sometimes a programmer might intentionally write an infinite loop. For example, an ATM machine would typically be controlled by a program with an infinite loop that handles deposits and withdrawals indefinitely. However, at this point in your programming, an infinite loop is likely to be an error.

The `for` Statement

A.61 When a counter controls the number of iterations in a `while` loop, you can replace the `while` statement with a **`for` statement**, or **`for` loop**. The `for` statement has the following general form:

```
for (initialize; test; update)
   statement;
```

Here *initialize* is an optional assignment of a value to a variable, *test* is a boolean expression, and *update* is an optional assignment that can change the value of *test*.

For example, the following `for` statement is exactly equivalent to the `while` statement in Segment A.59:

```
int count, number;
. . . // assign a value to number here
for (count = 1; count <= number; count++)
   System.out.println(count);
```

The first of the three expressions in parentheses, `count = 1`, initializes the counter before the loop body is executed for the first time. The second expression, `count <= number`, is a boolean expression that determines whether the loop should continue execution. This boolean expression is tested immediately after the first expression executes and again after each execution of the third expression. The third expression, `count++`, executes after each iteration of the loop body. Thus, the loop body is executed while `count <= number` is true.

In the previous example, we declared `count` before the `for` statement. After the loop completes its execution, `count` is still available as a variable. We could instead declare `count` within the `for` statement, as follows:

```
int number;
. . . // assign a value to number here
for (int count = 1; count <= number; count++)
   System.out.println(count);
```

In this case, `count` is defined only within the `for` loop and is not available after the loop completes its execution.

Programming Tip: Although declaring a variable within a `for` statement is convenient, realize that the variable's scope is then the `for` loop. The variable is not available after the loop completes its execution.

The counter in a `for` statement is not restricted to an integer type. It can have any primitive type. You can omit any of the expressions *initialize*, *test*, and *update* from a `for` statement, but you cannot omit their semicolons. Sometimes it is more convenient to write the *initialize* part before the for statement or to place the *update* part within the body of the loop. This is especially true when these parts are lengthy. Although you technically can omit the *test* from a for loop, you will get an infinite loop if you do.

Some Java programmers tend to favor the for statement over the while statement because in the for statement the initialization, testing, and incrementing of the counter all appear at the beginning of the loop.

Note: A for statement is basically another notation for a kind of while loop. Thus, like a while loop, a for statement might not execute its loop body at all.

A.62 **The comma in for statements.** A for loop can perform more than one initialization. To use a list of initialization actions, separate the actions with commas, as in the following example:

```
int n, product;
for (n = 1, product = 1; n <= 10; n++)
   product = product * n;
```

This for loop initializes n to 1 and product to 1. Note that you use a comma, not a semicolon, to separate the initialization actions.

You can have multiple update actions that are separated by commas. This can sometimes lead to a situation in which the for statement has an empty body and still does something useful. For example, we can rewrite the previous for statement in the following equivalent way:

```
for (n = 1, product = 1; n <= 10; product = product * n, n++);
```

In effect, we have made the loop body part of the update action. Notice the semicolon at the end of the statement. Since a semicolon at the end of a for statement is often the result of a programming error, a clearer way to write this loop makes the empty body explicit:

```
for (n = 1, product = 1; n <= 10; product = product * n, n++)
{
} // end for
```

However, the most readable style uses the update action only for variables that control the loop, as in the original version of this for loop.

Finally, you cannot have multiple boolean expressions to test for ending a for loop. However, you can string together multiple tests by using the && and || operators to form one larger boolean expression.

Programming Tip: If you have used other programming languages that have a general-purpose comma operator, be warned that the comma operator in Java can appear only in for statements.

A.63 **Using an enumeration with a for statement.** The for statement has another form when you want to repeat statements for each object in an enumeration. For example, if you define

```
enum Suit {CLUBS, DIAMONDS, HEARTS, SPADES}
```

the for loop

```
for (Suit nextSuit : Suit.values())
   System.out.println(nextSuit);
```

displays

```
CLUBS
DIAMONDS
HEARTS
SPADES
```

You declare a variable to the left of a colon in the `for` statement. To the right of the colon, you represent the values that the variable will have. For the enumeration `Suit`, the expression `Suit.values()` represents the four possible values `CLUBS`, `DIAMONDS`, `HEARTS`, and `SPADES`. As the loop executes, `nextSuit` takes on each of these values.

This kind of loop—called a **for-each loop**—can be used with other collections of data, as you will see.

The `do-while` Statement

A.64 The **`do-while` statement**, or **`do-while` loop**, is similar to the `while` statement, but the body of a `do-while` statement always executes at least once. As you saw earlier, the body of a `while` loop might not execute at all.

The general form of a `do-while` statement is

```
do
   statement;
while (expression);
```

The `do-while` loop starts with the reserved word `do`. The loop body is a statement, typically a compound statement enclosed in braces `{}`. The loop ends with the reserved word `while` followed by a boolean expression in parentheses and a semicolon.

Programming Tip: Be sure to include a semicolon at the end of a `do-while` statement.

The loop body executes and is repeated while the boolean expression is true. The loop body normally contains some action that can change the value of the boolean expression from true to false and so end the loop. The boolean expression is tested at the end of the loop, not at its beginning, as it is in a `while` statement. Thus, the loop body executes at least once even if the boolean expression starts out false.

The following `do-while` statement displays the integers from 1 to a given integer `number`:

```
int number;
. . . // assign a value to number here
int count = 1;
do
{
   System.out.println(count);
   count++;
} while (count <= number);
```

Again, let's suppose that `number` is 2. The variable `count` begins at 1 and is displayed. Next, `count` is incremented to 2. Since the expression `count <= number` is true at this point, the body of the loop executes again. The value of `count` (2) is displayed, and then `count` becomes 3. The expression `count <= number` is now false, so the `do-while` loop ends. Execution continues with the statement that follows the loop.

If `number` is zero or negative in the previous example, 1 is displayed, since the body of the loop executes at least once. If `number` can possibly be zero or negative, we should use either a `while` loop or a `for` loop here instead of a `do-while` loop.

Notice that we placed the ending brace and the `while` on the same line. Some programmers prefer to place them on different lines. Either form is fine, but be consistent.

To better understand a `do-while` loop, let's rewrite the previous example in the following way:

```
int number;
. . . // assign a value to number here
int count = 1;
{
   System.out.println(count);
   count++;
}
while (count <= number)
{
   System.out.println(count);
   count++;
}
```

When you compare the two versions, it is obvious that a `do-while` loop differs from a `while` loop in only one detail. With a `do-while` loop, the loop body is always executed at least once. With a `while` loop, the loop body might not execute at all.

Additional Loop Information

A.65 **Choosing a loop statement.** Suppose you decide that your program needs a loop. How do you decide whether to use a `while` statement, a `for` statement, or a `do-while` statement? You *cannot* use a `do-while` statement unless you are certain that the loop body should execute at least one time. If you are certain of this, a `do-while` statement is likely to be a good choice. However, more often than you might think, a loop requires the possibility that the body will not execute at all. In those cases, you must use either a `while` statement or a `for` statement. If it is a computation that changes some numeric quantity by some equal amount on each iteration, consider a `for` statement. If the `for` statement does not work well, use a `while` statement. The `while` statement is always a safe choice, since you can use it for any sort of loop. But sometimes one of the other alternatives is easier or clearer.

Programming Tip: A `while` loop can do anything that another loop can do.

A.66 **The `break` and `continue` statements in loops.** You can use the `break` statement in a `switch` statement or in any kind of loop statement. When the `break` statement executes in a loop, the immediately enclosing loop ends, and the remainder of the loop body is not executed. Execution continues with the statement after the loop.

Adding a `break` statement to a loop can make the loop more difficult to understand. Without a `break` statement, a loop has a simple, easy-to-understand structure. There is a test for ending the loop at the top (or bottom) of the loop, and every iteration will go to the end of the loop body. When you add a `break` statement, the loop might end because either the condition given at the top (or bottom) of the loop is false or the `break` statement has executed. Some loop iterations may go to the end of the loop body, but one loop iteration might end prematurely. Because of the complications they introduce, you should avoid `break` statements in loops. Some authorities contend that a `break` statement should never be used to end a loop, but virtually all programming authorities agree that they should be used at most sparingly.

The `continue` statement ends the current iteration of a loop. The loop continues with the next iteration. Using a `continue` statement in this way has the same problems as using a `break` statement.

However, replacing an empty loop body with a `continue` statement is acceptable. For example, you can revise the loop at the end of Segment A.62, as follows:

```
for (n = 1, product = 1; n <= 10; product = product * n, n++)
   continue;
```

Programming Tip: In general, do not use `break` or `continue` statements within the body of a loop.

The Class `String`

A.67 Strings of characters, such as `"Enter the amount:"`, do not have a primitive type in Java. However, Java does provide a class, called `String`, that you use to create and process strings of characters. The string constant `"Enter the amount:"`, in fact, is a value of type `String`. The class `String` is part of the package `java.lang` in the Java Class Library.

A variable of type `String` can name one of these string values. The statement

```
String greeting;
```

declares `greeting` to be the name of a `String` variable, and the following statement sets the value of `greeting` to the `String` value `"Hello!"`:

```
greeting = "Hello!";
```

These two statements are often combined into one, as follows:

```
String greeting = "Hello!";
```

We now can display `greeting` on the screen by writing

```
System.out.println(greeting);
```

The screen will show

```
Hello!
```

Characters Within Strings

A.68 Most programming languages use the **ASCII** character set, which assigns a standard number to each of the characters normally used on an English-language keyboard. Java, however, uses the Unicode character set instead. The Unicode character set includes all the ASCII characters plus many of the characters used in languages that have an alphabet different from English. As it turns out, this is not likely to be a big issue if you are using an English-language keyboard. Normally, you can just program as if Java were using the ASCII character set, since the codes for the ASCII characters are the same in Unicode. The advantage of the Unicode character set is that it allows you to easily handle languages other than English. The disadvantage of the Unicode character set is that it requires two bytes to store each character, whereas the ASCII character set requires one.

A.69 **Escape characters.** Suppose we want to display the following line on the screen:

The word “Java” names a language and a drink!

This string contains quotes, so the statement

```
System.out.println("The word "Java" names a language and a drink!");
```

will not work: It produces a compiler error message. The problem is that the compiler sees

```
"The word "
```

as a perfectly valid quoted string. Then the compiler sees

```
Java"
```

which is not valid in the Java language. The compiler does not know that we mean to include the quote character as part of the string unless we tell it that we want to do so. We tell the compiler this by placing a backslash \ before the troublesome character, like so:

```
System.out.println("The word \"Java\" names a language and a drink!");
```

Some other special characters also need a backslash in order to be included in strings. They are listed in Figure A-4. These are often called **escape characters** because they escape from the usual meaning of a character in Java, such as the usual meaning of the double-quote character.

It is important to note that each escape sequence is a single character, even though it is spelled with two symbols. So the string `"Say \"Hi\"!"` contains 9 characters, not 11.

To include a backslash in a string, you must write two backslashes. Displaying the string `"abc\\def"` on the screen would produce

abc\def

FIGURE A-4 Escape characters

`\"`	Double quote.
`\'`	Single quote (apostrophe).
`\\`	Backslash.
`\n`	New line. (Go to the beginning of the next line.)
`\r`	Carriage return. (Go to the beginning of the current line.)
`\t`	Tab. (Insert whitespace up to the next tab stop.)

Writing the string with only one backslash, as in `"abc\def"`, is likely to produce the error message “Invalid escape character,” because `\d` is invalid.

The escape sequence `\n` indicates that the string starts a new line at the `\n`. For example, the statement

```
System.out.println("The motto is\nGo for it!");
```

will write the following to the screen

```
The motto is
Go for it!
```

You can include a single quote (apostrophe) inside a quoted string, such as `"How's this?"`, but you cannot write a single quote within single quotes. Thus, to define a single-quote character, you use the escape sequence `\'`, as follows:

```
char singleQuote = '\'';
```

Concatenation of Strings

A.70 You can join two strings by using the + operator. Joining two strings together, end to end, to obtain a larger string is called **concatenation**. When + is used with strings, it is sometimes called the **concatenation operator**. For example, the statements

```
String greeting = "Hello";
String sentence = greeting + "my friend.";
System.out.println(sentence);
```

set the variable `sentence` to `"Hellomy friend."` and will write the following on the screen:

```
Hellomy friend.
```

No space separates the first two words, because no spaces are added when you concatenate two strings. If we want `sentence` to contain `"Hello my friend."`, we could change the assignment statement to

```
sentence = greeting + " my friend.";
```

Notice the space before the word "my."

You can concatenate any number of `String` objects by using the + operator. You can even concatenate a `String` object to any other type of object and get a `String` object as a result. Java can express any object as a string when you concatenate it to a string. For primitives like numbers, Java does the obvious thing. For example,

```
String solution = "The answer is " + 42;
```

will set the `String` variable `solution` to `"The answer is 42"`. This is so natural that it may seem as though nothing special is happening, but it does require a real conversion from one type to another. The Java literal `42` is an integer, whereas `"42"` is a string consisting of the two characters 4 and 2. Java converts the integer constant `42` to the string constant `"42"` and then concatenates the two strings `"The answer is "` and `"42"` to obtain the longer string `"The answer is 42"`.

Note: Every class has a method `toString` that Java uses to get a string representation of any object. If you do not define `toString` for a class that you write, the default `toString` will return a representation of an object's location in memory. Thus, you generally should provide your own `toString` method when you define a class. Appendices B and C discuss this method in more detail.

You can also concatenate a single character to a string by using +. For example,

```
String label = "mile";
String pluralLabel = label + 's';
```

sets `pluralLabel` to the string `"miles"`.

Segment A.73 will show you another way to concatenate strings.

String Methods

Readers who need to review Java methods should consult Appendix B before reading this section.

A.71 A `String` object has methods as well as a value. You use these methods to manipulate string values. A few of these `String` methods are described here. You invoke, or call, a method for a `String` object by writing the object name, a dot, and the name of the method, followed by a pair of parentheses. Some methods require nothing within the parentheses, while others require that you specify arguments. Let's look at some examples.

A.72 **The method `length`.** The method `length` gets the number of characters in a string. For example, suppose we declare two `String` variables as follows:

```
String command = "Sit Fido!";  // 9 characters
String answer = "bow-wow";     // 7 characters
```

Now `command.length()` has the value 9 (it returns 9), and `answer.length()` returns 7. Notice that you must include a pair of parentheses, even though there are no arguments to the method `length`. Also notice that spaces, special symbols, and repeated characters are all counted in computing the length of a string. All characters except the double quotes that enclose the string are counted.

You can use a call to the method `length` anywhere that you can use a value of type `int`. For example, all of the following are legal Java statements:

```
int count = command.length();
System.out.println("Length is " + command.length());
count = answer.length() + 3;
```

A.73 **The method `concat`.** You can use the method `concat` instead of the + operator to concatenate two strings. For example, if we declare the `String` variables

```
String one = "sail";
String two = "boat";
```

the expressions

```
one + two
```

and

```
one.concat(two);
```

are the same string, *sailboat*.

A.74 **Indices.** Some of the methods in the class `String` refer to the positions of the characters in the string. Positions in a string start with 0, not with 1. Thus, in the string `"Hi Mom"`, *H* is in position 0, *i* is in position 1, the blank character is in position 2, and so forth. A position is usually referred to as an **index**. So it would be more normal to say that *H* is at index 0, *i* is at index 1, and so on. Figure A-5 illustrates how index positions are numbered in a string.

FIGURE A-5 Indices 0 through 11 for the string `"Java is fun."`

0	1	2	3	4	5	6	7	8	9	10	11
J	a	v	a		i	s		f	u	n	.

A.75 **The methods `charAt` and `indexOf`.** The method `charAt` returns the character at the index given as its one argument. For example, the statements

```
String phrase = "Time flies like an arrow.";
char sixthCharacter = phrase.charAt(5);
```

assign the character *f* to the variable `sixthCharacter`, since the *f* in *flies* is at index 5. (Remember, the first index is 0, not 1.)

The method `indexOf` tests whether a string contains a given substring and, if it does, returns the index at which the substring begins. Thus, `phrase.indexOf("flies")` will return 5 because the substring *flies* begins at index 5 within `phrase`.

Note: Out-of-bounds index

`String` methods such as `charAt` that take an index as an argument will cause an error during execution if the index is negative or too large. Such an index is said to be **out of bounds**. The error causes a `StringIndexOutOfBoundsException`. Appendix E discusses exceptions.

A.76 **Changing case.** The method `toLowerCase` returns a string obtained from its argument string by replacing any uppercase letters with their lowercase counterparts. Thus, if `greeting` is defined by

```
String greeting = "Hi Mary!";
```

the expression

```
greeting.toLowerCase()
```

returns the string `"hi mary!"`. An analogous method, `toUpperCase`, converts any lowercase letters in a string to uppercase.

A.77 **The method `trim`.** The method `trim` trims off any leading and trailing white space, such as blanks. So the statements

```
String command = "   Sit Fido!    ";
String trimmedCommand = command.trim();
```

set `trimmedCommand` to the string `"Sit Fido!"`. The blanks between words are not affected.

A.78 **Comparing strings.** You use the method `compareTo` to compare two strings. Strings are ordered according to the Unicode values of their characters. This ordering—called **lexicographic ordering**—is analogous to alphabetic ordering. The expression

```
stringOne.compareTo(stringTwo)
```

returns a negative integer or a positive integer, depending on whether `stringOne` occurs before or after `stringTwo`. The expression returns zero if the two strings are equal.

The method `compareToIgnoreCase` behaves similarly to `compareTo`, except that the uppercase and lowercase versions of the same letter are considered to be equal. For example, the method `compareTo` places the string `"Hello"` before the string `"hello"`, but the method `compareToIgnoreCase` finds these strings to be equal.

Programming Tip: Do not use the operators ==, !=, <, <=, >, or >= to compare the contents of two strings.

If you want only to see whether two strings are equal—that is, contain the same values—you can use the method `equals`. Thus,

```
stringOne.equals(stringTwo)
```

is true if `stringOne` equals `stringTwo` and is false if they are not equal. The method `equalsIgnoreCase` behaves similarly to `equals`, except that the uppercase and lowercase versions of the same letter are equal. For example, the method `equals` finds the strings `"Hello"` and `"hello"` unequal, but the method `equalsIgnoreCase` finds them equal.

Programming Tip: When applied to two strings (or to any two objects), the operator == tests whether they are stored in the same memory location. Sometimes that is sufficient, but if you want to know whether two strings that are in different memory locations contain the same sequence of characters, use the method `equals`.

The Class `StringBuilder`

A.79 Once you create a string object of the class `String`, you cannot alter it. But sometimes you would like to. For example, we might define the string

```
String name = "rover";
```

and then decide that we want to capitalize its first letter. We cannot. We could, of course, write

```
name = "Rover";
```

but this statement creates a new string *Rover* and discards *rover*.

The class String has no method that modifies a String object. However, the class StringBuilder in the package java.lang has methods such as the following:

```
public StringBuilder append(String s)
```

Concatenates the string s to the end of this string and returns a reference to the result.

```
public StringBuilder delete(int start, int after)
```

Removes the substring of this string beginning at the index start and ending at either the index after - 1 or the end of the string, whichever occurs first, and returns a reference to the result. Throws StringIndexOutOfBoundsException if start is invalid.

```
public StringBuilder insert(int index, String s)
```

Inserts the string s into this string at the given index and returns a reference to the result. Throws StringIndexOutOfBoundsException if the index is invalid.

```
public StringBuilder replace(int start, int after, String s)
```

Replaces a substring of this string with the string s. The substring to be replaced begins at the index start and ends at either the index after - 1 or the end of the string, whichever occurs first. Returns a reference to the result. Throws StringIndexOutOfBoundsException if start is invalid.

```
public void setCharAt(int index, char character)
```

Sets the character at the given index of this string to a given character. Throws IndexOutOfBoundsException if the index is invalid.

If you are not familiar with exceptions, think of them as error messages for now. Appendix E will explain them to you.

StringBuilder has other versions of the methods append and insert that take data of any type as arguments. While the classes StringBuilder and String have some methods in common, several methods in String are not in StringBuilder.

A.80 **Examples.** If we have the following instance of StringBuilder

```
StringBuilder message = new StringBuilder("rover");
```

we can capitalize its first letter by writing

```
message.setCharAt(0, 'R');
```

Now the statement

```
message.append(", roll over!");
```

changes message to the string *Rover, roll over!*, and

```
message.insert(7, "Rover, ");
```

changes it to *Rover, Rover, roll over!* Next,

```
message.delete(0, 7);
```

changes message to *Rover, roll over!*, and

```
message.replace(7, 16, "come here");
```

changes message to *Rover, come here!*

Each of the previous methods except setCharAt returns the result of the operation. So, for example, if we write

```
newMessage = message.append(", roll over!");
```

both newMessage and message reference the same sequence of characters.

Using Scanner to Extract Pieces of a String

A.81 Segments A.32 through A.37 show how the class `Scanner` is used to read data that a user types at the keyboard. Methods in `Scanner`—such as `nextInt`, `nextDouble`, and `next`—read a group of contiguous characters, or **token**, as an integer, a real number, or a string, respectively. When more than one token appears in the input data, they are separated by one or more characters known as **delimiters**. By default, `Scanner` uses white space as the delimiter.

In addition to reading data from the keyboard, you can use `Scanner` to process a string that you define within your program. You simply use the string instead of `System.in` when creating a `Scanner` object.

For example, the statements

```
String phrase = "one potato        two     potato three potato four";
Scanner scan = new Scanner(phrase);
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
```

display the words

```
one
potato
two
potato
```

We essentially have read tokens from a string instead of from the keyboard. The tokens here are separated by white space, the default delimiter.

A.82 **Specifying delimiters.** Whether you read characters from an input device such as a keyboard or from a string defined in your program, you can specify the delimiters that `Scanner` will use. The `Scanner` method `useDelimiter` sets the delimiters to those indicated in its string argument. For example, to use a comma as a delimiter, you would write

```
scan.useDelimiter(",");
```

where `scan` is a `Scanner` object. Thus, the statements

```
String data = "one,potato,two,potato";
Scanner scan = new Scanner(data);
scan.useDelimiter(",");
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
```

display the words

```
one
potato
two
potato
```

A.83 **Example.** To extract the words *one*, *two*, *three*, and *four* from the string `phrase` that we defined in Segment A.81, you would describe the delimiter as one or more blanks, the word *potato*, and one or more blanks. You use the notation in Figure A-6 to describe the delimiters. For example, `\s+` denotes one or more white-space characters. You must duplicate each backslash character in this notation when it appears between the quotes of a string literal to distinguish it from escape characters such as `\n`.

FIGURE A-6 Some notation used to define the delimiters that `Scanner` uses

`\d`	Any digit 0 through 9
`\D`	Any character other than a digit
`\s`	Any white-space character
`\S`	Any character other than white space
`\w`	Any letter, digit, or underscore
`\W`	Any character other than a letter, digit, or underscore
`-`	Any character
X	One occurrence of *X*
X?	Zero or one occurrence of *X*
*X**	Zero or more occurrences of *X*
X+	One or more occurrences of *X*
X{*n*}	Exactly *n* occurrences of *X*
X{*n*,}	At least *n* occurrences of *X*

The following statement sets the delimiter of the `Scanner` object `scan` to the word *potato* with leading and trailing white-space characters:

```
scan.useDelimiter("\\s+potato\\s+");
```

Thus, the statements

```
String phrase = "one potato two potato three potato four";
Scanner scan = new Scanner(phrase);
scan.useDelimiter("\\s+potato\\s+");
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
```

display the words

```
one
two
three
four
```

A.84 **Example.** The statements

```
String phrase = "5 potato        6potato 7 potato more";
Scanner scan = new Scanner(phrase);
scan.useDelimiter("\\s*\\d\\s*");
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
```

display the words

```
potato
potato
potato more
```

The delimiter is a digit with optional leading and trailing white space.

If you now write

```
scan = new Scanner(phrase);
scan.useDelimiter("\\s*potato\\s*");
System.out.println(scan.nextInt());
System.out.println(scan.nextInt());
System.out.println(scan.next());
System.out.println(scan.next());
```

you will get the following output

```
5
6
7
more
```

Here the delimiter is the word *potato* with optional leading and trailing white space.

A.85 **Example.** The statements

```
String phrase = "one - two   -    three four";
Scanner scan = new Scanner(phrase);
scan.useDelimiter("\\s+-?\\s*");
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
System.out.println(scan.next());
```

display the words

```
one
two
three
four
```

The delimiter is at least one white space character optionally followed by a dash and more white space.

Arrays

A.86 In Java, an array is a special kind of object that stores a finite collection of items having the same data type. For example, you can create an array of seven numbers of type `double` as follows:

```
double[] temperature = new double[7];
```

The left side of the assignment operator declares `temperature` as an array whose contents are of type `double`. The right side uses the `new` operator to request seven memory locations for the array. This is like declaring the following strangely named variables to have type `double`:

```
temperature[0], temperature[1], temperature[2], temperature[3],
temperature[4], temperature[5], temperature[6]
```

Note that the numbering starts with 0, not 1. Each of these seven variables can be used just like any other variable of type `double`. For example, we can write

```
temperature[3] = 32;
temperature[6] = temperature[3] + 5;
System.out.println(temperature[6]);
```

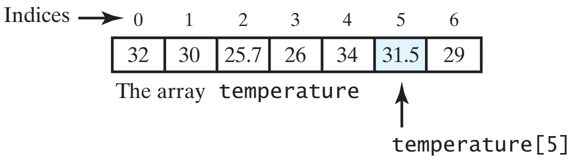
But these seven variables are more than just seven plain old variables of type `double`. The number in square brackets—called an **index**, or **subscript**—can be any arithmetic expression whose value is an integer. In this example, the index value must be between 0 and 6, because we declared that the array temperature should have seven memory locations. A variable such as temperature[3] is called either an **indexed variable**, a **subscripted variable**, or simply an **element** of the array. The value in an indexed variable is called an **entry**, and its data type is the array's **entry type**.

For example, the following statements read seven temperatures into an array and compare them with their average:

```
public static final int DAYS_PER_WEEK = 7;
Scanner keyboard = new Scanner(System.in);
. . .
double[] temperature = new double[DAYS_PER_WEEK];

System.out.println("Enter " + DAYS_PER_WEEK + " temperatures:");
double sum = 0;
for (int index = 0; index < DAYS_PER_WEEK; index++)
{
   temperature[index] = keyboard.nextDouble();
   sum = sum + temperature[index];
} // end for

double average = sum / DAYS_PER_WEEK;

System.out.println("The average temperature is " + average);
System.out.println("The temperatures are");
for (int index = 0; index < DAYS_PER_WEEK; index++)
{
   if (temperature[index] < average)
      System.out.println(temperature[index] + " below average.");
   else if (temperature[index] > average)
      System.out.println(temperature[index] + " above average.");
   else // temperature[index] == average
      System.out.println(temperature[index] + " average.");
} // end for
```

Figure A-7 illustrates the array temperature after seven values have been read into it.

FIGURE A-7 An array of seven temperatures



Each location in the array temperature contains a temperature. That is, the array is full. But arrays are not always full. You need to distinguish between the number of locations in an array—its **length**—and the number of items currently stored in the array.

An array has a data field length that contains the declared number of elements in the array. For example, if we create an array by writing

```
int[] age = new int[50];
```

then age.length is 50. Notice that length is not a method, so no parentheses follow it. If we place only 10 values into the first 10 locations of this array, age.length is still 50. If we need to know how many values we place into an array, we will need to keep track of that ourselves.

Note: An array has a data type and an entry type
The data type of the previously defined array `age` is `int[]`, but its entry type is `int`.

Array Parameters and Returned Values

Readers who need to review Java methods should consult Appendix B before reading this section.

A.87 **Array parameters.** You can pass an indexed variable as an argument to a method anyplace that you can pass an ordinary variable of the array's entry type. For example, if a method has the header

```
public double compute(double value)
```

and `temperature` is the array of `double` values that we defined earlier, we can invoke the method by writing

```
double result = compute(temperature[3])
```

An entire array can also be a single argument to a method. For example, the following method

```
public static void incrementArrayBy2(double[] array)
{
   for (int index = 0; index < array.length; index++)
      array[index] = array[index] + 2;
} // end incrementArrayBy2
```

will accept any array of `double` values as its single argument. We declare the parameter `array` in the method's header just as we would declare any other array: by specifying the type of the array entries followed by square brackets. We do not specify the length of the array.

The following statement is an example of how we would invoke this method:

```
incrementArrayBy2(temperature);
```

You use no square brackets when you pass an entire array as an argument to a method. Notice that the method can take any length array as an argument. The method `incrementArrayBy2` adds 2 to each entry in the argument array `temperature`. That is, the method actually changes the values in the argument array.

Note: A method can change the values in an argument array.

A.88 **Arrays as return values.** In Java, a method can return an array. For example, rather than modifying its array argument, the previous method `incrementArrayBy2` could return an array whose values are 2 more than the corresponding values in the array argument. You specify the method's return type in the same way that you specify a type for an array parameter. The method would then look like this:

```
public static double[] incrementArrayBy2(double[] array)
{
   double[] result = new double[array.length];
   for (int index = 0; index < array.length; index++)
      result[index] = array[index] + 2;
   return result;
} // end incrementArrayBy2
```

The following statements invoke this method:

```
double[] originalArray = new double[10];
< Statements that place values into originalArray >
. . .
```

```
double[] revisedArray = incrementArrayBy2(originalArray);
< At this point, originalArray is unchanged. >
```

Programming Tip: When a method returns an array that you want to assign to an array variable, you should declare the variable but not allocate memory for an array. For example, to invoke the method in the previous example, you do not write

```
double[] revisedArray = new double[10]; // WRONG!
revisedArray = incrementArrayBy2(originalArray);
```

The first statement allocates 10 locations for a new array, but these locations are discarded when the second statement executes.

Initializing Arrays

A.89 You can provide initial values for the elements in an array when you declare it. To do this, you enclose the values for the array in braces and place them after the assignment operator, as in the following example:

```
double[] reading = {3.3, 15.8, 9.7};
```

You do not explicitly state the array's length. Instead the length is the minimum number of locations that will hold the given values. This initializing declaration is equivalent to the following statements:

```
double[] reading = new double[3];
reading[0] = 3.3;
reading[1] = 15.8;
reading[2] = 9.7;
```

If you do not initialize the elements of an array, they are given initial default values according to their type. For example, if you do not initialize an array of integers, each element of the array will be initialized to zero. In an array of objects, each element is initialized to `null`. However, it is usually clearer to do your own explicit initialization, either when you declare the array or later by using a loop and assignment statements.

Array Index Out of Bounds

A.90 When programming with arrays, making a mistake with an index is easy. This is especially true if that index is an expression. If the array `temperature` has seven elements, but an index is some integer other than 0 through 6, the index is said to be out of bounds. An out-of-bounds index expression will compile without any error message, but will cause an error when you run your program. In particular, you will get an `IndexOutOfBoundsException`. As we mentioned in Segment A.75, a similar situation can occur when you work with strings. (See Appendix E for a discussion of exceptions.)

Use of = and == with Arrays

A.91 **The operator `=`.** Recall that a variable for an object really contains the memory address of the object. The same is true of arrays. All array locations are together in one section of memory so that one memory address can specify the location of the entire array. The assignment operator copies this memory address. For example, if `a` and `b` are arrays of the same size, the assignment `b = a` gives the array variable `b` the same memory address as the array variable `a`. In other words, `a` and `b` are two different names for the same array. These variables are **aliases**. Thus, when you change the value of `a[2]`, you are also changing the value of `b[2]`. Appendix B talks more about aliases and references.

If you want the array b to have the same values as the array a, but in separate memory locations, then instead of one assignment statement you must use something like the following:

```
for (int index = 0; index < a.length; index++)
   b[index] = a[index];
```

A.92 **The operator ==.** The equality operator == tests two arrays to see if they are stored in the same place in the computer's memory. It does not test whether the arrays contain the same values. To do so, you must compare the two arrays entry by entry. For example, if the arrays a and b contain primitive values and have the same length, the following code could be used:

```
boolean match = true;
int index = 0;
while (match && (index < a.length))
{
   if (a[index] != b[index])
      match = false;
   else
      index++;
} // end while

if (match)
   System.out.println("Arrays have the same contents");
else
   System.out.println("Arrays have different contents");
```

If the arrays contained objects instead of primitive values, we would use the boolean expression !a[index].equals(b[index]) instead of a[index] != b[index].

Note: Are arrays really objects?

Arrays behave very much like objects. On the other hand, arrays do not belong to any class. Because arrays were used by programmers for many years before classes and objects (as we have used them) were invented, arrays use a special notation of their own. Other features of objects do not apply to arrays, such as inheritance (which we discuss in Appendix C). So whether or not arrays should be considered objects is primarily an academic debate. Whenever Java documentation says that something applies to all objects, it also applies to arrays.

Note: Array types are reference types

A variable of an array type holds only the address where the array is stored in memory. This memory address is often called a **reference** to the array object in memory. For this reason, an array type is a reference type. A reference type is any type whose variables hold references—that is, memory addresses—as opposed to the actual item named by the variable. Array types and class types are both reference types. Primitive types are not reference types.

Arrays and the For-Each Loop

A.93 Earlier we used a for-each loop to process all the values in an enumeration. We can use a similar for-each loop to process all the values in an array. For example, the following statements compute the sum of the integers in an array:

```
int[] anArray = {1, 2, 3, 4, 5};
int sum = 0;
```

```
for (int integer : anArray)
   sum = sum + integer;
System.out.println(sum);
```

Similarly, the following statements display all the strings in an array:

```
String[] friends = {"Gavin", "Gail", "Jared", "Jessie"};
for (String name : friends)
   System.out.println(name);
```

Multidimensional Arrays

A.94 You can have an array with more than one index. For example, suppose we wanted to store the dollar amounts shown in Figure A-8 in some sort of array. The bold items are just labeling. There are 60 entries. If we use an array with one index, the array will have a length of 60, and keeping track of which entry goes with which index would be almost impossible. On the other hand, if we allow ourselves two indices, we can use one index for the row and one index for the column. This arrangement is illustrated in Figure A-9.

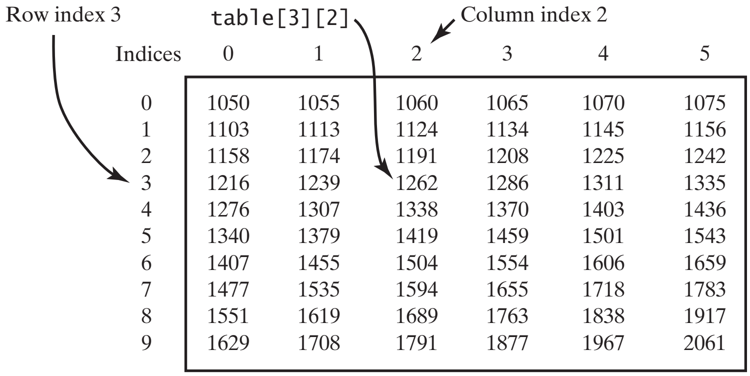
FIGURE A-8 A table of values

The effect of various interest rates on $1000 when compounded annually (rounded to whole dollars)

Year	5.00%	5.50%	6.00%	6.50%	7.00%	7.50%
1	$1050	$1055	$1060	$1065	$1070	$1075
2	$1103	$1113	$1124	$1134	$1145	$1156
3	$1158	$1174	$1191	$1208	$1225	$1242
4	$1216	$1239	$1262	$1286	$1311	$1335
5	$1276	$1307	$1338	$1370	$1403	$1436
6	$1340	$1379	$1419	$1459	$1501	$1543
7	$1407	$1455	$1504	$1554	$1606	$1659
8	$1477	$1535	$1594	$1655	$1718	$1783
9	$1551	$1619	$1689	$1763	$1838	$1917
10	$1629	$1708	$1791	$1877	$1967	$2061

Note that, as was true for the simple arrays you have already seen, you begin numbering indices with 0 rather than 1. If the array is named `table` and it has two indices, the Java notation `table[3][2]` specifies the entry in the fourth row and third column. By convention, we think of the first index as denoting the row and the second as denoting the column. Arrays that have exactly two indices can be displayed on paper as a two-dimensional table and are called **two-dimensional arrays**. More generally, an array is said to be an ***n*-dimensional array** if it has *n* indices. Thus, the ordinary one-index arrays that we have used up to now are **one-dimensional arrays**.

FIGURE A-9 Row and column indices for an array named `table`; `table[3][2]` is the element in the fourth row and third column



Indices	0	1	2	3	4	5
0	1050	1055	1060	1065	1070	1075
1	1103	1113	1124	1134	1145	1156
2	1158	1174	1191	1208	1225	1242
3	1216	1239	1262	1286	1311	1335
4	1276	1307	1338	1370	1403	1436
5	1340	1379	1419	1459	1501	1543
6	1407	1455	1504	1554	1606	1659
7	1477	1535	1594	1655	1718	1783
8	1551	1619	1689	1763	1838	1917
9	1629	1708	1791	1877	1967	2061

A.95 Arrays with multiple indices are handled much like arrays with one index. To declare and create the array `table` with 10 rows and 6 columns, we write

```
int[][] table = new int[10][6];
```

You can have arrays with any number of indices. To get more indices, you just use more square brackets in the declaration.

Indexed variables for multidimensional arrays are just like indexed variables for one-dimensional arrays, except that they have multiple indices, each enclosed in a pair of square brackets. For example, the following statements set all the elements in `table` to zero:

```
for (int row = 0; row < 10; row++)
   for (int column = 0; column < 6; column++)
      table[row][column] = 0;
```

Note that we used two `for` loops, one nested within the other. This is a common way of stepping through the indexed variables in a two-dimensional array. If we had three indices, we would use three nested `for` loops, and so forth for higher numbers of indices.

As was true of the indexed variables for one-dimensional arrays, indexed variables for multidimensional arrays are variables of the array's entry type and can be used anywhere that a variable of the entry type is allowed. For example, for the two-dimensional array `table`, an indexed variable such as `table[3][2]` is a variable of type `int` and can be used anyplace that an ordinary `int` variable can be used.

A multidimensional array can be a parameter of a method. For example, the following method header has a two-dimensional array as a parameter:

```
public static void clearArray(double[][] array)
```

A.96 Java implements multidimensional arrays as one-dimensional arrays. For example, consider the array

```
int[][] table = new int[10][6];
```

The array `table` is in fact a one-dimensional array of length 10, and its entry type is `int[]`. Thus, each entry in the array `table` is a one-dimensional array of length 6. In other words, a multidimensional array is an array of arrays.

Normally, you do not need to be concerned with this fact, since this detail is handled automatically by the compiler. However, sometimes you can profit from this knowledge. For example, consider the previous nested `for` loops that filled the two-dimensional array `table` with zeros. We used the constants 6 and 10 to control the `for` loops, but it would be better style to use the data field length instead. To do so, we need to think in terms of an array of arrays. For example, the following is a rewrite of the nested for loops:

```
for (int row = 0; row < table.length; row++)
   for (int column = 0; column < table[row].length; column++)
      table[row][column] = 0;
```

Here, table.length is the number of rows in table, and table[row].length is the number of columns.

Wrapper Classes

A.97 Java makes a distinction between the primitive types, such as int, double, and char, and the class types, such as String and the classes that you write. Java sometimes treats primitive types and class types differently. For example, an argument to a method and the assignment operator = behave differently for primitive types and class types. To make things uniform, Java provides a **wrapper class** for each of the primitive types that enables us to convert a value of a primitive type to an object of a corresponding class type.

For example, the wrapper class for the primitive type int is the predefined class Integer. If we want to convert an int value, such as 10, to an object of type Integer, we can do so in one of three ways, as the following statements demonstrate:

```
Integer ten = new Integer(10);
Integer fiftyTwo = new Integer("52");
Integer eighty = 80;
```

In the first way, you supply an int value as a literal, a variable, or an expression. In the second, you provide a string that contains an int value. The third way allows you to simply assign an int value without using the new operator.

A.98 Once you have defined Integer objects, you can compare them by using the methods compareTo and equals, much as you compare strings or the other objects we have discussed in this appendix. Do not use operators such as ==. Just as for strings and other objects, operators like == compare the memory addresses of objects, not their values.

If you need the value of an Integer object as a primitive, you can use methods such as intValue or doubleValue. For example, if ten is defined as in the previous segment, the expression

```
ten.intValue()
```

returns the int value 10, whereas

```
ten.doubleValue()
```

returns the double value 10.0. You also can simply assign the Integer object to an int variable, as in

```
int primitive10 = ten;
```

No type cast is necessary; however, using one is not an error.

A.99 **Boxing and unboxing.** When performing arithmetic with Integer objects, you can use the same operators that you use for arithmetic with primitives. You can also intermix primitive integers with Integer objects. Thus, you can write statements such as the following:

```
Scanner keyboard = new Scanner(System.in);
System.out.print("What is his age? ");
int hisAge = keyboard.nextInt();
System.out.print("What is her age? ");
Integer herAge = keyboard.nextInt();

Integer ageDifference = Math.abs(hisAge - herAge);
System.out.println("He is " + hisAge + ", she is " + herAge +
                   ": a difference of " + ageDifference + ".");
```

Java converts between int and Integer as necessary. The process of converting from a primitive type to a corresponding wrapper class is called **boxing**. **Unboxing** is the process used to convert in the other direction. Since these conversions happen automatically, they are often called **auto-boxing** and **auto-unboxing**. Realize that the previous statements are just a demonstration of what is possible. Using only primitive integers for this simple computation is certainly adequate.

A.100 The wrapper classes for the primitive types double, float, long, and char are Double, Float, Long, and Character, respectively. You create objects of these classes in a way analogous to how you create Integer objects. Except for Character, each wrapper class has methods that return a value in a variety of types. Integer has the methods doubleValue, floatValue, intValue, and longValue. Double, Float, and Long also have these same methods. The class Character has only the analogous method charValue.

Many of the classes that we study in this book represent collections of objects. If your data has a primitive type, an appropriate wrapper class enables you to represent the data as objects so that you can use these classes.

A.101 Wrapper classes also contain some useful static constants. For example, you can find the largest and smallest values of any of the primitive number types by using the associated wrapper class. The largest and smallest values of type int are

```
Integer.MAX_VALUE and Integer.MIN_VALUE
```

The largest and smallest values of type double are

```
Double.MAX_VALUE and Double.MIN_VALUE
```

A.102 Wrapper classes have static methods that can be used to convert a string to the corresponding number of type int, double, long, or float. For example, suppose your program needs to convert the string "199.98" to a double value (which will turn out to be 199.98, of course). The static method parseDouble of the wrapper class Double will convert a string to a value of type double. So if theString is a variable of type String whose value is "199.98",

```
Double.parseDouble(theString)
```

returns the double value 199.98. The other wrapper classes Integer, Long, and Float have the analogous methods parseInt, parseLong, and parseFloat.

If there is any possibility that the string named by theString has extra leading or trailing blanks, you should instead use

```
Double.parseDouble(theString.trim())
```

As we discussed in Segment A.77, the method trim, included in the class String, trims off leading or trailing white space, such as blanks. If the string is not a correctly formed number, the invocation of Double.parseDouble will cause an exception. The use of trim helps some in avoiding this problem.

A.103 Each of the numeric wrapper classes also has a static method called toString that will convert in the other direction—that is, it will convert from a primitive numeric value to a string representation of the numeric value. For example,

```
Integer.toString(42)
```

returns the string value "42", and

```
Double.toString(199.98)
```

returns the string value "199.98". Additionally, each wrapper class, like all other classes, has a nonstatic version of toString. For example, if we define n as follows:

```
Integer n = new Integer(198);
```

then n.toString() returns the string "198".

Note: Wrapper classes

Every primitive type has a wrapper class. Wrapper classes allow you to represent values of a primitive type as a class type. They also contain a number of useful predefined constants and methods.

A.104 `Character` is the wrapper class for the primitive type `char`. The following piece of code illustrates some of the basic methods for this class:

```
Character c1 = new Character('a');
Character c2 = new Character('A');
if (c1.equals(c2))
   System.out.println(c1.charValue() + " is the same as " + c2.charValue());
else
   System.out.println(c1.charValue() + " is not the same as " + c2.charValue());
```

This code displays

```
a is not the same as A
```

The `equals` method checks for equality of characters, so uppercase and lowercase letters are considered different.

Some of the static methods in the class `Character` follow:

`public static char toLowerCase(char ch)`
Returns the lowercase equivalent of `ch`, if `ch` is a letter; otherwise returns `ch`.

Examples:

`Character.toLowerCase('a')` returns `'a'`
`Character.toLowerCase('A')` returns `'a'`
`Character.toLowerCase('5')` returns `'5'`

`public static char toUpperCase(char ch)`
Returns the uppercase equivalent of `ch`, if `ch` is a letter; otherwise returns `ch`.

Examples:

`Character.toUpperCase('a')` returns `'A'`
`Character.toUpperCase('A')` returns `'A'`
`Character.toUpperCase('5')` returns `'5'`

`public static boolean isLowerCase(char ch)`
Returns true if `ch` is a lowercase letter.

Examples:

`Character.isLowerCase('a')` returns true
`Character.isLowerCase('A')` returns false
`Character.isLowerCase('5')` returns false

`public static boolean isUpperCase(char ch)`
Returns true if `ch` is an uppercase letter.

Examples:

`Character.isUpperCase('a')` returns false
`Character.isUpperCase('A')` returns true
`Character.isUpperCase('5')` returns false

`public static boolean isLetter(char ch)`
Returns true if `ch` is a letter.

Examples:

`Character.isLetter('a')` returns true
`Character.isLetter('A')` returns true
`Character.isLetter('5')` returns false

`public static boolean isDigit(char ch)`
Returns true if `ch` is a digit.

Examples:

`Character.isDigit('a')` returns false
`Character.isDigit('A')` returns false
`Character.isDigit('5')` returns true

`public static boolean isLetterOrDigit(char ch)`
Returns true if `ch` is either a letter or a digit.

Examples:

`Character.isLetterOrDigit('a')` returns true
`Character.isLetterOrDigit('A')` returns true
`Character.isLetterOrDigit('5')` returns true
`Character.isLetterOrDigit('%')` returns false

`public static boolean isWhitespace(char ch)`
Returns true if `ch` is a white-space character.

Examples:

`Character.isWhitespace('a')` returns false
`Character.isWhitespace(' ')` returns true

A.105 Java also has a wrapper class `Boolean`. This class has the two constants `Boolean.TRUE` and `Boolean.FALSE`. However, the Java reserved words `true` and `false` are much easier to use for these constants. So the constants in the class `Boolean` will not be of much help to us. The methods of the class `Boolean` are also not used very often. Although the class `Boolean` is not useless, it will be of little use to us in this text and we will discuss it no further.

Appendix

B

Java Classes

Contents

Prerequisites

Appendix A Java Essentials

This appendix reviews the use and creation of Java classes, methods, and packages. Even if you are familiar with this material, you should at least skim it to learn our terminology.

Objects and Classes

B.1 Appendix A introduced the basics of classes and objects. Recall that an object contains data and can perform certain actions. An object belongs to a class, which defines its data type. A class specifies the kind of data the objects of that class have. A class also specifies what actions the objects can take and how they accomplish those actions. **Object-oriented programming**, or **OOP**, views a program as a sort of world consisting of objects that interact with one another by means of actions. For example, in a program that simulates automobiles, each automobile is an object.

When you define a class in Java, the class is like a plan or a blueprint for constructing specific objects. As an example, Figure B-1 describes a class called `Automobile` and shows three `Automobile` objects. The class is a general description of what an automobile is and what it can do.

Each **instance**, or object, of the class `Automobile` is a particular automobile. You can name each object that you create, or **instantiate**. In Figure B-1, the names are `bobsCar`, `suesCar`, and `jakesTruck`. In a Java program, `bobsCar`, `suesCar`, and `jakesTruck` would be variables of type `Automobile`.

The definition of the `Automobile` class says that an `Automobile` object has data such as its model, its year, and how much fuel is in its tank. The class definition contains no actual data—no string and no numbers. The individual objects have the data, and the class simply specifies what kind of data they have.

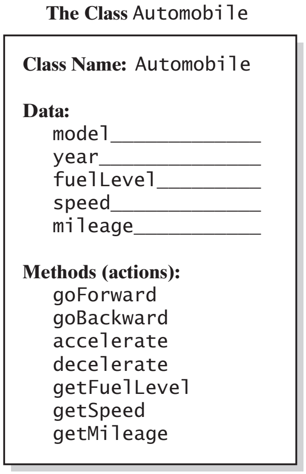
The `Automobile` class also defines methods such as `goForward` and `goBackward`. In a program that uses the class `Automobile`, the only actions an `Automobile` object can take are defined by those methods. All objects of a given class have exactly the same actions. The implementations of the methods indicate how the actions are performed and are included in the class definition. The objects themselves actually perform the method's actions, however.

The objects in a single class can have different characteristics. Even though these objects have the same types of data and the same actions, the individual objects can differ in the values of their data.

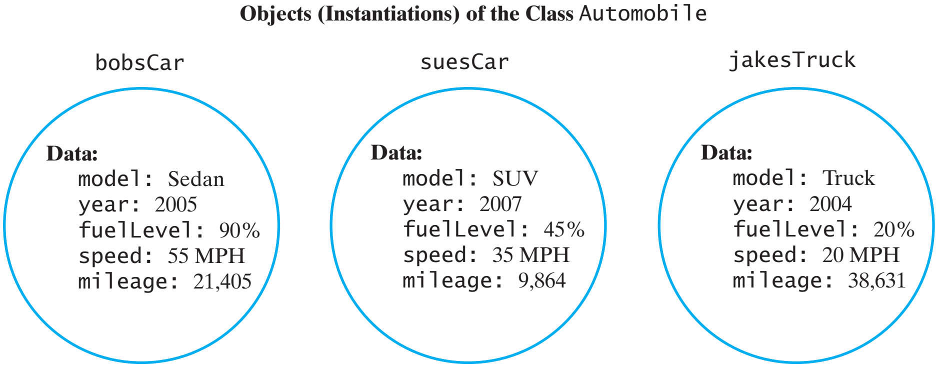
Note: An object is a program construct that contains data and performs actions. The objects in a Java program interact, and this interaction forms the solution to a given problem. The actions performed by objects are defined by methods.

Note: A class is a type or kind of object. All objects in the same class have the same kinds of data and the same actions. A class definition is a general description of what that object is and what it can do.

FIGURE B-1 An outline of a class and three of its instances



(Figure B-1 continued)



Note: You can view a class in several different ways when programming. When you instantiate an object of a class, you view the class as a data type. When you implement a class, you can view it as a plan or a blueprint for constructing objects—that is, as a definition of the objects' data and actions. At other times, you can think of a class as a collection of objects that have the same type.

Using the Methods in a Java Class

B.2 Let's assume that someone has written a Java class called `Name` to represent a person's name. We will describe how to use this class and, in doing so, we will show you how to use a class's methods. A program component that uses a class is called a **client** of the class.We will reserve the term "user" to mean a person who uses a program.

To declare a variable of data type `Name`, you would write, for example,

```
Name joe;
```

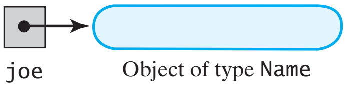
At this point, the variable `joe` contains nothing in particular; it is uninitialized. To create a specific object of data type `Name`—that is, to create an instance of `Name`—called `joe`, you write

```
joe = new Name();
```

The `new` operator creates an instance of `Name` by invoking a special method within the class, known as a **constructor**. The memory address of the new object is assigned to `joe`, as Figure B-2 illustrates. We will show you how to define constructors a bit later, in Segment B.17. Note that you can combine the previous two Java statements into one:

```
Name joe = new Name();
```

FIGURE B-2 A variable that references an object



B.3 Suppose that a person's name has only two parts: a first name and a last name. The data associated with the object `joe` then consists of two strings that represent the first and last names. Since you want to be able to **set**—that is, initialize or change—a person's name, the `Name` class should have methods that give you this capability. To set `joe`'s first and last names, you can use two methods from the class `Name`—`setFirst` and `setLast`—as follows:

```
joe.setFirst("Joseph");
joe.setLast("Brown");
```

You usually invoke a method by writing the name of the **receiving object** first, followed by a dot, the name of the method to be invoked, and finally a set of parentheses that contain **arguments**. In this example, `joe` is the receiving object, as it receives the call to perform an action, and the arguments are strings that represent inputs to the methods. The methods set the object's data fields to the specific values given as arguments.

The methods `setFirst` and `setLast` are examples of void methods, in that they do not return a value. As we mentioned in Segment A.2 of Appendix A, a second kind of method—the valued method—returns a single value. For example, the method `getFirst` returns a string that is the first name of the object that received the method call. Similarly, the method `getLast` returns the last name.

You can invoke a valued method anywhere that you can use a value of the type returned by the method. For example, `getFirst` returns a value of type `String`, and so you can use a method invocation such as `joe.getFirst()` anywhere that it is legal to use a value of type `String`. Such places could be in an assignment statement, like

```
String hisName = joe.getFirst();
```

or within a `println` statement, like

```
System.out.println("Joe's first name is " + joe.getFirst());
```

Notice that the methods `getFirst` and `getLast` have no arguments in their parentheses. Any method—valued or void—can require zero or more arguments.

Note: Valued methods return a single value; void methods do not return a value. For example, the valued method `getFirst` returns the string that represents the first name. The void method `setFirst` sets the first name to a given string but does not return a value. For now, you will distinguish valued methods and void methods by the description of what they do. Later, in Segments B.7 through B.9, you will see that their Java definitions distinguish one kind of method from another.

SELF-TEST

Question 1 Write Java statements that create an object of type `Name` to represent your name.

Question 2 Write a Java statement that uses the object you created in Question 1 to display your name in the form *last name, comma, first name*.

Question 3 Which methods of the class `Automobile`, as given in Figure B-1, are most likely valued methods, and which are most likely void methods?

References and Aliases

B.4 Java has eight primitive data types: `byte`, `short`, `int`, `long`, `float`, `double`, `char`, and `boolean`. A variable of a primitive type actually contains the primitive value. All other data types are reference, or class, types. The `String` variable `greeting` in

```
String greeting = "Hello";
```

is a reference variable. A reference variable contains the address in memory of an actual object. This address is called a **reference**. It is not important here to know that `greeting` contains a reference to the string `"Hello"` instead of the actual string. In such cases, it is easier to talk about the string `greeting`, when in fact this is not an accurate description of that variable. This book makes the distinction between an object and a reference to an object when it is important to do so.

Now suppose that you write

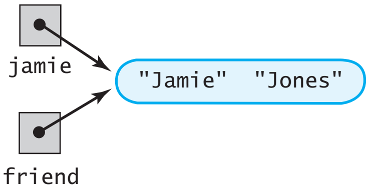
```
Name jamie = new Name();
jamie.setFirst("Jamie");
jamie.setLast("Jones");
Name friend = jamie;
```

The two variables `jamie` and `friend` reference the same instance of `Name`, as Figure B-3 shows. We say that `jamie` and `friend` are aliases, because they are two different names for the same object. You can use `jamie` and `friend` interchangeably when referencing the object.

For example, if you use the variable `jamie` to change Jamie Jones's last name, you can use the variable `friend` to access it. Thus, the statements

```
jamie.setLast("Smith");
System.out.println(friend.getLast());
```

FIGURE B-3 Aliases of an object



display *Smith*. Also note that the boolean expression `jamie == friend` is true, since both variables contain the same address.

Defining a Java Class

B.5 We now show you how to write the Java class `Name` that represents a person's name. You store a class definition in a file whose name is the name of the class followed by `.java`. Thus, the class `Name` should be in the file `Name.java`. Typically, you store only one class per file.

The data in a `Name` object consists of the person's first and last names as strings. The methods in the class will enable you to set and look at these strings. The class has the following form:

```
public class Name
{
   private String first; // first name
   private String last;  // last name

   < Definitions of methods are here >
   . . .
} // end Name
```

The word `public` simply means that there are no restrictions on where the class is used. That is, the class `Name` is available for use in any other Java class. The two strings `first` and `last` are called the class's **data fields** or **instance variables** or **data members**. Each object of this class will have these two data fields inside of it. The word `private` that precedes the declaration of each data field means that only the methods within the class can refer to the data fields by their names `first` and `last`. No other class will be able to do this. The words `public` and `private` are examples of an **access modifier** or **visibility modifier**, which specifies where a class, data field, or method can be used. A third access modifier, `protected`, is possible, as you will see in Appendix C.

Note: Access (visibility) modifiers
The words `public` and `private` are examples of access modifiers that specify where a class, method, or data field can be used. Any class can use a public method, but a private method can be used only by the class that defines it. Appendix C discusses the access modifier `protected`, and the section "Packages" of this appendix shows when you can omit the access modifier.

B.6 Since the data fields are private, how will a class that uses the class `Name` be able to change or look at their values? You can define methods in a class that look at or change the values of its data fields. You declare such methods to be public, so that anyone can use them. A method that enables you to look at the value of a data field is called an **accessor method** or **query method**. A method that changes the value of a data field is called a **mutator method**. Java programmers typically begin the names of accessor methods with `get` and the names of mutator methods with `set`. Because of this convention, accessor methods are sometimes called **get methods** or **getters**, and mutator methods are called **set methods** or **setters**. For example, the class `Name` will have methods that include `getFirst`, `getLast`, `setFirst`, and `setLast`.

You may think that accessor methods and mutator methods defeat the purpose of making data fields private. On the contrary, they give the class control over its data fields. For example, a mutator method can check that any change to a data field is appropriate and warn you if there is a problem. The class would be unable to make this check if its data fields were public, since anyone could alter the fields.

Note: An accessor (query) method enables you to look at the value of a data field. A mutator method changes the value of a data field. Typically, you begin the names of accessor methods with `get` and the names of mutator methods with `set`.

Programming Tip: You should make each data field in a class private by beginning its declaration with the access modifier `private`. You cannot make any direct reference to a private data field's name outside of the class definition. The programmer who uses the class is forced to manipulate the data fields only via methods in the class. The class then can control how a programmer accesses or changes the data fields. Within any of the class's method definitions, however, you can use the name of the data field in any way you wish. In particular, you can directly change the value of the data field.

SELF-TEST

Question 4 Is the method `setFirst` an accessor method or a mutator method?

Question 5 Should a typical accessor method be valued or void?

Question 6 Should a typical mutator method be valued or void?

Question 7 What is a disadvantage of making a data field in a class public?

Method Definitions

B.7 The definition of a method has the following general form:

```
access-modifier   use-modifier   return-type   method-name(parameter-list)
{
   method-body
}
```

The **use modifier** is optional and in most cases is omitted. When used, it can be either `abstract`, `final`, or `static`. Briefly, an abstract method has no definition and must be overridden in a derived class. A final method cannot be overridden in a derived class. A static method is shared by all instances of the class. You will encounter these use modifiers later.

Next comes the **return type**, which for a valued method is the data type of the value that the method returns. For a void method, the return type is `void`. You then write the name of the method and a pair of parentheses that contain an optional list of **formal parameters** and their data types. The formal parameters, or simply **parameters**, specify values or objects that are inputs to the method.

So far, we have described the first line of the method definition, which is called the method's **header** or **declaration**. After the header is the method's **body**—which is simply a sequence of Java statements—enclosed in curly braces.

B.8 As an example of a valued method, here is the definition of the method `getFirst`:

```
public String getFirst() ←—— Header
{
   return first;          } Body
} // end getFirst
```

This method returns the string in the data field `first`. The return type of this method is, therefore, `String`. A valued method must always execute a `return` statement as its last action. The data type of the value returned must match the data type declared as the return type in the method's header. Notice that this particular method does not have formal parameters.

B.9 Now let's look at an example of a void method. The void method `setFirst` sets the data field `first` to a string that represents a first name. The method definition is as follows:

```
public void setFirst(String firstName)
{
   first = firstName;
} // end setFirst
```

This method does not return a value, so its return type is `void`. The method has one formal parameter, `firstName`, that has the data type `String`. It represents the string that the method should assign to the data field `first`. The declaration of a formal parameter always consists of a data type and a parameter name. If you have more than one formal parameter, you separate their declarations with commas.

B.10 **The object `this`.** Notice that the bodies of the previous two method definitions refer to the data field `first` by name. This is perfectly legal. Exactly whose data field is involved here? Remember that each object of this class contains a data field `first`. The data field `first` that belongs to the object receiving the call to the method is the one involved. Java has a name for this object when you want to refer to it within the body of a method definition. It is simply `this`. For example, in the method `setFirst` you could write the statement

```
first = firstName;
```

as

```
this.first = firstName;
```

Some programmers use `this` in this way either for clarity or when they want to give the parameter the same name as the data field. For example, you could name `setFirst`'s parameter `first` instead of `firstName`. Clearly, the statement

```
first = first;
```

in the method's body would not work correctly, so instead you would write

```
this.first = first;
```

We typically will not use `this` for these reasons. Another situation, however, will occur where the use of `this` is essential, as you will see in Segment B.26.

Note: Members

Both the data fields and the methods of an object are sometimes called **members** of the object, because they belong to the object.

Note: Naming classes and methods

The normal convention when naming classes and methods is to start all class names with an uppercase letter and to start all method names with a lowercase letter. Use a noun or descriptive phrase to name a class. Use a verb or action phrase to name a method.

Note: Local variables

A variable declared within a method definition is called a **local variable**. The value of a local variable is not available outside of the method definition. If two methods each have a local variable of the same name, the variables are different, even though they have the same name.

B.11 Methods should be self-contained units. You should design methods separately from the incidental details of other methods of the class and separately from any program that uses the class. One incidental detail is the name of the formal parameters. Fortunately, formal parameters behave like local variables, and so their meanings are confined to their respective method definitions. Thus, you can choose the formal parameter names without any concern that they will be the same as some other identifier used in some other method. For team programming projects, one programmer can write a method definition while another programmer writes another part of the program that uses that method. The two programmers need not agree on the names they use for formal parameters or local variables. They can choose their identifier names completely independently, without any concern that some, all, or none of their identifiers might be the same.

Arguments and Parameters

B.12 Earlier you saw that an object of a class usually receives a call to a method defined within that class. For example, you saw that the statements

```
Name joe = new Name();
joe.setFirst("Joseph");
joe.setLast("Brown");
```

set the first and last names for the object `joe`. The strings `"Joseph"` and `"Brown"` are the arguments. These arguments must correspond to the formal parameters of the method definition. In the case of `setFirst`, for example, the formal parameter is the string `firstName`. The argument is the string `"Joseph"`. The argument is plugged in for the corresponding formal parameter. Thus, in the body of the method, `firstName` represents the string `"Joseph"` and behaves like a local variable.

A method invocation must provide exactly as many arguments as there are formal parameters in the corresponding method definition. In addition, the arguments in the invocation must correspond to the formal parameters in the method's definition with respect to both the order in which they occur and their data types. In some cases, however, Java will perform an automatic type conversion when the data types do not match.

Java does have a notation for formal parameters that allows a variable number of arguments. Since we really do not need this feature, we will not cover it.

Note: The arguments in the invocation of a method must correspond to the formal parameters in the method's definition with respect to number, order, and data type.

Aside: Use of the terms "parameter" and "argument"

The use of the terms "formal parameter" and "argument" in this book is consistent with common usage, but some people use the terms "parameter" and "argument" interchangeably. Some people use the term "parameter" for both what we call (formal) parameters and what we call arguments. Other people use the term "argument" for both what we call (formal) parameters and what we call arguments.

Passing Arguments

B.13 When a formal parameter has a primitive type, such as `int` or `char`, the parameter is initialized to the value of the corresponding argument in the method invocation. The argument in a method invocation can be a literal constant—like 2 or `'A'`—or it can be a variable or any expression that yields a value of the appropriate type. Note that the method cannot change the value of an argument that has a primitive data type. Such an argument serves as an input value only. This mechanism is described as **call-by-value**.

For example, suppose that the class `Name` provided for a middle initial by defining another data field and the method `setMiddleInitial`. Thus, the class might appear as follows:

```
public class Name
{
   private String first;
   private char   initial;
   private String last;
   . . .
   public void setMiddleInitial(char middleInitial)
   {
      initial = middleInitial;
   } // end setMiddleInitial
   . . .
```

A client of this class could contain the following statements:

```
char joesMI = 'T';
Name joe = new Name();
. . .
joe.setMiddleInitial(joesMI);
. . .
```

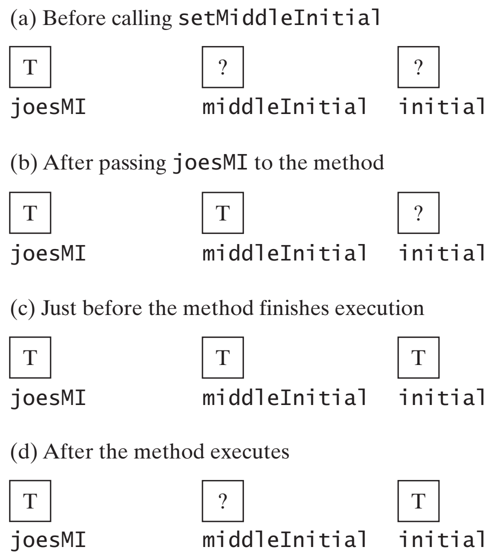
Figure B-4 shows the argument `joesMI`, the parameter `middleInitial`, and the data field `initial` as the method `setMiddleInitial` executes. (Although the data field has an initial value, it is not relevant, so the figure shows it as a question mark.)

If a method changes the value of its parameter, the corresponding argument will be unaffected. So if, for example, `setMiddleInitial` contained the statements

```
initial = middleInitial;
middleInitial = 'X'
```

the value of `middleInitial` in Figure B-4c would be *X*, but the rest of the figure would not change. In particular, the value of `joesMI` would not change.

FIGURE B-4 The effect of executing the method `setMiddleInitial` on its argument `joesMI`, its parameter `middleInitial`, and the data field `initial`



(a) Before calling `setMiddleInitial`

(b) After passing `joesMI` to the method

(c) Just before the method finishes execution

(d) After the method executes

B.14 When a formal parameter has a class type, the corresponding argument in the method invocation must be an object of that class type. The formal parameter is initialized to the memory address of that object.[1] Thus, the formal parameter will serve as an alternative name for the object. This implies that the method can change the data in the object, if the class has mutator methods. The method, however, cannot replace an object that is an argument with another object.

For example, if you adopt a child, you might give that child your last name. Suppose that you add the following method `giveLastNameTo` to the class `Name` that makes this change of name:

```
public void giveLastNameTo(Name child)
{
   child.setLast(last);
} // end giveLastNameTo
```

Notice that the formal parameter of this method has the type `Name`.

Now if Jamie Jones adopts Jane Doe, the following statements would change Jane's last name to Jones:

```
public static void main(String[] args)[2]
{
   Name jamie = new Name();
   jamie.setFirst("Jamie");
   jamie.setLast("Jones");

   Name jane = new Name();
   jane.setFirst("Jane");
   jane.setLast("Doe");

   jamie.giveLastNameTo(jane);
   . . .
} // end main
```

Figure B-5 shows the argument `jane` and the parameter `child` as the method `giveLastNameTo` executes.

B.15 What happens if you change the method definition to allocate a new name, as follows?

```
public void giveLastNameTo2(Name child)
{
   String firstName = child.getFirst();
   child = new Name();
   child.setFirst(firstName);
   child.setLast(last);
} // end giveLastNameTo2
```

With this change, the invoking statement

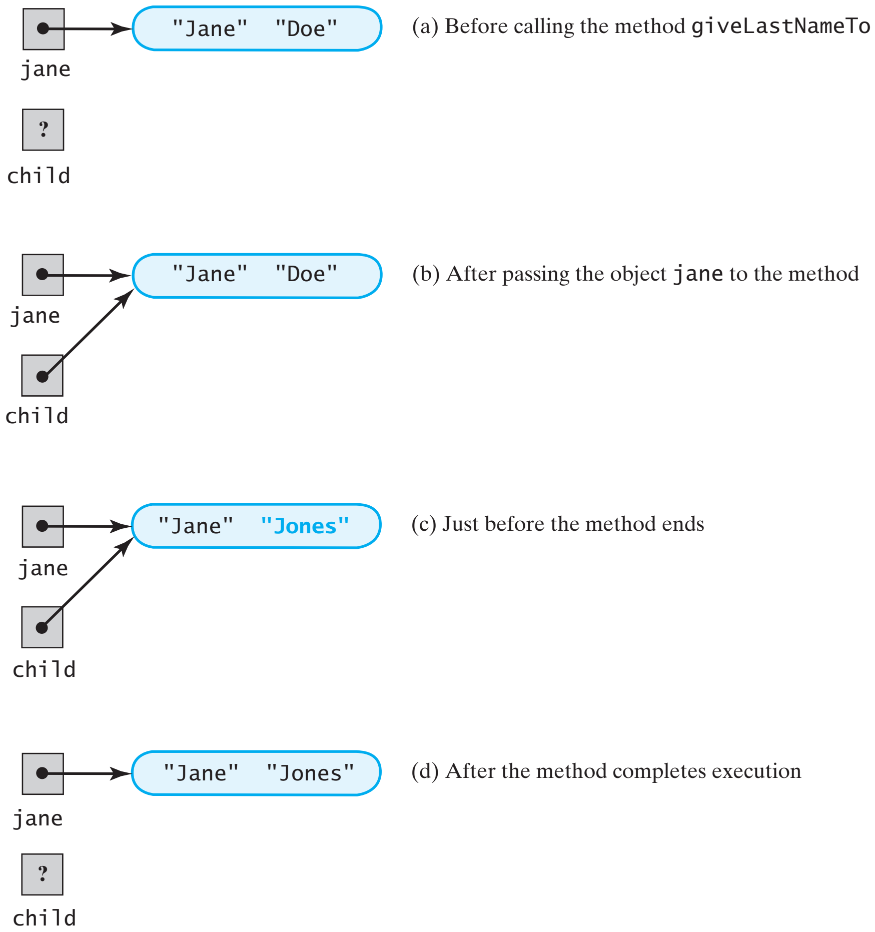
```
jamie.giveLastNameTo2(jane);
```

has no effect on `jane`, as Figure B-6 illustrates. The parameter `child` behaves like a local variable, so its value is not available outside of the method definition.

[1] The parameter mechanism for parameters of a class type is similar to **call-by-reference** parameter passing. If you are familiar with this terminology, be aware that parameters of a class type in Java behave a bit differently from call-by-reference parameters in other languages.

[2] If you are not familiar with `main` methods and application programs, consult the beginning of Appendix A.

FIGURE B-5 The method `giveLastNameTo` modifies the object passed to it as an argument



(a) Before calling the method `giveLastNameTo`

(b) After passing the object `jane` to the method

(c) Just before the method ends

(d) After the method completes execution

Question 8 Consider a method definition that begins with the statement

```
public void process(int number, Name aName)
```

If `jamie` is defined as in Segment B.14 and you invoke this method with the statement

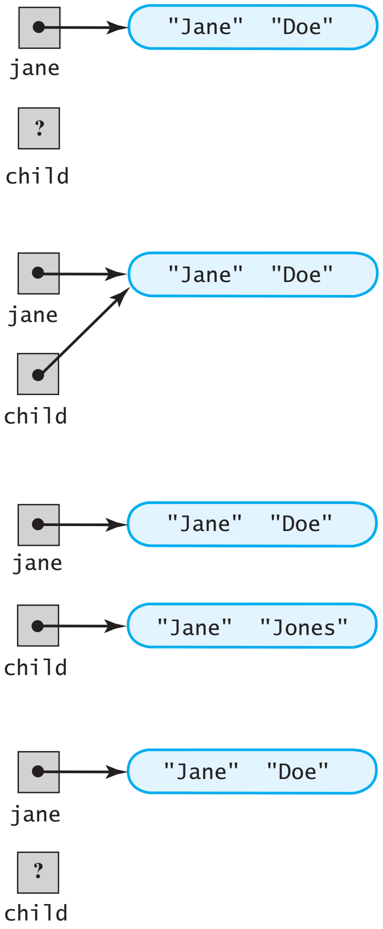
```
someObject.process(5, jamie);
```

what values are given to the parameters within the definition of the method?

Question 9 In Question 8, can the method `process` change the data fields in `jamie`?

Question 10 In Question 8, can the method `process` assign a new object to `jamie`?

FIGURE B-6 A method cannot replace an object passed to it as an argument



(a) Before calling the method

(b) After passing the object `jane` to the method

(c) Just before the method ends

(d) After the method completes execution

A Definition of the Class `Name`

B.16 A complete definition for the class `Name` appears in Listing B-1. We typically place data field declarations at the beginning of the class, but some people place them last. Although Java allows you to intermix method definitions and data field declarations, we prefer that you do not.

The sections that follow examine some other details of this class definition.

LISTING B-1 The class `Name`

```
public class Name
{
   private String first; // first name
   private String last;  // last name
   public Name()
   {
   } // end default constructor

   public Name(String firstName, String lastName)
   {
      first = firstName;
      last = lastName;
   } // end constructor

   public void setName(String firstName, String lastName)
   {
      setFirst(firstName);
      setLast(lastName);
   } // end setName

   public String getName()
   {
      return toString();
   } // end getName

   public void setFirst(String firstName)
   {
      first = firstName;
   } // end setFirst

   public String getFirst()
   {
      return first;
   } // end getFirst

   public void setLast(String lastName)
   {
      last = lastName;
   } // end setLast

   public String getLast()
   {
      return last;
   } // end getLast

   public void giveLastNameTo(Name aName)
   {
      aName.setLast(last);
   } // end giveLastNameTo

   public String toString()
   {
      return first + " " + last;
   } // end toString
} // end Name
```

Constructors

B.17 Segment B.2 mentioned that you create an object by using the `new` operator to invoke a special method called a constructor. A **constructor** allocates memory for the object and initializes the data fields. The method definition of a constructor has certain special properties. A constructor

- Has the same name as the class
- Has no return type, not even `void`
- Has any number of formal parameters, including no parameters

A class can have several constructors that differ in the number or type of parameters.

A constructor without parameters is called the **default constructor**. A class can have only one default constructor. The definition of the default constructor for `Name` is

```
public Name()
{
} // end default constructor
```

This particular default constructor has an empty body, but it need not be empty. It could explicitly initialize the data fields `first` and `last` to values other than the ones Java assigns by default.

For example, we could have defined the constructor as follows:

```
public Name()
{
   first = "";
   last = "";
} // end default constructor
```

Here we initialize the fields `first` and `last` to an empty string. If the constructor had an empty body, it would initialize these fields to `null` by default.

Note: In the absence of any explicit initialization within a constructor, data fields are set to default values: Reference types are `null`, primitive numeric types are zero, and boolean types are false.

Programming Tip: If a class depends on a data field's initial value, its constructor should set these values explicitly. Standard default values have been known to change.

Programming Tip: If a data field references a receiving object, initialize it to something other than `null`. Failure to do so can result in an execution-time error.

B.18 If you do not define any constructors for a class, Java will automatically provide a default constructor—that is, a constructor with no parameters. If you define a constructor that has parameters but you do not define a default constructor—one without parameters—Java will not provide a default constructor for you. Because classes are often reused again and again, and because eventually you might want to create a new object without specifying parameters, your classes typically should include a default constructor.

Note: Once you start defining constructors, Java will not define any other constructors for you. Most of the classes you define should include a default constructor.

B.19 The class `Name` contains a second constructor, one that initializes the data fields to values given as arguments when the client invokes the constructor:

```
public Name(String firstName, String lastName)
{
   first = firstName;
   last = lastName;
} // end constructor
```

This constructor has two parameters, `firstName` and `lastName`. You invoke it with a statement such as

```
Name jill = new Name("Jill", "Jones");
```

that passes first and last names as arguments.

B.20 After creating the object `jill`, you can change the values of its data fields by using the class's set (mutator) methods. You saw that this step was in fact necessary for the object `joe` in Segments B.2 and B.3, since `joe` was created by the default constructor and had default values—probably `null`—as its first and last names.

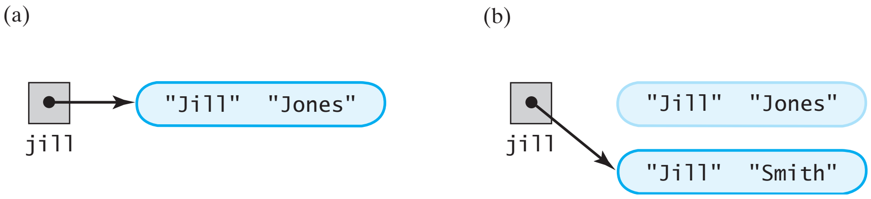
Let's see what would happen in the case of `jill` if you tried to use the constructor to change the values of `jill`'s data fields. After you created the object, the variable `jill` contained the memory address of that object, as Figure B-7a illustrates. If you now write the statement

```
jill = new Name("Jill", "Smith");
```

a new object is created, and `jill` contains its memory address. The original object is lost, because no program variable has its address, as shown in Figure B-7b.

What happens to a memory location when the variables in your program no longer reference it? Periodically, the Java run-time environment **deallocates** such memory locations by returning them to the operating system so that they can be used again. In effect, the memory is recycled. This process is called **automatic garbage collection**.

FIGURE B-7 An object (a) after its initial creation; (b) after its reference is lost



Note: Memory leak

If the Java run-time environment did not track and recycle memory that a program no longer references, a program could use all the memory available to it and subsequently fail. If you use another programming language—C++, for example—you would be responsible for returning unneeded memory to the operating system for reuse. A program that failed to return such memory would have what is known as a **memory leak**. Java programs do not have this problem.

Note: Classes without mutator methods
After you create an object of a class that has no set methods, you cannot change the values of its data fields. If they require change, you must use the constructor to create a new object. Chapter 30 discusses these classes further.

Question 11 What is a default constructor?

Question 12 How do you invoke a constructor?

Question 13 What happens if you do not define constructors for a class?

Question 14 What happens if you do not define a default constructor but you do define a constructor that has parameters?

Question 15 What happens when an object no longer has a variable that references it?

The Method `toString`

B.21 The method `toString` in the class `Name` returns a string that is the person's full name. You can use this method, for example, to display the name that the object `jill` represents by writing

```
System.out.println(jill.toString());
```

What is remarkable about `toString` is that Java will invoke it automatically when you write

```
System.out.println(jill);
```

For this reason, providing a class with a method `toString` is a good idea in general. If you fail to do so, Java will provide its own `toString` method, which produces a string that will have little meaning to you. Appendix C provides more detail about the `toString` method.

Methods That Call Other Methods

B.22 Notice the method `setName` in the class definition for `Name`. Although `setName` could use assignment statements to initialize `first` and `last`, it instead invokes the methods `setFirst` and `setLast`. Since these methods are members of the class, `setName` can invoke them without preceding the name with an object variable and a dot. If you prefer, you can use `this`, and write the invocation as

```
this.setFirst(firstName);
```

When the logic of a method's definition is complex, you should divide the logic into smaller pieces and implement each piece as a separate method. Your method can then invoke these other methods. Such helping methods, however, might be inappropriate for a client to use. If so, declare them as private instead of public so that only your class can invoke them.

Programming Tip: If a helping method is not appropriate for public use, declare it as private.

B.23 The method getName in the class Name also invokes another of Name's methods, namely toString. Here, we want both getName and toString to return the same string. Rather than writing the same statements in both methods, we have one method call the other. This ensures that both methods will always return the same values. If you later revise the definition of toString, you will automatically revise the string that getName returns.

Programming Tip: If you want two methods to have the same behavior, one of them should call the other.

B.24 Although it generally is a good idea for methods to call other methods to avoid repeating code, you need to be careful if you call public methods from the body of a constructor. For example, it is tempting to have the constructor mentioned in Segment B.19 call setName. But another class derived from your class could change the effect of setName and hence of your constructor. One solution is to define a private method that both the constructor and setName call. Another approach is given in Segment C.19 of the next appendix.

B.25 **Using this to invoke a constructor.** You can use the reserved word this to call a constructor from within the body of another constructor. For example, the class Name has two constructors. The default constructor, as given in Segment B.16, has an empty body. Segment B.17 suggested that it is a good idea to have the default constructor initialize the class's data fields explicitly, so we rewrote it, as follows:

```
public Name()
{
   first = "";
   last = "";
} // end default constructor
```

We could accomplish the same thing by revising the default constructor so that it initializes first and last by calling the second constructor, as follows:

```
public Name()
{
   this("", "");
} // end default constructor
```

The statement

```
this("", "");
```

calls the constructor that has two parameters. In this way, the initialization occurs in one place.

Programming Tip: Link the definitions of several constructors by using this to invoke one of them. Any use of this must be first in the body of the constructor's definition.

SELF-TEST

Question 16 A third constructor for the class Name could have the following header:

```
public Name(Name aName)
```

This constructor creates a Name object whose data fields match those of the object aName. Implement this new constructor by invoking one of the existing constructors.

Methods That Return an Instance of Their Class

B.26 The method `setName` in the class `Name` is a void method that sets both the first and last names of a `Name` object. We might use this method as follows:

```
Name jill = new Name();
jill.setName("Jill", "Greene");
```

Here, `setName` sets the first and last names of the invoking object `jill`.

Instead of defining `setName` as a void method, we could have it return a reference to the revised instance of `Name`, as follows:

```
public Name setName(String firstName, String lastName)
{
   setFirst(firstName);
   setLast(lastName);

   return this;
} // end setName
```

Here `this` represents the receiving object, whose first and last names were just set.

We can call this definition of `setName` just as we called its void version, or we could invoke it as follows:

```
Name jill = new Name();
Name myFriend = jill.setName("Jill", "Greene");
```

As before, `setName` sets the first and last names of the receiving object `jill`. But then it returns a reference to the receiving object. Here we used an assignment statement to retain this reference as an alias for `jill`. However, the invocation of `setName` could appear as an argument to another method.

Methods that return an instance of their class are not unusual within the classes of the Java Class Library.

Static Fields and Methods

B.27 **Static fields.** Sometimes you need a data field that does not belong to any one object. For example, a class could track how many invocations of the class's methods are made by all objects of the class. Such a data field is called a **static field**, **static variable**, or **class variable**. You declare a static field by adding the reserved word `static`. For example, the declaration

```
private static int numberOfInvocations = 0;
```

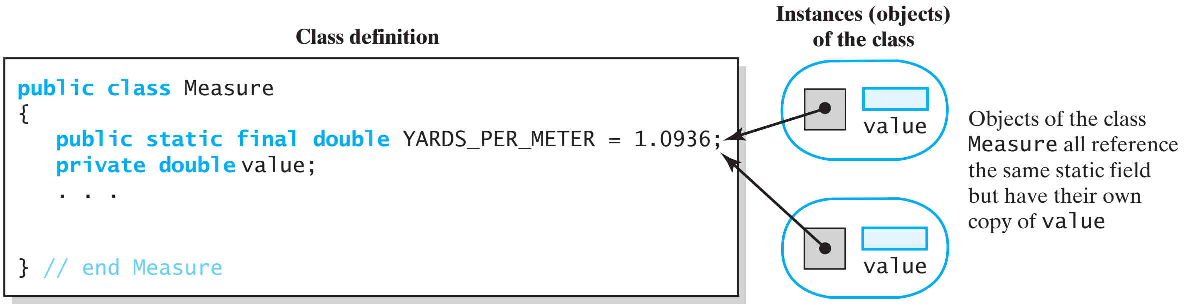
defines one copy of `numberOfInvocations` that every object of the class can access. Objects can use a static field to communicate with each other or to perform some joint action. In this example, each method increments `numberOfInvocations`. Such static fields normally should be private to ensure that access occurs only through appropriate accessor and mutator methods.

The definition of a named constant provides another example of a static field. The statement

```
public static final double YARDS_PER_METER = 1.0936;
```

defines a static field `YARDS_PER_METER`. The class has one copy of `YARDS_PER_METER`, rather than each object of the class having its own copy, as Figure B-8 illustrates. Since `YARDS_PER_METER` is also declared as final, its value cannot change, so we can safely make it public. But static fields in general can change value if you omit the modifier `final`.

FIGURE B-8 A static field `YARDS_PER_METER` versus a nonstatic field `value`



Class definition

```
public class Measure
{
   public static final double YARDS_PER_METER = 1.0936;
   private double value;
   . . .

} // end Measure
```

Objects of the class `Measure` all reference the same static field but have their own copy of `value`

> **Note: Static does not mean constant**
>
> A static field is shared by all objects of its class, but its value can change. If you want a field's value to remain constant, you must declare it as final. Although they often appear together, the modifiers `static` and `final` are not related. Thus, a field can be static, final, or both static and final.

B.28 **Static methods.** Sometimes you need a method that does not belong to an object of any kind. For example, you might need a method to compute the maximum of two integers or a method to compute the square root of a number. These methods have no obvious object to which they should belong. In these cases, you can define the method as static by adding the reserved word `static` to the header of the method.

A **static method** or **class method** is still a member of a class. However, you use the class name instead of an object name to invoke the method. For example, Java's predefined class `Math` contains several standard mathematical methods, such as `max` and `sqrt`. All of these methods are static, so you do not need—and in fact have no real use for—an object of the class `Math`. You call these methods by using the class name in place of an object name. Thus, you write statements such as

```
int maximum = Math.max(2, 3);
double root = Math.sqrt(4.2);
```

The definition of a static method cannot reference any data field in its class that is not static. It can, however, reference its class's static fields. Likewise, it cannot invoke a nonstatic method of the class, unless it creates a local object of the class and uses it to invoke the nonstatic method. However, a static method can call other static methods within its class. Since every application program's `main` method is static, these restrictions apply to `main` methods.

> **Programming Tip: Every class can have a `main` method**
>
> You can include a test of a class as a `main` method in the class's definition. Anytime you suspect something is wrong, you can easily test the class definition. Since you—and others—can see what tests you performed, flaws in your testing will become apparent. If you use the class as a program, the `main` method is invoked. When you use the class to create objects in another class or program, the `main` method is ignored.

Note: Constructors cannot be static

A constructor creates an object of its class, so it makes no sense to disassociate a constructor from such objects.

Question 17 What happens if you do not declare a constant data field as static?

Overloading Methods

B.29 Several methods within the same class can have the same name, as long as the methods do not have identical parameters. Java is able to distinguish among these methods since their parameters differ in number or data type. We say that these methods are **overloaded**.

For example, the class `Name` has the method `setName` whose header is

```
public void setName(String firstName, String lastName)
```

Imagine that we want another method that gives a `Name` object the same name as another `Name` object. The header for this method could be, for example,

```
public void setName(Name otherName)
```

The two versions of `setName` are not exactly the same, as they have different numbers of parameters.

We could then overload `setName` with a third method whose header is

```
public void setName(String firstName, Name otherName)
```

Imagine that this method sets the first name to the string `firstName` and the last name to `otherName`'s last name. Although two of the three versions of `setName` have two parameters each, the data types of the parameters are not exactly the same. The data types of the first parameter in each method match, but the data types of the second parameters do not.

Simply changing the names of the parameters is not enough to overload a method. The parameter names are not relevant. When two methods have the same name and the same number of parameters, at least one pair of corresponding parameters must differ in data type. Also, changing only the return type is insufficient. The compiler cannot distinguish between two methods that differ only in their return types.

Finally, note that by defining more than one constructor for a class, you are actually overloading them. Thus, their parameters must differ in either number or data type.

Note: Overloading a method definition

A method in a class overloads another method in the same class when both methods have the same name but differ in the number or types of parameters.

Note: The signature of a method

A method's **signature** consists of its name and the number, types, and order of its parameters. Thus, overloaded methods have the same name but different signatures.

Question 18 If a method overloads another method, can the two methods have different return types?

Enumeration as a Class

Appendix A introduced enumerations. As Segments A.54 and A.56 mentioned, the compiler creates a class when it encounters an enumeration. This section expands that discussion. Although you should consider using enumerations in your programs, they are not central to the presentation in this book.

B.30 When you define an enumeration, the class created has methods such as `toString`, `equals`, `ordinal`, and `valueOf`. For example, let's define a simple enumeration for the suits of playing cards, as follows:

```
enum Suit {CLUBS, DIAMONDS, HEARTS, SPADES}
```

We then can use these methods in the following ways:

- `Suit.CLUBS.toString()` returns the string *CLUBS*. That is, `toString` returns the name of its receiving object.
- `System.out.println(Suit.CLUBS)` calls `toString` implicitly, and so it displays *CLUBS*.
- `s.equals(Suit.DIAMONDS)` tests whether `s`, an instance of `Suit`, equals `DIAMONDS`.
- `Suit.HEARTS.ordinal()` returns 2, the ordinal position of `HEARTS` in the enumeration.
- `Suit.valueOf("HEARTS")` returns `Suit.HEARTS`.

B.31 You can define additional methods—including constructors—for any enumeration. By defining a private data field, you can assign values to each of the objects in the enumeration. Adding a get method will provide a way for the client to access these values. Listing B-2 contains a new definition for the enumeration `Suit` that shows how these ideas are realized.

LISTING B-2 The enumeration `Suit`

```
/** An enumeration of card suits. */
enum Suit
{
   CLUBS("black"), DIAMONDS("red"), HEARTS("red"), SPADES("black");

   private final String color;

   private Suit(String suitColor)
   {
      color = suitColor;
   } // end constructor

   public String getColor()
   {
      return color;
   } // end getColor
} // end Suit
```

We have chosen strings as the values for the enumerated objects. Notation such as `CLUBS("black")` invokes the constructor that we have provided and sets the value of `CLUBS`'s private data field `color` to the string *black*. Note that `color`'s value cannot change, since it is declared as final. Also observe that the constructor is private, so it is not available to the client. It is called only within the definition of `Suit`. The method `getColor` provides public access to the value of `color`.

Note: Constructors within enumerations must be private.

B.32 The class in Listing B-3 provides a simple demonstration of the enumeration `Suit` that appears in the previous segment. We use a for-each loop, described in Segment A.63 of Appendix A. In addition to the methods that we defined in `Suit`, the enumeration also has the methods `equals`, `ordinal`, and `valueOf` described in Segment B.30 of this appendix and Segment A.56 of Appendix A.

LISTING B-3 A class that demonstrates the enumeration `Suit` given in Listing B-2 of Segment B.31

```
/** A demonstration of the enumeration Suit. */
public class SuitDemo
{
   private enum Suit
   {
      . . . < See Listing B-2 >
   } // end Suit

   public static void main(String[] args)
   {
      for (Suit nextSuit : Suit.values())
      {
         System.out.println(nextSuit + " are " + nextSuit.getColor() +
                            " and have an ordinal value of " +
                            nextSuit.ordinal());
      } // end for
   } // end main
} // end SuitDemo
```

Output

```
CLUBS are black and have an ordinal value of 0
DIAMONDS are red and have an ordinal value of 1
HEARTS are red and have an ordinal value of 2
SPADES are black and have an ordinal value of 3
```

Note: Enumerations can have an access modifier such as `public` or `private`. If you omit the access modifier, the enumeration is private. You can define a public enumeration within its own file, just as you would define any other public class.

B.33 **Example**. Segment A.54 in Appendix A defined an enumeration for the letter grades A, B, C, D, and F. Here we expand that definition to include plus and minus grades as well as the quality-point values associated with each grade. As in the previous definition of `Suit`, we provide private data fields and a private constructor to represent and initialize the string representation and numeric value for each grade. We also provide accessor methods for the data fields and override the method `toString`.

Listing B-4 shows our new definition of the enumeration `LetterGrade`. We have made it public and will store it in the file `LetterGrade.java`.

LISTING B-4 The enumeration `LetterGrade`

```
public enum LetterGrade
{
   A("A", 4.0), A_MINUS("A-", 3.7), B_PLUS("B+", 3.3), B("B", 3.0),
   B_MINUS("B-", 2.7), C_PLUS("C+", 2.3), C("C", 2.0), C_MINUS("C-", 1.7),
   D_PLUS("D+", 1.3), D("D", 1.0), F("F", 0.0);

   private final String grade;
   private final double points;

   private LetterGrade(String letterGrade, double qualityPoints)
   {
      grade = letterGrade;
      points = qualityPoints;
   } // end constructor

   public String getGrade()
   {
      return grade;
   } // end getGrade

   public double getQualityPoints()
   {
      return points;
   } // end getQualityPoints

   public String toString()
   {
      return getGrade();
   } // end toString
} // end LetterGrade
```

If we define

```
LetterGrade myGrade = LetterGrade.B_PLUS;
```

then

- `myGrade.toString()` returns the string *B*+.
- `System.out.println(myGrade)` displays *B*+, since it calls `toString` implicitly.
- `myGrade.getGrade()` returns the string *B*+.
- `myGrade.getQualityPoints()` returns 3.3.

If we had not overriden the method `toString` with our own definition, `myGrade.toString()` would return the string *B_PLUS*.

Like the enumeration `Suit` given in Segment B.31, `LetterGrade` has the methods `equals`, `ordinal`, and `valueOf`.

Question 19 If `myGrade` is an instance of `LetterGrade` and is assigned the value `LetterGrade.B_PLUS`, what is returned by each of the following expressions?

a. `myGrade.ordinal()`
b. `myGrade.equals(LetterGrade.A_MINUS)`
c. `LetterGrade.valueOf("A_MINUS")`

Question 20 What does the following statement display?

```
System.out.println(LetterGrade.valueOf("A_MINUS"));
```

Packages

B.34 Using several related classes is more convenient if you group them together within a Java **package**. To identify a class as part of a particular package, you begin the file that contains the class with a statement like

```
package myStuff;
```

You then place all of the files within one directory or folder and give it the same name as the package.

To use a package in your program, you begin the program with a statement such as

```
import myStuff.*;
```

The asterisk makes all public classes within the package available to the program. You could, however, replace the asterisk with the name of a particular class in the package that you want to use. You probably have already used packages provided by Java, such as the package `java.util`.

Why did we just say “public classes”? What other kind of class is there? You can use an access modifier to control access to a class just as you can to control access to a data field or method. A public class—whether it is within a package or not—is available to any other class. If you omit the class’s access modifier entirely, the class is available only to other classes within the same package. This kind of class is said to have **package access**. Similarly, if you omit the access modifier on data fields or methods, they are available by name inside the definition of any class within the same package but not outside of the package. You can use package access in situations where you have a package of cooperating classes that act as a single encapsulated unit. If you control the package directory, you control who is allowed to access the package.

The Java Class Library

B.35 Java comes with a collection of many classes that you can use in your programs. For example, Segment B.28 mentioned the class `Math`, which contains several standard mathematical methods such as `sqrt`. This collection of classes is known as the **Java Class Library**, and sometimes as the **Java Application Programming Interface**, or **API**. The classes in this library are organized into several standard packages. For example, the class `Math` is a part of the package `java.lang`. Note that no `import` statement is needed when you use a class from this particular package.

You should become familiar with the documentation provided for the Java Class Library at `download.oracle.com/javase/7/docs/api/`.

Generic Data Types

B.36 **Generics** are a feature of Java that enables you to write a place holder instead of an actual class type within the definition of a class or interface. The place holder is a **generic data type**, or simply a **generic type** or a **type parameter**.

As you saw earlier in this appendix, the class `Object` is the ultimate ancestor of all other classes. Given a reference to an object of any type, you can assign the reference to a variable whose type is `Object`. Although you might be tempted to use `Object` as a generic class, you should not do so. Instead, you should use a generic data type to represent any one class type.

Imagine an array `A` of objects. If the data type of `A` was declared as `Object[]`, you could place strings, for example, into the array. Nothing, however, would stop you from placing objects of several other classes into the array along with the strings. While this might sound attractive, you likely would have a problem using such an array. For example, if you removed an object from the array, you would not know its dynamic type. Is it a string or some other object? Methods are available to retrieve an object's dynamic type, however, so such an array can have a use.

In contrast, an array or any other group whose entries are referenced by variables of a generic type can contain only objects of classes related by inheritance. Thus, generics enable you to restrict the types of entries in your collections. This restriction is not unusual, as it makes these collections easier to use.

B.37 To establish a generic type when you define a class, you write an identifier enclosed in angle brackets after the class name in the class's header. For example, a class could begin as follows:

```
public class MyClass<T>
```

The identifier `T`—which can be any identifier but usually is a single capital letter—represents a class type within the definition of `MyClass`. When you use `MyClass`, you supply an actual class type to replace `T`.

For example, to create an instance of `MyClass` by invoking its default constructor, you could write a statement such as

```
MyClass<String> item = new MyClass<String>();
```

Now, wherever `T` appears as a data type in the definition of `MyClass`, `String` is used. A generic type must be a reference type, not a primitive type.

B.38 **Example: Defining a generic class.** In mathematics, an *ordered pair* is a pair of values, *a* and *b*, denoted as (*a*, *b*). We say that the values in (*a*, *b*) are ordered because (*a*, *b*) does not equal (*b*, *a*) unless *a* equals *b*. For example, a point in a two-dimensional space is described by its *x*-coordinate and *y*-coordinate, that is, the ordered pair (*x*, *y*).

Let's create a class to represent ordered pairs of objects of the same class type. The class `OrderedPair`, given in Listing B-5, assumes that we care about the order in which the objects appear in the pair. The notation `<T>` follows the identifier *name* in the class's header. Within the definition, `T` represents the data type of the two private data fields, the data type of the two parameters of the constructor, the return type of the methods `getFirst` and `getSecond`, and the data type of the local variable `temp` in the method `changeOrder`.

LISTING B-5 The class `OrderedPair`

```
/**
   A class of ordered pairs of objects having the same data type.
   @author Frank M. Carrano
*/
```

```
public class OrderedPair<T>
{
   private T first, second;

   public OrderedPair(T firstItem, T secondItem) // NOTE: no <T> after
   {                                             // constructor name
      first = firstItem;
      second = secondItem;
   } // end constructor

   /** Returns the first object in this pair. */
   public T getFirst()
   {
      return first;
   } // end getFirst

   /** Returns the second object in this pair. */
   public T getSecond()
   {
      return second;
   } // end getSecond

   /** Returns a string representation of this pair. */
   public String toString()
   {
      return "(" + first + ", " + second + ")";
   } // end toString

   /** Interchanges the objects in this pair. */
   public void changeOrder()
   {
      T temp = first;
      first = second;
      second = temp;
   } // changeOrder
} // end OrderedPair
```

Note: Within the definition of a class *name*<T>, where T is a generic type parameter,

- <T> follows the identifier *name* in the class's header
- <T> does not follow the names of the constructors in their definitions
- T—not <T>—can be a data type of data fields, method parameters, and local variables, and it can be a return type of methods.

Note: A generic data type represents a class type, not a primitive type.

B.39 **Example: Creating objects of a generic class.** The type parameter T for the previous class OrderedPair represents the data type of the two objects in a pair. The actual class type associated with these objects is given when a specific instance of OrderedPair is instantiated. For example, you can use the class OrderedPair<String> to create a pair of String objects and the class OrderedPair<Name> to create a pair of Name objects, as shown in the following statements:

```
OrderedPair<String> fruit = new OrderedPair<String>("apples", "oranges");
System.out.println(fruit);
fruit.changeOrder();
System.out.println(fruit);

Name tweedleDee = new Name("Tweedle", "Dee");
Name tweedleDum = new Name("Tweedle", "Dum");
OrderedPair<Name> namePair = new OrderedPair<Name>(tweedleDee, tweedleDum);
System.out.println(namePair);
namePair.changeOrder();
System.out.println(namePair);
```

These statements produce the following output:

```
(apples, oranges)
(oranges, apples)
(Tweedle Dee, Tweedle Dum)
(Tweedle Dum, Tweedle Dee)
```

You also can create pairs of objects of any class that uses inheritance[3] to extend Name. For example, if the class FormalName extends Name and adds a title, such as Mr. or Ms., namePair could contain objects of both Name and FormalName.

Note: Within the client of a generic class *name<class-type>*,

- An expression of the form

  new *name<class-type>*(...)

  creates an object of the class

- The data type of objects of the class is *name<class-type>*, not *name*

Question 21 What method must a class such as String or Name define so that OrderedPair's method toString works correctly?

Question 22 Consider the class OrderedPair, as given in Listing B-5. Suppose that we did not use a generic type, but instead omitted <T> and declared the data types of the private fields, method parameters, and local variable to be Object instead of T. What would the effect of these changes be on the use of the class?

B.40 **More than one generic type.** In the previous example, the objects in a pair have either the same data type or data types related by inheritance. You can define more than one generic type within a class definition by writing their identifiers, separated by commas, within the angle brackets after the class's name, as in the class Pair shown in Listing B-6.

[3] Appendix C discusses inheritance.

LISTING B-6 The class `Pair`

```
public class Pair<S, T>
{
   private S first;
   private T second;

   public Pair(S firstItem, T secondItem)
   {
      first = firstItem;
      second = secondItem;
   } // end constructor

   public String toString()
   {
      return "(" + first + ", " + second + ")";
   } // end toString
} // end Pair
```

You can use this class to pair a name and a telephone number, for example, by writing the following statements:

```
Name joe = new Name("Joe", "Java");
String joePhone = "(401) 555-1234";
Pair<Name, String> joeEntry = new Pair<Name, String>(joe, joePhone);
System.out.println(joeEntry);
```

The output displayed is

```
(Joe Java, (401) 555-1234)
```

? SELF-TEST

Question 23 Can you use the class `OrderedPair`, as defined in Listing B-5, to pair two objects having different and unrelated data types? Why or why not?

Question 24 Can you use the class `Pair`, as defined in the previous segment, to pair two objects having the same data type? Why or why not?

Question 25 Using the class `Name`, as defined previously in this appendix, write statements that pair two students as lab partners.

Question 26 Using the class `Name`, as defined previously in this appendix, write statements that pair your name with the random sequence number given in the `int` variable `number`.

Exercises and Projects

A collection of exercises and projects for this appendix are available online at `pearsonhighered.com/carrano`.

Answers to Self-Test Questions

1.
```
Name myName = new Name();
myName.setFirst("Joseph");
myName.setLast("Brown");
```

2.
```
System.out.println(myName.getLast() + ", " + myName.getFirst());
```

3. Valued: getFuelLevel, getSpeed, getMileage.
Void: goForward, goBackward, accelerate, decelerate.

4. Mutator.

5. Valued.

6. Void.

7. A client could set the data field to an illegal value.

8. number is 5 and aName references jamie.

9. Yes, by using Name's set methods.

10. No.

11. A default constructor is a constructor that has no parameters.

12. You invoke a constructor by using the new operator.

13. If you do not define a constructor, the compiler defines a default constructor.

14. The compiler does not define additional constructors. Thus, the class will not have a default constructor.

15. An object that is not referenced is marked for garbage collection. Eventually, the Java run-time environment deallocates the object by returning its memory locations to the operating system so that they can be used again.

16.
```
public Name(Name aName)
{
   this(aName.getFirst(), aName.getLast());
} // end constructor
```

17. Each instance (object) of a class will have a copy of a constant data field that is not static.

18. Yes, as long as the methods also have different parameters. Return type alone is not sufficient to distinguish the methods.

19. **a.** 2
b. false
c. LetterGrade.A_MINUS

20. A-, because the method toString is called implicitly.

21. toString

22. The statements given in Segment B.39, for example, would begin with

```
OrderedPair fruit = new OrderedPair("apples", "oranges");
```

The disadvantage to this version of `OrderedPair` is that the compiler cannot warn you if you pair objects of different and unrelated types. Thus, you would be able to write

```
Name joe = new Name("Joe", "Java");
String joePhone = "(401) 555-1234";
OrderedPair joeEntry = new OrderedPair();
joeEntry.setPair(joe, joePhone);
```

23. No. The class defines only one generic type.

24. Yes. You can write the same data type twice to correspond to both `S` and `T`.

25.

```
Name kristen = new Name("Kristen", "Doe");
Name luci = new Name("Luci", "Lei");
OrderedPair<Name> labPartners = new OrderedPair<Name>(kristen, luci);
```

26.

```
Name kristen = new Name("Kristen", "Doe");
Integer seqN = number;
Pair<Name, Integer> aPair = new Pair<Name, Integer>(kristen, seqN);
```

Appendix C

Creating Classes from Other Classes

Contents

Prerequisites

Appendix B   Java Classes

A major advantage of object-oriented programming is the ability to use existing classes when defining new classes. That is, you use classes that you or someone else has written to create new classes, rather than reinventing everything yourself. We begin this appendix with two ways to accomplish this feat.

In the first way, you simply declare an instance of an existing class as a data field of your new class. In fact, you have done this already if you have ever defined a class that had a string as a data field. Since your class is composed of objects, this technique is called composition.

The second way is to use inheritance, whereby your new class inherits properties and behaviors from an existing class, augmenting or modifying them as desired. This technique is more complicated than composition, so we will devote more time to it. As important as inheritance is in Java, you should not ignore composition as a valid and desirable technique in many situations.

Both composition and inheritance define a relationship between two classes. These relationships are often called, respectively, *has a* and *is a* relationships. You will see why when we discuss them in this appendix.

Polymorphism is another key feature of object-oriented programming. In fact, object-oriented programming is usually described in terms of its main features: encapsulation, inheritance, and polymorphism. Used in conjunction with inheritance, polymorphism enables different objects that call methods having the same name to act appropriately.

Composition

C.1 Appendix B introduced you to the class `Name` to represent a person's name. It defined constructors, accessor methods, and mutator methods that involved the person's first and last names. The data fields in `Name` are instances of the class `String`. A class uses **composition** when it has a data field that is an instance of another class. And since the class `Name` has an instance of the class `String` as a data field, the relationship between `Name` and `String` is called a ***has a*** relationship.

Let's create another class that uses composition. Consider a class of students, each of whom has a name and an identification number. Thus, the class `Student` contains two objects as data fields: an instance of the class `Name` and an instance of the class `String`:

```
private Name   fullName;
private String id;
```

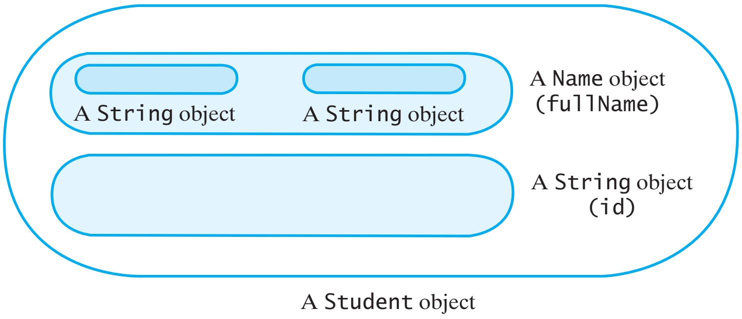
Figure C-1 shows an object of type `Student` and its data fields. Notice that the `Name` object has two `String` objects as its data fields. It is important to realize that these data fields actually contain references to objects, not the objects themselves.

For methods, we give the class `Student` constructors, accessors, mutators, and `toString`. Recall that `toString` is invoked when you use `System.out.println` to display an object, so it is a handy method to include in your class definitions.

Note: Composition (*has a*)

A class uses composition when it has objects as data fields. The class's implementation has no special access to such objects and must behave as a client would. That is, the class must use an object's methods to manipulate the object's data. Since the class "has a," or contains, an instance (object) of another class, the classes are said to have a *has a* relationship.

FIGURE C-1 A `Student` object is composed of other objects



C.2 Look at the definition of the class `Student` in Listing C-1, and then we will make a few more observations.

LISTING C-1 The class `Student`

```
public class Student
{
   private Name   fullName;
   private String id;      // identification number

   public Student()
   {
      fullName = new Name();
      id = "";
   } // end default constructor

   public Student(Name studentName, String studentId)
   {
      fullName = studentName;
      id = studentId;
   } // end constructor

   public void setStudent(Name studentName, String studentId)
   {
      setName(studentName); // or fullName = studentName;
      setId(studentId);     // or id = studentId;
   } // end setStudent

   public void setName(Name studentName)
   {
      fullName = studentName;
   } // end setName

   public Name getName()
   {
      return fullName;
   } // end getName

   public void setId(String studentId)
   {
      id = studentId;
   } // end setId

   public String getId()
   {
      return id;
   } // end getId

   public String toString()
   {
      return id + " " + fullName.toString();
   } // end toString
} // end Student
```

The method setStudent is useful when we create a student object by using the default constructor or if we want to change both the name and identification number that we gave to a student object earlier. Notice that the method invokes the other set methods from this class to initialize the data fields. For example, to set the field fullName to the parameter studentName, setStudent uses the statement

```
setName(studentName);
```

We could also write this statement as

```
this.setName(studentName);
```

where this refers to the instance of Student that receives the call to the method setStudent. Or we could write the assignment statement

```
fullName = studentName;
```

Implementing methods in terms of other methods is usually desirable. It might not be desirable when implementing constructors, however, as you will see later in this appendix.

Suppose that we want toString to return a string composed of the student's identification number and name. It must use methods in the class Name to access the name as a string. For example, toString could return the desired string by using either

```
return id + " " + fullName.getFirst() + " " + fullName.getLast();
```

or, more simply,

```
return id + " " + fullName.toString();
```

The data field fullName references a Name object whose private fields are not accessible by name in the implementation of the class Student. We can access them indirectly via the accessor methods getFirst and getLast or by invoking Name's toString method.

SELF-TEST

Question 1 What data fields would you use in the definition of a class Address to represent a student's address?

Question 2 Add a data field to the class Student to represent a student's address. What new methods should you define?

Question 3 What existing methods need to be changed in the class Student as a result of the added field that Question 2 described?

Question 4 What is another implementation for the default constructor that uses this, as described in Segment B.25 of Appendix B?

Adapters

C.3 Suppose that you have a class, but the names of its methods do not suit your application. Or maybe you want to simplify some methods or eliminate others. You can use composition to write a new class that has an instance of your existing class as a data field and defines the methods that you want. Such a new class is called an **adapter class**.

For example, suppose that instead of using objects of the class `Name` to name people, we want to use simple nicknames. We could use strings for nicknames, but like `Name`, the class `String` has more methods than we need. The class `NickName` in Listing C-2 has an instance of the class `Name` as a data field, a default constructor, and set and get methods. Arbitrarily, we use the first-name field of the class `Name` to store the nickname.

LISTING C-2 The class `NickName`

```
public class NickName
{
   private Name nick;

   public NickName()
   {
      nick = new Name();
   } // end default constructor

   public void setNickName(String nickName)
   {
      nick.setFirst(nickName);
   } // end setNickName

   public String getNickName()
   {
      return nick.getFirst();
   } // end getNickName
} // end NickName
```

Notice how this class uses the methods of the class `Name` to implement its methods. A `NickName` object now has only `NickName`'s methods, and not the methods of `Name`.

SELF-TEST

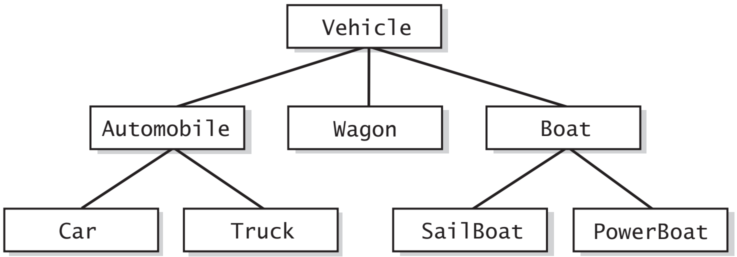
Question 5 Write statements that define `bob` as an instance of `NickName` to represent the nickname *Bob*. Then, using `bob`, write a statement that displays *Bob*.

Inheritance

C.4 **Inheritance** is a major aspect of object-oriented programming that enables you to organize classes. The name comes from the notion of inherited traits like eye color, hair color, and so forth, but it is perhaps clearer to think of inheritance as a classification system. Inheritance allows you to define a general class and then later to define more specialized classes that add to or revise the details of the older, more general class definition. This saves work, because the specialized class inherits all the properties of the general class and you need only program the new or revised features.

For example, you might define a class for vehicles and then define more specific classes for particular types of vehicles, such as automobiles, wagons, and boats. Similarly, the class of automobiles includes the classes of cars and trucks. Figure C-2 illustrates this hierarchy of classes. The `Vehicle` class is the superclass for the subclasses, such as `Automobile`. The `Automobile` class is the superclass for the subclasses `Car` and `Truck`. Another term for superclass is base class, and another term for subclass is derived class.

FIGURE C-2 A hierarchy of classes



As you move up in the diagram, the classes are more general. A car is an automobile and therefore is also a vehicle. However, a vehicle is not necessarily a car. A sailboat is a boat and is also a vehicle, but a vehicle is not necessarily a sailboat.

C.5 Java and other programming languages use inheritance to organize classes in this hierarchical way. A programmer can then use an existing class to write a new one that has more features. For example, the class of vehicles has certain properties—like miles traveled—that its data fields record. The class also has certain behaviors—like going forward—that its methods define. The classes `Automobile`, `Wagon`, and `Boat` have these properties and behaviors as well. Everything that is true of all `Vehicle` objects, such as the ability to go forward, is described only once and inherited by the classes `Automobile`, `Wagon`, and `Boat`. The subclasses then add to or revise the properties and behaviors that they inherit. Without inheritance, descriptions of behaviors like going forward would have to be repeated for each of the subclasses `Automobile`, `Wagon`, `Boat`, `Car`, `Truck`, and so on.

Note: Inheritance

Inheritance is a way of organizing classes so that common properties and behaviors can be defined only once for all the classes involved. Using inheritance, you can define a general class and then later define more specialized classes that add to or revise the details of the older, more general class definition.

Since the `Automobile` class is derived from the `Vehicle` class, it inherits all the data fields and public methods of that class. The `Automobile` class would have additional fields for such things as the amount of fuel in the fuel tank, and it would also have some added methods. Such fields and methods are not in the `Vehicle` class, because they do not apply to all vehicles. For example, wagons have no fuel tank.

Inheritance gives an instance of a subclass all the behaviors of the superclass. For example, an automobile will be able to do everything that a vehicle can do; after all, an automobile *is a* vehicle. In fact, inheritance is known as an ***is a*** relationship between classes. Since the subclass and the superclass share properties, you should use inheritance only when it makes sense to think of an instance of the subclass as also being an instance of the superclass.

Note: An *is a* relationship

With inheritance, an instance of a subclass is also an instance of the superclass. Thus, you should use inheritance only when this *is a* relationship between classes is meaningful.

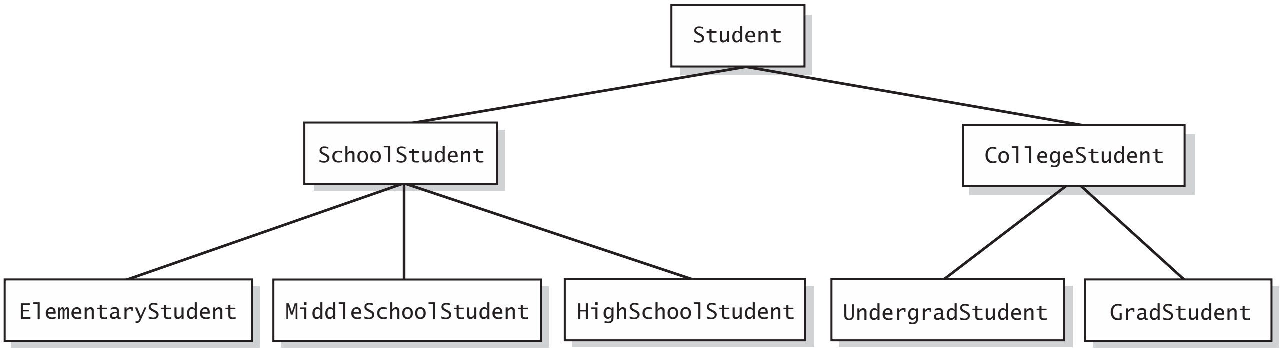
Question 6 Some vehicles have wheels and some do not. Revise Figure C-2 to organize vehicles according to whether they have wheels.

C.6 **Example.** Let's construct an example of inheritance within Java. Suppose we are designing a program that maintains records about students, including those in grade school, high school, and college. We can organize the records for the various kinds of students by using a natural hierarchy that begins with students. College students are then one subclass of students. College students divide into two smaller subclasses: undergraduate students and graduate students. These subclasses might further subdivide into still smaller subclasses. Figure C-3 diagrams this hierarchical arrangement.

A common way to describe subclasses is in terms of family relationships. For example, the class of students is said to be an **ancestor** of the class of undergraduate students. Conversely, the class of undergraduate students is a **descendant** of the class of students.

Although our program may not need any class corresponding to students in general, thinking in terms of such classes can be useful. For example, all students have names, and the methods of initializing, changing, and displaying a name will be the same for all students. In Java, we can define a class that includes data fields for the properties that belong to all subclasses of students. The class likewise will have methods for the behaviors of all students, including methods that manipulate the class's data fields. In fact, we have already defined such a class—`Student`—in Segment C.2.

FIGURE C-3 A hierarchy of student classes



C.7 Now consider a class for college students. A college student is a student, so we use inheritance to derive the class `CollegeStudent` from the class `Student`. Here, `Student` is the existing superclass and `CollegeStudent` is the new subclass. The subclass inherits—and therefore has—all the data fields and methods of the superclass. In addition, the subclass defines whatever data fields and methods we wish to add.

To indicate that `CollegeStudent` is a subclass of `Student`, we write the phrase `extends Student` on the first line of the class definition. Thus, the class definition of `CollegeStudent` begins

```
public class CollegeStudent extends Student
```

When we create a subclass, we define only the added data fields and the added methods. For example, the class `CollegeStudent` has all the data fields and methods of the class `Student`, but we do not mention them in the definition of `CollegeStudent`. In particular, every object of the class `CollegeStudent` has a data field called `fullName`, but we do not declare the data field `fullName` in the definition of the class `CollegeStudent`. The data field is there, however. But because `fullName` is a private data field of the class `Student`, we cannot reference `fullName` directly by name within `CollegeStudent`. We can, however, access and change this data field by using `Student`'s methods, since the class `CollegeStudent` inherits all of the public methods in the superclass `Student`.

For example, if `cs` is an instance of `CollegeStudent`, we can write

```
cs.setName(new Name("Joe", "Java"));
```

even though `setName` is a method of the superclass `Student`. Since we have used inheritance to construct `CollegeStudent` from the class `Student`, every college student *is a* student. That is, a `CollegeStudent` object "knows" how to perform `Student` behaviors.

C.8 A subclass, like `CollegeStudent`, can also add some data fields and/or methods to those it inherits from its superclass. For example, `CollegeStudent` adds the data field `year` and the methods `setYear` and `getYear`. We can set the graduation year of the object `cs` by writing

```
cs.setYear(2016);
```

Suppose that we also add a data field that represents the degree sought and the methods to access and change it. We could also add fields for an address and grades, but to keep it simple, we will not. Let's look at the class as given in Listing C-3 and focus on the constructors first.

LISTING C-3 The class `CollegeStudent`

```
public class CollegeStudent extends Student
{
   private int    year;    // year of graduation
   private String degree; // degree sought

   public CollegeStudent()
   {
      super();     // must be first
      year = 0;
      degree = "";
   } // end default constructor

   public CollegeStudent(Name studentName, String studentId,
                         int graduationYear, String degreeSought)
   {
      super(studentName, studentId); // must be first
      year = graduationYear;
      degree = degreeSought;
   } // end constructor

   public void setStudent(Name studentName, String studentId,
                          int graduationYear, String degreeSought)
   {
      setName(studentName); // NOT fullName = studentName;
      setId(studentId);     // NOT id = studentId;
// or setStudent(studentName, studentId); (see Segment C.17)

      year = graduationYear;
      degree = degreeSought;
   } // end setStudent
   < The methods setYear, getYear, setDegree, and getDegree go here. >

   . . .

   public String toString()
   {
      return super.toString() + ", " + degree + ", " + year;
   } // end toString
} // end CollegeStudent
```

Invoking Constructors from Within Constructors

C.9 **Calling the superclass's constructor.** Constructors typically initialize a class's data fields. In a subclass, how can the constructor initialize data fields inherited from the superclass? One way is to call the superclass's constructor. The subclass's constructor can use the reserved word `super` as a name for the constructor of the superclass.

Notice that the default constructor in the class `CollegeStudent` begins with the statement

```
super();
```

This statement invokes the default constructor of the superclass. Our new default constructor must invoke the superclass's default constructor to properly initialize the data fields that are inherited from the superclass. Actually, if you do not invoke `super`, Java will do it for you. In this book, we will always invoke `super` explicitly, to make the action a bit clearer. Note that the call to `super` must occur first in the constructor. You can use `super` to invoke a constructor only from within another constructor.

In like fashion, the second constructor invokes a corresponding constructor in the superclass by executing the statement

```
super(studentName, studentId);
```

If you omit this statement, Java will invoke the default constructor, which is not what you want.

Note: Calling the constructor of the superclass

You can use `super` within the definition of a constructor of a subclass to call a constructor of the superclass explicitly. When you do, `super` always must be the first action taken in the constructor definition. You cannot use the name of the constructor instead of `super`. If you omit `super`, each constructor of a subclass automatically calls the default constructor of the superclass. Sometimes this action is what you want, but sometimes it is not.

Note: Constructors are not inherited

A constructor of a class `C` creates an object whose type is `C`. It wouldn't make sense for this class to have a constructor named anything other than `C`. But that is what would happen if a class like `CollegeStudent` inherited `Student`'s constructors: `CollegeStudent` would have a constructor named `Student`.

Even though `CollegeStudent` does not inherit `Student`'s constructors, its constructors do call `Student`'s constructors, as you have just seen.

C.10 **Reprise: Using `this` to invoke a constructor.** As you saw in Segment B.25, you use the reserved word `this` much as we used `super` here, except that it calls a constructor of the same class instead of a constructor of the superclass. For example, consider the following definition of a constructor that we might add to the class `CollegeStudent` in Segment C.8:

```
public CollegeStudent(Name studentName, String studentId)
{
   this(studentName, studentId, 0, "");
} // end constructor
```

The one statement in the body of this constructor definition is a call to the constructor whose definition begins

```
public CollegeStudent(Name studentName, String studentId,
                      int graduationYear, String degreeSought)
```

As with `super`, any use of `this` must be the first action in a constructor definition. Thus, a constructor definition cannot contain both a call using `super` and a call using `this`. What if you want both a call with `super` and a call with `this`? In that case, you would use `this` to call a constructor that has `super` as its first action.

Private Fields and Methods of the Superclass

C.11 **Accessing inherited data fields.** The class `CollegeStudent` has a `setStudent` method with four parameters, `studentName`, `studentId`, `graduationYear`, and `degreeSought`. To initialize the inherited data fields `fullName` and `id`, the method invokes the inherited methods `setName` and `setId`:

```
setName(studentName); // NOT fullName = studentName
setId(studentId);     // NOT id = studentId
```

Recall that `fullName` and `id` are private data fields defined in the superclass `Student`. Only a method in the class `Student` can access `fullName` and `id` directly by name from within its definition. Although the class `CollegeStudent` inherits these data fields, none of its methods can access them by name. Thus, `setStudent` cannot use an assignment statement such as

```
id = studentId; // ILLEGAL in setStudent
```

to initialize the data field `id`. Instead it must use some public mutator method such as `setId`.

Note: A data field that is private in a superclass is not accessible by name within the definition of a method for any other class, including a subclass. Even so, a subclass inherits the data fields of its superclass.

The fact that you cannot access a private data field of a superclass from within the definition of a method of a subclass seems wrong to people. To do otherwise, however, would make the access modifier `private` pointless: Anytime you wanted to access a private data field, you could simply create a subclass and access it in a method of that class. Thus, all private data fields would be accessible to anybody who was willing to put in a little extra effort.

C.12 **Private methods of the superclass.** A subclass cannot invoke a superclass's private methods directly. This should not be a problem, since you should use private methods only as helpers within the class in which they are defined. That is, a class's private methods do not define behaviors. Thus, we say that a subclass does not inherit the private methods of its superclass. If you want to use a superclass's method in a subclass, you should make the method either protected or public. We discuss protected methods in the next segment.

Suppose that superclass `B` has a public method `m` that calls a private method `p`. A class `D` derived from `B` inherits the public method `m`, but not `p`. Even so, when a client of `D` invokes `m`, `m` calls `p`. Thus, a private method in a superclass still exists and is available for use, but a subclass cannot call it directly by name.

Note: A subclass does not inherit and cannot invoke by name a private method of the superclass.

Protected Access

C.13 You know that you control access to a class's data fields and methods by using an access modifier like `public` or `private`. As you saw in Appendix B, you can omit the access modifier entirely when the class is within a package and you want the class to be available only to other classes in the package. You also have one other choice for controlling access: You can use the access modifier `protected` for methods and data fields.

A method or data field that is modified by `protected` can be accessed by name only within

- Its own class definition `C`
- Any class derived from `C`
- Any class within the same package as `C`

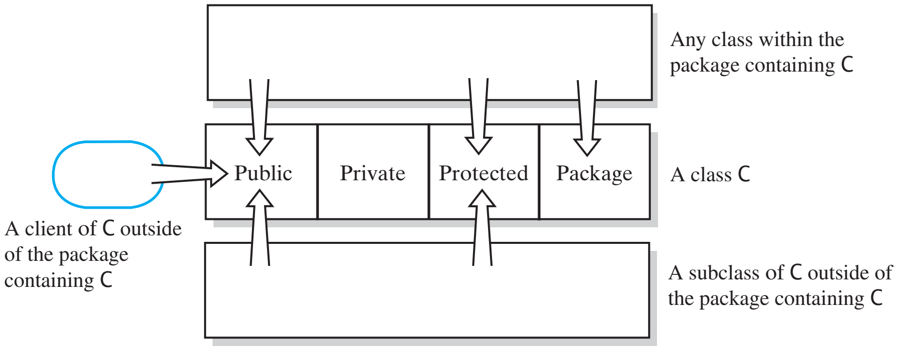
That is, if a method is marked `protected` in class `C`, you can invoke it from within any method definition in a class derived from class `C`. However, with classes that are not derived from `C` or that are not in the same package as `C`, a protected method behaves as if it were private.

You should continue to declare all data fields as private. If you want a subclass to have access to a data field in the superclass, define protected accessor or mutator methods within the superclass.

Note that package access is more restricted than protected access and gives more control to the programmer defining the classes. If you control the package directory, you control who is allowed package access.

Figure C-4 illustrates the various kinds of access modifiers.

FIGURE C-4 Public, private, protected, and package access of the data fields and methods of class `C`


Any class within the package containing C
Public
Private
Protected
Package
A class C
A client of C outside of the package containing C
A subclass of C outside of the package containing C

Overriding and Overloading Methods

C.14 The set and get methods of the class `CollegeStudent` are straightforward, so we will not bother to look at them. However, we have provided the class with a method `toString`. Why did we do this, when our new class inherits a toString method from its superclass Student? Clearly, the string that the superclass's toString method returns can include the student's name and identification number, but it cannot include the year and degree that are associated with the subclass. Thus, we need to write a new method toString.

But why not have the new method invoke the inherited method? We can do this, but we'll need to distinguish between the method that we are defining for CollegeStudent and the method inherited from Student. As you can see from the class definition in Segment C.8, the new method toString contains the statement

```
return super.toString() + ", " + degree + ", " + year;
```

Since Student is the superclass, we write

```
super.toString()
```

to indicate that we are invoking the superclass's toString. If we omitted super, our new version of toString would invoke itself. Here we are using super as if it were an object. In contrast, we used super with parentheses as if it were a method within the constructor definitions.

If you glance back at Segment C.2, you will see that Student's toString method appears as follows:

```
public String toString()
{
   return id + " " + fullName.toString();
} // end toString
```

This method calls the toString method defined in the class Name, since the object fullName is an instance of the class Name.

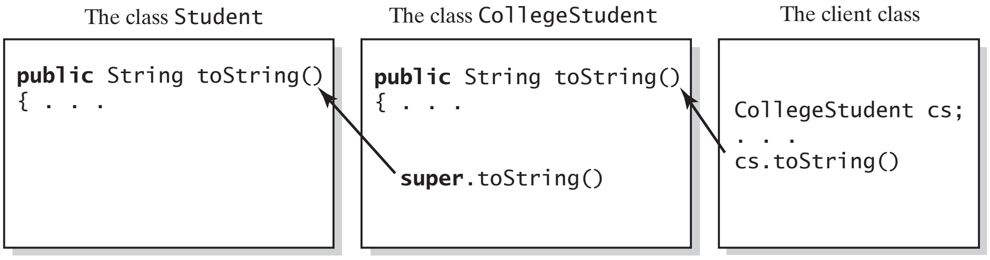
C.15 **Overriding a method.** In the previous segment, you saw that the class CollegeStudent defines a method toString and also inherits a method toString from its superclass Student. Both of these methods have no parameters. The class, then, has two methods with the same name, the same parameters, and the same return type.

When a subclass defines a method with the same name, the same number and types of parameters, and the same return type as a method in the superclass, the definition in the subclass is said to **override** the definition in the superclass. Objects of the subclass that invoke the method will use the definition in the subclass. For example, if cs is an instance of the class CollegeStudent,

```
cs.toString()
```

uses the definition of the method toString in the class CollegeStudent, not the definition of toString in the class Student, as Figure C-5 illustrates. As you've already seen, however, the definition of toString in the subclass can invoke the definition of toString in the superclass by using super.

FIGURE C-5 The method toString in CollegeStudent overrides the method toString in Student



Note: Overriding a method definition

A method in a subclass overrides a method in the superclass when both methods have the same name, the same number and types of parameters, and the same return type. Since a method's signature is its name and parameters, a method in a subclass overrides a method in the superclass when both methods have the same signature and return type.

Note: Overriding and access

An overriding method in a subclass can have either public, protected, or package access according to the access of the overridden method in the superclass, as follows:

Access of the overridden method in the superclass	Access of the overriding method in the subclass
public	public
protected	protected or public
package	package, protected, or public

A private method in a superclass cannot be overridden by a method in a subclass.

Note: You can use `super` in a subclass to call an overridden method of the superclass.

Question 7 Question 5 asked you to create an instance of `NickName` to represent the nickname *Bob*. If that object is named `bob`, do the following statements produce the same output? Explain.

```
System.out.println(bob.getNickName());
System.out.println(bob);
```

C.16 **Covariant return types (Optional).** A class cannot define two methods that have different return types but the same signatures—that is, the same name and parameters. However, if the two methods are in different classes, and one class is a subclass of the other, this can be possible. In particular, when a method in a subclass overrides a method in the superclass, their signatures are the same. But the return type of the method in the subclass can be a subclass of the return type of the method in the superclass. Such return types are said to be **covariant**.

For example, in Segment C.8 the class `CollegeStudent` was derived from the class `Student` defined in Segment C.2. Now imagine a class `School` that maintains a collection of `Student` objects. (This book will give you the tools to actually do this.) The class has a method `getStudent` that returns a student given his or her ID number. The class might appear as follows:

```
public class School
{
   . . .
   public Student getStudent(String studentId)
   {
      . . .
   } // end getStudent
} // end School
```

Now, consider a class `College` that has a collection of college students. We can derive `College` from `School` and override the method `getStudent`, as follows:

```
public class College extends School
{
   . . .
   public CollegeStudent getStudent(String studentId)
   {
      . . .
    } // end getStudent
} // end College
```

The method `getStudent` has the same signature as `getStudent` in `School`, but the return types of the two methods differ. In fact, the return types are covariant—and therefore legal—because `CollegeStudent` is a subclass of `Student`.

C.17 **Reprise: Overloading a method.** Segment B.29 of Appendix B discussed overloaded methods within the same class. Such methods have the same name but different signatures. Java is able to distinguish between these methods since their parameters are not identical.

Suppose that a subclass has a method with the same name as a method in its superclass, but the methods' parameters differ in number or data type. The subclass would have both methods—the one it defines and the one it inherits from the superclass. The method in the subclass overloads the method in the superclass.

For example, the superclass `Student` and the subclass `CollegeStudent` each have a method named `setStudent`. The methods are not exactly the same, however, as they have a different number of parameters. In `Student`, the method's header is

```
public void setStudent(Name studentName, String studentId)
```

whereas in `CollegeStudent` it is

```
public void setStudent(Name studentName, String studentId,
                       int graduationYear, String degreeSought)
```

An instance of the class `Student` can invoke only `Student`'s version of the method, but an instance of `CollegeStudent` can invoke either method. Again, Java can distinguish between the two methods because they have different parameters.

Within the class `CollegeStudent`, the implementation of `setStudent` can invoke `Student`'s `setStudent` to initialize the fields `fullName` and `id` by including the statement

```
setStudent(studentName, studentId);
```

instead of making calls to the methods `setName` and `setId`, as we did in Segment C.8. Since the two versions of `setStudent` have different parameter lists, we do not need to preface the call with `super` to distinguish the two methods. However, we are free to do so by writing

```
super.setStudent(studentName, studentId);
```

Note: Overloading a method definition

A method in a class overloads another method in either the same class or its superclass when both methods have the same name but differ in the number or types of parameters. Thus, overloaded methods have the same name but different signatures.

Although the terms "overloading" and "overriding" are easy to confuse, you should distinguish between the concepts, as they both are important.

C.18 **Multiple use of `super`.** As we have already noted, within the definition of a method of a subclass, you can call an overridden method of the superclass by prefacing the method name with `super` and a dot. However, if the superclass is itself derived from some other superclass, you cannot repeat the use of `super` to invoke a method from that superclass.

For example, suppose that the class `UndergradStudent` is derived from the class `CollegeStudent`, which is derived from the class `Student`. You might think that you can invoke a method of the class `Student` within the definition of the class `Undergraduate`, by using `super.super`, as in

```
super.super.toString(); // ILLEGAL!
```

As the comment indicates, this repeated use of `super` is not allowed in Java.

Note: `super`

Although a method in a subclass can invoke an overridden method defined in the superclass by using `super`, the method cannot invoke an overridden method that is defined in the superclass's superclass. That is, the construct `super.super` is illegal.

SELF-TEST

Question 8 Are the two definitions of the constructors for the class `Student` (Segment C.2) an example of overloading or overriding? Why?

Question 9 If you add the method

```
public void setStudent(Name studentName, String studentId)
```

to the class `CollegeStudent` and let it give some default values to the fields `year` and `degree`, are you overloading or overriding `setStudent`? Why?

C.19 **The `final` modifier.** Suppose that a constructor calls a public method `m`. For simplicity, imagine that this method is in the same class `C` as the constructor, as follows:

```
public class C
{
   . . .
   public C()
   {
      m();
      . . .
   } // end default constructor

   public void m()
   {
      . . .
   } // end m
   . . .
```

Now imagine that we derive a new class from `C` and we override the method `m`. If we invoke the constructor of our new class, it will call the superclass's constructor, which will call our overridden version of the method `m`. This method might use data fields that the constructor has not yet initialized, causing an error. Even if no error occurs, we will, in effect, have altered the behavior of the superclass's constructor.

To specify that a method definition cannot be overridden with a new definition in a subclass, you make it a **final method** by adding the `final` modifier to the method header. For example, you can write

```
public final void m()
```

Note that private methods are automatically final methods, since you cannot override them in a subclass.

Programming Tip: If a constructor invokes a method in its class, declare that method to be final so that no subclass can override the method and hence change the behavior of the constructor.

Constructors cannot be final. Since a subclass does not inherit, and therefore cannot override, a constructor in the base case, final constructors are unnecessary.

You can declare an entire class as a **final class**, in which case you cannot use it as superclass to derive any other class from it. Java's `String` class is an example of a final class.

Note: `String` cannot be the superclass for any other class because it is a final class.

Programming Tip: When you design a class, consider the classes derived from it, either now or in the future. They might need access to your class's data fields. If your class does not have public accessor or mutator methods, provide protected versions of such methods. Keep the data fields private.

Multiple Inheritance

C.20 Some programming languages allow one class to be derived from two different superclasses. That is, you can derive class `C` from classes `A` and `B`. This feature, known as **multiple inheritance**, is not allowed in Java. In Java, a subclass can have only one superclass. You can, however, derive class `B` from class `A` and then derive class `C` from class `B`, since this is not multiple inheritance.

A subclass can implement any number of interfaces—which we describe in Appendix D—in addition to extending any one superclass. This capability gives Java an approximation to multiple inheritance without the complications that arise with multiple superclasses.

Type Compatibility and Superclasses

C.21 **Object types of a subclass.** Previously, you saw the class `CollegeStudent`, which was derived from the class `Student`. In the real world, every college student is also a student. This relationship holds in Java as well. Every object of the class `CollegeStudent` is also an object of the class `Student`. Thus, if we have a method that has a formal parameter of type `Student`, the argument in an invocation of this method can be an object of type `CollegeStudent`.

Specifically, suppose that the method in question is in some class and begins as follows:

```
public void someMethod(Student scholar)
```

Within the body of `someMethod`, the object `scholar` can invoke public methods that are defined in the class `Student`. For example, the definition of `someMethod` could contain the expression `scholar.getId()`. That is, `scholar` has `Student` behaviors.

Now consider an object joe of CollegeStudent. Since the class CollegeStudent inherits all the public methods of the class Student, joe can invoke those inherited methods. That is, joe can behave like an object of Student. (It happens that joe can do more, since it is an object of CollegeStudent, but that is not relevant right now.) Therefore, joe can be the argument of someMethod. That is, for some object o, we can write

```
o.someMethod(joe);
```

No automatic type casting[1] has occurred here. As an object of the class CollegeStudent, joe is also of type Student. The object joe need not be, and is not, type-cast to an object of the class Student.

We can take this idea further. Suppose that we derive the class UndergradStudent from the class CollegeStudent. In the real world, every undergraduate is a college student, and every college student is also a student. Once again, this relationship holds for our Java classes. Every object of the class UndergradStudent is also an object of the class CollegeStudent and so is also an object of the class Student. Thus, if we have a method whose formal parameter is of type Student, the argument in an invocation of this method can be an object of type UndergradStudent. Thus, an object can actually have several types as a result of inheritance.

Note: An object of a subclass has more than one data type. Everything that works for objects of an ancestor class also works for objects of any descendant class.

C.22 Because an object of a subclass also has the types of all of its ancestor classes, you can assign an object of a class to a variable of any ancestor type, but not the other way around. For example, since the class UndergradStudent is derived from the class CollegeStudent, which is derived from the class Student, the following are legal:

```
Student amy = new CollegeStudent();
Student brad = new UndergradStudent();
CollegeStudent jess = new UndergradStudent();
```

However, the following statements are all illegal:

```
CollegeStudent cs = new Student();           // ILLEGAL!
UndergradStudent ug = new Student();         // ILLEGAL!
UndergradStudent ug2 = new CollegeStudent(); // ILLEGAL!
```

This makes perfectly good sense. For example, a college student is a student, but a student is not necessarily a college student. Some programmers find the phrase “is a” to be useful in deciding what types an object can have and what assignments to variables are legal.

Programming Tip: Because an object of a subclass is also an object of the superclass, do not use inheritance when an *is a* relationship does not exist between your proposed class and an existing class. Even if you want class C to have some of the methods of class B, use composition if these classes do not have an *is a* relationship.

[1] Segment A.21 of Appendix A reviews type casts.

SELF-TEST

Question 10 If HighSchoolStudent is a subclass of Student, can you assign an object of HighSchoolStudent to a variable of type Student? Why or why not?

Question 11 Can you assign an object of Student to a variable of type HighSchoolStudent? Why or why not?

The Class Object

C.23 As you have already seen, if you have a class A and you derive class B from it, and then you derive class C from B, an object of class C is of type C, type B, and type A. This works for any chain of subclasses no matter how long the chain is.

Java has a class—named Object—that is at the beginning of every chain of subclasses. This class is an ancestor of every other class, even those that you define yourself. Every object of every class is of type Object, as well as being of the type of its class and also of the types of all the other ancestor classes. If you do not derive your class from some other class, Java acts as if you had derived it from the class Object.

Note: Every class is a descendant class of the class Object.

The class Object contains certain methods, among which are toString, equals, and clone. Every class inherits these methods, either from Object directly or from some other ancestor class that ultimately inherited the methods from the class Object.

The inherited methods toString, equals, and clone, however, will almost never work correctly in the classes you define. Typically, you need to override the inherited method definitions with new, more appropriate definitions. Thus, whenever you define the method toString in a class, for example, you are actually overriding Object's method toString.

C.24 **The toString method.** The method toString takes no arguments and is supposed to return all the data in an object as a String. However, you will not automatically get a nice string representation of the data. The inherited version of toString returns a value based upon the invoking object's memory address. You need to override the definition of toString to cause it to produce an appropriate string for the data in the class being defined. You might want to look again at the toString methods in Segments C.2 and C.8.

C.25 **The equals method.** Consider the following objects of the class Name that we defined in Appendix B:

```
Name joyce1 = new Name("Joyce", "Jones");
Name joyce2 = new Name("Joyce", "Jones");
Name derek = new Name("Derek", "Dodd");
```

Now joyce1 and joyce2 are two distinct objects that contain the same name. Typically, we would consider these objects to be equal, but in fact joyce1.equals(joyce2) is false. Since Name does not define its own equals method, it uses the one it inherits from Object. Object's equals method compares the addresses of the objects joyce1 and joyce2. Because we have two distinct objects, these addresses are not equal. However, joyce1.equals(joyce1) is true, since we are comparing an object with itself. This comparison is an **identity**. Notice that identity and equality are different concepts.

The method `equals` has the following definition in the class `Object`:

```
public boolean equals(Object other)
{
   return (this == other);
} // end equals
```

Thus, the expression `x.equals(y)` is `true` if `x` and `y` reference the same object. We must override `equals` in the class `Name` if we want it to behave more appropriately.

As you will recall, `Name` has two data fields, `first` and `last`, that are instances of `String`. We could decide that two `Name` objects are equal if they have equal first names and equal last names. The following method, when added to the class `Name`, detects whether two `Name` objects are equal by comparing their data fields:

```
public boolean equals(Object other)
{
   boolean result = false;

   if (other instanceof Name)
   {
      Name otherName = (Name)other;
      result = first.equals(otherName.first) &&
               last.equals(otherName.last);
   } // end if

   return result;
} // end equals
```

To ensure that the argument passed to the method `equals` is a `Name` object, you use the Java operator `instanceof`. For example, the expression

```
other instanceof Name
```

is true if `other` references an object of either the class `Name` or a class derived from `Name`. If `other` references an object of any other class, or if `other` is `null`, the expression will be false.

Given an appropriate argument, the method compares the data fields of the two objects. Notice that we first must cast the type of the parameter `other` from `Object` to `Name` so that we can access `Name`'s data fields. To compare two strings, we use `String`'s `equals` method. The class `String` defines its own version of `equals` that overrides the `equals` method inherited from `Object`.

? SELF-TEST

Question 12 If `sue` and `susan` are two instances of the class `Name`, what `if` statement can decide whether they represent the same name?

C.26 **The `clone` method.** Another method inherited from the class `Object` is the method `clone`. This method takes no arguments and returns a copy of the receiving object. The returned object is supposed to have data identical to that of the receiving object, but it is a different object (an identical twin or a "clone"). As with other methods inherited from the class `Object`, we need to override the method `clone` before it can behave properly in our class. However, in the case of the method `clone`, there are other things we must do as well. A discussion of the method `clone` appears in Chapter 30.

Abstract Classes and Methods

C.27 The class `Student` defined in Segment C.2 is a superclass for other classes such as `CollegeStudent`. We really do not need to create objects of type `Student`, although it is certainly legal to do so. We might, however, want to prevent a client from creating objects of type `Student`. To do so, we can declare the class to be an **abstract class** by including the reserved word `abstract` in the header of the class definition, as follows:

```
public abstract class Student
{
  . . .
```

Note: An abstract class will be the superclass of another class. Thus, an abstract class is sometimes called an **abstract superclass**.

Often when programmers define an abstract class, they declare one or more methods that have no body. The intention in doing so is to require that every subclass implement such methods in an appropriate way for that class. For example, we might want every subclass of `Student` to implement a method `display`. We certainly cannot write such a method for a future class that is not yet defined, but we can require one. To do so, we declare `display` as an **abstract method** by including the reserved word `abstract` in the header of the method, as follows:

```
public abstract void display();
```

Note that the method header is followed by a semicolon; the method has no body.

Note: An abstract method declaration within an abstract class consists of the method's header followed by a semicolon. The header must include the reserved word `abstract`. An abstract method cannot be private, static, or final.

C.28 If a class has at least one abstract method, Java requires that you declare the class itself as abstract. This makes sense, for otherwise you could create an object of an incomplete class. In our example, the object would have a method `display` without an implementation.

What if the subclass of an abstract class does not implement all of the abstract methods? Java will treat the subclass as abstract and prevent you from creating an object of its type. For example, if the class `CollegeStudent`, which is derived from `Student`, did not implement `display`, `CollegeStudent` would have to be abstract.

Note: A class with at least one abstract method must be declared as an abstract class. Thus, abstract methods can appear only within an abstract class.

Even after we've made the class `Student` abstract by adding the abstract method `display`, not all of its methods are abstract. All the method definitions, except for the method `display`, are exactly the same as in our original definition. They are full definitions that do not use the reserved word `abstract`. When it makes sense to implement a method in an abstract class, you should do so. In this way, you include as much detail as possible in the abstract class, detail that need not be repeated in subclasses.

Note: **Constructors cannot be abstract**

Since a class cannot override a constructor in its superclass, if the constructor were abstract, it could not be implemented. Thus, constructors are never abstract.

C.29 **Example.** Let's add another method to the class `Student`, one that invokes the abstract method `display`. Before you complain about invoking a method that has no body, remember that `Student` is an abstract class. When we finally derive a class from `Student` that is not abstract, `display` will be implemented.

The method we have in mind serves mainly as an example, rather than doing anything useful. It simply skips the specified number of lines before displaying an object:

```
/** Displays the object after skipping numberOfLines lines. */
public void displayAt(int numberOfLines)
{
   for (int count = 0; count < numberOfLines; count++)
      System.out.println();
   display();
} // end displayAt
```

The method `displayAt` invokes the abstract method `display`. Here the abstract method serves as a placeholder for a method that will be defined in a future subclass. If `display` were not abstract, we would have to give it a body that really would be useless, since every subclass would override it.

SELF-TEST

Question 13 Suppose that you change the name of the previous method `displayAt` to `display`. Does the resulting method overload or override the method `display`? Why?

Polymorphism

C.30 The term "polymorphism" comes from a Greek word meaning "many forms." Polymorphism as a concept is actually common in English. For example, the English instruction "Play your favorite sport" means different things to different people. To one person it means to play baseball. To another person it means to play soccer. In Java, **polymorphism** allows the same program instruction to mean different things in different contexts. In particular, one method name, used as an instruction, can cause different actions depending on the kind of object performing the action.

Originally, overloading a method name was considered polymorphism. However, the modern usage of the term refers to an object determining at execution time which action of a method it will use for a method name that is overridden either directly or indirectly.

Note: Polymorphism

One method name in an instruction can cause different actions according to the kinds of objects that receive the method invocation.

C.31 **Example.** For example, a method named `display` can display the data in an object. But the data it displays and how much it displays depend on the kind of object that you use to invoke the method. Let's add the method `display` to the class `Student` of Segment C.2 and assume that neither the method nor the class is abstract. Thus, `display` has an implementation within the class `Student`. Now add to the class the method `displayAt` as it appears in Segment C.29.

If the only class around were `Student`, these changes would not be exciting. But we derived the class `UndergradStudent` from the class `CollegeStudent`, which we derived from the class `Student`. The class `UndergradStudent` inherits the method `displayAt` from the class `Student`. In addition, `UndergradStudent` overrides the method `display` defined in `Student` by providing its own implementation. So what? you might be wondering.

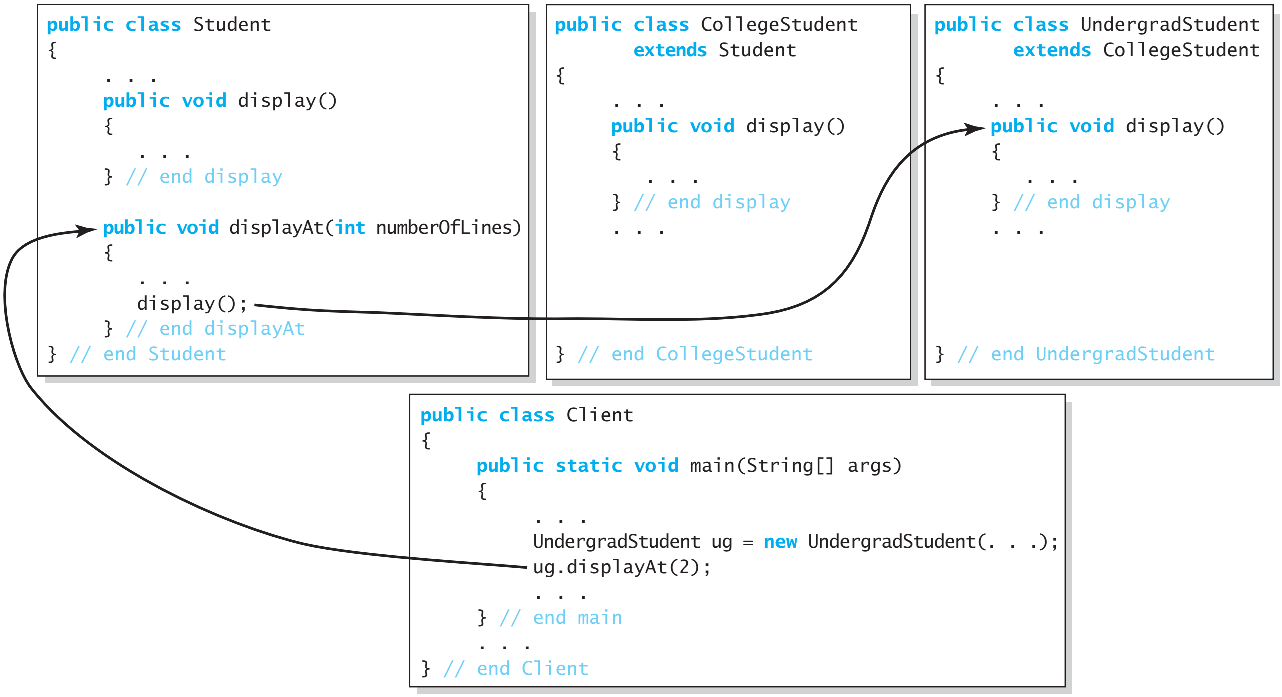
Well, look at the poor compiler's job when it encounters the following Java statements (we are ignoring the constructor's arguments):

```
UndergradStudent ug = new UndergradStudent(. . .);
ug.displayAt(2);
```

The method `displayAt` was defined in the class `Student`, but it calls the method `display` that is defined in the class `UndergradStudent`, as Figure C-6 illustrates. The code for `displayAt` could have been compiled with the class `Student` *before* the class `UndergradStudent` was even written. In other words, this compiled code could use a definition of the method `display` that was not even written at the time that `displayAt` was compiled. How can that be?

When the code for `displayAt` is compiled, the call to `display` produces an annotation that says, "use the appropriate definition of `display`." Then, when we invoke `ug.displayAt(2)`, the compiled code for `displayAt` reaches this annotation and replaces it with an invocation of the version of `display` that goes with `ug`. Because in this case `ug` is of type `UndergradStudent`, the version of `display` that is used will be the definition in the class `UndergradStudent`.

FIGURE C-6 The method `displayAt` calls the correct version of `display`



```
public class Student
{
     . . .
     public void display()
     {
        . . .
     } // end display

     public void displayAt(int numberOfLines)
     {
        . . .
        display();
     } // end displayAt
} // end Student
```

```
public class CollegeStudent
       extends Student
{
     . . .
     public void display()
     {
        . . .
     } // end display
     . . .

} // end CollegeStudent
```

```
public class UndergradStudent
       extends CollegeStudent
{
     . . .
     public void display()
     {
        . . .
     } // end display
     . . .

} // end UndergradStudent
```

```
public class Client
{
     public static void main(String[] args)
     {
          . . .
          UndergradStudent ug = new UndergradStudent(. . .);
          ug.displayAt(2);
          . . .
     } // end main
     . . .
} // end Client
```

C.32 The decision as to which method definition to use depends on the receiving object's place in the inheritance chain, not on the type of the variable naming the object. For example, consider the following code:

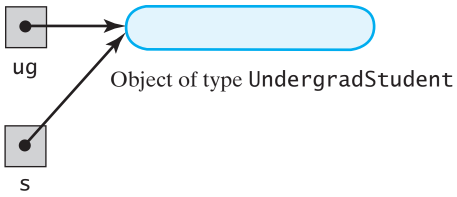
```
UndergradStudent ug = new UndergradStudent(. . .);
Student s = ug;
s.displayAt(2);
```

As we noted in Segment C.22, assigning an object of the class `UndergradStudent` to a variable of type `Student` is perfectly legal. Here, the variable `s` is just another name for the object that ug references, as Figure C-7 illustrates. That is, s and ug are aliases. But the object still remembers that it was created as an UndergradStudent. In this case, s.displayAt(2) ultimately will use the definition of display given in UndergradStudent, not the definition of display given in Student.

A variable's **static type** is the type that appears in its declaration. For example, the static type of the variable s is Student. The static type is fixed and determined when the code is compiled. The type of object that a variable references at a point in time during execution is called its **dynamic type**. A variable's dynamic type can change as execution progresses. When the assignment s = ug executes in the previous code, the dynamic type of s is UndergradStudent. A variable of a reference type is called a **polymorphic variable**, since its dynamic type can differ from its static type and change during execution.

For our example, Java decides which definition of display to use by seeing which constructor created the object. That is, Java uses the dynamic type of the variable s to make this determination.

FIGURE C-7 The variable s is another name for an undergraduate object



Note: Java uses an object's dynamic type, not its name, to see which method to invoke.

This way of handling a call to a method that might be overridden later is called **dynamic binding** or **late binding**, because the *meaning* of the method invocation is not bound to the *location* of the method invocation until you run the program. If Java did not use dynamic binding when you ran the preceding code, you would not see the data for an undergraduate student. Instead you would see only what the method display of the class Student provided.

Note: Dynamic binding

Dynamic binding is the process that enables different objects to use different method actions for the same method name.

C.33 Java is so good at figuring out which definition of a method to use that even a type cast will not fool it. Recall that you use a type cast to change the type of a value to some other type. The meaning of s.displayAt(2) in the previous segment will always be appropriate for an UndergradStudent, even if we use a type cast to change the type of ug to the type Student, as in the following statements:

```
UndergradStudent ug = new UndergradStudent(. . .);
Student s = (Student) ug;
s.displayAt(2);
```

Despite the type cast, `s.displayAt(2)` will use the definition of `display` given in `UndergradStudent`, not the definition of `display` given in `Student`. An object's dynamic type, not its name, is the determining factor in choosing the correct method to invoke.

To see that dynamic binding really is a big deal, consider the following code:

```
UndergradStudent ug = new UndergradStudent(. . .);
Student s = ug;
s.displayAt(2);
GradStudent g = new GradStudent(. . .);
s = g;
s.displayAt(2);
```

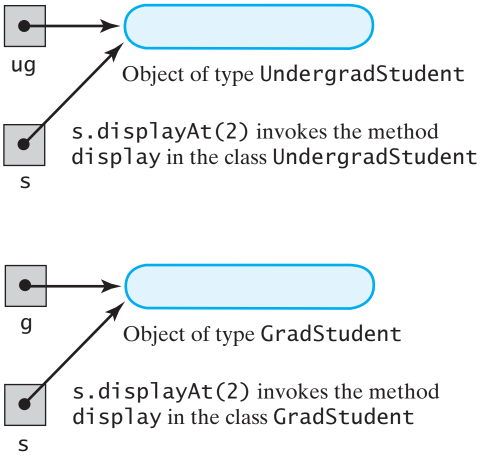
The two highlighted lines are identical, yet each one invokes a different version of `display`. The first line displays an `UndergradStudent` and the second displays a `GradStudent`, as Figure C-8 illustrates. An object remembers what method definitions it had when the `new` operator created it. You can place the object in a variable of a different (but ancestor) class type, but that has no effect on which method definition the object uses for an overridden method.

Let's pursue this process a bit more to see that it is even more dramatic than it may appear at first glance. Note that objects of the classes `UndergradStudent` and `GradStudent` inherit the method `displayAt` from the class `Student` and do not override it. Thus, the text of the method definition is even the same for objects of the classes `UndergradStudent` and `GradStudent`. It is the method `display`, invoked in the definition of `displayAt`, that is overridden.

Note: Objects know how they are supposed to act

When an object receives a call to either an overridden method or a method that calls an overridden method, the action of that method is the one defined in the class whose constructor created the object. The choice of action is not affected by the static type of the variable naming the object. A variable of any ancestor class can reference an object of a descendant class, but the object always remembers which method actions to use for every method name, because Java uses dynamic binding.

FIGURE C-8 An object, not its name, determines its behavior


ug
Object of type UndergradStudent
s.displayAt(2) invokes the method
display in the class UndergradStudent
s
g
Object of type GradStudent
s.displayAt(2) invokes the method
display in the class GradStudent
s

C.34 **Type checking and dynamic binding.** You need to be aware of how dynamic binding interacts with Java's type checking. For example, if `UndergradStudent` is a subclass of the class `Student`, we can assign an object of type `UndergradStudent` to a variable of type `Student`, as in

```
Student s = new UndergradStudent();
```

But that is not the end of the story.

Although we can assign an object of type `UndergradStudent` to a variable `s` of type `Student`, we cannot use `s` to invoke a method that is only in the class `UndergradStudent`. However, if the method is overridden in the definition of the class `UndergradStudent`, the version of the method defined in `UndergradStudent` will be used. In other words, the variable determines what method names can be used, but the object determines which definition of the method name will be used. If we want to use a method name that was first introduced in the class `UndergradStudent` with the object named by the variable `s` of type `Student`, we must use a type cast.

C.35 **Example.** For example, recall that `Student` is not abstract and implements the method `display`. Also remember that `UndergradStudent` is a subclass of `Student`. The following statements are legal:

```
Student s = new UndergradStudent(. . .);
s.setName(new Name("Jamie", "Jones"));
s.display();
```

The definition of `display` given in the class `UndergradStudent` is used. *Remember, the object, not the variable, determines which definition of a method will be used.*

On the other hand, the following is illegal:

```
s.setDegree("B.A."); // ILLEGAL
```

because `setDegree` is not the name of a method in the class `Student`. *Remember, the variable determines which method names can be used.*

The variable `s` is of type `Student`, but it references an object of type `UndergradStudent`. That object can still invoke the method `setDegree`, but the compiler does not know this. To make the invocation legal, we need a type cast, such as the following:

```
UndergradStudent ug = (UndergradStudent)s;
ug.setDegree("B.A."); // LEGAL
```

You may think this is all just a silly exercise, because you would never assign an object of type `UndergradStudent` to a variable of type `Student`. Not so. You might not often make such an assignment directly, but you frequently will do so unwittingly. Recall that we can have an argument of type `UndergradStudent` for a method parameter of type `Student` and that a formal parameter behaves like a local variable that is assigned the value of its corresponding argument. In this case an object of type `UndergradStudent` (the argument in the method invocation) is assigned to a variable of type `Student` (the formal parameter in the method definition).

C.36 **Example.** Since each of the classes `Student` and `Name` has an appropriate version of the method `toString`, we can display an object of this class as follows:

```
Name joe = new Name("Joe", "Student");
Student s = new Student(joe, "5555");
System.out.println(s.toString());
```

But thanks to dynamic binding, we do not even need to write `toString` in our invocation of `System.out.println`. The method invocation `System.out.println(s)` will work just as well and will produce exactly the same output. Let's see why.

The object System.out has the method println. One definition of the method println has a single parameter of type Object. The definition is equivalent to the following:

```
public void println(Object theObject)
{
   System.out.println(theObject.toString());
} // end println
```

The method println invoked inside the braces is a different, overloaded definition of the method println that has a parameter of type String, not Object.

These definitions of println existed before the class Student was defined. Yet the invocation

```
System.out.println(s);
```

with an object s of type Student—and hence also of type Object—uses Student's toString, not Object's toString. Dynamic binding is what makes this work.

Question 14 Is a method display with no parameters that is defined explicitly in each of the classes Student, CollegeStudent, and UndergradStudent an example of overloading or overriding? Why?

Question 15 Is overloading a method name an example of polymorphism?

Question 16 In the following code, will the two invocations of displayAt produce the same output?

```
Student s = new UndergradStudent(. . .);
s.displayAt(2);
s = new GradStudent(. . .);
s.displayAt(2);
```

Exercises and Projects

A collection of exercises and projects for this appendix are available online at pearsonhighered.com/carrano.

Answers to Self-Test Questions

1. Some possibilities are roomNumber and dorm, or street, city, state, zip.

2. `private Address residence;`
Add the methods setAddress and getAddress.

3. The constructors, setStudent, and toString.

4.
```
public Student()
{
   this(new Name(), "");
} // end default constructor
```

5.
```
NickName bob = new NickName();
bob.setNickName("Bob");
System.out.println(bob.getNickName());
```

6. The Vehicle class has two subclasses, WheeledVehicle and WheellessVehicle. The subclasses of WheeledVehicle are Automobile and Wagon. Boat is a subclass of WheellessVehicle. The remaining subclasses are the same as given in the figure.

7. No. Since `getNickName` returns a string, the first statement implicitly calls the method `toString` defined in the class `String`. Thus, *Bob* is displayed. Since the class `NickName` does not define its own version of `toString`, the second statement invokes `Object`'s `toString`. The output involves the memory address of the object referenced by `bob`.

8. Overloading.The constructors have the same name but different signatures.

9. Overriding. The revised version of `setStudent` in `CollegeStudent` has the same signature and return type as the version in the superclass `Student`.

10. Yes. You can assign an object of a class to a variable of any ancestor type. An object of type `HighSchoolStudent` can do anything that an object of type `Student` can do.

11. No. The `Student` object does not have all the behaviors expected of a `HighSchoolStudent` object.

12. `if (sue.equals(susan))`

13. Overload. The two methods have the same name but different signatures: One has a parameter, one does not.

14. Overriding. The methods have the same signatures and return types.

15. At one time, overloading was an example of polymorphism. Today, polymorphism describes a situation in which an object determines at execution time which action of a method it will use for a method name that is overridden either directly or indirectly.

16. No. Each call to `displayAt` will invoke the correct version of `display`. The first invocation calls `display` in `UndergradStudent`; the second invocation calls `display` in `GradStudent`.

Appendix D

Designing Classes

Contents

Prerequisites

Appendix A	Java Essentials
Appendix B	Java Classes
Appendix C	Creating Classes from Other Classes
Appendix G	Documentation and Programming Style

Object-oriented programming embodies three design concepts: encapsulation, inheritance, and polymorphism. We have already discussed inheritance and polymorphism.

Now, building on our earlier discussion of classes, this appendix introduces encapsulation as a way to hide the details of an implementation during the design of a class. It then goes on to emphasize the importance of specifying how a method should behave before you implement it and of expressing your specifications as comments in your program.

We introduce Java interfaces as a way to separate the declarations of a class's behavior from its implementation. Finally, we present, at an elementary level, some techniques for identifying the classes necessary for a particular solution.

Encapsulation

D.1 What is the most useful description of an automobile, if you want to learn to drive one? It clearly is not a description of how its engine goes through a cycle of taking in air and gasoline, igniting the gasoline/air mixture, and expelling exhaust. Such details are unnecessary when you want to learn to drive. In fact, such details can get in your way. If you want to learn to drive an automobile, the most useful description of an automobile has such features as the following:

- If you press your foot on the accelerator pedal, the automobile will move faster.
- If you press your foot on the brake pedal, the automobile will slow down and eventually stop.
- If you turn the steering wheel to the right, the automobile will turn to the right.
- If you turn the steering wheel to the left, the automobile will turn to the left.

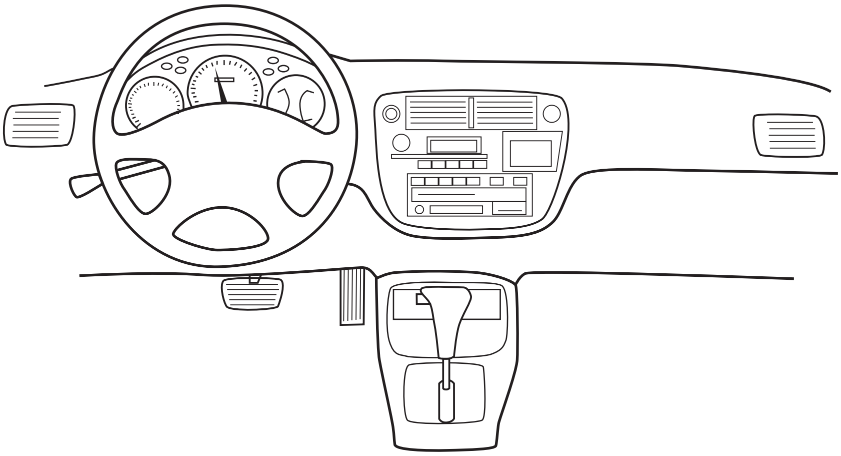
Just as you need not tell somebody who wants to drive a car how the engine works, you need not tell somebody who uses a piece of software all the fine details of its Java implementation. Likewise, suppose that you create a software component for another programmer to use in a program. You should describe the component in a way that tells the other programmer how to use it but that spares the programmer all the details of how you wrote the software.

D.2 **Encapsulation** is one of the design principles of object-oriented programming. The word "encapsulation" sounds as though it means putting things into a capsule, and that image is indeed correct. Encapsulation hides the fine detail of what is inside the "capsule." For this reason, encapsulation is often called **information hiding**. But not everything should be hidden. In an automobile, certain things are visible—like the pedals and steering wheel—and others are hidden under the hood. In other words, the automobile is encapsulated so that the details are hidden, and only the controls needed to drive the automobile are visible, as Figure D-1 shows. Similarly, you should encapsulate your Java code so that details are hidden and only the necessary controls are visible.

Encapsulation encloses data and methods within a class and hides the implementation details that are not necessary for using the class. If a class is well designed, its use does not require an understanding of its implementation. A programmer can use the class's methods without knowing the details of how they are coded. The programmer must know only how to provide a method with appropriate arguments, leaving the method to perform the right action. Stated simply, the programmer need not worry about the internal details of the class definition. The programmer who uses encapsulated software to write more software has a simpler task. As a result, software is produced more quickly and with fewer errors.

Note: **Encapsulation** is a design principle of object-oriented programming that encloses data and methods within a class, thereby hiding the details of a class's implementation. A programmer receives only enough information to be able to use the class. A well-designed class can be used as though the body of every method was hidden from view.

FIGURE D-1 An automobile's controls are visible to the driver, but its inner workings are hidden



D.3 **Abstraction** is a process that asks you to focus on *what* instead of *how*. When you design a class, you practice **data abstraction**. You focus on what you want to do with or to the data without worrying about how you will accomplish these tasks and how you will represent the data. Abstraction asks you to focus on what data and operations are important. When you abstract something, you identify the central ideas. For example, an abstract of a book is a brief description of the book, as opposed to the entire book.

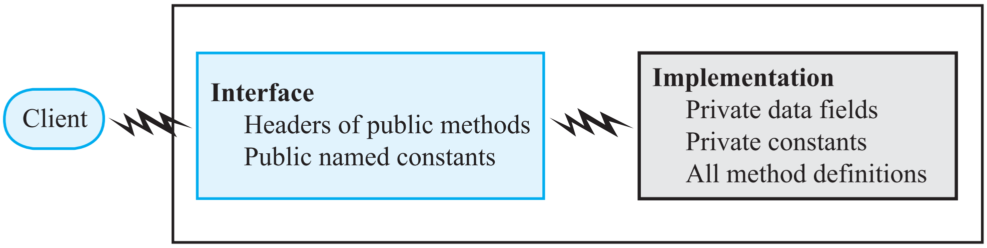
When designing a class, you should not think about any method's implementation. That is, you should not worry about *how* the class's methods will accomplish their goals. This separation of specification from implementation allows you to concentrate on fewer details, thereby making your task easier and less error-prone. Detailed, well-planned specifications facilitate an implementation that is more likely to be successful.

Note: The process of abstraction asks you to focus on *what* instead of *how*.

D.4 When done correctly, encapsulation divides a class definition into two parts, which we will call the **client interface** and the **implementation**. The client interface describes everything a programmer needs to know to use the class. It consists of the headers for the public methods of the class, the comments that tell a programmer how to use these public methods, and any publicly defined constants of the class. The client interface part of the class definition should be all you need to know to use the class in your program.

The implementation consists of all data fields and the definitions of all methods, including those that are public, private, and protected. Although you need the implementation to run a client (a program that uses the class), you should not need to know anything about the implementation to write the client. Figure D-2 illustrates an encapsulated implementation of a class and the client interface. Although the implementation is hidden from the client, the interface is visible and provides a well-regulated means for the client to communicate with the implementation.

FIGURE D-2 An interface provides well-regulated communication between a hidden implementation and a client



The client interface and implementation are not separated in the definition of a Java class. They are mixed together. You can, however, create a separate interface in Java as a companion to your class. A later section of this appendix describes how to write such an interface, and we will write several of them in this book.

? SELF-TEST

Question 1 How does a client interface differ from a class implementation?

Question 2 Think of an example, other than an automobile, that illustrates encapsulation. What part of your example corresponds to a client interface and what part to an implementation?

Specifying Methods

Separating the purpose of a class and its methods from their implementations is vital to a successful software project. You should specify what each class and method does without concern for its implementation. Writing descriptions enables you to capture your ideas initially and to develop them so that they are clear enough to implement. Your written descriptions should reach the point where they are useful as comments in your program. You need to go beyond a view that sees comments as something you add after you write the program to satisfy an instructor or boss.

Comments

Let's focus on comments that you write for a class's methods. Although organizations tend to have their own style for comments, the developers of Java have specified a commenting style that you should follow. If you include comments written in this style in your program, you can run a utility program called `javadoc` to produce documents that describe your classes. This documentation tells people what they need to know to use your class but omits all the implementation details, including the bodies of all method definitions.

The program `javadoc` extracts the header for your class, the headers for all public methods, and comments that are written in a certain form. Each such comment must appear immediately before a public class definition or the header of a public method and must begin with `/**` and end with `*/`. Certain **tags** that begin with the symbol `@` appear within the comments to identify various aspects of the method. For example, you use `@param` to identify a parameter, `@return` to identify a return value, and `@throws` to indicate an exception that the method throws. You will see some examples of these tags within the comments in this appendix. Appendix G provides the details for writing comments acceptable to `javadoc`.

Rather than talk further about the rules for `javadoc` here, we want to discuss some important aspects of specifying a method. First, you need to write a concise statement of the method's purpose or task. Beginning this statement with a verb will help you to avoid many extra words that you really do not need.

In thinking about a method's purpose, you should consider its input parameters, if any, and describe them. You also need to describe the method's results. Does it return a value, does it cause some action, or does it affect the state of an argument? In writing such descriptions, you should keep in mind the following ideas.

Preconditions and Postconditions

D.5 A **precondition** is a statement of the conditions that must be true before a method begins execution. The method should not be used, and cannot be expected to perform correctly, unless the precondition is satisfied. A precondition can be related to the description of a method's parameters. For example, a method that computes the square root of x can have $x \geq 0$ as a precondition.

A **postcondition** is a statement of what is true after a method completes its execution, assuming that the precondition was met. For a valued method, the postcondition will describe the value returned by the method. For a void method, the postcondition will describe actions taken and any changes to the calling object. In general, the postcondition describes all the effects produced by a method invocation.

Thinking in terms of a postcondition can help you to clarify a method's purpose. Notice that going from precondition to postcondition leaves out the *how*—that is, we separate the method's specification from its implementation.

Programming Tip: A method that cannot satisfy its postcondition, even though its precondition is met, can throw an exception. (See Appendix E for a discussion of exceptions.)

D.6 **Responsibility.** A precondition implies responsibility for guaranteeing that certain conditions are met. If the client is responsible for meeting the conditions before calling the method, the method need not check the conditions. On the other hand, if the method is responsible for enforcing the conditions, the client does not check them. A clear statement of who must check a given set of conditions increases the probability that someone will do so and avoids duplication of effort.

For example, you could specify the square root method that we mentioned in the previous segment by writing the following comments before its header:

```
/** Computes the square root of a number.
    @param x  a real number >= 0
    @return the square root of x
*/
```

In this case, the method assumes that the client will provide a nonnegative number as an argument.

On the other hand, the method could assume responsibility for checking the argument. In that case, its comments could read as follows:

```
/** Computes the square root of a number.
    @param x  a real number
    @return the square root of x if x >= 0
    @throws ArithmeticException if x < 0
*/
```

Although we've integrated the precondition and postcondition into the previous comments, we could instead identify them separately.

Programming Tip Specify each public method fully in comments placed before the method's header. State whether a method or its client is responsible for ensuring that the necessary conditions are met for the successful execution of the method. In this way, checking is done but not duplicated. During debugging, however, a method should check that its precondition has been met.

D.7 When you use inheritance and polymorphism to override a method in a superclass, the method in the subclass could be inconsistent with the method in the superclass. Preconditions and postconditions will help you to avoid this problem. A postcondition must apply to all versions of a method throughout the subclasses. An overriding method can add to a postcondition—that is, it can do more—but it should not do less. However, an overriding method cannot augment its precondition. In other words, it cannot require more than a version of the method in a base class requires.

SELF-TEST

Question 3 Assume that the class `Square` has a data field `side` and the method `setSide` to set the value of `side`. What header and comments can you write for this method? Keep in mind a precondition and postcondition as you do this.

Assertions

D.8 An **assertion** is a statement of truth about some aspect of your program's logic. You can think of it as a boolean expression that is true, or that at least should be true, at a certain point. Preconditions and postconditions, for example, are assertions made about conditions at the beginning and end of a method. If one of these assertions is false, something is wrong with your program.

You can state assertions as comments within your code. For example, if at some point in a method's definition, you know that the variable `sum` should be positive, you could write the following comment:

```
// Assertion: sum > 0
```

Such comments point out aspects of the logic that might not be clear. Additionally, they provide places for you to check the accuracy of your code during debugging.

SELF-TEST

Question 4 Suppose that you have an array of positive integers. The following statements find the largest integer in the array. What assertion can you write as a comment after the `if` statement in the following loop?

```
int max = 0;
for (int index = 0; index < array.length; index++)
{
   if (array[index] > max)
      max = array[index];
  // Assertion:
} // end for
```

D.9 **The `assert` statement.** Java enables you to do more than simply write a comment to make an assertion. You can enforce the assertion by using an `assert` statement, such as

```
assert sum > 0;
```

If the boolean expression that follows the reserved word `assert` is true, the statement does nothing. If it is false, an **assertion error** occurs and program execution terminates. An error message such as the following is displayed:

```
Exception in thread “main” java.lang.AssertionError
```

You can clarify this error message by adding a second expression to the `assert` statement. The second expression must represent a value, since its representation as a string is displayed within the error message. For example, the statement

```
assert sum > 0 : sum;
```

adds the value of `sum` to the error message in case `sum` ≤ 0. For example, the error message might be

```
Exception in thread “main” java.lang.AssertionError: -5
```

By default, `assert` statements are disabled at execution time. Thus, you can leave `assert` statements in your program after you have finished it without wasting execution time. When you run a program, you must enable the `assert` statements if you want them to execute. Exactly how you enable them depends on your programming environment.[1]

Note: Assertions within a program identify aspects of your logic that must be true. In Java, you can use an `assert` statement to make an assertion. It has the following form:

`assert` *boolean_expression* `:` *valued_expression*;

The value of the optional second expression appears in the error message that occurs if the first expression is false.

Programming Tip: Using the `assert` statement is a simple but effective way to find errors in your program’s logic. After serving this purpose, assertions left in your program document its logic for those who want to revise or expand its capability. Remember, Java ignores `assert` statements unless the user of your program specifies otherwise.

Programming Tip: Use an `assert` statement during debugging to enforce that a method’s precondition has been met. However, an `assert` statement is not a substitute for an `if` statement. You should use `assert` statements as a programming aid, not as part of a program’s logic.

Java Interfaces

D.10 Earlier in this appendix, we spoke in general terms about the client interface, which tells you all you need to know to use a particular class in your program. Although a Java class intermixes its interface with its implementation, you can write a separate interface.

A **Java interface** is a program component that declares a number of public methods and can define public named constants. Such an interface should include comments that specify the methods, in order to provide a programmer with the necessary information to implement them. Some interfaces describe all the public methods in a class, while others specify only certain methods.

[1] If you use the Java Development Kit (JDK) from Oracle, the command `java -ea MyProgram` executes `MyProgram` with assertions enabled. Further details about enabling assertions when using the JDK are available at the following URL: `download.oracle.com/javase/7/docs/technotes/guides/language/assert.html`

When you write a class that defines the methods declared in an interface, we say that the class **implements** the interface. A class that implements an interface must define a body for every method that the interface specifies. The interface, however, might not declare every method defined in the class.

You can write your own interfaces, and you can use those that are in the Java Class Library. When you write a Java interface, you place it in its own file. That is, the interface and the class that implements it are in two separate files.

Writing an Interface

D.11 A Java interface begins like a class definition, except that you use the word `interface` instead of `class`. That is, an interface begins with the statement

`public interface` *interface-name*

rather than

`public class` *class-name*

The interface can contain any number of public method headers, each followed by a semicolon. An interface does not declare the constructors for a class and cannot declare static or final methods. Note that methods within an interface are public by default, so you can omit `public` from their headers. The interface can also define any number of public named constants.

D.12 **Example.** Imagine objects such as circles, squares, or plots of land that have both a perimeter and an area. Suppose that we want the classes of these objects to have get methods that return these quantities. If various programmers implemented these classes, they likely would not name or specify these get methods in the same way. To ensure that these classes define our methods in a uniform way, we write the interface shown in Listing D-1. This interface provides a programmer with a handy summary of the methods' specifications. The programmer should be able to use these methods without looking at the class that implements them.

LISTING D-1 An interface `Measurable`

```
/** An interface for methods that return
    the perimeter and area of an object.
*/
public interface Measurable
{
   /** Gets the perimeter.
       @return the perimeter */
   public double getPerimeter();

   /** Gets the area.
       @return the area */
   public double getArea();
} // end Measurable
```

You store an interface definition in a file with the same name as the interface, followed by `.java`. For example, the previous interface is in the file `Measurable.java`.

Programming Tip: A Java interface is a good place to provide comments that specify each method's purpose, parameters, precondition, and postcondition. In this way, you can specify a class in one file and implement it in another.

Note: An interface can declare data fields, but they must be public. By convention, a class's data fields are private, so any data fields in an interface should represent named constants. Thus, they should be public, final, and static.

Note: Methods declared within an interface cannot be static and cannot be final. However, such methods can be declared as final within a class that implements the interface.

D.13 **Example.** Suppose that you eventually want to define a class of people's names. You might begin by writing the Java interface given in Listing D-2 to specify the methods for such a class. We have included comments for only the first two methods, to save space. This interface provides specifications of the desired methods for an entire class. You could use it when implementing a class such as `Name`. Additionally, you should be able to write a client for the class just by looking at the interface.

LISTING D-2 The interface `NameInterface`

```
/** An interface for a class of names. */
public interface NameInterface
{
   /** Sets the first and last names.
       @param firstName  a string that is the desired first name
       @param lastName   a string that is the desired last name */
   public void setName(String firstName, String lastName);

   /** Gets the full name.
       @return a string containing the first and last names */
   public String getName();

   public void setFirst(String firstName);
   public String getFirst();

   public void setLast(String lastName);
   public String getLast();

   public void giveLastNameTo(NameInterface aName);

   public String toString();
} // end NameInterface
```

Notice that the parameter of the method `giveLastNameTo` has `NameInterface` as its data type instead of `Name`. We will talk about interfaces as data types beginning with Segment D.16. For now, simply be aware that an interface should not restrict the name of the class or classes that might implement it.

Note: Naming an interface
Interface names, particularly those that are standard in Java, often end in “able,” such as `Measurable`. That ending does not always provide a good name, so endings such as “er” or “Interface” are also used. Just as Java’s exception names end in “Exception,” we will usually end our interface names with “Interface.”

Implementing an Interface

D.14 Any class that implements an interface must state this at the beginning of its definition by using an `implements` clause. For example, if a class `Circle` implemented the interface `Measurable`, it would begin as follows:

```
public class Circle implements Measurable
```

The class then must provide a definition for each method declared in the interface. In this example, the class `Circle` must implement at least the methods `getPerimeter` and `getArea`.

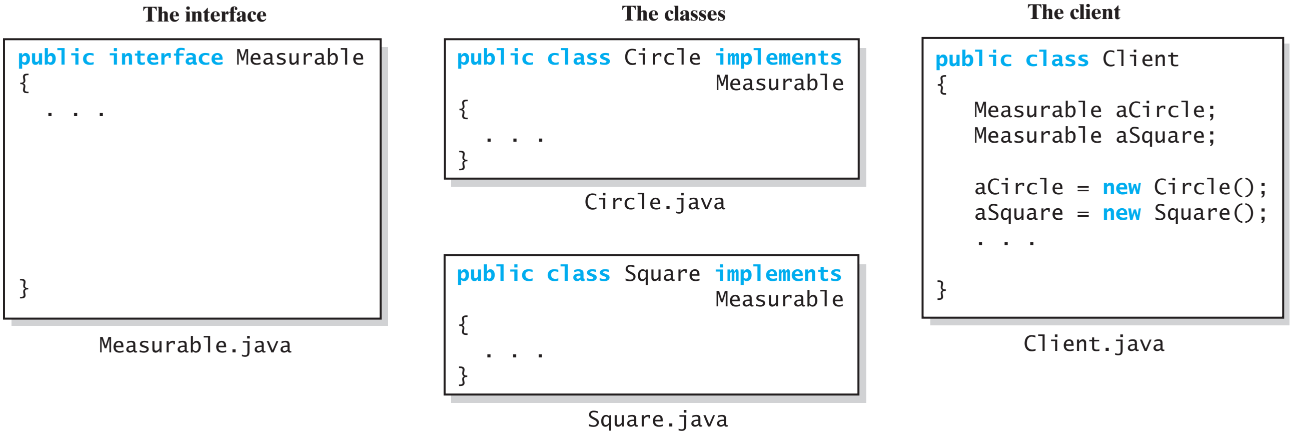
If we wrote a class `Square` that implemented `Measurable`, the class would begin as

```
public class Square implements Measurable
```

and would define at least the methods `getPerimeter` and `getArea`. Clearly, the definitions of these two methods would differ from those in the class `Circle`.

Figure D-3 illustrates the files that contain `Measurable`, `Circle`, `Square`, and their client.

FIGURE D-3 The files for an interface, a class that implements the interface, and the client



```
public interface Measurable
{
   . . .



}
```

```
public class Circle implements
                    Measurable
{
   . . .
}
```

```
public class Square implements
                    Measurable
{
   . . .
}
```

```
public class Client
{
   Measurable aCircle;
   Measurable aSquare;

   aCircle = new Circle();
   aSquare = new Square();
   . . .
}
```

Note: Writing an interface is a way for a class designer to specify methods for another programmer. Implementing an interface is a way for a programmer to guarantee that a class has defined certain methods.

Note: Several classes can implement the same interface, perhaps in different ways. For example, many classes can implement the interface `Measurable` and provide their own version of the methods `getPerimeter` and `getArea`.

D.15 **Multiple interfaces.** A class can implement more than one interface. If it does, you simply list all the interface names, separated by commas. If the class is derived from another class, the `implements` clause always follows the `extends` clause. Thus, you could write

```
public class C extends B implements Measurable, AnotherInterface
```

To remember this order, note that the reserved words `extends` and `implements` appear alphabetically in the heading of the class.

A class that implements several interfaces must define each method declared in the interfaces. If the same method header appears in more than one interface that a class implements, the class defines only one corresponding method.

You cannot derive a class from more than one base class. This restriction avoids the possibility of inheriting conflicting implementations. But a Java interface contains method specifications, not implementations. A class can implement these specifications regardless of whether they appear in one interface or are spread among several interfaces. By allowing a class to implement any number of interfaces, Java approximates multiple inheritance without the complications it can cause.

SELF-TEST

Question 5 Write a Java interface that specifies and declares methods for a class of students.

Question 6 Begin the definition of a class that implements the interface that you wrote in answer to the previous questions. Include data fields, a constructor, and at least one method definition.

An Interface as a Data Type

D.16 You can use a Java interface as you would a data type when you declare a variable, a data field, or a method's parameter. For example, the method `giveLastNameTo` in Segment D.13 has a parameter whose type is `NameInterface`:

```
public void giveLastNameTo(NameInterface aName);
```

Any argument that you pass to this method must be an object of a class that implements `NameInterface`.

Why didn't we declare `aName`'s type to be `Name`, as we did in Appendix B? We want the interface to be independent of any class that implements it, since more than one class can implement an interface. By using `NameInterface` as the parameter's type, you ensure that the method's argument will be able to invoke all of the methods declared in `NameInterface`. In general, you can be sure that a method's parameter can invoke particular methods, namely those declared in an interface, if its data type is the interface. Additionally, the parameter can invoke only those methods.

What if a class `C` does not begin with the phrase `implements NameInterface`, yet still implements the methods in the interface? You could not pass an instance of `C` to `giveLastNameTo`.

Note: By using an interface as a variable's type, you indicate that the variable can invoke a certain set of methods and only those methods.

D.17 A variable declaration such as

```
NameInterface myName;
```

makes `myName` a reference variable. Now `myName` can reference any object of any class, such as `Name`, that implements `NameInterface`. So if you have

```
myName = new Name("Coco", "Puffs");
```

then `myName.getFirst()` returns a reference to the string `"Coco"`. If the class `AnotherName` also implements `NameInterface`, and you later write

```
myName = new AnotherName("April", "MacIntosh");
```

then `myName.getFirst()` returns a reference to the string `"April"`.

Appendix C introduced polymorphic variables when discussing inheritance. In that discussion, you saw that you could write

```
A item = new B();
```

if class `B` is a subclass of class `A`. The variable `item` is polymorphic, since its dynamic type can differ from its static type. Here you see that the variable `myName` also is polymorphic. Thus, polymorphic variables can occur as a result of using either inheritance or interfaces.

Question 7 What revision(s) should you make to both the interface you wrote for Question 5 and the class `Student` that implements it to make use of `NameInterface`?

Generic Types Within an Interface

D.18 Imagine a class of pairs of objects whose data types are the same and are of any one class type. We can use a generic type in the definition of the class. For example, a method `setPair`—which assigns two objects to a pair—can have parameters of the same generic type.

Now imagine an interface `Pairable` that declares this method. We use a notation for the generic type that is analogous to the notation we use for a class, as follows:

```
public interface Pairable<S>
{
   public void setPair(S firstItem, S secondItem);
} // end Pairable
```

A class that implements this interface could begin with the statement

```
public class OrderedPair<T> implements Pairable<T>
```

In this example, the data type that we pass to the interface in the `implements` clause is the generic type `T` declared for the class. In general, one could pass the name of an actual class to the interface that appears in an `implements` clause. You will see an example of this situation in Segment D.21.

Note that the definition of the interface `Pairable` could have used `T` instead of `S` for its generic type, even though we use `T` for the same purpose when defining the class that implements the interface.

D.19 **The method `compareTo`.** Recall from Appendix A the method `compareTo` for the class `String`. This method returns an integer as a result of comparing two strings. For example, if `s` and `t` are strings, `s.compareTo(t)` is

- Negative if `s` comes before `t`
- Zero if `s` and `t` are equal
- Positive if `s` comes after `t`

Other classes can have their own `compareTo` method that behaves in an analogous way.

Note: The method compareTo compares two objects and returns a signed integer that indicates the result of the comparison. For example, if x and y are two instances of the same class that implements the interface Comparable, x.compareTo(y) returns

- A negative integer if x is less than y
- Zero if x equals y
- A positive integer if x is greater than y

If x and y have different types, x.compareTo(y) throws the exception ClassCastException.

Note: Enumerations have a compareTo method. The order in which the enumerated objects appear in the enumeration determines the result of a comparison. For example, if we have

```
enum Coin {PENNY, NICKEL, DIME, QUARTER}
```

and myCoin is an instance of Coin, myCoin.compareTo(Coin.DIME) decides whether myCoin is before, after, or equal to Coin.DIME. In particular, if myCoin has the value Coin.PENNY, the result of the comparison with Coin.DIME is a negative integer.

D.20 **The interface Comparable.** All classes that define the method compareTo implement the standard interface Comparable, which is in the Java Class Library in the package java.lang. This interface, which is shown in Listing D-3, uses a generic type T to represent the class that implements the interface. Thus, by invoking compareTo, you compare two objects of the class T.

LISTING D-3 The interface java.lang.Comparable

```
package java.lang;
public interface Comparable<T>
{
   public int compareTo(T other);
} // end Comparable
```

D.21 Let's create the class Circle, giving it the methods equals, compareTo, and the methods in the interface Measurable, as given in Listing D-1. The class implements two interfaces, so we begin it as follows:

```
public class Circle implements Comparable<Circle>, Measurable
{
   private double radius;

   < Definitions of constructors and methods are here >
   . . .
```

The name of the class appears in brackets after the interface name Comparable. Thus, Circle corresponds to T in the interface and is therefore the data type of compareTo's parameter.

The method compareTo has the following implementation within the class:

```
public int compareTo(Circle other)
{
   int result;
   if (this.equals(other))
      result = 0;
   else if (radius < other.radius)
      result = -1;
   else
      result = 1;

   return result;
} // end compareTo
```

This version of `compareTo` assumes that `Circle` has its own `equals` method. While `compareTo` need not invoke `equals`, these two methods usually should return consistent results. That is, if `object1.equals(object2)` is true, `object1.compareTo(object2)` should return zero.

D.22 Although the previous version of `compareTo` returns either -1 or +1 for unequal objects, the specification of `compareTo` does not insist on these values. Only the sign of the result must be correct. Thus, when the comparison involves integers, a simple subtraction often produces a suitable return value. For example, if the data field `radius` in the class `Circle` were an integer instead of a real value, `compareTo` could have had the following simple definition:

```
// assumes radius is an integer
public int compareTo(Circle other)
{
   return radius - other.radius;
} // end compareTo
```

D.23 You might wonder why `compareTo` does not belong to the class `Object`. The reason is that not all classes should have a `compareTo` method. Classes of objects without a natural ordering are possible and not at all unusual. For example, consider a class of mailing addresses. Deciding whether two addresses are equal should be simple, but what does it mean for one address to be less than another?

Note: Not all classes should implement the interface `Comparable`.

Question 8 Define a class `Name` that implements the interface `NameInterface`, as given in Listing D-2, and the interface `Comparable`.

Extending an Interface

D.24 Once you have an interface, you can derive another interface from it by using inheritance. In fact, you can derive an interface from several interfaces, even though you cannot derive a class from several classes.

When an interface extends another interface, it has all the methods of the inherited interface. Thus, you can create an interface that consists of the methods in an existing interface plus some new methods. For example, consider classes of pets and the following interface:

```
public interface Nameable
{
   public void setName(String petName);
   public String getName();
} // end Nameable
```

We can extend `Nameable` to create the interface `Callable`:

```
public interface Callable extends Nameable
{
   public void come(String petName);
} // end Callable
```

A class that implements `Callable` must implement the methods `come`, `setName`, and `getName`.

D.25 You also can combine several interfaces into a new interface and add even more methods if you like. For example, suppose that in addition to the previous two interfaces, we define the following interfaces:

```
public interface Capable
{
   public void hear();
   public void respond();
} // end Capable

public interface Trainable extends Callable, Capable
{
   public void sit();
   public void speak();
   public void lieDown();
} // end Trainable
```

A class that implements `Trainable` must implement the methods `setName`, `getName`, `come`, `hear`, and `respond`, as well as the methods `sit`, `speak`, and `lieDown`.

Note: A Java interface can be derived from several interfaces, even though you cannot derive a class from several classes.

SELF-TEST

Question 9 Imagine a class `Pet` that contains the method `setName`, yet does not implement the interface `Nameable` of Segment D.24. Could you pass an instance of `Pet` as the argument of the method with the following header?

```
void enterShow(Nameable petName)
```

Interfaces Versus Abstract Classes

The purpose of an interface is similar to that of an abstract class. However, an interface is not a class. When should you use an interface and when should you use an abstract class? Use an abstract class if you want to provide a method definition or declare a private data field that your classes will have in common. Otherwise, use an interface. Remember that a class can implement several interfaces but can extend only one abstract class.

Let's look at two examples. One uses an interface and the other an abstract class.

D.26 **Example: An interface.** Imagine classes for various geometric forms like circles, spheres, and cylinders. Each of these forms has a radius. We could define the following interface that our classes would implement:

```
public interface Circular
{
   public void setRadius(double newRadius);
   public double getRadius();
} // end Circular
```

This interface recognizes that a radius will exist, and so declares both set and get methods for it. However, it cannot declare a field for the radius. The class that implements the interface will do that.

A class Circle that implements this interface could appear as follows:

```
public class Circle implements Circular
{
   private double radius;

   public void setRadius(double newRadius)
   {
      radius = newRadius;
   } // end setRadius

   public double getRadius()
   {
      return radius;
   } // end getRadius

   public double getArea()
   {
      return Math.PI * radius * radius;
   } // end getArea
} // end Circle
```

The class defines a private data field radius, and implements the methods setRadius and getRadius that the interface Circular declares. An interface cannot contain a data field like radius, since it is private.

Note: A class can define more methods than are declared in the interfaces it implements. For example, the class Circle defines the method getArea, which is not in the interface Circular.

D.27 **Example: An abstract class.** Instead of using an interface in the implementation of a class like Circle, let's define an abstract class:

```
public abstract class CircularBase
{
   private double radius;

   public void setRadius(double newRadius)
   {
      radius = newRadius;
   } // end setRadius

   public double getRadius()
   {
      return radius;
   } // end getRadius

   public abstract double getArea();
} // end CircularBase
```

This class declares the data field radius that descendant classes will inherit. Since the data field radius is private, the class CircularBase must implement set and get methods so that its descendant classes can access it. If CircularBase simply declared setRadius and getRadius as abstract—omitting their implementations—a descendant class would be unable to implement them because it would be unable to access radius.

If the definition of CircularBase stopped here, it would not need to be abstract, but it still would be a useful base class. However, this class also declares the abstract method getArea, which its descendant classes must implement in their own way.

The following class is derived from the class `CircularBase`. It implements the abstract method `getArea`, invoking the inherited method `getRadius` to access the inherited data field `radius`. `Circle` cannot reference the data field `radius` by name.

```
public class Circle extends CircularBase
{
   public double getArea()
   {
      double radius = getRadius();
      return Math.PI * radius * radius;
   } // end getArea
} // end Circle
```

In this method, `radius` is simply a local variable.

Programming Tip: If you want to define a method or declare a private data field that your classes will have in common, use an abstract class. Otherwise, use an interface.

Named Constants Within an Interface

An interface can contain named constants, that is, public data fields that you initialize and declare as final. If you want to implement several classes that share a common set of named constants, you can define the constants in an interface that the classes implement. You also could define your constants in a separate class instead of an interface. We will look at both ways in this section. Whichever way you choose, you have only one set of constants to keep current.

Imagine several classes that must convert measurements to the metric system. We can define conversion factors as constants that these classes can share. Let's place the constants in an interface.

D.28 **An interface of constants.** The following interface defines three named constants:

```
public interface ConstantsInterface
{
   public static final double INCHES_PER_CENTIMETER = 0.39370079;
   public static final double FEET_PER_METER = 3.2808399;
   public static final double MILES_PER_KILOMETER = 0.62137119;
} // end ConstantsInterface
```

Any interface can define constants in addition to declaring methods, but this interface contains only constants.

To use these constants in a class, you write an `implements` clause in the class definition. The constants then will be available by name throughout the class. For example, consider the following simple class:

```
public class Demo implements ConstantsInterface
{
   public static void main(String[] args)
   {
      System.out.println(FEET_PER_METER);
      System.out.println(ConstantsInterface.MILES_PER_KILOMETER);
   } // end main
} // end Demo
```

Qualifying the constants with the name of the interface is optional. However, if the same named constant is defined in more than one interface that a class implements, the class must qualify the constant with the name of the interface.

D.29 **A class of constants.** Instead of defining constants in an interface, you can define them in a class just for that purpose:

```
public class Constants
{
   private Constants()
   {
   } // end private default constructor

   public static final double INCHES_PER_CENTIMETER = 0.39370079;
   public static final double FEET_PER_METER = 3.2808399;
   public static final double MILES_PER_KILOMETER = 0.62137119;
} // end Constants
```

Notice the private constructor. Since we provide a constructor, Java will not. And since our constructor is private, a client cannot create instances of the class.

Using this class is simple, as the following example shows:

```
public class Demo
{
   public static void main(String[] args)
   {
      System.out.println(Constants.FEET_PER_METER);
      System.out.println(Constants.MILES_PER_KILOMETER);
   } // end main
} // end Demo
```

Since the constants are static, you must precede their names with the name of the class and a period. This can be an advantage, as readers of your program will see immediately the source of the constant. If doing so becomes an annoyance, you can always define a local copy of the constant, such as

```
final double FEET_PER_METER = Constants.FEET_PER_METER;
```

and use it instead.

Design Decision: Should you define constants in an interface or in a class?

Programmers seem to disagree about the answer to this question. Even the Java Class Library contains examples of both techniques. Generally, constant definitions are an implementation detail that should appear within a class. Interfaces declare methods and so are in the realm of specification, not implementation. Reserving interfaces solely for methods is a reasonable guideline.

Choosing Classes

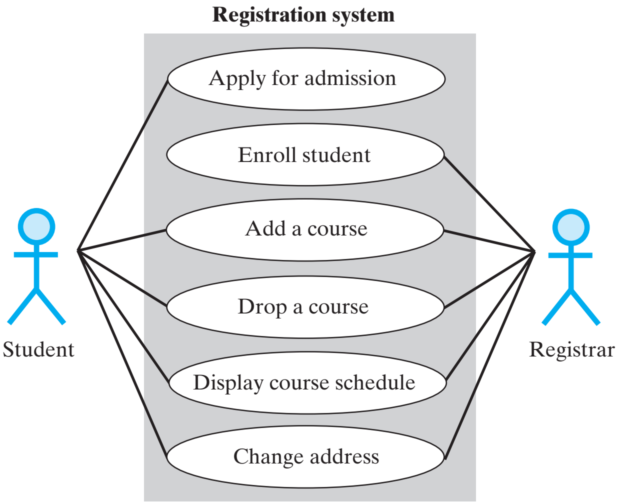
We have talked about specifying classes and implementing classes, but up to now, we have described the class to specify or implement. If you must design an application from scratch, how will you choose the classes you need? In this section, we introduce you to some techniques that software designers use in choosing and designing classes. Although we will mention these techniques from time to time throughout the book, our intent is simply to expose you to these ideas. Future courses will cover ways to select and design classes in more depth.

D.30 Imagine that we are designing a registration system for your school. Where should we begin? A useful way to start would be to look at the system from a functional point of view, as follows:

- **Who or what will use the system?** A human user or a software component that interacts with the system is called an **actor**. So a first step is to list the possible actors. For a registration system, two of the actors could be a student and the registrar.
- **What can each actor do with the system?** A **scenario** is a description of the interaction between an actor and the system. For example, a student can add a course. This basic scenario has variations that give rise to other scenarios. For instance, what happens when the student attempts to add a course that is closed? Our second step, therefore, is to identify scenarios. One way to do this is to complete the question that begins "What happens when...".
- **Which scenarios involve common goals?** For example, the two scenarios we just described are related to the common goal of adding a course. A collection of such related scenarios is called a **use case**. Our third step, then, is to identify the use cases.

You can get an overall picture of the use cases involved in a system you are designing by drawing a **use case diagram**. Figure D-4 is a use case diagram for our simple registration system. Each actor—the student and the registrar—appears as a stick figure. The box represents the registration system, and the ovals within the box are the use cases. A line joins an actor and a use case if an interaction exists between the two.

FIGURE D-4 A use case diagram for a registration system



Some use cases in this example involve one actor, and some involve both. For example, only the student applies for admission, and only the registrar enrolls a student. However, both the student and the registrar can add a course to a student's schedule.

Note: Use cases depict a system from the actors' points of view. They do not necessarily suggest classes within the system.

Identifying Classes

D.31 Although drawing a use case diagram is a step in the right direction, it does not identify the classes that are needed for your system. Several techniques are possible, and you will probably need to use more than one.

One simple technique is to describe the system and then identify the nouns and verbs in the description. The nouns can suggest classes, and the verbs can suggest appropriate methods within the classes. Given the imprecision of natural language, this technique is not foolproof, but it can be useful.

For example, we could write a sequence of steps to describe each use case in Figure D-4. Figure D-5 gives a description of the use case for adding a course from the point of view of a student. Notice the alternative actions taken in Steps 2a and 4a when the system does not recognize the student or when a requested course is closed.

FIGURE D-5 A description of a use case for adding a course

System: Registration

Use case: Add a course

Actor: Student

Steps:

1. Student enters identifying data.
2. System confirms eligibility to register.
   a. If ineligible to register, ask student to enter identification data again.
3. Student chooses a particular section of a course from a list of course offerings.
4. System confirms availability of the course.
   a. If course is closed, allow student to return to Step 3 or quit.
5. System adds course to student's schedule.
6. System displays student's revised schedule of courses.

What classes does this description suggest? Looking at the nouns, we could decide to have classes to represent a student, a course, a list of all courses offered, and a student's schedule of courses. The verbs suggest actions that include confirming whether a student is eligible to register, seeing whether a course is closed, and adding a course to a student's schedule. One way to assign these actions to classes is to use CRC cards, which we describe next.

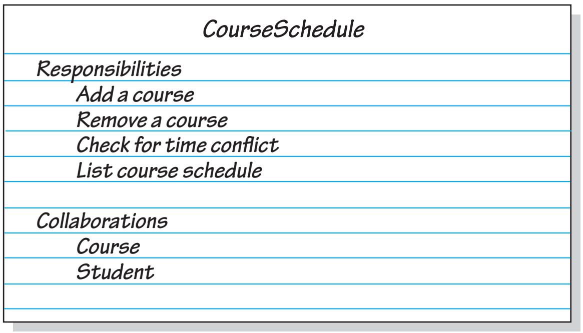
CRC Cards

D.32 A simple technique for exploring the purpose of a class uses index cards. Each card represents one class. You begin by choosing a descriptive name for a class and writing it at the top of a card. You then list the actions that represent the class's **responsibilities**. You do this for each class in the system. Finally, you indicate the interactions, or **collaborations**, among the classes. That is, you write on each class's card the names of other classes that have some sort of interaction with the class. Because of their content, these cards are called **class-responsibility-collaboration**, or **CRC**, **cards**.

For example, Figure D-6 shows a CRC card for the class `CourseSchedule` that represents the courses in which a student has enrolled. Notice that the small size of each card forces you to write brief notes. The number of responsibilities must be small, which suggests that you think at a high level and consider small classes. The size of the cards also lets you arrange them on a table and move them around easily while you search for collaborations.

Question 10 Write a CRC card for the class `Student` given in Appendix C.

FIGURE D-6 A class-responsibility-collaboration (CRC) card

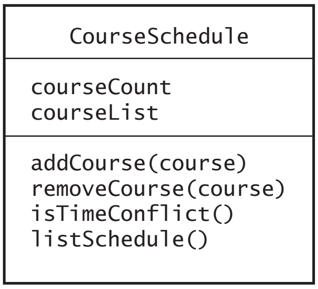


The Unified Modeling Language

D.33 The use case diagram in Figure D-4 is part of a larger notation known as the **Unified Modeling Language**, or **UML**. Designers use the UML to illustrate a software system's necessary classes and their relationships. The UML gives people an overall view of a complex system more effectively than either a natural language or a programming language can. English, for example, can be ambiguous, and Java code provides too much detail. Providing a clear picture of the interactions among classes is one of the strengths of the UML.

Besides the use case diagram, the UML provides a **class diagram** that places each class description in a box analogous to a CRC card. The box contains a class's name, its **attributes** (data fields), and **operations** (methods). For example, Figure D-7 shows a box for the class `CourseSchedule`. Typically, you omit from the box such common operations as constructors, get methods, and set methods.

FIGURE D-7 A class representation that can be a part of a class diagram



As your design progresses, you can provide more detail when you describe a class. You can indicate the visibility of a field or method by preceding its name with + for public, - for private, and # for protected. You also can write the data type of a field, parameter, or return value after a colon that follows the particular item. Thus, in Figure D-7 you can write the data fields as

```
-courseCount: integer
-courseList: List
```

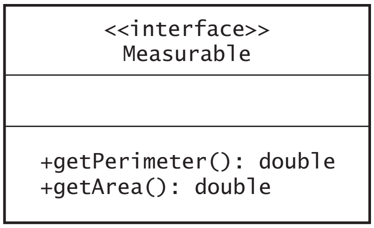
and the methods as

```
+addCourse(course: Course): void
+removeCourse(course: Course): void
```

```
+isTimeConflict(): boolean
+listSchedule(): void
```

You represent an interface in UML much as you represent a class, but you precede its name with `<<interface>>`. Figure D-8 shows the notation for the interface `Measurable` that appears in Segment D.12.

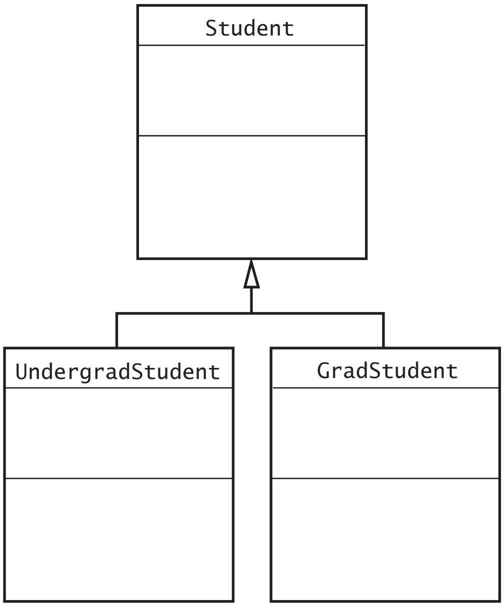
FIGURE D-8 UML notation for the interface `Measurable`



Question 11 How would the class `Name`, given in Appendix B, appear in a class diagram of the UML?

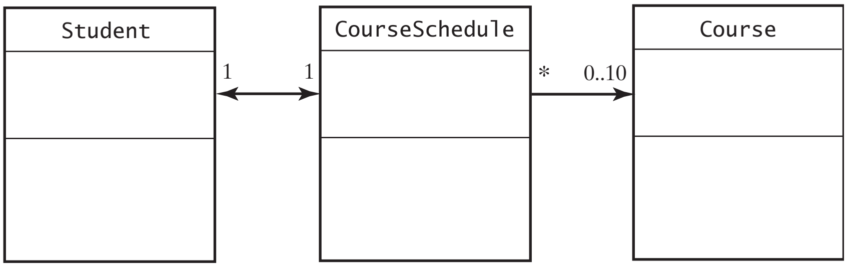
D.34 In a class diagram, lines join the class boxes to show the relationships among the classes, including any inheritance hierarchy. For example, the class diagram in Figure D-9 shows that the classes `UndergradStudent` and `GradStudent` are each derived from the class `Student`. An arrow with a hollow head points to the superclass. Within the UML, the superclass `Student` is said to be a **generalization** of `UndergradStudent` and `GradStudent`. If a class implements an interface, you draw an arrow having a dotted shaft and hollow head from the class to the interface.

FIGURE D-9 A class diagram showing the base class `Student` and two subclasses



An **association** is a line that represents a relationship between instances of two classes. Basically, an association represents what a CRC card calls a collaboration. For example, relationships exist among the classes `Student`, `CourseSchedule`, and `Course`. Figure D-10 shows how the UML pictures these relationships. The association (line) between the classes `CourseSchedule` and `Course`, for example, indicates a relationship between objects of the class `CourseSchedule` and objects of the class `Course`. This association has an arrow pointing toward `Course`. The arrow indicates responsibilities. Thus, a `CourseSchedule` object should be able to tell us the courses it contains, but a `Course` object need not be able to tell us to which schedules it belongs. The UML calls this aspect of the notation the **navigability**.

FIGURE D-10 Part of a UML class diagram with associations



This particular arrow is said to be **unidirectional**, since it points in one direction. An association with arrowheads on both ends is called **bidirectional**. For example, a `Student` object can find its course schedule, and a `CourseSchedule` object can discover the student to which it belongs. You can assume that the navigability of an association without arrowheads is unspecified at the present stage of the design.

At the ends of each association are numbers. At the end of the line beginning at `CourseSchedule` and extending to `Course`, you see the notation 0..10. This notation indicates that each `CourseSchedule` object is associated with between zero and ten courses. If you follow the line in the other direction, you encounter an asterisk. It has the same meaning as the notation 0..infinity. Each `Course` object can be associated with many, many course schedules—or with none at all. The figure also indicates a relationship between one `Student` object and one `CourseSchedule` object. This notation on the ends of an association is called the association's **cardinality** or **multiplicity**.

? SELF-TEST

Question 12 Combine Figures D-9 and D-10 into one class diagram. Then add a class `AllCourses` that represents all courses offered this semester. What new association(s) do you need to add?

Reusing Classes

D.35 When you first start to write programs, you can easily get the impression that each program is designed and written from scratch. On the contrary, most software is created by combining already existing components with new components. This approach saves time and money. In addition, the existing components have been used many times and so are better tested and more reliable.

For example, a highway simulation program might include a new highway object to model a new highway design, but it would probably model automobiles by using an automobile class that had already been designed for some other program. As you identify the classes that you need for your project, you should see whether any of the classes exist already. Can you use them as is, or would they serve as a good base class for a new class?

D.36 As you design new classes, you should take steps to ensure that they are easily reusable in the future. You must specify exactly how objects of that class interact with other objects. This is the principle of encapsulation that we discussed in the first section of this appendix. But encapsulation is not the only principle you must follow. You must also design your class so that the objects are general and not tailored too much for one particular program. For example, if your program requires that all simulated automobiles move only forward, you should still include a reverse in your automobile class. Some other simulation may require automobiles to back up.

Admittedly, you cannot foresee all the future uses of your class. But you can and should avoid dependencies that will restrict its use later. Chapter 17 describes the design of a class with its future use in mind.

Using the principles that this appendix discusses to design a reusable class with an interface that has comments suitable for `javadoc` takes work. Hacking together a solution to your specific problem would take less time. But the payback for your effort will come later on, when you or another programmer needs to reuse an interface or a class. If you planned for the future when you wrote those components, every use of them will be faster and easier. Actual software developers use these principles to save time over the long term, because saving time saves them money. You should use them, too.

Exercises and Projects

A collection of exercises and projects for this appendix are available online at `pearsonhighered.com/carrano`.

Answers to Self-Test Questions

1. A client interface describes how to use the class. It contains the headers for the class's public methods, the comments that tell you how to use these methods, and any publicly defined constants of the class. The implementation consists of all data fields and the definitions of all methods, including those that are public, private, and protected.

2. A television is one example. The remote control and the controls on the TV form the client interface. The implementation is inside the TV itself.

3. Here are three possibilities:

```
/** Sets the side of the square to a new value.
    @param newSide  a real number >= 0 */
public void setSide(double newSide)

/** Sets the side of the square to a new value.
    @param newSide  a real number
    @return true if the side is set, or
            false if newSide is < 0 */
public boolean setSide(double newSide)

/** Sets the side of the square to a new value.
    @param newSide  a real number
    @throws IllegalArgumentException if newSide is < 0 */
public void setSide(double newSide)
```

4.
```
// Assertion: max is the largest of array[0],..., array[index]
```

5.
```
public interface StudentInterface
{
   public void setStudent(Name studentName, String studentId);
   public void setName(Name studentName);
   public Name getName();
   public void setId(String studentId);
   public String getId();
   public String toString();
} // end StudentInterface
```

6.
```
public class Student implements StudentInterface
{
   private Name   fullName;
   private String id;      // identification number

   public Student()
   {
      fullName = new Name();
      id = "";
   } // end default constructor

   public Student(Name studentName, String studentId)
   {
      fullName = studentName;
      id = studentId;
   } // end constructor

   public void setStudent(Name studentName, String studentId)
   {
      setName(studentName); // or fullName = studentName;
      setId(studentId);     // or id = studentId;
   } // end setStudent

   public void setName(Name studentName)
   {
      fullName = studentName;
   } // end setName

   public Name getName()
   {
      return fullName;
   } // end getName

   public void setId(String studentId)
   {
      id = studentId;
   } // end setId

   public String getId()
   {
      return id;
   } // end getId

   public String toString()
   {
      return id + " " + fullName.toString();
   } // end toString
} // end Student
```

7. In the interface and in the class, replace `Name` with `NameInterface` in the methods `setStudent`, `setName`, and `getName`. Additionally in the class, replace `Name` with `NameInterface` in the declaration of the data field `fullName` and in the parameterized constructor.

8.

```
/**
    A class that represents a person's name.
    @author Frank M. Carrano
*/
public class Name implements NameInterface, Comparable<Name>
{
   private String first; // first name
   private String last;  // last name

   public Name()
   {
      first = "";
      last = "";
   } // end default constructor

   public Name(String firstName, String lastName)
   {
      first = firstName;
      last = lastName;
   } // end constructor

   public void setName(String firstName, String lastName)
   {
      setFirst(firstName);
      setLast(lastName);
   } // end setName

   public String getName()
   {
      return toString();
   } // end getName

   public void setFirst(String firstName)
   {
      first = firstName;
   } // end setFirst

   public String getFirst()
   {
      return first;
   } // end getFirst

   public void setLast(String lastName)
   {
      last = lastName;
   } // end setLast

   public String getLast()
   {
      return last;
   } // end getLast

   public void giveLastNameTo(NameInterface aName)
   {
      aName.setLast(last);
   } // end giveLastNameTo
   public String toString()
   {
      return first + " " + last;
   } // end toString

   public int compareTo(Name other)
   {
      int result = last.compareTo(other.last);

      // if last names are equal, check first names
      if (result == 0)
         result = first.compareTo(other.first);
      return result;
   } // end compareTo
} // end Name
```

9. No. The class `Pet` must state that it implements `Nameable` in an `implements` clause.

10.

Student
Responsibilities
Set name and ID
Set name
Set ID
Get name
Get ID
Get a string that represents a student
Collaborations
`String`
`Name`

11.

```
Name
-first: String
-last: String
+setName(firstName: String, lastName: String): void
+getName(): String
+setFirst(firstName: String): void
+getFirst(): String
+setLast(lastName: String): void
+getLast(): String
+giveLastNameTo(aName: Name): void
+toString(): String
```

12. Add a unidirectional association (arrow) from `AllCourses` to `Course` with a cardinality of 1 on its tail and * on its head.

Appendix E

Handling Exceptions

Contents

Prerequisites

Appendix A	Java Essentials
Appendix B	Java Classes
Appendix C	Creating Classes from Other Classes

An **exception** is an object that is created when an unusual circumstance or event occurs during the execution of a method, thereby interrupting program execution. Some exceptions indicate mistakes in your code. By correcting those mistakes, you avoid the exceptions and no longer have to worry about them. In fact, your final code gives no indication that an exception could occur. Furthermore, if your code is entirely correct, an exception will not occur.

On the other hand, a programmer can intentionally cause an exception to occur under certain conditions. In fact, the programmers who wrote the code for the Java Class Library did so. If you peruse the documentation for this library, you will see lists of the exceptions that might occur during the execution of certain methods. We need to know about exceptions so we can use these methods. What should we do when such an exception occurs? Should we ever intentionally cause an exception in our own programs, and if so, how would we do so? These are some of the questions that this appendix will answer. This knowledge will be particularly important to us when we talk about file input and output in Appendix F.

The Basics

E.1 When a method creates an exception object, we say that the method **throws** the exception. An exception is a signal to the rest of the program that something unexpected has happened. Our code can react appropriately to the exception based on its class type and what the exception, as an object, can tell us via its methods. We **handle** the exception when we detect and react to it.

Exceptions belong to various classes, but all of these classes have the standard class `Exception` as an ancestor. `Exception` is in the Java Class Library and is available to us without an `import` statement. Exceptions are classified into two groups:

- Checked exceptions, which must be handled
- Unchecked, or runtime, exceptions, which need not be—and usually are not—handled

E.2 **Checked exceptions** are the result of a serious occurrence during program execution. For example, if a program is reading data from a disk—as the next appendix will describe—and the system cannot find the file that contains the data, a checked exception will occur. The name of the class to which this exception belongs is `FileNotFoundException`. This name, like the names of all exception classes in the Java Class Library, is meant to describe the cause of the exception. A common practice is to describe an exception by its class name. For example, we might say that a `FileNotFoundException` has occurred.

Note: **Checked exceptions in the Java Class Library**
The following classes in the Java Class Library represent some of the checked exceptions that you might encounter:

```
ClassNotFoundException
FileNotFoundException
IOException
NoSuchMethodException
WriteAbortedException
```

E.3 **Unchecked exceptions** often are called by their alternate name, **runtime exceptions**, so that is what we will call them. The cause of a runtime exception usually is a logical error in the program. For example, an out-of-bounds array index causes an exception of the class `ArrayIndexOutOfBounds`. A division by zero causes an `ArithmeticException`. Although we could add code that would handle a runtime exception, we usually just need to fix the mistakes in our program.

Note: **Runtime (unchecked) exceptions in the Java Class Library**
The following classes in the Java Class Library represent some of the runtime exceptions that you are likely to encounter:

```
ArithmeticException
ArrayIndexOutOfBoundsException
ClassCastException
IllegalArgumentException
IllegalStateException
IndexOutOfBoundsException
NoSuchElementException
NullPointerException
StringIndexOutOfBoundsException
UnsupportedOperationException
```

Note: All exception classes have the standard class Exception as an ancestor
The standard class RuntimeException extends Exception, and all classes for runtime exceptions extend RuntimeException. Thus, a runtime exception is an object of either the class RuntimeException or one of its descendants. The classes of checked exceptions extend Exception but do not have RuntimeException as an ancestor.

Note: Many exception classes are in the package java.lang, and so do not need to be imported. Some exception classes, however, are in another package, and these do need to be imported. For example, when we use the class IOException in a program, we will need the import statement

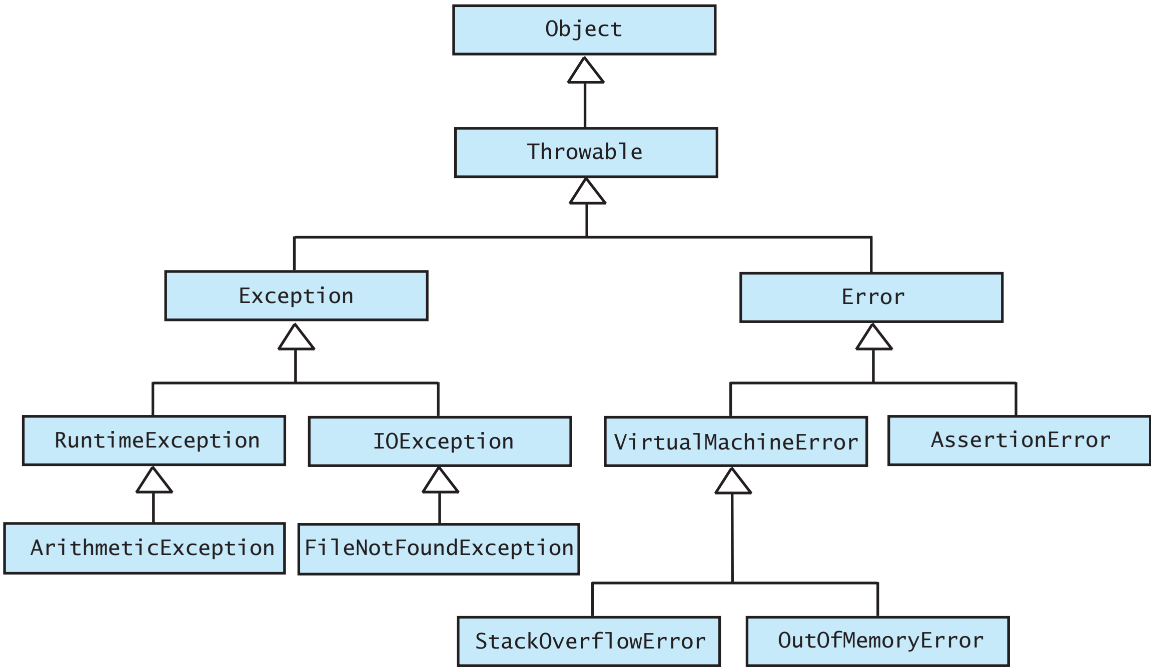
```
import java.io.IOException;
```

We will encounter this exception in the next appendix.

E.4 An **error** is an object of either the standard class Error or one of its descendant classes. Since the class Error is not derived from the class Exception, an error is not an exception, even though errors appear to be similar to exceptions. In general, an error indicates the occurrence of an abnormal situation, such as running out of memory. If your program uses more memory than is available, you must either revise your program to make it more efficient in its use of memory, change a setting to let Java access more memory, or buy more memory for your computer. These situations are too serious for a typical program to handle. Hence, errors need not be handled, even though doing so is legal.

Figure E-1 shows the hierarchy of some exception and error classes. Runtime exceptions, such as ArithmeticException, are descended from RuntimeException. Checked exceptions, such as

FIGURE E-1 The hierarchy of some standard exception and error classes



IOException, are descended from Exception but not RuntimeException. An assertion error, which Segment D.9 of Appendix D defined, is an object of the class AssertionError, which has the class Error as its superclass. When we discuss recursion in Chapter 7, we mention a stack overflow error. This error belongs to the class StackOverflowError. Both StackOverflowError and OutOfMemoryError are derived from the abstract class VirtualMachineError, which has Error as its superclass. Right now, all that is important to us is knowing that StackOverflowError, OutOfMemoryError, and AssertionError have Error, instead of Exception, as an ancestor class.

Handling an Exception

When a checked exception might occur, it must be handled somewhere. For a method that might cause a checked exception, you have two choices: Handle the exception within the method or tell the method's client to do so.

Postpone Handling: The throws Clause

E.5 Imagine a method that returns the string it reads from a disk. Since we will learn how to write such a method in the next appendix, let's not worry about how this method accomplishes its task. However, something might go wrong while reading from the disk. That something could generate an IOException. Since an IOException is a checked exception, it must be handled. We could handle the exception within the method's body. Sometimes, however, a programmer is not sure what action is best for a client when an exception occurs. Should execution end, or would another action make more sense? When you're not sure what action to take, you can leave the handling of the exception to the method's client. As long as the exception is handled at some point, you need not handle it within the method itself.

A method that can cause but does not handle a checked exception must declare that fact in its header. For example, if the method readString might throw an IOException but does not handle it, its header would look like this:

```
public String readString(. . .) throws IOException
```

The highlighted portion is a **throws clause**. It frees the method readString of the responsibility of handling any exceptions of type IOException that might occur during its execution. If, however, another method calls readString, it must deal with the exception. That invoking method can either handle the IOException itself or tell its client to handle the exception by including it in a throws clause in its header. Eventually, every thrown, checked exception should be handled somewhere in the program.

You can list more than one checked exception in a throws clause by separating the exceptions with commas.

Note: The throws clause

A method's header can contain a throws clause that lists the possible exceptions the method can throw but will not handle. The clause has the following syntax:

throws *exception-list*

The exception names listed in *exception-list* are separated by commas. Their order here is unimportant.

Note: If a method can throw a checked exception, you must either declare the exception in the method's header by writing a `throws` clause or handle the exception within the method. Failure to do so will cause a syntax error.

If a method can throw an unchecked exception, you can declare it in a `throws` clause or handle it, but neither action is required.

Note: A method's execution ends if it throws but does not handle an exception
If a method throws an exception but does not handle it, the method's execution ends. For example, if the previous method `readString` throws an `IOException`, its execution ends immediately. Program execution continues, however, and the exception is passed to `readString`'s client.

Programming Tip

When defining a method that can throw a checked exception, if you cannot provide a reasonable reaction to the exception, pass it on to the method's client by writing a `throws` clause in the method's header. Avoid using `Exception` in a `throws` clause, as doing so provides another programmer with little if any useful information about calling the method. Instead, use as specific an exception as you can.

Handle It Now: The `try-catch` Blocks

E.6 To handle an exception, we first must identify the Java statements that can cause it. We also must decide which exception to look for. A method's documentation and `throws` clause will tell us which checked exceptions might occur. It is those exceptions that we will handle.

The code to handle an exception consists of two pieces. The first piece, the **`try` block**, contains the statements that might throw an exception. The second piece consists of one or more `catch` blocks. Each **`catch` block** contains code to react to, or **catch**, a particular type of exception. Thus, the code to handle an `IOException` as a result of invoking the method `readString` would have the following form:

```
try
{
   < Possibly some code >
   anObject.readString(. . .); // might throw an IOException
   < Possibly some more code >
}
catch (IOException e)
{
   < Code to react to the exception, probably including the following statement: >
   System.out.println(e.getMessage());
}
```

E.7 The statements within the `try` block execute just as they would if the block was not there. If no exception occurs and the `try` block completes execution, execution continues with the statement after the `catch` block. However, if an `IOException` occurs within the `try` block, execution immediately transfers to the `catch` block. The exception now has been caught.

The syntax for a `catch` block resembles that of a method definition. The identifier `e` is called a **`catch` block parameter**; it represents the object of `IOException` that the `catch` block will handle.

Although a catch block is not a method definition, throwing an exception within a try block is like calling a catch block, in that the parameter e represents an actual exception.

As an object, every exception has the accessor method getMessage, which returns a descriptive string created when the exception is thrown. By displaying this string, we provide a programmer with an indication of the nature of the exception.

E.8 After the catch block executes, the statements after it execute. But what if the problem is serious, and the best reaction to it is to terminate the program? The catch block can end the program by calling the exit method as follows:

```
System.exit(0);
```

The number 0 given as the argument to System.exit indicates a normal termination of the program. Although we have encountered a serious problem, we intentionally terminate the program, which, in the view of the operating system, is normal.

Note: If you do not handle a checked exception or declare it in a throws clause, the compiler will complain. You can handle some of a method's exceptions within its definition and declare some in its throws clause. Generally, you do not handle or declare runtime (unchecked) exceptions, since they indicate a bug in your program. Such exceptions terminate program execution when they are thrown.

Note: A catch block whose parameter has the type C can catch exceptions of the class C and any of C's descendant classes.

Multiple catch Blocks

E.9 The statements within a single try block can throw any one of a number of different types of exceptions. For example, suppose that the code within the try block in Segment E.6 could throw more than one type of checked exception. The catch block after this try block can catch exceptions of the class IOException and any class derived from IOException. To catch exceptions of other types, we can write more than one catch block after the try block. When an exception is thrown, the order in which catch blocks appear is significant. Execution continues with the first catch block—in order of appearance—whose parameter matches the exception in type.

E.10 **A poor order for catch blocks.** For example, the following sequence of catch blocks is poor, because the catch block for FileNotFoundException never executes:

```
catch (IOException e)
{
  . . .
}
catch (FileNotFoundException e)
{
  . . .
}
```

With this ordering, any I/O exception will be caught by the first catch block. Because FileNotFoundException extends IOException, a FileNotFoundException is a kind of IOException and will match the parameter of the first catch block. Fortunately, this ordering likely will receive a warning from the compiler.

E.11 **A good order for catch blocks.** The correct ordering places the more specific exception before its ancestor class, as follows:

```
catch (FileNotFoundException e)
{
   . . .
}
catch (IOException e) // handle all other IOExceptions
{
   . . .
}
```

Programming Tip

Since all exception classes have `Exception` as an ancestor, avoid using `Exception` in a `catch` block. Instead, catch as specific an exception as you can, and catch the most specific one first.

Note: The `try-catch` blocks have the following syntax:

```
try
{
   < Statements that can cause an exception >
}
catch (exceptionType e)
{
   < Code to react to the exception, probably including the following: >
   System.out.println(e.getMessage());
}
< Possibly other catch blocks >
```

Programming Tip: Avoid nested try-catch blocks, if possible

Although nesting `try-catch` blocks within either a `try` block or a `catch` block is legal, you should avoid doing so if possible. First see whether you can organize your logic differently to avoid the nesting. Failing that, move the inner blocks to a new method that you call within what was an outer block.

If you must nest `try-catch` blocks, the following guidelines apply. When a `catch` block appears within another `catch` block, they must use different identifiers for their parameters. If you plan to nest `try-catch` blocks within a `try` block, you could omit the inner `catch` blocks if the outer `catch` blocks deal with the relevant exceptions appropriately. In such a case, an exception thrown within an inner `try` block is caught in the outer `try` block.

Note: **Where to next?**

At this point in our use of Java, handling an exception thrown by an invoked library method is our greatest concern. This ability will be essential to learning how to perform input and output with external files, as you will see in Appendix F. If you want, you can skip the rest of this appendix for now and read the next one.

Throwing an Exception

Although the ability to handle an exception is most useful, knowing how to throw an exception and how to define a class of exceptions is also important. This section looks at how exceptions are thrown. You should throw an exception within a method only in unusual or unexpected situations that you cannot address in a reasonable way.

E.12 **The `throw` statement.** A method intentionally throws an exception by executing a `throw` statement. Its general form is

```
throw exception_object;
```

Rather than creating the exception object in a separate step, programmers usually create the object within the `throw` statement, as in the following example:

```
throw new IOException();
```

This statement creates a new object of the class `IOException` and throws it. Just as we should catch as specific an exception as possible, the exceptions we throw should be as specific as possible.

Although we can invoke the default constructor of the exception class, as in the previous example, we also can provide the constructor with a string as an argument. The resulting object will contain that string in a data field, and both the object and this string will be available to the `catch` block that handles the exception. The `catch` block then can use the exception's method `getMessage` to retrieve the string, as you saw earlier. The default constructor provides a default value for such a string.

Note: The `throw` statement has the following syntax:

```
throw exception_object;
```

where *exception_object* is an instance of a class of exceptions, typically created by one of the following invocations of the class's constructor:

```
new class_name()
```

or

```
new class_name(message)
```

Either the string provided by the default constructor or the string *message* is available to the code that catches the exception, via the exception's method `getMessage`.

Design Decision: **If an unusual situation occurs, should I throw an exception?**

- If you can resolve the unusual situation in a reasonable manner, you likely can use a decision statement instead of throwing an exception.
- If several resolutions to an abnormal occurrence are possible, and you want the client to choose one, you should throw a checked exception.
- If a programmer makes a coding mistake by using your method incorrectly, you can throw a runtime exception. However, you should not throw a runtime exception simply to enable a client to avoid handling it.

You should throw only exceptions, never errors.

Programming Tip

If a method contains a `throw` statement to throw an exception, add a `throws` clause to its header rather than catching the exception within the method's body. In general, throwing an exception and catching one should occur in separate methods.

Programmer-Defined Exception Classes

You can define your own exception classes by extending existing exception classes. An existing superclass could be one in the Java Class Library or one of your own. The constructors in an exception subclass are the most important—and often the only—methods you need to define. Other methods are inherited from the superclass.

E.13 **A sample definition.** For example, consider the method `sqrt` provided by Java's class `Math` to compute the square root of a real number. Since `sqrt` returns a `double` value, it computes the square root of only nonnegative numbers. If we give the method a negative number, it returns the special value `NaN`, which stands for "not a number." If displayed, this value appears as `NaN`. If it is involved in arithmetic, the result is `NaN`.

Imagine, instead, a square root method that requires a nonnegative argument. Let's treat the passing of a negative number to the method as a programming mistake and throw a runtime exception. The method certainly could throw an instance of `RuntimeException`, but a more specific exception would be better. So let's define our own class, `SquareRootException`, as shown in Listing E-1. Because we want a runtime exception, our class extends `RuntimeException`. Each of the two constructors for this class uses `super` to invoke `RuntimeException`'s constructor, passing it a string as a message. The default constructor passes a default message, but the second constructor passes the message provided as its argument when it is called. Most programmer-defined exception classes are as straightforward as `SquareRootException`.

LISTING E-1 The exception class `SquareRootException`

```
/**
    A class of runtime exceptions thrown when an attempt
    is made to find the square root of a negative number.
    @author Frank M. Carrano
*/
public class SquareRootException extends RuntimeException
{
   public SquareRootException()
   {
      super("Attempted square root of a negative number.");
   } // end default constructor

   public SquareRootException(String message)
   {
      super(message);
   } // end constructor
} // end SquareRootException
```

E.14 Note that SquareRootException's default constructor could use `this` instead of `super`, as follows:

```
public SquareRootException()
{
   this("Attempted square root of a negative number.");
} // end default constructor
```

By using `this`, the constructor calls the second constructor, which in turn calls RuntimeException's constructor. In contrast, the default constructor shown in Listing E-1 uses `super` to call RuntimeException's constructor directly. Although the version in Listing E-1 is more direct and might seem better, using `this` to link one constructor to another is usually preferable, as the technique tends to reduce mistakes.

E.15 **Using our own exception class.** We have imagined a square root method that throws a runtime exception when given a negative number as its argument. Now that we have an appropriate class of exceptions, let's define this method within a class of static methods, as shown in Listing E-2, that is much like the class `Math`. We have numbered the lines in this listing to make the output in the next listing easier for you to understand.

The header for the method `squareRoot` is similar to the header of `Math.sqrt` but also includes a `throws` clause to indicate that the method might throw a `SquareRootException`. Notice the tag `@throws` in the `javadoc` comment preceding the header. It provides documentation of the exception that might occur. If the method could throw more than one exception, we would include a separate `@throws` tag for each one and list the exceptions alphabetically by name. Since `SquareRootException` is a runtime exception, listing it in a `throws` clause and documenting it in `javadoc` are optional.

Within the body of the method `squareRoot` is a `throw` statement. It throws a `SquareRootException` if the method's argument is negative. If the argument is not negative, the method simply returns the square root as computed by the method `Math.sqrt`.

LISTING E-2 The class `OurMath` and its static method `squareRoot`

```
 1 /**
 2     A class of static methods to perform various mathematical
 3     computations, including the square root.
 4     @author Frank M. Carrano
 5 */
 6 public class OurMath
 7 {
 8    /** Computes the square root of a nonnegative real number.
 9        @param value  a real value whose square root is desired
10        @return the square root of the given value
11        @throws SquareRootException if value < 0 */
12    public static double squareRoot(double value)
13           throws SquareRootException
14    {
15       if (value < 0)
16          throw new SquareRootException();
17       else
18          return Math.sqrt(value);
19    } // end squareRoot

      < Other methods not relevant to this discussion are here. >

99 } // end OurMath
```

Programming Tip: Do not confuse the keywords `throw` and `throws`

You use the Java reserved word `throws` in a method's header to declare the exceptions that the method might throw. The reserved word `throw` is used within the body of a method to actually throw an exception.

Note: The `javadoc` tag `@throws`

A `javadoc` comment that precedes a method's header should contain a separate line for each exception the method might throw. Each of these lines begins with the tag `@throws`, and they should be ordered alphabetically by the names of the exceptions. All checked exceptions must be documented.

Documenting runtime exceptions is optional and is generally not done. However, a designer can document those runtime exceptions that a client might reasonably want to handle. In fact, you will encounter some documented runtime exceptions in the Java Class Library. Realize, however, that your use of a method might cause a runtime exception that is undocumented. If you decide to document runtime exceptions, they must not depend on how the method is defined. Thus, identifying the exceptions a method might throw should be done as part of its design and specification, and not its implementation.

E.16 A demonstration of the class `OurMath` is given in Listing E-3. Note the message displayed as a result of the exception. Also note that execution stops when the exception occurs.

LISTING E-3 A driver for the class `OurMath`

```java
/**
    A demonstration of a runtime exception using the class OurMath.
*/
public class OurMathDriver
{
   public static void main(String[] args)
   {
      System.out.print("The square root of 9 is ");
      System.out.println(OurMath.squareRoot(9.0));

      System.out.print("The square root of -9 is ");
      System.out.println(OurMath.squareRoot(-9.0));

      System.out.print("The square root of 16 is ");
      System.out.println(OurMath.squareRoot(16.0));
   } // end main
} // end OurMathDriver
```

Output

```
The square root of 9 is 3.0
The square root of -9 is Exception in thread "main" SquareRootException:
Attempted square root of a negative number.
         at OurMath.squareRoot(OurMath.java:16)
         at OurMathDriver.main(OurMathDriver.java:12)
```

E.17 Imagine that our class `OurMath` is widely available to other programmers. Joe wants to use our class to compute the square root, but he doesn't want to receive an error message when `squareRoot` encounters a negative argument. Instead, he wants the method to return a complex[1] number involving *i*, which is an abbreviation for the square root of –1. For example, the square root of –9 is 3*i*, because

$$\sqrt{-9} = \sqrt{9(-1)} = \sqrt{9}\sqrt{-1} = 3i$$

To accommodate results that involve *i* and those that do not, Joe has his method return a string. Thus, Joe envisions a method that would return the string `"3i"` as the square root of –9 and would return the string `"3"` as the square root of 9.

Joe's method will invoke `OurMath.squareRoot`. As this invocation must appear within a `try` block, Joe writes

```
String result = "";
try
{
   Double temp = OurMath.squareRoot(value);
   result = temp.toString();
}
```

All is fine, as long as `value` is not negative, but if it is negative, a `SquareRootException` is thrown. Instead of displaying an error message, as the driver in Listing E-3 does, Joe wants his method to return the correct value for a negative argument. He writes the following pseudocode to get his ideas on paper:

```
// assume value is negative
catch (SquareRootException e)
{
   Double temp = the square root of -value
   result = temp.toString() followed by "i"
}
```

Joe then translates this pseudocode into the following `catch` block:

```
catch (SquareRootException e)
{  // Assertion: value is negative
   Double temp = OurMath.squareRoot(-value);
   result = temp.toString() + "i";
}
```

E.18 Listing E-4 shows Joe's method `squareRoot` within his class `JoeMath`, and Listing E-5 provides a demonstration of the class.

LISTING E-4 The class `JoeMath`

```
/**
    A class of static methods to perform various mathematical
    computations, including the square root.
*/
```

[1] Only a rudimentary knowledge of complex numbers is needed here. Complex numbers build on real numbers by adding an imaginary part involving *i*. Every complex number has the form $a + b\,i$, where *a* and *b* are real numbers. To represent a real number in this notation, you would have *b* be zero.

```
public class JoeMath
{
   /** Computes the square root of a real number.
       @param value  a real value whose square root is desired
       @return a string containing the square root of the given value */
   public static String squareRoot(double value)
   {
      String result = "";
      try
      {
         Double temp = OurMath.squareRoot(value);
         result = temp.toString();
      }
      catch (SquareRootException e)
      {
         Double temp = OurMath.squareRoot(-value);
         result = temp.toString() + "i";
      }

      return result;
   } // end squareRoot

   < Other methods not relevant to this discussion could be here. >

} // end JoeMath
```

LISTING E-5 A driver for the class `JoeMath`

```
/**
    A demonstration of a runtime exception using the class JoeMath.
*/
public class JoeMathDriver
{
   public static void main(String[] args)
   {
      System.out.print("The square root of 9 is ");
      System.out.println(JoeMath.squareRoot(9.0));

      System.out.print("The square root of -9 is ");
      System.out.println(JoeMath.squareRoot(-9.0));

      System.out.print("The square root of 16 is ");
      System.out.println(JoeMath.squareRoot(16.0));

      System.out.print("The square root of -16 is ");
      System.out.println(JoeMath.squareRoot(-16.0));
    } // end main
} // end JoeMathDriver
```

Output

```
    The square root of 9 is 3.0
    The square root of -9 is 3.0i
    The square root of 16 is 4.0
    The square root of -16 is 4.0i
```

Inheritance and Exceptions

E.19 Imagine a class whose method `someMethod` has a `throws` clause in its header. Can we override `someMethod` in a subclass and list additional checked exceptions in its `throws` clause? No, Java will not let us; we will get a syntax error if we do.

For example, consider the following superclass and subclass:

```
public class SuperClass
{
   public void someMethod() throws Exception1
   { . . .
   } // end someMethod
} // end SuperClass
public class SubClass extends SuperClass
{
   public void someMethod() throws Exception1, Exception2 // ERROR!
   { . . .
   } // end someMethod
} // end SubClass
```

The `throws` clause in the overriding method will be flagged as syntactically incorrect. Let's think about why this is an error.

Suppose a program creates an instance of `SubClass`, assigns the object to a variable of `SuperClass`—let's call it `superObject`—and places the call `superObject.someMethod()` within a `try` block, as follows:

```
public class Driver
{
   public static void main(String[] args)
   {
      SuperClass superObject = new SubClass();
      try
      {
         superObject.someMethod();
      }
      catch (Exception1 e)
      {
         System.out.println(e.getMessage());
      }
   } // end main
} // end Driver
```

Since `superObject` references an instance of `SubClass`, `SubClass`'s version of `someMethod` is called. But since `superObject`'s static type is `SuperClass`, the compiler sees only `SuperClass`'s definition for someMethod. Thus, it checks only that Exception1 is caught. If the throws clause in SubClass was legal, we could call SubClass's someMethod without catching Exception2.

E.20 The rule governing which exceptions can appear in a throws clause of an overriding method is relaxed somewhat if the exceptions are related by inheritance. For example, if Exception2 extends Exception1, the following is legal:

```
public class SuperClass
{
   public void someMethod() throws Exception1
   { . . .
   } // end someMethod
} // end SuperClass

public class SubClass extends SuperClass
{
   public void someMethod() throws Exception2 // OK, assuming Exception2
   { . . .                                    //     extends Exception1
   } // end someMethod
} // end SubClass
```

Note: An overriding method in a subclass cannot list exceptions in a throws clause that aren't listed in a throws clause of the overridden method in the superclass, unless they are derived from the exception classes listed in the overridden method. However, an overriding method can list fewer exceptions in its throws clause or none at all.

The finally Block

E.21 If you have code that must execute regardless of whether an exception occurs, you could place it at the end of the try block and at the end of each catch block. An easier way to accomplish this, however, is to place one copy of the code in question within a finally block that follows the last catch block. Code within a finally block executes after either the try block or an executing catch block ends. Although optional, the finally block is a good way to provide cleanup services, such as closing a file or releasing system resources.

The following code shows the placement of the finally block:

```
try
{
   < Code that might throw an exception, either by executing a throw statement
     or by calling a method that throws an exception >
}
catch (AnException e)
{
   < Code that handles exceptions of type AnException or a subclass of AnException >
}

< Possibly other catch blocks to handle other types of exceptions >

finally
{
   < Code that executes after either the try block or an executing catch block ends >
}
```

Note: Statements within a `finally` block execute regardless of whether an exception occurs, but they do not execute if either the `try` block or a `catch` block calls `System.exit`. If no exception takes place, the `finally` block executes after its corresponding `try` block completes its execution. (If the `try` block contains a `return` statement, the `finally` block executes before the `return`.) However, if an exception occurs, and it is caught by one of the `catch` blocks, the `finally` block executes after that `catch` block executes.

E.22 **Example.** Imagine that you open the refrigerator door and reach for the milk. Whether you find milk or not, you should close the door. In the following code, the method `takeOutMilk` will throw an exception if no milk is found. Whether an exception occurs or not, `closeRefrigerator` is called within the `finally` block.

```
try
{
   openRefrigerator();
   takeOutMilk();
   pourMilk();
   putBackMilk();
}
catch (NoMilkException e)
{
   System.out.println(e.getMessage());
}
finally
{
   closeRefrigerator();
}
```

E.23 Let's explicitly demonstrate the behavior of a `finally` block by executing the code given in the previous example. The program in Listing E-6 provides simple definitions for the methods called in this example. All but `takeOutMilk` simply display an appropriate message. The method `takeOutMilk`, however, displays a message some of the time at random but throws a `NoMilkException` the rest of the time.

In the first sample output shown in Listing E-6, no exception occurs. Each method within the `try` block executes in turn, as the output indicates. Lastly, the method `closeRefrigerator` within the `finally` block executes. In the second sample output, `openRefrigerator` executes normally, but then `takeOutMilk` throws an exception. After the exception is caught by the `catch` block, the `finally` block executes as expected.

LISTING E-6 A demonstration of a `finally` block

```
/**
    Demonstrates the behavior of a finally block.
*/
public class GetMilk
{
   public static void main(String[] args)
   {
      try
      {
         openRefrigerator();
         takeOutMilk();
         pourMilk();
         putBackMilk();
      }
      catch (NoMilkException e)
      {
         System.out.println(e.getMessage());
      }
      finally
      {
         closeRefrigerator();
      }
   } // end main

   public static void openRefrigerator()
   {
      System.out.println("Open the refrigerator door.");
   } // end openRefrigerator

   public static void takeOutMilk() throws NoMilkException
   {
      if (Math.random() < 0.5)
         System.out.println("Take out the milk.");
      else
         throw new NoMilkException("Out of Milk!");
   } // end openRefrigerator

   < The methods pourMilk, putBackMilk, and closeRefrigerator are analogous to
     openRefrigerator and are here. >
   . . .
} // end GetMilk
```

Sample Output 1 (no exception is thrown)

```
Open the refrigerator door.
Take out the milk.
Pour the milk.
Put the milk back.
Close the refrigerator door.
```

Sample Output 2 (exception is thrown)

```
Open the refrigerator door.
Out of milk!
Close the refrigerator door.
```

E.24 **Nested try-finally blocks.** Some programmers find the try-catch-finally sequence confusing and prefer to place the finally block after a try block instead of after a catch block. Doing so, however, requires the try-finally blocks to be nested within another try block. For example, the program in Listing E-7 revises the try-catch-finally blocks shown in Segment E.22.

If takeOutMilk throws an exception, the finally block executes and then the exception propagates to the outer try block and is caught by the catch block. The output shown in Listing E-7 demonstrates this behavior. Thus, when an exception is thrown, the finally block executes before the exception is caught, whereas the reverse is true for the code shown in Listing E-6.

LISTING E-7 A demonstration of a nested finally block

```
/**
    Demonstrates the behavior of a nested finally block.
*/
public class GetMilk2
{
   public static void main(String[] args)
   {
      try
      {
         try
         {
            openRefrigerator();
            takeOutMilk();
            pourMilk();
            putBackMilk();
         }
         finally
         {
            closeRefrigerator();
         }
      }
      catch (NoMilkException e)
      {
         System.out.println(e.getMessage());
      }
   } // end main

   < The remaining methods are identical to those given in Listing E-6. >
   . . .
} // end GetMilk
```

Sample Output 1 (no exception is thrown) ← Same as the output shown in Listing E-6

```
Open the refrigerator door.
Take out the milk.
Pour the milk.
Put the milk back.
Close the refrigerator door.
```

Sample Output 2 (exception is thrown) ← Different from the output shown in Listing E-6

```
Open the refrigerator door.
Close the refrigerator door.
Out of milk!
```

Note: Despite our earlier advice about not nesting `try-catch` blocks, nesting `try-finally` blocks can be a reasonable choice. However, the unnested sequence of a `try` block followed by a `finally` block followed by one or more `catch` blocks is illegal.

Appendix F

File Input and Output

Contents

Prerequisites

Appendix A	Java Essentials
Appendix D	Designing Classes
Appendix E	Handling Exceptions

Input to a program can come from a keyboard or a mouse, and its output can be displayed on a screen. These forms of input and output occur in real time and are temporal. When the program ends, the input and output vanish.

However, program input can be taken from a file, and its output can be sent to a file. A **file** is simply data on a particular storage medium, such as a disk. In this appendix, we explain how you can read input from a file and send output to another file, and why you would want to do so.

Preliminaries

We begin by looking at some generalities about files.

Why Files?

F.1 You own files such as Java programs, songs, photos, and so on. Each of these files was created by a program. You might have run the program yourself—such as when you used a text editor to write a Java program—or obtained the result of someone else's program in the form of a song or picture. In any event, you clearly do not want the output of the program to disappear when program execution stops. You want the data to last, to be persistent. By "persistent," we mean "to last beyond program execution." The contents of a file last until a program changes them or they become damaged. We want to use our files—that is, we want to run our programs, listen to our music, and watch our videos. The reasons for creating files should be obvious.

Would you ever use a file as input to a program? You have done so if you have ever edited a photo or revised an essay you wrote yesterday. Java programs—like the ones we have written—can read data from an input file rather than the keyboard. Thus, files provide a convenient way to deal with large data sets.

Note: Reasons why a program creates a file

- A program creates and saves a file so it can be used over and over by other programs.
- A program creates a file for its own use as temporary storage for numerous intermediate results. The program writes data into the file and later reads the data, but does not save the file.

Streams

F.2 In Java, all input and output of data—including reading a file or writing a file—involves streams. A **stream** is an object that represents a flow of data. The data is a collection of eight-bit bytes representing numbers, characters, music, and so on. A stream either

- Sends data from your program to a destination, such as a file or the screen—in which case it is called an **output stream**—or
- Takes data from a source, such as a file or the keyboard, and delivers it to your program—in which case it is called an **input stream**

For example, the object `System.out` is an output stream that moves data from a program to a display. If an output stream is connected to a file, data will move from the program to the file. That is, the program will write the file. Likewise, `System.in` is an input stream that moves data from the keyboard to a program. If an input stream is connected to a file, the program will read data from the file.

Note: Input and output are done from the perspective of the program. Thus, "input" means that data moves *into* your program from an input device such as a disk or keyboard. The word "output" means that data moves *out of* your program to an output device such as a disk or the screen.

Note: Streams in Java are objects of certain classes in the Java Class Library. In particular, these classes are in the package `java.io`. Thus, to use these classes, you must import them from `java.io`.

Before you can read or write a file, you must connect the file to an appropriate stream and associate it with your Java program. You accomplish these steps, or **open** the file, when you create the stream by invoking the constructor of the stream's class. After you are finished with the file you must **close** it, or disconnect it from the stream, and hence from your program, by calling a particular stream method named `close`. We will examine the details of these steps shortly.

The Kinds of Files

F.3 All files are written as representations of ones and zeros, that is, binary digits, or bits. Java, however, treats files as either text files or binary files. A **text file** represents a collection of characters. The streams associated with text files provide methods that interpret the file's binary contents as characters. The files that contain your Java programs are most likely text files. A text editor can read a text file and make it appear to you as a sequence of characters. Any file other than a text file is called a **binary file**. For example, your song and picture files are binary files.

A Java program can create or read text files and binary files. The steps for processing—that is, writing or reading—a text file are analogous to those for a binary file. The kind of file determines which stream classes we use to perform the input or output.

Programming Tip: Choosing the kind of file

Your Java program should use a text file if you will use a text editor to

- Create files that the program will read
- Read or edit files that the program will create

If you will not use a text editor to create or read a file, consider using a binary file, as such files typically require less disk space than text files.

File Names

F.4 Although Java does not specify the characters that can make up a file name, your operating system does. Typically, you use letters, digits, and a dot in the name of a data file, ending it with a suffix, such as `.txt`. This suffix is simply a convention, not a Java rule, although your operating system can give it meaning. This book uses the suffix `.txt` to indicate a text file.

Within a Java program, the name of a file is a string. Although our examples will use a `String` constant as a file name, our programs could have asked the user for the file's name and stored it in a `String` variable.

Text Files

We begin by exploring text files.

Creating a Text File

F.5 **The contents of a text file.** A text file contains a sequence of characters in which each character is represented by the system's default encoding. Java uses the Unicode character set, which includes many letters in natural languages that are quite different from English. The Unicode representation of each character requires two bytes. Many text editors, operating systems, and programming languages other than Java use the ASCII character set. The ASCII character set is a subset of the Unicode character set and contains the characters normally used for English and typical Java programs. The representation of each character in ASCII requires one byte.

A typical text file is organized as lines, each ending with a special end-of-line character. The lines in a text file are analogous to the lines you see when a program displays its output. In reality, however, the file is a sequence of data. For this reason, we say that a text file offers **sequential access** to its contents. That is, before you can read the n^{th} line of the file, you start at the file's beginning and read through its first $n - 1$ lines.

F.6 **Opening a text file for output.** The standard class `PrintWriter`, which is in the package `java.io` of the Java Class Library, has the familiar methods `print` and `println`. As you will see, these methods behave like `System.out.print` and `System.out.println`. Thus, we will use this class to create an output stream for a text file.

Before you can write to a text file, you must open it by writing a statement like

```
PrintWriter toFile = new PrintWriter(fileName);
```

The call to `PrintWriter`'s constructor creates an output stream and connects it to the file named by the `String` variable `fileName`. The **stream variable** `toFile` references this stream. Once the stream is created, your program always refers to the file by using the stream variable instead of its actual file name.

When you connect a file to an output stream in this way, your program always starts with an empty text file. If the file named by `fileName` did not exist before the constructor was called, a new, empty file is created and given that name. However, if the file named by `fileName` already exists, its old contents will be lost.

F.7 `PrintWriter`'s constructor can throw a checked exception—`FileNotFoundException`—while attempting to open a file. For this reason, its invocation must appear within either a `try` block that is followed by an appropriate `catch` block or a method whose header lists this exception in a `throws` clause. For example, we could write the following statements to open the text file `data.txt` for output:

```
String fileName = "data.txt";
PrintWriter toFile = null;
try
{
   toFile = new PrintWriter(fileName);
}
catch (FileNotFoundException e)
{
   System.out.println("PrintWriter error opening the file " + fileName);
   System.out.println(e.getMessage());
   < Possibly other statements to deal with this exception. >
}
```

Notice that we declared the variable `toFile` outside of the `try` block so that `toFile` is available outside of this block. Remember, anything declared in a block—even a `try` block—is local to the block. Declaring `toFile` in this way enables us to use it to write to the file, as you will see.

A `FileNotFoundException` does not necessarily mean that the file was not found. After all, if you are creating a new file, it doesn't already exist. In that case, an exception is thrown if the file could not be created because, for example, the file name is already used as a folder (directory) name.

Note: A `FileNotFoundException` will occur if a file cannot be opened for output, either because it does not exist and cannot be created or because it is inaccessible.

F.8 **Writing a text file.** The methods `println` and `print` of the class `PrintWriter` work the same for writing to a text file as the respective methods `System.out.println` and `System.out.print` work for writing to the screen. Thus, when a program writes a value to a text file, the number of characters written is the same as if it had written the value to the screen. For example, writing the `int` value 12345 to a text file places five characters in the file. In general, writing an integer of type `int` places between 1 and 11 characters in a text file.

The following statements write four lines to the text file we created in the previous segment:

```
for (int counter = 1; counter <= 4; counter ++)
   toFile.println("Line " + counter);
```

Notice that we use the stream variable `toFile` when calling `println`, not the actual name of the file. Additionally, we did not place the call to `println` within a `try` block, as `println` does not throw any checked exceptions. The same is true of `print`.

Note: `System.out` is an object of the standard class `PrintStream`. Because both `PrintStream` and `PrintWriter` define the same `print` and `println` methods, objects of `PrintWriter` have the same `print` and `println` methods as `System.out`. However, `System.out` directs its output stream to a different destination than `PrintWriter` objects do.

F.9 **Buffering.** Instead of sending output to a file immediately, `PrintWriter` waits to send a larger packet of data. Thus, the output from `println`, for example, is not sent to the output file right away. Instead, it is saved and placed into a portion of memory called a **buffer**, along with the output from other invocations of `print` and `println`. When the buffer is too full to accept more output, its contents are written to the file. Thus, the output from several `print` and `println` statements is written at the same time, instead of each time one statement executes. This technique is called **buffering**, and it saves execution time.

F.10 **Closing a text file.** When we are finished using a file, we must disconnect it from the stream. Every stream class, including `PrintWriter`, has a method named `close` to accomplish this task. Thus, to close the file associated with the stream variable `toFile`, we write

```
tofile.close();
```

When closing a file, the system writes any data still left in the buffer to the file and releases any resources it used to connect the stream to the file. Note that `close` will not throw an exception.

Programming Tip:

If you do not close a file, Java will close it for you, but only if your program ends normally. To avoid any possible loss of data or damage to a file, you should close it as soon as possible after you are finished using it.

F.11 **Flushing an output text file.** You can force any pending output that is currently in a buffer to be written to its destination file by calling `PrintWriter`'s method `flush`, as follows:

```
toFile.flush();
```

The method `close` automatically calls the method `flush`, so for most simple applications, you do not need to call `flush` explicitly. However, if you continue to program, you will eventually encounter situations in which you will have to use `flush`.

F.12 **Example: Creating a text file of user data.** Let's create a text file of data that a user enters at the keyboard. We'll provide this operation as a static method within a class that reads and writes text files. If the method accepts as arguments the file name and number of lines that the user will enter, its header could be as follows:

```
public static boolean createTextFile(String fileName, int howMany)
```

The only checked exception that can occur is during the opening of the file, so our method can return a boolean value to indicate whether the operation succeeds.

Listing F-1 shows the definition of this method within a class `TextFileOperations`.

LISTING F-1 The static method `createTextFile` in the class `TextFileOperations`

```
import java.io.FileNotFoundException;
import java.io.PrintWriter;
import java.util.Scanner;
public class TextFileOperations
{
   /** Writes a given number of lines to the named text file.
       @param fileName  the file name as a string
       @param howMany  the positive number of lines to be written
       @return true if the operation is successful */
   public static boolean createTextFile(String fileName, int howMany)
   {
      boolean fileOpened = true;
      PrintWriter toFile = null;
      try
      {
         toFile = new PrintWriter(fileName);
      }
      catch (FileNotFoundException e)
      {
         fileOpened = false; // error opening the file
      }

      if (fileOpened)
      {
         Scanner keyboard = new Scanner(System.in);
         System.out.println("Enter " + howMany + " lines of data:");
         for (int counter = 1; counter <= howMany; counter++)
         {
            System.out.print("Line " + counter + ": ");
            String line = keyboard.nextLine();
            toFile.println(line);
         } // end for

         toFile.close();
      } // end if
      return fileOpened;
   } // end createTextFile
} // end TextFileOperations
```

F.13 **Appending to a text file: Getting ready.** When you add a line to the end of an existing text file, you **append** it to the file. Since we actually will write a new line to the file, we want to use the class `PrintWriter`, as we did to create the file. When we consult the documentation for `PrintWriter`, however, we find no constructor that will open an existing file and append data to it. As we mentioned before, when you open a file for output, you ordinarily lose any existing data.

Although `PrintWriter` does not have an appropriate constructor, it is still the class we want to use to write the file. What we need is another class to help us open the file in the way we want. Such a class is `FileWriter`, which is in the package `java.io` of the Java Class Library. The appropriate `FileWriter` constructor is declared as follows:

```
public FileWriter(String fileName, boolean append)
```

This constructor opens the text file named by the `String` variable `fileName` for output. If the argument `append` is true, any data written to the file will be appended to its end. Otherwise, if `append` is false, any existing data in the file is lost.

Conveniently, `PrintWriter` has a constructor that accepts a `FileWriter` object as its argument. Thus, we can use `FileWriter` to open the text file so that it will accept additional output, and then use `PrintWriter` to provide methods such as `print` and `println` to write the output. Note that `FileWriter` does not have such methods.

F.14 **Appending to a text file: Writing new lines.** Most of `FileWriter`'s constructors, including the one given in the previous segment, can throw an `IOException` if they cannot open the designated file. The reasons for such an occurrence are the same as we encountered before: Either no such file exists and one cannot be created; `fileName` names an existing text file, but it cannot be opened; or `fileName` is actually the name of a folder instead of a text file.

Since the constructors of both `FileWriter` and `PrintWriter` can throw an exception, we invoke them within a `try` block. For example, to append another line to the existing text file `CollegeFile.txt`, we could open it as follows:

```
try
{
   FileWriter fw = new FileWriter(fileName, true);// IOException?
   toFile = new PrintWriter(fw);                  // FileNotFoundException?
}
catch (FileNotFoundException e)
{
   System.out.println("PrintWriter error opening the file " + fileName);
   System.out.println(e.getMessage());
   System.exit(0);
}
catch (IOException e)
{
   System.out.println("FileWriter error opening the file " + fileName);
   System.out.println(e.getMessage());
   System.exit(0);
}
```

We now can add one or more lines of data to the end of the file by using statements of the form

```
toFile.println(. . .);
```

Note: Why do we need PrintWriter? Isn't FileWriter enough?
Although the class FileWriter has the constructor we need to append to a file, it provides only basic support for text files. The class PrintWriter lacks the necessary constructor, but it has useful methods such as println. Using both classes provides an appropriate constructor and convenient methods.

Reading a Text File

You know how to use the class Scanner to read data from the keyboard. We can use this same class to read data from a text file. Let's see how. Realize, however, that Scanner is not a stream class.

F.15 **Opening a text file for input.** Previously, we invoked PrintWriter's constructor to open a text file for output. In an analogous way, we might invoke Scanner's constructor to open a text file for input. You might guess that we would accomplish this step by using a statement such as

```
Scanner fromFile = new Scanner(fileName);// does NOT open a text file
```

where fileName is a String variable representing the file's name. Unfortunately, this statement has nothing to do with any text file! Instead, the Scanner object fromFile extracts portions of the string fileName, as Appendix A describes, beginning with Segment A.81.

Note: A Scanner object is not a stream
We will use Scanner to get data from a text file, but Scanner is not a stream class. In fact, it belongs to the package java.util, not java.io. According to the constructor we use to create it, a Scanner object can process a string

- Entered at the keyboard or
- Read from a text file or
- Given to its constructor as an argument

The Scanner objects we will use to read a text file actually contain a stream object that reads the file. The Scanner methods then translate, or parse, the input string, convert the data to the desired data type, and pass it to our program.

Scanner has several constructors, one of which takes an instance of the standard class File. This latter class, which is in the package java.io, represents a file in a system-independent and abstract way. Moreover, one of File's constructors accepts a string that is the file's name as its argument. Thus, to access the file whose name is referenced by the String variable fileName, we would write

```
Scanner fileData = new Scanner(new File(fileName));
```

Although the Scanner object fileData is not a stream object, it creates an input stream, thus opening the file for input. We chose the name fileData as a reminder that it is not a stream variable.

Unlike other constructors of Scanner that we use when either reading from the keyboard or processing a string, this new constructor can throw a FileNotFoundException. Thus, its invocation must appear within either a try block that is followed by an appropriate catch block or a method whose header lists this exception in a `throws` clause. For example, the following statements ultimately open the text file named `data.txt` for input:

```
String fileName = "data.txt";
Scanner fileData = null;
try
{
   // can throw FileNotFoundException
   fileData = new Scanner(new File(fileName));
}
catch (FileNotFoundException e)
{
   System.out.println("Scanner error opening the file " + fileName);
   System.out.println(e.getMessage());
   < Possibly other statements that react to this exception. >
}
```

Note: The `Scanner` constructor, whose parameter is a `File` object, will throw a `FileNotFoundException` if it cannot open a file for input because the file either does not exist or is inaccessible.

Note: Input from a text file

Opening a text file for input enables you to read data from it sequentially, starting from the file's beginning. Note that other standard classes exist that let you open a text file directly, rather than indirectly by using `Scanner`. However, `Scanner` offers you a convenient way to read from a text file.

F.16 **Reading a text file.** All of `Scanner`'s methods are available to you when reading a text file. If you do not know the format of the data in the file, you can use the `Scanner` method `nextLine` to read it line by line. For example, the following statements read and display the lines in the existing text file `data.txt`:

```
while (fileData.hasNextLine())
{
   String line = fileData.nextLine();
   System.out.println(line);
} // end while
```

If `nextLine` were to read beyond the end of a file, it would throw a runtime (unchecked) exception, `NoSuchElementException`. We use `Scanner`'s method `hasNextLine` to prevent `nextLine` from reading beyond the end of the file. Thus, the `while` loop ends when the end of the file is reached.

F.17 **Example.** Listing F-2 adds the static method `displayFile` to our previous class `TextFileOperations`, as shown in Listing F-1. Notice the previous code fragments in the context of this method. After the text file is read, we use the `Scanner` method `close` to indirectly close the file. Also, notice that the `try` block here opens, reads, and closes the file, whereas the `try` block in Listing F-1 only opens the file. However, that code requires an `if` statement to skip further file processing if the file cannot be opened.

LISTING F-2 The static method `displayFile` in the class `TextFileOperations`

```
import java.io.File;
import java.io.FileNotFoundException;
import java.io.PrintWriter;
import java.util.Scanner;
/**
    A class of methods that create and display a text file of
    user-supplied data.
    @author Frank M. Carrano
*/
public class TextFileOperations
{
   < The method createTextFile, as given in Listing F-1, appears here. >

   /** Displays all lines in the named text file.
       @param fileName  the file name as a string
       @return true if the operation is successful */
   public static boolean displayFile(String fileName)
   {
      boolean fileOpened = true;
      try
      {
         Scanner fileData = new Scanner(new File(fileName));
         System.out.println("The file " + fileName +
                            " contains the following lines:");
         while (fileData.hasNextLine())
         {
            String line = fileData.nextLine();
            System.out.println(line);
         } // end while
         fileData.close();
      }
      catch (FileNotFoundException e)
      {
         fileOpened = false; // error opening the file
      }

      return fileOpened;
   } // end displayFile
} // end TextFileOperations
```

Note: Other methods in Scanner

The previous example simply reads and displays entire lines of a text file. However, `Scanner` has other convenient methods that read data in various formats. Segment A.33 of Appendix A mentioned `nextInt` and `nextDouble` in conjunction with reading from the keyboard. You can use these and other similar methods to read data from a text file as well. Note that none of `Scanner`'s methods throw checked exceptions, and so they need not appear within a `try` block. However, `Scanner`'s constructor can throw a checked exception.

Programming Tip: When you read a value at the end of a line using any `Scanner` method other than `nextLine`, the character or characters marking the end of the line remain as the next to be read. Only `nextLine` reads past these characters. So regardless of whether you read from the keyboard or from a text file using `Scanner`, you must read beyond these ending characters—by using `nextLine`—before trying to read the next string.

Note: The class File

Segment F.15 introduced the standard class `File`, whose constructor accepts as its argument a string that is a file's name. Because `Scanner` has no constructor that accepts a file name as an argument, but `File` does, we can open a text file for input by using a statement such as

```
Scanner fileData = new Scanner(new File(fileName));
```

This `Scanner` constructor accepts a `File` object as its argument and instantiates an input stream connected to the file. Although this stream belongs to the `Scanner` object `fileData` and is hidden from us, we can use `fileData` to read from the file.

The class `File` is useful in other ways as well. For example, you can

- Test whether a file exists, using the method `exists`
- Test whether a file exists and can be read, using the method `canRead`
- Test whether a file exists and can be written, using the method `canWrite`
- Change the name of a file, using the method `renameTo`
- Delete a file, using the method `delete`

The next segment will use `File` to perform some of these tasks. For more details, you should consult the online documentation for this class in the Java Class Library.

Note: Reading a file more than once

Reading a file can be time consuming, particularly when it is quite large. Therefore, you should not read a file more than once, if at all possible. Some algorithms, however, do require you to process data more than once. Doing so is not a problem when the data is in memory, but accommodating the data from a large file in an array, for example, might be impossible. In those cases, you might want to read the file several times.

After the file is read completely, you close it by closing the `Scanner` object. You then open it again so that you can read the file from its beginning.

Note: Sorting the data in a file

Searching a data set for a particular entry is a common task. As Chapter 18 shows, searching sorted data can be faster than searching unsorted data. Chapters 8 and 9 discuss sorting the entries in an array. When your data is in a file, however, it might not fit entirely into an array: The file is simply too large for the computer's memory to accommodate it.

The merge sort, which Chapter 9 presents, can be adapted to sort the data in a file. Although the details of this modification are beyond our scope, its basic idea is not hard to understand. The sort reads a portion of the file into an array, sorts the array, and writes the sorted data to another file. This process is repeated for successive portions of the original file until all data has been processed. In the next phase, pairs of sorted blocks of data are read from the new file and merged into a larger block, which is then written to the original file. This merging continues until the second file is processed and is then repeated on the original file, and so on until all the data has been sorted. Note that merging involves small portions of sorted data at a time, so that only a small amount of memory is needed to sort a large file.

Changing Existing Data in a Text File

F.18 Once we have created a text file, some of its data might need to be updated or corrected. Although we can add lines at the end of an existing text file, we cannot add them anywhere else. Moreover, we cannot delete lines, and in general, we cannot modify lines. However, as we read a text file, we can copy lines unchanged to another text file, skip lines that we no longer want to keep in the file, and write additional or modified lines to the new file. Changing the data in a file, therefore, is like reading one file and writing another. Most of the logic in a program that makes the changes must decide where and what changes are needed.

When the new file is complete, we can delete the old file and—if we like—give the new file the name of the old file. Although we can use the operating system to rename or delete a file after the program has run, our Java program can perform these tasks for us by using the class `File`. For example, to change the name of the existing file `MyData.txt`, we could write statements such as

```
File originalFile = new File("MyData.txt");
File newFile = new File("MyUpdatedData.txt");
originalFile.renameTo(newFile);
```

The `File` method `renameTo` changes the name of the file and returns a boolean value to indicate whether the change was successful. Although the previous example ignores this returned value, we could have written something like

```
if (originalFile.renameTo(newFile))
   System.out.println("Name change successful.");
else
   System.out.println("Attempted name change unsuccessful.");
```

Note that `renameTo` has a `File` object as its argument instead of a string containing the new name.

The `File` method `delete` deletes an existing file and returns either true or false to indicate whether the operation was successful. For example, to delete the previous file, we could write

```
originalFile.delete();
```

Programming Tip: Avoid a silent program

Without the `println` statements in the previous segment, the user would not know whether the name of the file was changed. Worse than this code is an entire program that processes a file without any visible output. Such a program is called a **silent program** and can bewilder its user. Did the program actually do anything? Always provide at least one message to the user to indicate the program's status.

Defining a Method to Open a Stream

F.19 Imagine that we want to write a method that opens a file. We will open a text file for output, but the idea is also applicable to opening it for input and to a binary file. Our method has a `String` parameter that represents the file name. The client of this method could either read the file name from the user or use a literal for the file name. The method that follows creates an output stream, connects it to the given file, and returns the stream to the client.

```
public static PrintWriter openOutputTextFile(String fileName)
                          throws FileNotFoundException, IOException
{
   PrintWriter toFile = new PrintWriter(fileName);
   return toFile;
} // end openOutputTextFile
```

We could invoke this method as follows:

```
PrintWriter toFile = null;
try
{
   PrintWriter toFile = openOutputTextFile("data.txt");
}
< appropriate catch blocks are here >
```

and go on to use `toFile` to write to the file.

What if we had written the method as a void method, so that instead of returning an output stream, it had the stream as a parameter? The following method looks reasonable, but it has a problem:

```
// This method does not do what we want it to do.
public static void openFile(String fileName, PrintWriter stream)
                   throws FileNotFoundException, IOException
{
   stream = new PrintWriter(fileName);
} // end openFile
```

Let's consider, for example, the following statements that invoke the method:

```
PrintWriter toFile = null;
try
{
   openFile("data.txt", toFile);
}
```

After this code is executed, the value of `toFile` is still `null`. The file that was opened by the method `openFile` went away when the method ended. The problem has to do with how Java handles arguments of a class type. These arguments are passed to the method as a memory address that cannot be changed. The object at the memory address normally can be changed, but the memory address itself cannot be changed. Thus, you cannot change `toFile`.

This observation applies only to arguments of methods. If the stream variable is either a data field or declared locally within the body of the method, you can open a file and connect it to the stream and this problem will not occur. Once a stream is connected to a file, however, you can pass the stream variable as an argument to a method, and the method can change the file.

Binary Files

F.20 Text files and binary files share some similarities. For example, before you can read or write any file, you open it, and when you are finished, you close it. The standard classes used for text files, however, differ from those we will use here for binary files, with one exception—the class `File`. This class, which is introduced in Segment F.15, can be used with binary files as well as text files.

Anything that you can write to a text file can be written to a binary file. Binary files store a value of a primitive type in the same format and the same number of bytes as it is stored in the computer's primary memory. For example, every `int` value occupies four bytes, and every `double` value requires eight bytes. This is one reason why a binary file can be more efficient in its use of time and space than a text file.

The most commonly used stream classes for processing binary files are `DataInputStream` and `DataOutputStream`. Each class has methods to read or write data one byte at a time. These streams can also convert numbers and characters to bytes that can be stored in a binary file. They allow your program to write data to or read data from a binary file as if the data were not just bytes but either strings or items of any of Java's primitive data types. If you do not need to access your files via an editor, the easiest and most efficient way to read and write data to files is to use `DataInputStream` and `DataOutputStream` with a binary file.

Binary files also offer features not possible with a text file. For example, you can write entire objects to a binary file, without knowledge of their data, and read them back again. In contrast, writing an object to a text file requires you to write each of its data components. This task becomes even more complicated when a data field is itself an object.

While some binary files are read only via sequential access, just like text files, certain binary files offer **random access**. That is, you can access their data without having to first read all the data preceding it, as you would with a sequential-access file. In other words, you can access the data at position *n* much as you can access the data in an array at index *n*. However, we will not cover random-access files in this book.

In this appendix, we will use the suffix `.bin` when naming a sequential-access binary file.

Note: You can use a Java program to create a binary file on one computer and have it read by a Java program on another computer. Normally, you cannot use a text editor to read binary files.

Creating a Binary File of Primitive Data

Would you ever create or use a binary file of primitive data? Yes; files of statistical or scientific data, such as the hourly outdoor temperatures for a year in a certain location, are common. Let's begin by writing primitive data to a sequential-access binary file. When a binary file will contain only data of a primitive type, you typically will use the standard class `DataOutputStream` to create it.

F.21 **Opening a binary file for output.** To write primitive data to a binary file, we'll use a stream of the class `DataOutputStream`. Like most of the classes we use for files, this one belongs to the package `java.io` of the Java Class Library. To open a binary file for output, we invoke the constructor of `DataOutputStream`. Like some of the classes we encountered earlier for text files, `DataOutputStream` has no constructor that accepts a file name as an argument. However, it does have a constructor whose argument can be an object of the class `FileOutputStream`, and a constructor of `FileOutputStream` does accept a file name as an argument. Thus, if the name of a binary file is in the `String` variable `fileName`, we can open the file for output by writing statements such as

```
FileOutputStream fos = new FileOutputStream(fileName);
DataOutputStream toFile = new DataOutputStream(fos);
```

These statements create an output stream and connect it to the named file. The stream variable `toFile` references this stream. As is true of text files, when you open a binary file for output, your program always starts with an empty file. If the named file does not already exist, a new, empty file is created. However, if the named file already exists, its old contents will be lost.

F.22 **Example.** `FileOutputStream`'s constructor will throw a `FileNotFoundException` if the file either does not exist and cannot be created or exists but cannot be opened. Since this exception is a checked exception, the constructor's invocation must appear within either a `try` block that is followed by an appropriate `catch` block or a method whose header lists this exception in a `throws` clause. Note that `DataOutputStream`'s constructor does not throw any exceptions. For example, we could write the following statements to open the binary file `data.bin` for output:

```
String fileName = "data.bin";
DataOutputStream toFile = null;
try
{
   FileOutputStream fos = new FileOutputStream(fileName);
   toFile = new DataOutputStream(fos);
}
catch (FileNotFoundException e)
{
   System.out.println("Cannot find, create, or open the file " + fileName);
   System.out.println(e.getMessage());
   System.exit(0);
}
```

We declare the variable `toFile` outside of the `try` block so that `toFile` is available outside of the `try` block.

F.23 **Using the class `File` when opening a binary file.** Earlier, in Segment F.15, we introduced the class `File`. Recall that you can use methods from this class to test whether a file exists, can be read, or can be written. We can use `File` to make the same tests of a binary file. If a particular binary file exists and can be written, for example, we can use the `File` object that we created for the tests as the argument for a constructor of `FileOutputStream`. Thus, we might modify the previous statements as follows:

```
< Declarations of and assignments to fileName and toFile >
boolean okToWrite = true;
try
{
   File aFile = new File(fileName);
   if (aFile.canWrite())
   {
      FileOutputStream fos = new FileOutputStream(aFile);
      toFile = new DataOutputStream(fos);
   }
   else
      okToWrite = false;
}
< catch block is here >
< Statements that test okToWrite and act accordingly >
```

F.24 **Writing primitive data to a binary file.** `DataOutputStream` has an output method for each primitive data type, as follows:

- `writeByte`—Writes the low-order 8 bits of its `int` argument
- `writeChar`—Writes the low-order 16 bits of its `int` argument as a Unicode character
- `writeShort`—Writes the low-order 16-bit integer of its `int` argument
- `writeInt`—Writes the 32-bit integer in its `int` argument
- `writeLong`—Writes the 64-bit integer in its `long` argument
- `writeFloat`—Writes the 32-bit real value in its `float` argument
- `writeDouble`—Writes the 64-bit real value in its `double` argument
- `writeBoolean`—Writes the boolean value in its `boolean` argument

Each method is a void method, has one parameter, and can throw an `IOException` in case an error occurs while writing a value to the file. Note that each of the methods `writeByte`, `writeChar`, `writeShort`, and `writeInt` has an `int` parameter, even though the first three write a smaller value to the file.

F.25 **Closing a binary file.** You use the method `close` to close a binary file after writing it. Unlike the `close` method for a text file, this method can throw an `IOException` if an error occurs while closing the file.

F.26 **Appending to an existing binary file.** To add a program's output to the end of the data already in an existing binary file—that is, to append data to the file—you would revise the statements given in Segment F.21 that open the file by calling `FileOutputStream`'s constructor as follows:

```
FileOutputStream fos = new FileOutputStream(fileName, true);
```

The second parameter of the constructor is a boolean value that indicates whether to append data to an existing file. If the named file does not exist already, Java will create an empty file of that name and write the output to this empty file. However, if the file does exist, the program's output will be placed after the old contents of the file.

F.27 **Example: Creating a binary file of random integers.** Let's define a class of static methods that deal with binary files. One of these methods can create a file of random integers. Suppose that the client of our method passes it the name of the file and the desired number of random values the file should contain. The method's header then can have the following form:

```
public static returnType createBinaryFile(String fileName, int howMany)
```

As we noted previously in this appendix, we must concern ourselves with checked exceptions when we create this file. We must either handle the exceptions within the body of the method or add a `throws` clause to the method's header.

Design Decision: Should the method handle the exceptions or pass them on to its client?

If we pass the exceptions on to the method's client, our method will be simpler to write and can be a void method. However, the client will have to worry about the exceptions. Let's handle the exceptions ourselves but tell the client what has happened by returning a code. The resulting method definition will be more involved, but its use will be convenient.

Listing F-3 shows the definition of the static method `createBinaryFile` within a class `BinaryFileOperations`. Notice how we close the file within a `finally` block, which Appendix E introduced. Its use not only ensures that the file is closed but also allows us to have two separate `catch` blocks for an `IOException`. In this way, we can tell whether an `IOException` is caused by the method `writeInt` or the method `close`.

LISTING F-3 The static method `createRandomIntegers` in the class `BinaryFileOperations`

```
import java.io.DataOutputStream;
import java.io.FileNotFoundException;
import java.io.FileOutputStream;
import java.io.IOException;
import java.util.Random;

public class BinaryFileOperations
```

```
{
   /** Writes a given number of random integers to the named binary
       file.
       @param fileName  the file name as a string
       @param howMany  the positive number of integers to be written
       @return an integer code indicating the outcome of the operation
   */
   public static int createBinaryFile(String fileName, int howMany)
   {
      int resultCode = 0;
      Random generator = new Random();
      DataOutputStream toFile = null;
      try
      {
         FileOutputStream fos = new FileOutputStream(fileName);
         toFile = new DataOutputStream(fos);

         for (int counter = 0; counter < howMany; counter++)
         {
            toFile.writeInt(generator.nextInt());
         } // end for
      }
      catch (FileNotFoundException e)
      {
         resultCode = 1;     // error opening file
      }
      catch (IOException e)
      {
         resultCode = 2;     // error writing file
      }
      finally
      {
         try
         {
            if (toFile != null)
               toFile.close();
         }
         catch (IOException e)
         {
            resultCode = 3; // error closing file
         }

         return resultCode;
      } // end finally
   } // end createBinaryFile
} // end BinaryFileOperations
```

Reading a Binary File of Primitive Data

F.28 Once we have created a binary file of primitive values by using an output stream of the class `DataOutputStream`, we will use the class `DataInputStream` to read the file. The details of opening and closing this file for input are like the ones we saw earlier. For each method in `DataOutputStream` that we used to write a primitive value to the file, `DataInputStream` has an analogous method to read the value from the file. Those methods are as follows:

- `readByte`—Reads and returns the next byte in the file as a `byte` value
- `readChar`—Reads and returns the next two bytes in the file as a `char` value
- `readShort`—Reads and returns the next two bytes in the file as a `short` value
- `readInt`—Reads and returns the next four bytes in the file as an `int` value
- `readLong`—Reads and returns the next eight bytes in the file as a `long` value
- `readFloat`—Reads and returns the next four bytes in the file as a `float` value
- `readDouble`—Reads and returns the next eight bytes in the file as a `double` value
- `readBoolean`—Reads and returns the next `boolean` value in the file

Each of these methods can throw either an `EOFException` when the end of the file is encountered or an `IOException` if an error occurs during the read operation. These checked exceptions must be handled.

F.29 **Example: Reading a binary file of integers.** Listing F-4 shows the definition of the static method `displayBinaryFile` within the class `BinaryFileOperations`. The method is similar to `createBinaryFile`, as given in Listing F-3, with respect to its return value and how it opens the file, closes the file, and handles exceptions. This method has only one parameter, the file name as a string, and displays all of the integers in the file. Notice that `createBinaryFile` did not write a sentinel after it wrote the last integer to the file.

LISTING F-4 The static method `displayBinaryFile` in the class `BinaryFileOperations`

```
import java.io.DataInputStream;
import java.io.DataOutputStream;
import java.io.EOFException;
import java.io.FileInputStream;
import java.io.FileNotFoundException;
import java.io.FileOutputStream;
import java.io.IOException;
import java.util.Random;

/**
    A class of methods that create and display a binary file of
    random integers.
    @author Frank M. Carrano
*/
public class BinaryFileOperations
{
   < The method createBinaryFile, as given in Listing F-3, appears here. >

   /** Displays all integers in the named binary file.
       @param fileName  the file name as a string
       @return an integer code indicating the outcome of the operation
   */
   public static int displayBinaryFile(String fileName)
   {
      int resultCode = 0;
      DataInputStream fromFile = null;
      try
      {
         FileInputStream fis = new FileInputStream(fileName);
         fromFile = new DataInputStream(fis);

         while (true)
         {
            int number = fromFile.readInt();
            System.out.println(number);
         } // end while
      }
      catch (FileNotFoundException e)
      {
         resultCode = 1;     // error opening file
      }
      catch (EOFException e)
      {
         // normal occurrence since entire file is read
      }
      catch (IOException e)
      {
         resultCode = 2;     // error reading file
      }
      finally
      {
         try
         {
            if (fromFile != null)
               fromFile.close();
         }
         catch (IOException e)
         {
            resultCode = 3; // error closing file
         }

         return resultCode;
      } // end finally
   } // end displayBinaryFile
} // end BinaryFileOperations
```

When you read a binary file of primitive values, you can detect the end of the file without checking for a sentinel at its end. You do so by catching the `EOFException` that the read methods throw when the end of the file is reached. You treat the exception as an expected occurrence, not as a mistake.

For example, the method `displayBinaryFile` contains an infinite loop to read all of the integers in the file. The entire loop—highlighted in the listing—is within a `try` block, so we can handle the exceptions. `DataInputStream`'s method `readInt` can throw either an `EOFException` when it attempts to read beyond the end of the file or an `IOException` if an error occurs during a read operation. Since the end of the file is an expected and normal occurrence in this example, one of the `catch` blocks after the previous `try` block is

```
catch (EOFException e)
{
   // normal occurrence since the entire file is read; ignore exception
}
```

Thus, we catch but ignore an `EOFException`.

Strings in a Binary File

Although strings are objects, you can use `DataOutputStream` to write one to a binary file as a sequence of characters and `DataInputStream` to read these characters from the file as a string.

F.30 **Writing strings to a binary file.** `DataOutputStream` has another method, `writeUTF`, that writes a string to a file as a sequence of characters in a machine-independent way:

- `writeUTF`—Writes the characters in its `String` argument using an encoding called **UTF**, or **Unicode Transformation Format**

Like the other methods of `DataOutputStream` that we introduced previously, `writeUTF` is a void method, has one parameter, and can throw an `IOException`.

Note: What is UTF?

Recall that Unicode is a representation of characters by unique integers. However, Unicode does not specify how many bytes one should use to store each integer in memory. UTF makes this specification. Several UTF encoding schemes exist. **UTF-8** represents each character as a sequence of between one and four bytes. UTF-8 uses one byte for each ASCII character, which is sufficient for the characters in the English language. Unicode, however, is more universal and provides many more characters. For such characters, UTF-8 uses two, three, or four bytes each. The method `writeUTF` uses a slight variation of UTF-8, known as modified UTF-8. Thus, the method `writeUTF` can represent all Unicode characters, but it saves file space when ASCII characters are written.

UTF-16 uses either two or four bytes to represent a character. Java uses UTF-16, but a `char` value occupies only two bytes, and the method `writeChar` writes two bytes to a file. For strings and `char` arrays, characters requiring four bytes each are represented as pairs of `char` values.

UTF-32 uses four bytes for each character. Although a fixed number of bytes per character is convenient, UTF-32 is space inefficient.

F.31 **Reading strings from a binary file.** To read a value previously written to the file by the method `writeUTF`, we use the following method of `DataInputStream`:

- `readUTF`—Reads and returns the next UTF string in the file as a `String` object

This method can throw one of two possible checked exceptions: a `UTFDataFormatException` if the bytes read are not a UTF string, or an `IOException` if an error occurs during the read operation.

Programming Tip: A binary file can contain data of differing types, and you can use `DataInputStream` to read such a file. You must be careful, however, to match the data with the appropriate read methods. If a data item in the file is not of the type expected by the reading method, the result is likely to be wrong. For example, if your program writes an integer using `writeInt`, any program that reads that integer should read it using `readInt`. If you instead use `readDouble`, for example, your program will misbehave. This and subsequent read operations will most likely not match the file and will be incorrect.

Programming Tip: Binary files and text files encode their data in different ways. A stream that expects to read a binary file, such as a stream in the class `DataInputStream`, will have problems reading a text file. If you attempt to read a text file with a stream in the class `DataInputStream`, your program either will read "garbage values" or will encounter some other error condition.

Programming Tip: Check for the end of a file

When reading from a file, your program should check for the end of the file and do something appropriate when it reaches it. If your program tries to read beyond the end of a file, it may enter an infinite loop or end abnormally. Even if you think your program will not read past the end of the file, you should provide for this eventuality just in case things do not go exactly as you planned.

Programming Tip: Ways to check for the end of a file

Here are some possible ways to test for the end of a file:

- Catch an `EOFException`, if the read method that you use throws one. When reading from a binary file, the methods in `DataInputStream` throw an `EOFException` when they try to read beyond the end of a file.
- Test for a special value—the sentinel—if one has been written at the end of the file. Your program then can stop reading when it reads the sentinel value. For example, you could use a negative integer as a sentinel value at the end of a file of nonnegative integers. Using a sentinel value, however, restricts your data to values other than the sentinel.

Note: Path names

When passing a file name as an argument to a constructor of classes like `File`, you can use a simple file name. Java assumes that the file is in the same directory (folder) as the one that contains the program. You also can use a full or relative path name. A full path name gives not only the name of the file, but also tells what directory (folder) the file is in. A relative path name gives the path to the file starting in the directory that contains your program. Paths depend on your operating system rather than the Java language.

Note that Java 7 provides an interface `Path` and a class `Paths` whose instances are like path names, but cannot be passed to `File`'s constructors.

Object Serialization

F.32 You have seen how to write primitive values and strings to a file, and how to read them again. How would you write and read objects other than strings? You could, of course, write an object's data fields to a file and invent some way to reconstruct the object when you read the file. When you consider that a data field could be another object that itself could have an object as a data field, completing this task sounds formidable.

Fortunately, Java provides a way—called **object serialization**—to represent an object as a sequence of bytes that can be written to a binary file. This process will occur automatically for any object that belongs to a class that implements the interface `Serializable`. This interface, which is in the package `java.io`, is empty, so you have no additional methods to implement. Adding only the words `implements Serializable` to the class's definition is enough.

For example, we could begin a class `Student` as follows:

```
import java.io.Serializable;
public class Student implements Serializable
{
   . . .
```

The `Serializable` interface tells the compiler that `Student` objects can be serialized. Since `Serializable` appears only once in the definition of the class `Student`, you could conveniently omit the `import` statement and begin the class as follows:

```
public class Student implements java.io.Serializable
```

To serialize an object and write it to a binary file, you use the method `writeObject` from the class `ObjectOutputStream`. To read a serialized object from a binary file, you use the method `readObject` from the class `ObjectInputStream`.

F.33 **Example.** To serialize the `Student` object `aStudent` and write it to a binary file, we would write the following statements within one or more `try` blocks:

```
FileOutputStream fos = new FileOutputStream(fileName);
ObjectOutputStream toFile = new ObjectOutputStream(fos);
. . .
toFile.writeObject(aStudent);
```

Any objects that are data fields of `aStudent` must also belong to a class that implements `Serializable`. Such objects are serialized when `aStudent` is serialized. Many classes in the Java Class Library—including `String`—implement `Serializable`.

To read the `Student` object from the binary file, we would write

```
FileInputStream fis = new FileInputStream(fileName);
ObjectInputStream fromFile = new ObjectInputStream(fis);
. . .
Student joe = (Student)fromFile.readObject();
```

within one or more `try` blocks.

F.34 We will call a class serializable if all of the following are true:

- It implements the interface `Serializable`
- Its data fields are either primitive values or objects of a serializable class
- Its direct superclass, if any, is serializable and defines a default constructor

Any subclass of a serializable class is serializable.

We will call an object serializable if it belongs to a class that is either serializable or a subclass of a serializable class.

Note: Object serialization
As we mentioned, object serialization is the process Java uses to represent an entire object as a sequence of bytes. Any serialized object receives a serial number as it is written to a binary file. If later we write the same object to the file, only its serial number is written. This approach saves space on the file, as the object's data is not written again. Moreover, when the first instance of the object is read from the file, both its data and serial number are read and a new object is created within the program. However, when a duplicate serial number is read, Java creates a reference to the object—that is, an alias—instead of a duplicate object.

Shouldn't all objects be serialized to save file space? Clearly, the designers of Java did not think so. Some objects simply should not be saved in a file. Perhaps more importantly, serialized objects are easy to access, and for security reasons, easy access is not always a good idea. Thus, the interface `Serializable` gives programmers the option of allowing or preventing the serialization of objects of their classes.

If an object is not serializable, you cannot use `writeObject` to write it to a binary file. Instead, you must write each of its data fields to the file.

Note: Strings are serializable objects
We know that strings are objects of the class `String`. Since `String` implements the interface `Serializable`, we can use the method `writeObject` instead of `writeUTF` to write strings to a binary file.

F.35 **Arrays.** Java treats arrays as objects, and they are serializable. You can use `writeObject` to write an array to a binary file, and you can use `readObject` to read it from the file. For example, suppose that `group` is an array of `Student` objects. If `toFile` is an instance of `ObjectOutputStream` that is associated with a binary file, we can write the array to that file by executing the statement

```
toFile.writeObject(group);
```

After creating the file, we can read the array by using the statement

```
Student[] myArray = (Student[])fromFile.readObject();
```

where `fromFile` is an instance of `ObjectInputStream` that is associated with the file that we just created.

Note: The classes `ObjectOutputStream` and `ObjectInputStream` have methods to write and read primitive values when working with a binary file. You can consult the online documentation for the Java Class Library to learn about these methods.

Note: Checked exceptions thrown by a selection of classes and methods in the Java Class Library

Class and Methods	**Exceptions** (All are in the package java.io, except for ClassNotFoundException)
DataInputStream	
Constructor	None
close	IOException
readInt, other read methods	EOFException, IOException
DataOutputStream	
Constructor	None
close, flush, writeInt, other write methods	IOException
File	
Constructor, canRead, canWrite, delete, exists, getName, getPath, length	None
FileInputStream	
Constructor	FileNotFoundException
close	IOException
FileOutputStream	
Constructor	FileNotFoundException
close	IOException
ObjectInputStream	
Constructor	IOException, StreamCorruptedException
close	IOException
readObject	java.lang.ClassNotFoundException, InvalidClassException, OptionalDataException, StreamCorruptedException, IOException
ObjectOutputStream	
Constructor, close	IOException
writeObject	InvalidClassException, IOException, NotSerializableException
PrintWriter	
Constructor	FileNotFoundException
close, flush	IOException
print, println	None
Scanner	
Constructor	FileNotFoundException
Other methods	None

Appendix G

Documentation and Programming Style

Contents

Prerequisite

Appendix A Java Essentials

Most programs are used many times and are changed either to fix bugs or to accommodate new demands by the user. If the program is not easy to read and to understand, it will not be easy to change. It might even be impossible to change without heroic efforts. Even if you use your program only once, you should pay some attention to its readability. After all, you will have to read the program to debug it.

In this appendix, we discuss three techniques that can help make your program more readable: meaningful names, indenting, and comments.

Naming Variables and Classes

G.1 Names without meaning are almost never good variable names. The name you give to a variable should suggest what the variable is used for. If the variable holds a count of something, you might name it `count`. If the variable holds a tax rate, you might name it `taxRate`.

In addition to choosing names that are meaningful and legal in Java, you should follow the normal practice of other programmers. That way it will be easier for them to read your code and to combine your code with their code, should you work on a project with more than one person. By convention, each variable name begins with a lowercase letter, and each class name begins with an uppercase letter. If the name consists of more than one word, use a capital letter at the beginning of each word, as in the variable `numberOfTries` and the class `StringBuffer`.

Use all uppercase letters for named constants to distinguish them from other variables. Use the underscore character to separate words, if necessary, as in `INCHES_PER_FOOT`.

Indenting

G.2 A program has a structure: Smaller parts are within larger parts. You use indentation to indicate this structure and thereby make your program easier to read. Although Java ignores any indentation you use, indenting consistently is essential to good programming style.

Each class begins at the left margin and uses braces to enclose its definition. For example, you might write

```
public class CircleCalculation
{
   . . .
} // end CircleCalculation
```

The data fields and methods appear indented within these braces, as illustrated in the following simple program:

```
public class CircleCalculation
{
   public static final double PI = Math.PI;

   public static void main(String[] args)
   {
      double radius; // in inches
      double area;   // in square inches
      . . .
   } // end main
} // end CircleCalculation
```

Within each method, you indent the statements that form the method's body. These statements in turn might contain compound statements that are indented further. Thus, the program has statements nested within statements.

Each level of nesting should be indented from the previous level to show the nesting more clearly. The outermost structure is not indented at all. The next level is indented. The structure nested within that is double indented, and so on. Typically, you should indent two or three spaces at each level of indentation. You want to see the indentation clearly, but you want to be able to use most of the line for the Java statement.

If a statement does not fit on one line, you can write it on two or more lines. However, when you write a single statement on more than one line, you should indent the successive lines more than the first line, as in the following example:

```
System.out.println("The volume of a sphere whose radius is " +
                   radius + " inches is " + volume +
                   " cubic inches.");
```

Ultimately, you need to follow the rules for indenting—and for programming style in general—given by your instructor or project manager. In any event, you should indent consistently within any one program.

Comments

G.3 The documentation for a program describes what the program does and how it does it. The best programs are **self-documenting**. That is, their clean style and well-chosen names make the program's purpose and logic clear to any programmer who reads the program. Although you should strive for such self-documenting programs, your programs will also need a bit of explanation to make them completely clear. This explanation can be given in the form of **comments**.

Comments are notations in your program that help a person understand the program, but that are ignored by the compiler. Many text editors automatically highlight comments in some way, such as showing them in color. In Java, there are several ways of forming comments.

Single-Line Comments

G.4 To write a comment on a single line, begin the comment with two slashes `//`. Everything after the slashes until the end of the line is treated as a comment and is ignored by the compiler. This form is handy for short comments, such as

```
String sentence; // Spanish version
```

If you want a comment of this kind to span several lines, each line must contain the symbols `//`.

Comment Blocks

G.5 Anything written between the matching pair of symbols `/*` and `*/` is a comment and is ignored by the compiler. This form is not typically used to document a program, however. Instead, it is handy during debugging to temporarily disable a group of Java statements. Java programmers do use the pair `/**` and `*/` to delimit comments written in a certain form, as you will see in Segment G.7.

When to Write Comments

G.6 It is difficult to explain just when you should write a comment. Too many comments can be as bad as too few. Too many comments can hide the really important ones. Too few comments can leave a reader baffled by things that were obvious to you. Just remember that you also will read your program. If you read it next week, will you remember what you did just now?

Every program file should begin with an explanatory comment. This comment should give all the important information about the file: what the program does, the name of the author, how to contact the author, the date that the file was last changed, and in a course, what the assignment is. Every method should begin with a comment that explains the method.

Within methods, you need comments to explain any nonobvious details. Notice the poor comments on the following declarations of the variables `radius` and `area`:

```
double radius; // the radius
double area;   // the area
```

Because we chose descriptive variable names, these comments are obvious. But rather than simply omitting these comments, can we write something that is not obvious? What units are used for the radius? Inches? Feet? Meters? Centimeters? We will add a comment that gives this information, as follows:

```
double radius; // in inches
double area;   // in square inches
```

Java Documentation Comments

G.7 The Java language comes with a utility program named **`javadoc`** that will generate HTML documents that describe your classes. These documents tell people who use your program or class how to use it, but they omit all the implementation details.

The program `javadoc` extracts the header for your class, the headers for all public methods, and comments that are written in a certain form. No method bodies and no private items are extracted.

For javadoc to extract a comment, the comment must satisfy two conditions:

- The comment must occur immediately before a public class definition or the header of a public method.
- The comment must begin with /** and end with */.

Segment G.12 contains an example of a comment in this style.

You can insert HTML commands in your comments so that you gain more control over javadoc, but that is not necessary and we have not done so in this book.

G.8 **Tags.** Comments written for javadoc usually contain special **tags** that identify such things as the programmer and a method's parameters and return value. Tags begin with the symbol @. We will describe only four tags in this appendix.

The tag @author identifies the programmer's name and is required of all classes and interfaces. The other tags of interest to us are used with methods. They must appear in the following order within a comment that precedes a method's header:

```
@param
@return
@throws
```

We will describe each of these tags next.

G.9 **The @param tag.** You must write a @param tag for every parameter in a method. You should list these tags in the order in which the parameters appear in the method's header. After the @param tag, you give the name and description of the parameter. Typically, you use a phrase instead of a sentence to describe the parameter, and you mention the parameter's data type first. Do not use punctuation between the parameter name and its description, as javadoc inserts one dash when creating its documentation.

For example, the comments

```
@param code      the character code of the ticket category
@param customer  the string that names the customer
```

will produce the following lines in the documentation:

```
code - the character code of the ticket category
customer - the string that names the customer
```

G.10 **The @return tag.** You must write a @return tag for every method that returns a value, even if you have already described the value in the method's description. Try to say something more specific about this value here. This tag must come after any @param tags in the comment. Do not use this tag for void methods and constructors.

G.11 **The @throws tag.** Next, if a method can throw a checked exception, you name it by using a @throws tag, even if the exception also appears in a throws clause in the method's header. You can list unchecked exceptions if a client might reasonably catch them. Include a @throws tag for each exception, and list them alphabetically by name.

G.12 **Example.** Here is a sample javadoc comment for a method. We usually begin such comments with a brief description of the method's purpose. This is our convention; javadoc has no tag for it.

```
/** Adds a new entry to a roster.
    @param newEntry     the object to be added to the roster
    @param newPosition  the position of newEntry within the roster
    @return true if the addition is successful
    @throws RosterException if newPosition < 1 or newPosition > 1 + the length
                               of the roster */
public boolean add(Object newEntry, int newPosition) throws RosterException
```

The documentation that `javadoc` prepares from the previous comment appears as follows:

add

```
public boolean add(java.lang.Object newEntry,
                   int newPosition)
            throws RosterException
```

Adds a new entry to a roster.

Parameters:
`newEntry` - the object to be added to the roster
`newPosition` - the position of newEntry within the roster
Returns:
true if the addition is successful
Throws:
`RosterException` - if newPosition < 1 or newPosition > 1 + the length of the roster

To save space in this book, we sometimes omit portions of a comment that we would include in our actual programs. For example, some methods might have only a description of their purpose, and some might have only a `@return` tag. Note that `javadoc` accepts these abbreviated comments.

Running javadoc

G.13 You run `javadoc` on an entire package. However, if you want to run it on a single class, you can make the class into a package simply by inserting the following at the start of the file for the class:

package *package_name*;

Remember that the package name should describe a relative path name for the directory or folder containing the files in the package.

To run `javadoc`, you must be in the folder that contains the package's folder, but not in the package folder itself. Then you execute the following command:

`javadoc -d` *document_folder* *package_name*

Replace *document_folder* with the name of the folder in which you want `javadoc` to place the HTML documents it produces. The folder must already exist; `javadoc` will not create it for you.

For example, suppose you want to use `javadoc` to generate documentation for the class `MyClass`. Create a folder to hold a package; for instance, you might call the folder—and the package—`MyStuff`. Place the file `MyClass.java` in the folder `MyStuff`, and place the following at the start of the file `MyClass.java`:

```
package MyStuff;
```

The package `MyStuff` now contains the class `MyClass`.

Next, create a folder to receive the HTML documents. For example, you might call this folder `MyDocs`. Place `MyDocs` in the folder containing `MyStuff`. Do not place it within `MyStuff`.

Finally, use the command `cd` to change to the directory that contains both `MyStuff` and `MyDocs`, and give the following command:

```
javadoc -d MyDocs MyStuff
```

If you then look in the folder `MyDocs`, you will see a number of HTML documents whose names end in `.html`. You can view these files by using your browser. The HTML documents will describe the package `MyStuff`, including the class `MyClass`.

If you wish, you can use the folder `MyStuff` in place of `MyDocs`, so that both the source file `MyClass.java` and the HTML documents end up in the same folder.

Further details about `javadoc` are available at `www.oracle.com/technetwork/java/javase/documentation/index-137868.html`.

Index